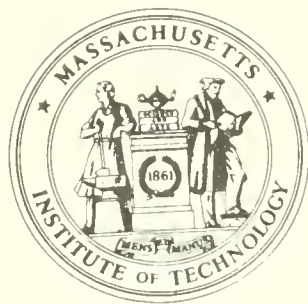


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OPTIMAL TIMING OF BOND REFUNDING*

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H. Martin Weingartner

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OPTIMAL TIMING OF BOND REFUNDING

H. Martin Weingartner

Introduction

Bonds frequently contain a provision whereby the issuer can call the bonds before they have matured. Exercise of this privilege requires notice on the order of 30 to 45 days, and payment of a premium above par specified in the bond itself. Refunding is the operation whereby a bond is called and redeemed with the proceeds of a new issue. Although not exclusively so, it is usually carried out in order to reduce the interest cost of borrowed funds. It is the purpose of this paper to examine the decision whether to call a bond or to leave it outstanding.

Basically, the refunding decision must weigh the alternatives of continuing payment of the interest on the outstanding bonds, or to make a present payment in the amount of flotation costs and the call premium in order to achieve lower interest costs when these become available. In making this evaluation, it is clear that the stream of future payments must be discounted; it is less clear what discount rate should be applied to it. Additionally, it is not obvious what the streams are which must be compared.

Assumptions

Initially, it is assumed that future interest rates and bond yields are either known with certainty, or that the decision maker is willing to act on the basis of forecasts as if they were certain. It is

further assumed that the schedule of call premiums is a known function of the original bond coupon rate and of the age of the bond at time of call, but that it is otherwise invariant over time. Similarly, it is assumed that the costs of flotation of a bond issue of given size, including all expenses and commissions, are either constant over time or a function of the coupon rate of the new issue.

Refunding as a Policy Decision

Discussions of refunding decisions usually take as the period over which interest savings are to be considered the term to maturity of the outstanding bond. If the interest cost reduction more than offsets the call premium and flotation costs, refunding is indicated; otherwise not. Seldom is the possibility of further refunding considered. Even when it is, the assumed horizon is still the remaining life of the old bond.¹

Refunding may be attractive when it is necessary or advisable to stretch the maturity of an outstanding issue, or when it is possible to spread the fixed flotation costs over a larger issue at a time when additional debt capital is raised, possibly also removing some restrictions in the indenture agreement of the old bonds. Such opportunities represent special situations which should not distract from the need for making refunding decisions routinely as part of the implementation of an optimal financial policy, and not every refunding should be regarded as an isolated occurrence. Refunding should involve consideration of providing for the capital requirements currently met by the outstanding bonds, and by its successors. It seems more reasonable, therefore, to

consider refunding in terms of a policy of maintaining the given amount of debt over a fixed period of time, not necessarily (or generally) the life of the currently outstanding bond. Alternatively, it may be more appropriate to regard debt as a permanent component of the company's capital structure, and hence to consider the refunding decision as an effort to minimize interest cost on borrowed funds essentially forever.

In either case, the only decision which is to be made today is whether to continue to pay interest on the outstanding bond for another period, or whether to refund it today. Nothing else needs to be implemented today. Although it is obvious that future events may have to be taken into account in order to arrive at the appropriate decision, and even that future actions, e.g., future refundings, may be implied by one or both alternatives, only the current action must be determined, and future actions requiring no implementation today are only forecasts of actions, and not decisions.² In the present context, a decision to keep the old bonds may be predicated on calling them before paying the coupon after the impending one. Keeping is therefore the current decision. Refunding next period is not a commitment; it is a planned action which will be carried out only if expectations about the future which the next period reveals are actually met.

Discrete Time

For analytic convenience time is assumed to be divided into discrete periods. More precisely, we assume that a decision to refund can only be made after given intervals of time. It is not necessary that the decision periods be the same as the interest periods, conventionally

every six months. Hence, the periods could be arbitrarily small. However, little generality is lost and considerable simplicity and clarity of the models is gained if the decision period is assumed to be of the same length as the interest period.

