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Sanborn C. Brown

Abstract

A phenomenological description of high-frequency gas discharge breakdown is given. The effects of superimposing on the a-c field, a small d-c field, and a magnetic field are also discussed.



HIGH-FREQUENCY GAS DISCHARGE BREAKDOWN

Introduction

Knowledge of the breakdown of a gas discharge under the action of a high-frequency field has greatly increased in the last few years. The present report is written to describe the similarities and differences between these discharges and the more familiar d-c type of breakdown, discussing the physical phenomena that occur, and showing the type of information that may be obtained from these relatively simple types of breakdown.

In a high-frequency gas discharge breakdown, the primary ionization due to the electron motion is the only production phenomenon which controls the breakdown. For this reason, high-frequency studies are much simpler than the d-c type of breakdown which must always have a source of electrons present, often supplied by secondary electron effects, to make up for the electrons which are continuously swept out by the field. If one calculates the maximum kinetic energy in the oscillatory motion of an electron at the minimum field intensities for breakdown experimentally determined, one finds that this energy corresponds to about 10^{-3} electron volt. It is therefore obvious that the energy of oscillation is insufficient to account for breakdown.

A free electron in a vacuum under the action of an alternating field oscillates with its velocity 90° out of phase with the field, and thus takes no power, on the average, from the applied field. The electrons can gain energy from the field only by suffering collisions with the gas atoms, and they do so by having their ordered oscillatory motion changed to random motion on collision. The electrons gain random energy on each collision until they are able to make inelastic (exciting or ionizing) collisions with gas atoms. The fact that the electron can continue to gain energy in the field, on the average, despite the fact that the field changes sign, can be seen by showing that the energy absorbed is proportional to the square of the electric field and hence independent of its sign. The rate of gain of energy of the electron from the electric field E_{rms} is $P = eEv$, where e is the electronic charge and v is the average drift velocity resulting from the applied electric field.

If the electrons are acted on by an electric field, oscillating with a radian frequency ω , $E = E_0 \exp(j\omega t)$ in the presence of a gas in which collisions act as a viscous damping force, Lorenz (1) showed that the equation of motion may be written

$$m(dv/dt) + (m\nu_c)v = -eE_0 \exp(j\omega t) \quad . \quad (1)$$

The collision frequency, ν_c , is the reciprocal of the mean free time between collisions and is considered to be constant. The electron velocity determined from this equation of motion can be written

$$v = eE_0 \exp(j\omega t)/(j\omega m + m\nu_c) \quad . \quad (2)$$

The average drift velocity of an electron is proportional to the electric field in which it travels and the proportionality constant is called the mobility μ . Thus $v = \mu E$ which may be written in the form of an a-c mobility

$$\mu = e/m(j\omega + \nu_c) \quad . \quad (3)$$

This expression can be compared with the case of d-c electric fields where $\omega = 0$, or with the case of high-pressure discharges where the collision frequency is much larger than the radian frequency of an oscillating electric field, where one of the standard expressions for the mobility takes the form $\mu = e/m\nu_c$.

The current density of n electrons per unit volume is $J = -nev$ where the velocity is given by Eq. 2. Separating the real and imaginary parts of the current, we may write

$$J = \frac{ne^2E}{m\omega} \frac{\nu_c/\omega}{(\nu_c/\omega)^2 + 1} - j \frac{ne^2E}{m\omega} \frac{1}{(\nu_c/\omega)^2 + 1} \quad . \quad (4)$$

The rate of energy gain of the electrons from the field is the real power going into the gas per unit volume

$$P = J_r E = (ne^2E^2/m\nu_c) \left[\nu_c^2/(\nu_c^2 + \omega^2) \right] \quad . \quad (5)$$

It is convenient to write the electric field in terms of an effective field which would produce the same energy transfer as a d-c field

$$E_e^2 = E^2 \left[\nu_c^2/(\nu_c^2 + \omega^2) \right] \quad (6)$$

so that the rate of energy gain of the electrons from the field is

$$P = ne^2E_e^2/m\nu_c \quad . \quad (7)$$

Thus we see that the rate of energy gain of the electrons in the field is proportional to the square of the electric field, and hence in a manner independent of the sign of that field.

1. Diffusion of Electrons

A gas discharge breakdown occurs when the gain in electron density due to the ionization of the gas becomes equal to the loss of electrons. In a d-c discharge the electron loss is due primarily to the mobility motion caused by the steady electric field. In the a-c case, the electrons are not thus swept from the discharge and their loss can be due to such phenomena as diffusion, recombination and attachment. Under all experimental conditions so far studied of high-frequency breakdown, the loss has been due entirely to diffusion.