Discrete time appears to be a natural way to handle this problem. The schedule of call premiums indicates a fixed call price of the bond over an interest period, or over several interest periods. It therefore usually pays to refund just before a coupon is due for payment. Additionally, considerable lead times are involved. Negotiations with underwriters must take place; an issue may have to be registered with a regulatory agency such as the Securities and Exchange Commission. Finally, the management review process takes time. Hence a constant review process is apt to prove costly.³

Discounting

The interest savings which a refunding operations is expected to yield take place over time. They must therefore be discounted to arrive at their present value. While this much is clear, it is less obvious what the appropriate discount rate should be. The discount rate used should express the opportunity cost on funds employed--in this case for incurring the fixed flotation costs and the call premium.

It is easy to rule out the cost of capital as the correct rate, despite recommendation for its use in a number of well known texts [3,7,8]. This quantity is a measure of the earning power and riskiness of the assets of a firm and, presumably, these are unaffected by the substitution of one form of debt, a bond with a new coupon rate, for another, the old bond.⁴

The appropriate discount rate raises complex problems because it depends itself on the decision taken. The fixed charges for calling the old and floating the new bonds constitute a discontinuity in the cost function in which shadow prices may be indeterminate.⁵ Another problem is related to the term which the new bond is expected to remain outstanding, which differs, generally, from the term remaining for the existing bond. By confining ourselves to a given level of debt, it is safe to assert that the correct discount rate is related to a rate on debt. However, if refunding is undertaken, the rate for discounting must be the rate related to the new bonds and to the term these bonds will be outstanding. If the decision at this moment is to retain the old bonds, the appropriate rate for discounting is a quantity related to the old bonds and their actual life. Furthermore, calculations involving future switches would also require utilization of the rates that are expected to apply then.⁶

These comments are admittedly ambiguous. A more precise definition of the discount rates to be used must be postponed until after the decision model has been developed. A further discussion of the discount rate will be taken up in the section dealing with uncertainty. In the meantime, it is necessary to emphasize that although the computations are carried out with a number of different discount rates for any one period only a single rate will turn out to be the correct one. While it is impossible to specify in advance what that rate will be since it hinges on the decision itself, it will come out of the computations for the model. Perhaps more important, this treatment of the discount rate imposes no substantial computational burden on the model proposed below for the

determination of the refunding decision. Exactly how this discounting scheme operates will become clear from the description of the model.

Yield Information

Regardless of the model used to arrive at a refunding decision, whether formal or informal, some information about future yields on bonds must be obtained. The models proposed here are more demanding in such information than simpler ones. However, a substantial amount of information is available in the form of the yield curve. This curve is a schedule of bond yields on bonds with maturities (i.e., remaining lives) running from the shortest to the longest [4]. From such schedules it is possible to infer what the financial market's current belief is about future bond yields.⁷

More precisely, the required information is derived from the term structure of interest rates, which in turn is derived from the yield curve. The needed information consists of yields of bonds selling at par (and for which the coupon rate is equal to the yield), which are expected to apply to bonds issued in the future as well as at the present. The yield curve as commonly drawn does not make this correction. Further, for the derivation of future bond yields an intermediate step is required to take into account the effect of the intervening coupon payments implied by a bond contract. This most easily makes use of the term structure of interest rates as the schedule of rates of contracts with a single outlay and a single payment which includes interest compounded annually, at the end of a fixed term.⁸ The derivation of currently expected future interest rates requires some simple algebraic manipulations involving the relation that the n-period rate is the geometric mean of its component one period rates, or of other components within that term, appropriately weighted.⁹

A second, less readily described procedure, is to relate the schedule of yields on the lowest risk class of bonds typically incorporated in the yield curve to the bonds of the given firm with its own degree of risk. This point will,

without doubt, continue to require the judgement of knowledgeable people in the underwriting field, aided, in all probability, by systematic statistical work.

Tax Consequences

The tax consequences of a refunding operation are important in the decision and in themselves are quite complex. To the extent that marginal tax rates will differ for the firm in the future, either due to changing profitability of the firm or due to changes in tax laws, the incorporation of tax consequences affords additional complications difficult to handle. The difficulties are, of course, no less than for the traditional ways of evaluating the refunding decision. With assumed stable tax rates, incorporating the tax effects explicitly into the analysis has the primary effect of distracting from the basic principles of the models employed, and hence tax considerations will be omitted here.

Notation and Development of the Models

Whether a finite horizon or an indefinite one is more appropriate, a dynamic programming approach analogous to that formulated for the equipment replacement problem [1] may be employed for solving the refunding decision problem. The notation can be kept considerably simpler by expressing all costs in dollars per dollar of existing debt, leaving the size of the bond issue outside the problem.¹⁰ Similarly, we assume that bonds are issued at par, which implies that coupon rates are identical with yields. Although this condition may not be met in practice, a relationship between coupon rates and yields exhibited in [15] may be utilized. We also express all interest rates in terms of dollars per interest period. This would be equivalent to ordinary interest rates if coupons are annual, and it may be best to think of bonds with a single annual coupon for interpretation of the models. However, the model itself is general.¹¹

Taking the simplest of the models first, primarily to fix ideas and develop the notation, we take as the horizon a fixed period (year), T . We denote calendar time by t and call today, the decision date, t_0 . The age of a bond, i.e., the number of periods since its issuance, is denoted by j ; the outstanding bond is of age j_0 . For this first model we also assume that bonds which might be issued in a refunding operation, either now or at some later date, will have a maturity fixed by convention for this firm or industry, and call this conventional maturity, m_c . It is not necessary that the old bond also has an original term-to-maturity of m_c .

We let r_t^m denote the coupon rate for a bond of original maturity m issued in year t . The call premium is a declining function of age and original term-to-maturity, as well as of the original coupon rate. It is not necessary that this be an analytic function: a table will be sufficient. It is necessary, however, that this function remain constant over time. Thus, for example, it is typical to begin with a call premium equal to a year's coupon.¹² Usually, every interest period the call premium declines by one over the original maturity times the coupon.¹³ We then write the call premium for a bond with an original maturity of m and of age j in year t as $c_j(r_{t-j}^m)$.¹⁴ We let F be the fixed costs of floating an issue. Since the latter may be tied to the coupon rate of a new issue, we may wish to write this quantity as $F(r_t^m)$. We require only that this quantity is known and can be projected into the future.

To aid in understanding what follows, recall that often dynamic programming works by "backward optimization." A complex decision tree is analyzed by traversing it backward, looking at the terminal decision, then the decision in the period preceding the terminal one, and so on until the current decision is reached. In the process not all branches of the tree need to be examined. Indeed, the efficacy of the method lies in its ability to rule out a large fraction of the astronomical number of combinations of which the tree is comprised. Thus we define $f_t(j)$ to be the discounted cost associated with the liability of bonds of age j in year t when an optimal refunding policy is followed for the remainder of the process.

The Basic Fixed Horizon Model

The basic model for the refunding decision is expressed in terms of a fixed horizon, denoted by T , the period in which this debt is to be retired. We assume that all bonds issued in a refunding will be of conventional maturity, m_c , except possibly near the end of the process, at which time bonds may be issued to mature at the horizon. This term to maturity is denoted by the symbol $[x, m_c]$, where x , to be discussed in detail below, is the remaining time to the horizon, and for the two quantities separated by the comma in the bracket read "the smaller of x and m_c ."

Utilizing the above assumptions, we may express the recursive relations which determine the optimal refunding decision as follows (leaving the discounting rates, r_t^{k*} , momentarily undefined):

$$(1) \quad f_{t_0}(j_0) = \text{Min} \left\{ \begin{array}{l} \text{Keep:} \quad \frac{r_{t_0-j_0}^{m_c} + f_{t_0+1}(j_0+1)}{1 + r_{t_0-j_0}^{k^*}} \\ \text{Refund:} \quad c_{j_0}(r_{t_0-j_0}) + F + \frac{r_{t_0}^{m_c} + f_{t_0+1}(1)}{1 + r_{t_0}^{k^*}} \end{array} \right.$$

For $t_0 < t \leq T - m_c$

$$(2) \quad f_t(j) = \text{Min} \left\{ \begin{array}{l} \text{Keep:} \quad \frac{r_{t-j}^{m_c} + f_{t+1}(j+1)}{1 + r_{t-j}^{k^*}} \\ \text{Refund:} \quad c_j(r_{t-j}) + F + \frac{r_t^{m_c} + f_{t+1}(1)}{1 + r_t^{k^*}} \end{array} \right.$$