Diffusion occurs whenever a particle concentration gradient exists. The total flow of particles out of a volume of high concentration may be written from ordinary kinetic theory considerations (2) as

$$\Gamma = -D\nabla n \quad (8)$$

where D is the diffusion coefficient for electrons, n the electron density, and Γ the

electron current density in electrons per second per unit area. The diffusion coefficient is proportional to the average velocity of the electrons and in terms of the mean free path, ℓ , may be written as $D = \ell v/3$.

We will develop the breakdown conditions for a region bounded by walls which absorb electrons. A radioactive source near the discharge tube provides a small amount of ionization, S , in the tube. A detailed study of the build-up of the discharge is obtained from considering the continuity equation for electrons

$$\partial n/\partial t = \nu_i n + S - \nabla \cdot \Gamma \quad (9)$$

or in terms of Eq. 8

$$\partial n/\partial t = D \nabla^2 n + \nu_i n + S \quad (10)$$

In this equation the term $D \nabla^2 n$ describes the loss of electrons by diffusion. The term $\nu_i n$ is the rate of gain of electrons by ionization and S is the rate at which electrons are produced by an external source. For the case of infinite parallel plates

$$\partial n/\partial t = D(\partial^2 n/\partial x^2) + \nu_i n + S \quad (11)$$

Assuming that the approach to breakdown is so slow in time that $\partial n/\partial t$ may be neglected

$$-S = D(\partial^2 n/\partial x^2) + \nu_i n \quad (12)$$

This is a characteristic value problem which may be solved under the conditions that S is uniform throughout the cavity, and that the boundary condition on the electron density is that it is zero at the walls. The solution of this equation tells us that the electron density before breakdown at any distance x from the median plane between parallel plates of separation L may be written

$$n = S \cos(\pi x/L) / [D(\pi/L)^2 - \nu_i] \quad (13)$$

Breakdown can be defined by the condition that the electron density goes to infinity, which occurs when $\nu_i = D(\pi/L)^2$.

If we write $\nu_i/D = (\pi/L)^2 = 1/\Lambda^2$, for parallel plates, the quantity Λ is known as the characteristic diffusion length and is very useful in describing the shape of the gas container when discussing diffusion problems. One other example of a very useful boundary condition is the case of a cylinder of height h and radius r , with flat ends. This geometry leads to the relation that $1/\Lambda^2 = (\pi/h)^2 + (2.4/r)^2$, where the diffusion to the end plates is given by the first term on the right, and the second term describes the diffusion to the cylindrical walls.

2. Ionization Coefficients

Gas discharge phenomena under the action of d-c fields are often described in terms of Townsend ionization coefficients. If one considers that electrons in a d-c field create a new electrons in a path one centimeter long in the field direction, the increase of electrons, dn , produced by n electrons in a distance dx will be $dn = an dx$, and

$n = n_0 \exp(ax)$ where n_0 is the initial electron concentration. The quantity a is called the first Townsend coefficient. This first Townsend coefficient may also be written as the ionization produced by an electron falling through a potential difference of one volt rather than travelling one centimeter. This coefficient is given the symbol η and is related to a by $\eta = a/E$.

These Townsend first ionization coefficients may be given in terms of an "ionization" collision frequency. Since a is the number of electrons produced by the collisions of the primary electrons travelling one centimeter, one can write $a = \nu_i/v$, where ν_i is the frequency of ionization and v is the average drift velocity of the electrons in the field. The average drift velocity $v = \mu E$ and one may write $a = \nu_i/\mu E$ or

$$\eta = \nu_i/\mu E^2 \quad (14)$$

By analogy with the first Townsend coefficient for d-c ionization where the electron loss is controlled by mobility, we may define a coefficient for high-frequency discharges (3) where the loss is by diffusion as

$$\zeta = \nu_i/DE^2 \quad (15)$$

From our previous discussion of diffusion we saw that at breakdown $\nu_i/D = 1/\Lambda^2$, and thus we may measure the a-c ionization coefficient by measuring the breakdown field in tubes of known size, since $\zeta = 1/\Lambda^2 E^2$. Measurements of ζ as a function of E/p , the ratio of the rms electric field and the pressure for various gases is shown in Fig. 1.

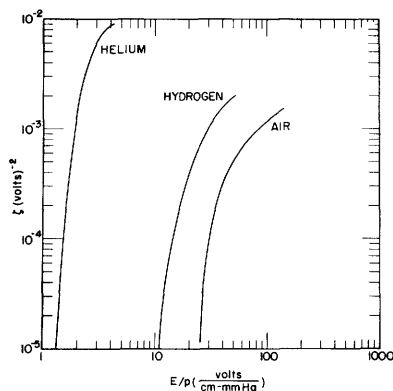


Fig. 1

A-c ionization coefficient for helium, hydrogen and air.