For $T - m_c < t < T - 1$

$$(3) \quad f_t(j) = \text{Min} \left\{ \begin{array}{l} \text{Keep:} \quad \frac{r_{t-j}^{[T-t+j, m_c]} + f_{t+1}(j+1)}{1 + r_{t-j}^{k^*}} \\ \text{Refund:} \quad c_j(r_{t-j}^{T-t}) + F + \frac{r_t^{T-t} + f_{t+1}(1)}{1 + r_t^{k^*}} \end{array} \right.$$

Finally, for $t=T-1$

$$(4) \quad f_{T-1}(j) = \text{Min} \left\{ \begin{array}{l} \text{Keep:} \quad \frac{i_{T-j}^{j+1}}{1 + r_{T-j}^{k*}} \\ \text{Refund:} \quad c_j (r_{T-j-1}^{T-j}) + F + \frac{r_{T-1}^1}{1 + r_{T-1}^1} \end{array} \right.$$

Interpretation of these equations requires two additional comments.

First, when $j=m$, i.e., when a bond has matured, the "Keep" option is no longer open, and the "Refund" option is mandatory. At this time the call premium has disappeared. Thus

$$(5) \quad f_t(m) = F + \frac{r_t^m + f_{t+1}(1)}{1 + r_t^{k*}}$$

In this expression the maturity was purposely written as m and not as m_c , the conventional maturity which we have assumed will be utilized--when applicable. The latter qualification is the second point which needs to be clarified.

As indicated earlier, with a fixed horizon, it makes little sense to issue bonds of longer maturity than the remaining time to the horizon, only to call the bonds at the horizon. Put differently, we can rule out the advantage for such an alternative a priori in the present context.¹⁵ With this in mind, we must write the maturity of the

bond explicitly, as well as the yields which apply to it, whenever the time to the horizon from date of issue is less than m_c . In equations (3) and (4) this has been indicated by the superscript on coupon rates of the new bonds in the "Refund" strategy in unambiguous fashion. However, the notation requires also that the coupon rate of the outstanding bonds in the "Keep" strategy, as well as the call premium under the "Refund" strategy, be qualified to indicate what maturity the bond had at issue. This will be the remaining time to the horizon at time of issue, $T-(t-j)$, or the conventional maturity, m_c , whichever is smaller, and has been denoted by $[T-t+j, m_c]$.

The computations implied by this model can usually be reduced to a considerable extent by use of dominance relationships. Leaving discussion of these until after the more general models have been developed, we may nevertheless give a brief verbal description of the process of computation for this model.

For each possible age of an outstanding bond at the date of the last decision point, i.e., the period before the horizon, the decision to keep the bonds or to issue new ones is based on a comparison of their relative costs. Keeping implies paying interest of r_{T-j}^{j+1} , which is the rate which applied to a bond with an original maturity of $j+1$ years, when issued in year $T-j$. This interest payment comes at the end of the period, hence it must be discounted to its beginning, utilizing for discounting the rate which has been denoted by r_{T-j}^{k*} . Refunding in that year implies paying the call premium on bonds of age j issued with the indicated coupon rate, plus the fixed costs of flotation, plus the coupon on the newly issued

one-year "bonds" paid at the end of the year, but discounted to its beginning.¹⁶
The rate used for discounting is the new rate itself.

Thus, for each possible age of bond at the date $T-1$ one of the two strategies may be crossed off, and the cost of the more favorable one is noted. Backing up to the previous period, the costs just arrived at are entered in their respective places of the Keep and Refund strategies of that period. The decision to keep or refund at this stage is computed for a bond of each age, and is based on a comparison of the following two alternatives. For the "Keep" strategy add the interest cost for that period, discounted to its beginning, to the cost of the most favorable continuation from there on to the horizon, also discounted to the beginning of the period. For the "Refund" strategy add the call premium on the outstanding bond, the flotation costs of the new, the coupon for one period for the new bond, plus the cost of the best continuation with this new bond, the latter two discounted to the beginning of the period. Having eliminated one of the choices for a bond of every age possible, the cost of the remaining choice is noted, and the period prior to that just considered is computed. This process is repeated until date t_0 , the present, is reached. It will be noted that at this point in time only a bond of age j_0 is relevant, and the decision to keep or refund depends only on the comparison of costs for the bond of this age.¹⁷