There is a very close physical relation between the a-c and the d-c ionization coefficients. If one divides Eq. 14 by Eq. 15 there results the relation $\eta/\zeta = D/\mu$. Townsend showed that the ratio of D/μ was a measure of the average energy of the electrons (4) and determined this average energy as a function of E/p experimentally. Thus, in principle, one can determine η from ζ or vice versa from these Townsend-like measurements. There is difficulty in carrying this out exactly, however, for the actual values depend on how the averaging of the energy is carried out. Since the electron energy distribution functions are different for the a-c and the d-c cases, one might expect mathematical complications to arise if this were tried in actual cases.

For the one case of hydrogen gas, where considerable simplifications are possible in the mathematics, it has been shown (5) that the d-c Townsend coefficient can be determined from the a-c ionization coefficient.

3. High-Frequency Breakdown

Typical of the behavior of the breakdown field at high frequency with changes in gas pressure are the curves shown in Fig. 2. At first sight these curves look similar to corresponding data taken with d-c fields; that is, as the pressure is decreased the breakdown field first decreases and then increases again at low pressures. In the low-

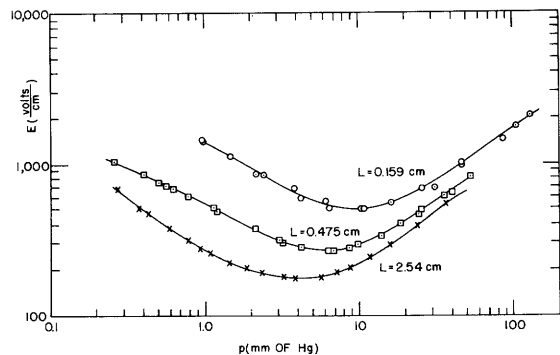


Fig. 2

Typical breakdown curves for hydrogen at 3000 Mc frequency.

the factor of the effective field given in Eq. 6. In the high-pressure region, the reason for the rising breakdown field with increasing pressure in high-frequency discharges is the same as in the d-c case. As the pressure increases, the electron mean free path decreases and the energy per mean free path decreases. Since to cause ionization the electron must gain energy corresponding to the ionization potential, more and more electric field must be applied to do this as the pressure increases. The minimum corresponds essentially to the point where the frequency of collision between electrons and gas atoms is equal to the frequency of the applied r-f field. Thus at low pressure, where the electron makes many oscillations per collision, its behavior is governed by strictly a-c considerations. At high pressure, where the electrons make many collisions per oscillation, their behavior is the same as in a d-c discharge.

The remarkable feature of the breakdown curves, for those used to d-c phenomena, is the fact that the greater the electrode spacing, the easier it becomes to cause a breakdown. This, of course, is a necessary result of the breakdown condition of the balance between energy gained from the field and electron loss by diffusion. As the electrode spacing becomes less, the diffusion loss becomes greater and hence the field must increase to make up for the increased loss.

Curves of gas discharge breakdown as a function of pressure are often plotted in d-c work as Paschen curves in which, for a particular gas, the breakdown voltage V is found to be a function of pd , independent of the magnitude of the electrode spacing d . The same type of quantities may be introduced in the high-frequency case, where for the breakdown voltage we write $E\Lambda$, the field times the diffusion length, and for pd we use $p\Lambda$. In the case of high-frequency phenomena we have one more variable than for the d-c case,

namely the frequency, and this may be introduced as the variable $p\lambda$, where λ is the wavelength of the applied field. In terms of these three variables, all high-frequency breakdown data may be put on a three dimensional surface. This is illustrated for the case of hydrogen in Fig. 3. In this kind of a plot, breakdown data taken for a given size tube at constant applied frequency as that of Fig. 2, fall on lines at 45° in the $p\Lambda$ - $p\lambda$ plane. All the higher-pressure data taken above the minimum breakdown field corre-

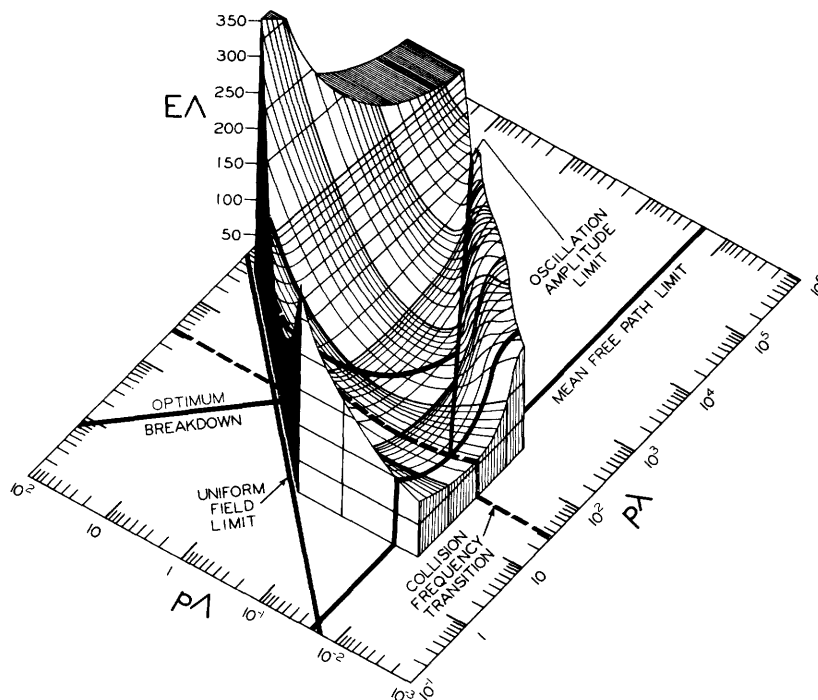


Fig. 3 A surface constructed from the experimental determinations indicated in Fig. 2, correlating the breakdown voltage measurements in terms of variables derived from dimensional analysis.

spond to the case of an electron making many collisions per oscillation. In this case the frequency of the applied field makes no difference; that is, the data are independent of the variable $p\lambda$ and all the $E\Delta$ versus $p\lambda$ curves have the same shape, corresponding to the Paschen law curves for d-c breakdown. This is shown in Fig. 4.

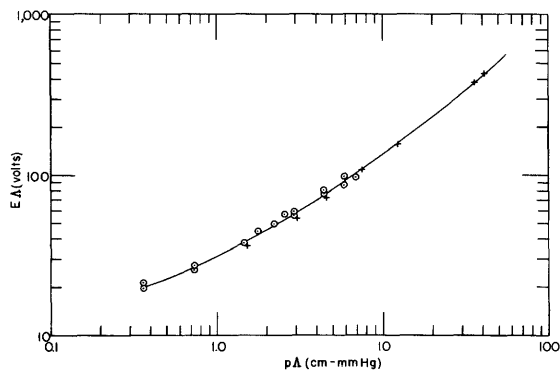


Fig. 4

High-pressure a-c breakdown data showing similarity to d-c Paschen law when electron makes many collisions per oscillation. Crosses are data at 10 cm wavelength (7) and circles are wavelengths of 16,700; 10,600; 8,050; 4,740; 460; and 389 cm (10).

4. Breakdown Theory

To derive a more quantitative description of the breakdown behavior and avoid mathematical complexity, one must choose a gas in which simplifying assumptions are most likely to apply. The two most important of these assumptions are, first, that the frequency of collision be constant in energy, an assumption we have already used in Eq. 1, and, second, that all inelastic collisions be ionizing, that is that there be no excitation. There turns out to be no such gas in nature, but it is possible to make such a gas by adding small amounts of mercury vapor to helium. Let us coin a name of Heg for this gas. The helium gas by itself has the property, in the region of electron energy in which breakdown occurs, of behaving such that $\nu_c = \text{constant}$. Helium has a metastable level at 19.8 volts since transitions from this level to the ground state by radiation are forbidden. Since the metastable states have mean lives of the order of thousands of microseconds, practically every helium atom which reaches an energy of 19.8 volts will collide with a mercury atom and lose its energy by ionizing the mercury. Therefore, each inelastic collision will produce an ionization and the effective ionization potential will be $u_1 = 19.8$ volts.

Having thus found a gas to which we can apply a simple theory, let us first consider what happens at high pressure. Here the power which goes into the electrons from the electric field is dissipated in elastic collisions between the electrons and the gas molecules. This region corresponds to the lowest values of E/p measured experimentally. The data on either the Townsend first ionization coefficient or the a-c ionization coefficient, such as in Fig. 1, show that here E/p is nearly constant for a wide variation of these coefficients and is equal in Heg to 1 rms volt/cm/mm Hg. Thus, for an Heg discharge, in which nearly all the loss goes into nonionizing collisions, the field and pressure are related by the equation $E = p$. It can be seen that the high-pressure breakdown measurements tend to approach this line. This is shown in Fig. 5.

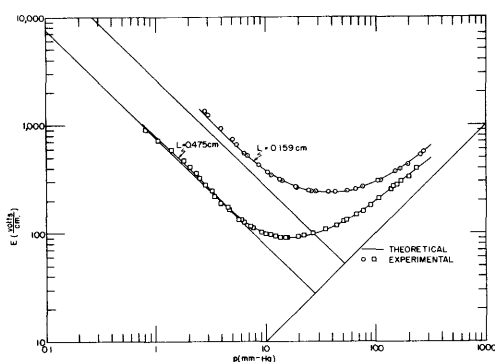


Fig. 5

Experimental breakdown electric fields compared with a simplified theory.