With this picture in mind, it is finally possible to define the elusive discount rates. At each step, and for each age of bond, as one or the other strategy is crossed off, information is obtained about the age at which refunding for a future issue is indicated. Thus, if at

stage t a bond of age k is refunded, it will have had a total life of k years beginning in year $t-k$. At this point it is possible to give the discount rate as the k -year yield beginning in $t-k$, and this rate applies to all "Keep" strategies for the years between $t-k$ and t . Since these strategies are considered after year t in the backward optimization, the correct rate will be available for these latter evaluations. Once the refund strategy has been chosen for this age of bond, its retirement age is denoted by k^* , and the discount rate by $r_{t-k}^{k^*}$. Thus, for the "Keep" strategy, the discount rate will already have been set by a subsequent (determined earlier) refunding decision.

The computations begin with year $T-1$, the year before the horizon and final retirement of debt. If refunding is the chosen strategy at this point, the coupon payment at the end of that period is discounted at its own interest rate. If, however, the choice is to keep the then outstanding bonds, it is implied that the bonds will be retired at the horizon, and the discount rate of this and prior "Keep" strategies is the rate that pertains to this bond life. Similarly, when a bond matures without first being called, the discount rate which applied during its lifetime is its coupon rate, in which case the quantity denotes by $r_t^{k^*}$ in the denominator is r_t^m , the rate at issue.

Variable Maturity, Fixed Horizon Model

Generalization of the basic fixed horizon model to include the possibility of issuing bonds of any of a number of maturities at any time requires only a few additional definitions. The model itself resembles the basic one to a sufficient extent to make it unnecessary to present explicitly any but the general expression.

In place of $f_t(j)$ we substitute $f_t(j,m)$ which is defined as the discounted cost in period t of having bonds of age j with an original maturity of m periods (years) when an optimal policy is followed for the remainder of the process. We further define M as the set of maturities which are to be considered,¹⁸ and let m' denote the maturity of the new bond after refunding. We also denote by $F(r_t^{m'})$ the fixed costs associated with issuing bonds of maturity m' in period t . The general expression then becomes,¹⁹

$$(6) \quad f_t(j,m) = \text{Min} \left\{ \begin{array}{l} \text{Keep:} \quad \frac{r_{t-j}^m + f_{t+1}(j+1,m)}{1 + r_{t-j}^{k^*}} \\ \text{Refund:} \quad \text{Min}_{m' \in M} \left[c_j(r_{t-j}^m) + F(r_t^{m'}) + \frac{r_t^{m'} + f_{t+1}(1,m')}{1 + r_t^{k^*(m')}} \right] \end{array} \right.$$

In this equation the first computation finds the best of the eligible maturities if the choice should be to refund and the associated discounted cost which includes the cost of the best continuation. This cost is then compared with the cost of keeping the then outstanding bond for another period and the cost of its best continuation, discounted to the beginning of the period.

Completion of the model would require separate expressions for t_0 and for dates before the horizon, following the reasoning employed earlier. To conserve space, they will be omitted. Our attention is focussed instead on the infinite horizon model.

Permanent Debt

In many instances, the assumption that debt will be a permanent component of the enterprise's capital structure is the more realistic one. Nevertheless, the dynamic programming approach can still be employed with an infinite horizon under the condition, which is generally correct, that the yield curve levels off after some date. That is, we are able to incorporate all relevant costs for an indefinite future once the date has been reached after which the yield curve is flat--yields are the same, regardless of term to maturity. We call this date period (year) T.

At the first refunding after year T, the cost consists of the discounted value of interest payments at the constant rate $r_T^{m_c}$ plus the periodic flotation costs F every m_c years.²⁰ The total cost of debt service and refunding, from the date of this first refunding after the horizon, and discounted to that date of refunding is

$$(7) \quad r_T^{m_c} \sum_{i=1}^{\infty} \left(\frac{1}{1+r_T^{m_c}} \right)^i + F \sum_{i=0}^{\infty} \left[\left(\frac{1}{1+r_T^{m_c}} \right)^{m_c} \right]^i.$$

The first term consists of the periodic coupon payments, beginning at the end of the period and continuing forever, and discounted to the date of the first refunding after the year T. The second term has been written to show the flotation costs incurred every m_c years, starting at that date, and discounted similarly. The first term reduces to unity.²¹ The second term may be summed, and the total discounted cost of interest plus refunding from the date of the first refunding after the horizon may be expressed as

$$(8) \quad 1 + \frac{F}{1 - (1+r_T^{m_c})^{-m_c}} .$$

At year $t=T+m_c$, i.e., m_c years after the yield curve first becomes level, the bond outstanding must necessarily carry a coupon of $r_T^{m_c}$, having been issued in year T or subsequently. If it is kept, it will not pay to refund it until it has matured. Thus, if the old bond is j years old at this date, $j < m_c$, refunding will take place m_c-j years later. That is, the alternative of keeping versus refunding comes down to deferring the flotation cost cycles by m_c-j years or incurring them immediately by refunding. For this year, then, the cost of the optimal strategy for each age of bond is given by²²

$$(9) \quad f_{T+m_c}(j) = \text{Min} \left\{ \begin{array}{l} \text{Keep:} \quad 1 + \left[\frac{F}{1 - (1+r_T^{m_c})^{-m_c}} \right] (1+r_T^{m_c})^{j-m_c} \\ \text{Refund:} \quad 1 + \frac{F}{1 - (1+r_T^{m_c})^{-m_c}} \end{array} \right.$$

For years t_0 equation (1) holds; for years t_0+1 through year $T-1$ equation (2) applies. For years T to $T+m_c-1$ we need

$$(10) \quad f_t(j) = \text{Min} \left\{ \begin{array}{l} \text{Keep:} \quad \frac{r_{t-j}^{m_c} + f_{t+1}(j+1)}{1 + r_{t-j}^{k^*}} \\ \\ \text{Refund:} \quad c_j(r_{t-j}) + 1 + \frac{F}{1 - (1+r_T^{m_c})^{-m_c}} \end{array} \right.$$

Of course, the "Keep" strategy is omitted whenever $j=m_c$. Similarly, note that the call premium has been omitted from refunding for year $t=T+m_c$, calling in advance of maturity being unfavorable from that date on, if not before.

Although refunding is assumed to continue indefinitely after year $T+m_c$, the cost of the two possible strategies no longer depends on the decision taken then. That is, equation (9) does not contain a term $f_{T+m_c+1}(j+1)$ on the right side. Thus a starting point for the backward optimization has been obtained.

Dominance Relations to Reduce Computation

The computations for dynamic programming problems of this type can require considerable computer time and large high-speed memories. Fortunately simplifications are available to reduce the amount of computational effort. These resemble devices developed in another context in [16] and employ dominance relations which may be used to rule out a substantial number of the possible strategies. Our purpose here is only to indicate generally the form of these relations, and this task will be accomplished by means of an illustration.

In the computations for the indefinite horizon model, the first step requires finding $f_{T+m_c}(j)$, using equation (9) for each value of j , $j=1, \dots, m_c$. Since the right hand sides of equation (9) do not involve values of $f_{T+m_c+1}(j)$, as previously pointed out, it is possible to substitute numerical values as calculated from the data. For simplicity, let

$$(11) \quad a_j = 1 + \left[\frac{F}{m_c^{m_c} (1+r_T)^{-m_c}} \right] (1 + r_T)^{j-m_c}$$

and

$$(12) \quad a_{m_c} = 1 + \frac{F}{m_c^{m_c} (1+r_T)^{-m_c}}$$

Since keeping is not an alternative for $j=m_c$, it follows that $f_{T+m_c}(m_c) = a_{m_c}$. It is also clear that the "Keep" strategy is the optimal one for $j < m_c$, i.e., $f_{T+m_c}(j) = a_j$, since this alternative defers the refunding cycle and its fixed outlays, with consequently smaller discounted cost.²³