In the low-pressure region, the electrons make many oscillations per collision and the breakdown field may be determined by equating the number of oscillations to ionize to the number of collisions to diffuse out of the tube. Since all inelastic collisions are ionizing ones, all the input power (which is the rate of transfer of energy) goes into

ionization, this is to say that the power is the frequency of ionizing collisions, ν_i , times the energy to ionize a gas atom, eu_i . Thus we may write from Eq. 7

$$\nu_i = P/eu_i = eE_e^2/mu_i\nu_c \quad (16)$$

Since we are discussing the low-pressure region, we may assume that $\omega^2 \gg \nu_c^2$ in Eq. 6 for E_e^2 . In the diffusion section we found that $\nu_i = D/\Lambda^2$ and that $D = \ell v/3$. This leads to a relation for the frequency of ionization of the form $\nu_i = \ell v/3\Lambda^2$. If we multiply numerator and denominator of this expression for ν_i by the velocity, and combine with Eq. 16 we obtain

$$\nu_i = v^2/3\Lambda^2\nu_c = eE_e^2\nu_c/ u_i m \omega^2 \quad (17)$$

We solve this expression for the electric field. For ω we may write $2\pi c/\lambda$, where c is the velocity of light. The electron energy is related to the velocity by the equation $eu = mv^2/2$. Combining these, we obtain

$$E = 2\pi c \sqrt{2uu_i/3} / \Lambda \lambda \nu_c \quad (18)$$

The collision frequency, ν_c , has been measured for many gases and the results have been summarized by Brode (6). Brode gives his results in terms of the probability of collision, P_c , which may be defined as $1/p\ell$. The relation between the probability of collision and the frequency of collision is $\nu_c = p\nu P_c$ since elementary considerations show that ν_c may be written as v/ℓ . From Brode's data we find $\nu_c = 2.37 \times 10^9 p$. For Heg the ionization potential $u_i = 19.8$ volts, and since we are assuming a very low pressure where all the power goes into ionization, the average energy u also equals 19.8 volts. Calculating the electric field under these approximations leads to a relation $E = 1284/p\Lambda\lambda$. Calculating the electric field from this relation for the two different size cavities for which data are given in Fig. 5 shows that the experimental values do approach these theoretical lines at very low pressures.

In this report we have treated the breakdown problem in a qualitative manner, discussing the behavior of an average electron. This gives us less accurate agreement in detail than would a more rigorous treatment but allows us to discuss the mechanisms in a correct and more easily understood fashion. An accurate description of the breakdown phenomena can be given theoretically only by taking into account the electron energy distribution. The electron energy distribution function may be determined by setting up the electron continuity equation, accounting for production and loss of electrons by ionization and diffusion, and expressing the results in terms of the high-frequency ionization coefficient ζ . The precision with which such a treatment predicts the experimental observations gives us confidence in the validity of the approach, but the mathematical complexity of this type of theory largely obscures a clear picture of the mechanisms involved. For the more rigorous treatment, the reader is referred to the original papers on the subject (3, 7).

5. The Limits of Diffusion-Controlled Breakdown

Certain basic assumptions have been made in the calculations of breakdown as a balance between the ionization rate and the loss of electrons by diffusion. Let us examine the limits of experimental parameters beyond which diffusion is no longer the controlling phenomenon. These limits can be discussed in terms of the variables $p\Lambda$ and $p\lambda$, and we will choose the case of hydrogen, since more quantitative work has been done with this gas than any other.

At low frequencies the experimental measurements of breakdown are always taken in vessels whose dimensions are small compared to the wavelength of the exciting power. For this case, an assumption of uniform field between the electrodes is very good. At very high frequencies there exists a limit to the size of the discharge tube consistent with the assumption of the diffusion theory that there be no region of zero electric field except at the walls of the tube. This can be written in terms of the size of the tube allowable to sustain a single loop of a standing wave of the electric field. The relation between the parallel plate separation, L , and the wavelength is that $\lambda/2 = L$. Since in terms of the diffusion length $L = \pi\Lambda$, the Uniform Field Limit shown in Fig. 3 is calculated from

$$p\lambda = 2\pi(p\Lambda) \quad . \quad (19)$$

The diffusion theory will not apply where the electron mean free path becomes comparable to the tube size. In the limiting case, this can be expressed as the mean free path, l , being equal to the diffusion length Λ . We have seen that the probability of collision P_c is equal to $1/p\lambda$. To plot this condition in Fig. 3, we write

$$p\Lambda = 1/P_c \quad . \quad (20)$$

The value of P_c is not a constant, but depends upon the electron energy. Assuming that the average electron has an energy equal to one-third of the ionization potential, the average electron energy would be 5 volts for hydrogen. Using Brode's measured value for the probability of collision in hydrogen for the average electron, $P_c = 49 \text{ (cm-mm Hg)}^{-1}$. With this value, we obtain the horizontal line in Fig. 3 marked Mean Free Path Limit.

Within the limits of experimental conditions in which diffusion theory adequately explains the breakdown behavior, several different phenomena may occur. One of the important phenomenological changes is the transition from many collisions per oscillation of the electron to many oscillations per collision. This can be written as the condition that $\nu_c = \omega$, where ν_c , the collision frequency, is the ratio of the average velocity to the mean free path, and ω is the radian frequency of the applied field. From Brode's data, we can obtain the relation $\nu_c = 5.93 \times 10^9 p$. Putting this in terms of the proper variables, we obtain

$$p\lambda = 32 \quad . \quad (21)$$

This relation is plotted in Fig. 3 as the dotted line marked as the Collision Frequency Transition.

We can calculate a line on the $p\Lambda$ - $p\lambda$ plane corresponding to the minimum breakdown field for any given container size. At low pressure we have seen that the breakdown field approaches the condition given by Eq. 18. For hydrogen $\nu_c = 5.93 \times 10^9 p$, $\dot{u}_1 = 15.4$ volts and we again consider $u = u_1$. This leads to a value of $E = 400/\Lambda p$. The limiting value of E/p for the ionization coefficient in hydrogen is 8 volts/cm/mm Hg, and therefore at high pressures $E = 8p$. Eliminating the field between these two equations will allow us to calculate the pressure at which breakdown will occur most easily. In terms of the variables of Fig. 3 this leads to the equation

$$p\lambda = 50/p\Lambda \quad . \quad (22)$$

This relation is plotted in Fig. 3 as the line marked the Optimum Breakdown.

When the amplitude of the electron oscillation (8) in the electric field is sufficiently high, the electrons can travel completely across the tube and collide with the walls on every half cycle. We have already seen that under the action of the field, an electron attains a velocity $v = eE/m\nu_c$. Putting in the sinusoidal variation of the field with time

$$v = (eE_p/m\nu_c) \sin \omega t \quad (23)$$

$$x = (eE_p/m\omega\nu_c) \cos \omega t$$

where E_p is the peak value of the field. The limiting case on the diffusion mechanism in which all of the electrons will hit the walls would be calculated by setting the oscillation amplitude equal to one-half of the electrode separation. Thus the oscillation amplitude becomes equal to $eE_p/m\omega\nu_c = L/2$. Substituting λ in terms of ω , ν/l in place of ν_c , and $1/pP_c$ for l we obtain

$$p\lambda = (\pi m c \nu P_c / e) pL / (E_p / p) \quad . \quad (24)$$

From Brode's data again we use $\nu P_c = 5.93 \times 10^9 p(\text{sec-mm Hg})^{-1}$, and putting in this numerical value with the parallel plate relation that $L = \pi\Lambda$, one has

$$p\lambda = 10^6 p\Lambda / (E_p / p) \quad . \quad (25)$$

This equation can be solved numerically where the experimental values of the breakdown field are available. These have been measured and the calculation yields the Oscillation Amplitude Limit of Fig. 3.

6. Phenomena Outside Diffusion Controlled Breakdown

Many experimenters have studied r-f breakdown outside the limits set by the diffusion theory. The phenomena in these regions are complicated by secondary effects similar to that found in d-c discharges. For example, when the oscillation amplitude limit is crossed, the work of Githens (9) and Thomson (10) gives the sudden increase in the breakdown voltage $E\Lambda$ shown in Fig. 3. The increase probably results from the increase in electron loss due to the amplitude of the oscillation forcing the electrons

against the walls and hence out of the discharge. This hump shown in Fig. 3 was found by them to flatten off, and later work by Gill and von Engel (11) showed that the breakdown field thereafter decreased. This decrease being a function of electrode material is due to a new source of electrons entering the discharge from secondary electron production by the electron bombardment.

Some work has also been carried out measuring the breakdown at high frequencies below the mean free path limit (12). Here again the phenomena become complicated by the increased loss and the introduction of secondary effects, and the problem again approaches the difficulty of a d-c discharge.

7. Effect of a Superimposed D-C Field

The gas in a tube will break down when the losses of electrons to the walls of the tube are replaced by ionization in the body of the gas. When an a-c field alone is applied, electrons are lost by diffusion. When a small d-c sweeping field is applied, electrons are lost both by diffusion and mobility and the a-c breakdown field will increase. The breakdown condition can be formulated mathematically by a consideration of these processes (5).