Backing up one step to the previous decision point, we may utilize equations (11) and (12) as follows. From (11) it is possible to derive that, for $j < m_c$, $a_j < a_{j+1}$; in fact,

$$(13) \quad \Delta a_j \equiv a_{j+1} - a_j = (a_j - 1) \{ (1 + r_T)^{m_c} - 1 \}.$$

Also, $c_j > c_{j+1}$, i.e., the call premium falls as the bond age increases. Frequently,

$$(14) \quad -\Delta c_j \equiv c_j - c_{j+1} = \frac{1}{m_c} r_T$$

where r_T is the coupon on the bond. Equation (13) shows Δa_j to increase with j while $-\Delta c_j$ remains the same. Hence if the "Keep" strategy is indicated for $j=1$, then there is a value $j=j^*$ such that for all $j > j^*$ the refunding strategy is optimal and once j^* is found, further evaluations of (10) for other values of j is unnecessary.

Extension of the analysis above for values of t smaller than $T+m_c-1$ is also straightforward, albeit somewhat more complex. It may prove to be useful given computational limitations.

Approaches to Uncertainty

The analysis so far has been based on the assumption that the term structure of interest rates expresses the market's current expectations about future interest rates, or that these can be derived from the term structure by separating out any risk premiums and also removing any differential between the rate on callable bonds and bonds without this feature. The decision-maker in this analysis is not **betting** against the market. He is using the market data, with which he agrees, to evaluate his alternatives. One consequence which follows from this is that with complete freedom with respect to future bond maturities, the expectation is now that future bonds will not be called before they mature. This follows since all maturities are assumed to be available, and calling involves incurring an avoidable cost. However, without a completely unconstrained choice of maturities, calling bonds to be issued in the future may be planned today. For example, calling a future issue and refunding it may be the best way to avoid high rates expected to prevail when the bonds mature.

Incorporation of the call feature in a bond issue may be legally required. For example, this applies to many regulated utilities. If it is not required, the call privilege is not costless, and its adoption is in the

nature of a hedge against future interest rate declines. Estimation of the value of the call privilege is not the central concern here, although the models used can be employed for this purpose. In doing so one must clearly differentiate between two kinds of hedges which may be relevant for the re-funding decision.

The first, mostly ignored, might be called a "data hedge." Although a full Bayesian analysis would incorporate it, the decision maker may wish to express his state of belief about the course of future interest rates in terms of several term structures, each with a subjective probability attached. The procedure of the certainty models could then be repeated for each term structure, and if the decision with respect to the current refunding is altered, the choice is made so as to minimize expected cost or to maximize expected utility. Clearly, if the decisions agree, no further evaluation needs to be made.

To the extent that the decision maker evaluates the likelihood of future events and their economic impact differently from the way the market is currently doing, which is expressed by the market term structure of interest rates, the forecaster must be prepared to state when the market will correctly anticipate these events and how the term structure will change as a result. Consider, for example, that the private belief is that next period the term structure will have a different shape from that implied by the current one. The current rates are still applicable for equation (1) in which adopting the refunding strategy implies issuing bonds with a coupon of $r_{t_0}^{m_c}$ and the strategy to keep the old bonds implies a continuation of coupon payments in the amount of $r_{t_0-j}^m$. However, in order to make the evaluation about refunding or keeping the outstanding bonds next year requires using the curve being projected for the market next year, and not the rates the market expects today to apply to contracts written next year.

In principle, a full treatment of interest rate speculation within the context of the refunding decision requires a separate forecast of the term structure of interest rates for each future decision point, to the horizon. In practice it will likely be the case that much less is needed than this formidable amount of information. Where forecasts are predicated on future events, belief in whose occurrence is not certain, forecasts of term structures conditional on the happening of these events may be necessary, together with the probabilities attached to them. Again, the analysis is made to see if the current decision is the same for all of these, and if not, a decision is taken corresponding to a criterion such as minimizing expected cost or maximizing utility. The latter would be required should a linear approximation to the utility function be deemed inadequate for the problem at hand. If the choice of maturity for future bonds is completely free, the amount of forecast data is considerably reduced. In that case, only the market's expectations of future one-year rates needs to be forecast for each future decision point since the relevant rates will all be functions of these.