Equation 8 described the particle current flowing through a unit area due to the phenomenon of diffusion. When the electron current is due not only to diffusion but also to the motion due to an electric field, one may write similar equations for the electron loss by adding a mobility motion term. Thus we can have

$$\Gamma = -D\nabla n - \mu E_{dc} n \quad (26)$$

and

$$\nabla \cdot \Gamma = -D\nabla^2 n - \mu E_{dc} (\partial n / \partial z) \quad (27)$$

Equivalent to Eq. 10 we may write

$$\partial n / \partial t = D\nabla^2 n + \mu E_{dc} (\partial n / \partial z) + v_i n + S \quad (28)$$

In a fashion similar to the solution obtained for the electron density in Eq. 13, Eq. 28 may be solved for the case of a cylinder of axial height L and axial coordinate z, radius R and radial coordinate r by the method of separation of variables to yield

$$n = \text{constant } J_0(2.4r/R) \left[\sin \pi z / L \right] \exp(-\mu E_{dc} z / 2D) \quad (29)$$

In this expression the term in the zero order Bessel function represents diffusion to the cylindrical walls of the tube, the sine function represents diffusion to the end walls, and the exponential represents the deformation of the sine caused by the sweeping of electrons by the d-c field. This solution is subject to the condition $v_i / D = 1 / \Lambda_{dc}^2$, where Λ_{dc} defines a modified diffusion length given by the relation

$$1 / \Lambda_{dc}^2 = 1 / \Lambda^2 + \left[E_{dc} / (2D / \mu) \right]^2 \quad (30)$$

For this case, the characteristic diffusion length is given by $1 / \Lambda^2 = (\pi / L)^2 + (2.4 / R)^2$ which is the diffusion length previously discussed for cylindrical tubes.

The only difference between the breakdown condition in the ac-dc case and the pure a-c case is the substitution of a modified diffusion length Λ_{dc} for the characteristic diffusion length Λ . It will be noted that the modified diffusion length of a cavity is smaller than the characteristic diffusion length. A cavity whose electron losses are increased by a d-c sweeping field is equivalent to a smaller cavity without a sweeping field (5).

8. Superimposed Magnetic Field

The discharge breakdown at high frequencies in the presence of a constant magnetic field has been studied by a number of workers (13, 14, 15) in the diffusion controlled region. Let us consider the motion of an electron between collisions under the influence of an electric field along the x-axis, $E = E_0 \exp(j\omega t)$, and a constant magnetic field B along the z-axis. The equation of motion similar to Eq. 1 is then

$$m(dv/dt) + eBv = -eE_0 \exp(j\omega t) \quad . \quad (31)$$

The solution of this equation corresponds to the superposition of a circular helical motion and a plane elliptical motion. For the helical motion whose axis is along the magnetic field, the velocity oscillates at the cyclotron frequency $\omega_b = eB/m$ and the energy of this motion is constant. For the elliptical motion the velocity oscillates at the frequency of the applied field and the energy is determined by the magnitude and frequency of the applied field.

The mean energy gain between collisions of the electrons with the gas atoms can be obtained as before, considering the input power to an electron as $P = eEv$. By a suitable averaging of the velocity and averaging the energies over the collision times, one can determine the mean energy gain between collisions as

$$u = (eE_0^2/4m) \left\{ 1/ [(\omega + \omega_b)^2 + \nu_c^2] + 1/ [(\omega - \omega_b)^2 + \nu_c^2] \right\} \quad . \quad (32)$$

At low pressures where the collision frequency approaches zero, the energy approaches twice the energy of the elliptical motion of the electron. At higher pressures, such that there are many collisions per oscillation, the energy of the elliptic motion loses its meaning and the collision energy becomes $eE_0^2/2m\nu_c^2$. One can use Eq. 32 to define an effective field E_e which is the rms field at high pressure, and Eq. 32 may be written $u = eE_e^2/m\nu_c^2$. This concept is useful when the collision frequency, ν_c , is independent of velocity, since this single function takes into account the effects of frequency and magnetic field on the energy. At low pressures the effective field has a maximum at resonance with the cyclotron frequency as shown in Fig. 6.