FOOTNOTES

1. See [6].
2. See [11], for example, for a discussion of the difference between decisions and plans.
3. The lead times in particular rule out taking advantage of yield fluctuations which are expected to last only a very short time. In any case, yields on long-term bonds do not change drastically in very short spans of time. This will be true even when short-term interest rates fluctuate materially. See, for example [15].
4. See [12]. This point was also stressed in [2], which appeared after this paper was first presented, although what follows will also differ from the position taken there.
5. See [5] and [14], Chapter 5. This difficulty does not arise when only a one-time refunding decision at a given time is evaluated, as is universally done in the literature. However, essential aspects of the problem are completely overlooked thereby, as can be seen by considering the alternatives of refunding now versus refunding one period later.
6. Those whose reflex is to use the cost of capital do not succeed in avoiding these complications but rather increase the difficulties. Estimation of the cost of capital either by the "Harvard method" [7, p. 413ff.] or by the theoretically more sound Modigliani-Miller approach [12] for present purposes comes down to the same thing. Cost of capital is a weighted average of the cost of equity and the cost of debt. Although the weights according to these two approaches are quite different and are determined in a different way, the effect of a weighted average is still to require use of the correct rate on the debt component, making the cost of capital itself a changing quantity when the cost of debt changes.
7. This statement is probably too strong at this time. Considerable work of recent date has shed much light on the relation between the yield curve and expected yields in the future, as discussed, for example, by Meiselman [10], Kessel [9], and Modigliani and Sutch [13]. At issue for our purpose is not the question whether the expectations hypothesis is valid, or whether the market does, in fact, add a liquidity premium on longer maturities. What matters here is that by deducting a liquidity premium, if necessary, some information about future yields may be obtained.

8. For a discussion of the difference between the yield curve, which includes the periodic coupon payments, and the term structure of interest rates, see [15].
9. These problems fall somewhat outside our present interests, and will not be repeated here. Among the issues is the effect of coupon rates and the shape of the yield curve on the inferred future interest rates. These are also treated in [15].
10. As indicated before, the refunding decision is viewed here as motivated by potential interest savings, not as an attempt to stretch maturities, per se, or to increase or decrease the amount of debt.
11. If the review period is shorter than an interest period, the models will require modification, though in an obvious way.
12. Since all computations are on a basis of \$1 par value, coupons and coupon rates are identical. The same applies to the call premium.
13. Where this quantity is too cumbersome, slight departures from linear declines are made. In any case, a table is presented in the bond offering.
14. That is, the call premium depends on the age of the bond and on its coupon rate.
15. Clearly, under uncertainty, the horizon itself is a quantity subject to revision and this remark may not hold. Before giving weight to such considerations, however, see the discussion on infinite horizons, below, and the explicit consideration of uncertainty.
16. When the horizon comes close, other forms of financing, e.g., bank loans, are more realistically substituted for bonds.
17. For this reason it is also possible to eliminate from consideration all but two values for j at $t=t_0+1$, namely $j=1$ and $j=j_0+1$; all but $j=1, 2, j_0+2$ at $t=t_0+2$, etc.
18. Since, in effect, the computation for the model is being enlarged by an order of magnitude, it is decidedly worthwhile to omit from consideration those maturities which are either too long or too short to be likely candidates, and also to omit maturities regarded as oddities by the financial community, as, perhaps, bonds of 23 years to maturity at issue.
19. The discount rate for the refunding strategy is now written as $r_t^{k^*(m')}$, i.e., as a function of the term m' of the replacement bonds.

20. Since the coupon rate is assumed to remain constant from the horizon on, refunding prior to maturity is not economical for bonds issued after T. Therefore it is optimal to spread the flotation costs over the longest practical or acceptable maturity, which is denoted by m_c . The rate for discounting is then also the coupon rate, $r_T^{m_c}$.
21. The present value of a perpetuity of \$1 at an interest r is $\$1/r$. The present value of a perpetuity of \$r at an interest rate of r is $\$r/r=1$.
22. To focus on the difference between this and the fixed horizon models, the assumption of a fixed conventional maturity is made again.
23. This may be seen from equation (9) in which the quantity $\frac{F}{1 - (1+r_T^{m_c})^{-m_c}}$ is multiplied by $(1+r_T^{m_c})^{j-m_c}$ which is less than unity.

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