The electrons produced by ionization have initially very little energy, but this increases in steps of u until the energy reaches the ionization energy u_i , disregarding excitation collisions. The number N of free times to ionize is $N = u_i/u$ when ν_c is constant. The electrons thus double their number by ionization every N collisions and unless some equally effective process exists which removes electrons, their number will

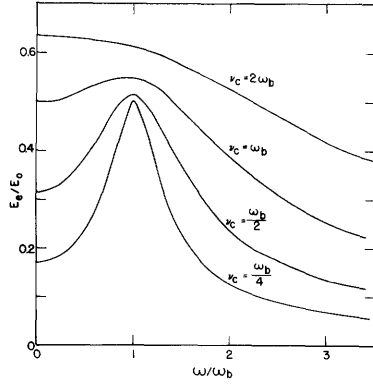


Fig. 6 The effective field as a function of the frequency showing the resonance in the presence of a magnetic field.

increase exponentially. Here we are considering electron diffusion to the walls of the discharge tube to be the balancing process. In the absence of the magnetic field, the random-walk theory (16) gives the mean square distance $\Lambda^2 = N\ell^2/3$ reached in N free paths of mean square length ℓ^2 , so that if the average electron reaches the wall in a distance Λ the diffusion process will just balance ionization. This is the condition for breakdown and we can write it

$$u/u_i = \ell^2/3\Lambda^2 = v^2/3\Lambda^2 v_c^2 \quad (33)$$

If there is a magnetic field, u will be altered according to Eq. 32. At the same time the diffusion theory must be altered to take into account the curved paths between collisions. This may be done by appropriately decreasing the mean free path or increasing the diffusion length. We shall adopt the latter and denote the new length by Λ_e , where

$$\Lambda_e^2 = (\omega_b^2 + v_c^2/v_c^2) \Lambda^2 \quad (34)$$

The effect of a magnetic field is to make the dimensions of the tube at right angles to the field appear larger to an electron.

When the mean free path is much smaller than Λ_e , the intercollision energy gain, u , is correspondingly smaller than the ionization potential. From Eq. 33 we see that breakdown should occur at the same effective field if the ratio of the mean free path to the effective diffusion length is the same, that is, the effective field for breakdown is a function of $p\Lambda_e$ only. Combining Eqs. 32 and 33 we obtain

$$E_e^2 = 2uu_i/3\Lambda_e^2 \quad (35)$$

We saw in Eq. 34 that the effect of a magnetic field is to make the dimensions of the tube at right angles to the field appear larger to an electron. Equation 35 shows that this should reduce the effective field for breakdown in the same proportion.

Experimental data for breakdown in Heg, which again agree with our simplifying assumptions of constant v_c and no loss of energy to excitation, are shown in Fig. 7. The result of plotting the effective field in place of the actual field would be to remove the resonance effect of the magnetic field.

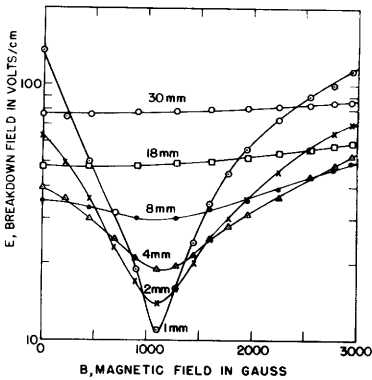


Fig. 7 Breakdown of helium in transverse electric and magnetic fields.

We have just been discussing the case in which the electric field and the magnetic field have been mutually perpendicular. If the two fields are oriented in the same direction, the effect is one of changing the diffusion only. Experimental breakdown curves in Heg gas for this case are shown in Fig. 8.

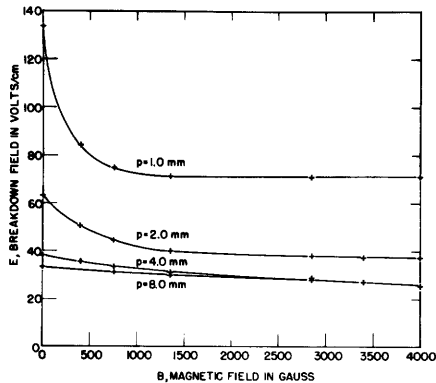


Fig. 8 Breakdown of helium in parallel electric and magnetic fields in a cylindrical cavity.

9. Conclusion

In summary it can be said that high-frequency gas discharge breakdown is much simpler than d-c discharge breakdown because of the absence of necessary secondary effects. In all physical cases so far studied, the governing loss mechanism has been diffusion of electrons to the boundaries of the tubes. Although detailed calculations of the diffusion processes lead to considerable mathematical complexity, simplified theories based on the behavior of the average electron are quite adequate for many cases. Superposition of d-c and magnetic fields on the high-frequency field affect the breakdown in general as they modify the diffusion loss of the electrons.

Limits can be set on the application of the diffusion process, and when breakdown is studied outside these limits, the discharge becomes complicated by the same secondary phenomena as are present in a d-c discharge even if the electric field is oscillating at a radio frequency.

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