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On the Connections Between Activity Based Costing Models and Optimization Models for Decision Support

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Abstract

A numerical example from the Activity-Based Costing (ABC) literature is used to illustrate the extension of an ABC model to an optimization model for decision support. The paper continues with detailed discussions of the similarities and differences between the two types of models and computer systems based on them. The paper concludes with a discussion of issues surrounding organizational adaptation to the use of such models and systems.

ON THE CONNECTIONS BETWEEN ACTIVITY BASED COSTING MODELS AND OPTIMIZATION MODELS FOR DECISION SUPPORT

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1. Introduction

In manufacturing firms, traditional accounting methods determine unit product costs from allocations of functional categories that have little to do with the firms' value adding activities. Activity-based costing (ABC) methods seek instead to identify cost drivers and relationships that more accurately describe how costs of manufacturing activities are incurred (Kaplan [1983], Cooper [1988]). Some costs may vary directly with the volume of a cost driver, whereas others may not vary with volume, or may vary in a non-linear or non-smooth manner.

For example, direct labor costs may be accurately described as a linear function of direct labor hours as the cost driver. By contrast, indirect plant overhead costs may be accurately described as the sum of a lumpy (fixed) cost term and a variable cost term based on the number of employees working at the plant who are not directly involved in production.

Although we will focus on manufacturing applications in this paper, ABC methods are also pertinent to service organizations and service functions within manufacturing firms (see Chapter 7 in Cooper and Kaplan [1991]). Banker and Johnston [1993] report on an extensive analysis of cost drivers and cost relationships for U. S. airline companies. Lewis [1991] discusses the development of ABC models to describe marketing costs in manufacturing firms.

The development of ABC models involves both science and intuition (Cooper [1989]). Statistical regression and exploratory data analysis techniques may be used to identify cost drivers and to develop mathematical relationships that describe costs as a function of these drivers (Novin [1992], Datar et al [1993], Banker and Johnston [1993]). Turney [1992] provides practical guidelines for developing ABC models and convincing management to use them.

Managerial judgment about likely drivers and the extent to which they are volume based can greatly speed up the process of constructing an ABC model. Human judgment is also required to balance accuracy of an ABC model against its parsimony and ease of use. Babad and Balachandran [1993] address this issue from an information theoretic viewpoint by developing an integer programming model that optimally balances savings in information processing costs against accuracy.

It is not our purpose here to review ABC in detail. Rather, we wish to discuss several important but largely ignored connections between ABC models and optimization models to support managerial decision making. The oversight is curious since, as Dopuch [1993; p. 615] points out,

> "In the 1960's, teaching and research in managerial accounting were heavily influenced by the growing popularity of operations research and management science approaches to business problems. These approaches were largely normative in nature, consisting basically of optimization techniques that, if implemented, promised to improve managerial decision making."

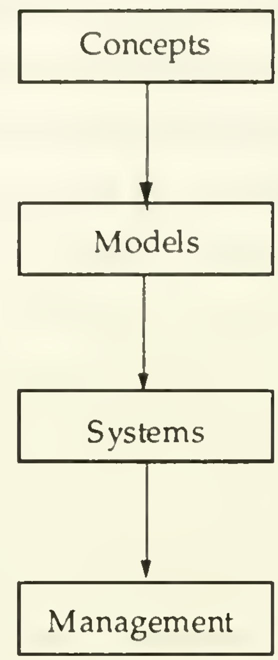
The implication is that ABC grew out of this early interest in analyzing quantitative data to improve decision making. Yet today, when computer systems for ABC and optimization models are becoming much more widely used, the conceptual and practical affinity of the two disciplines has not been recognized or exploited.

Dopuch goes on to question the empirical evidence that new systems, whether they are managerial accounting systems or decision support systems, can and will improve decision making to an extent that justifies investment in them. There is little doubt today that properly conceived and executed decision support systems based on optimization models can lead to large and sometimes enormous cost savings or increases in profits (for example, see Jack, Kai and, Shulman [1992], Moore, Warmke and Gorban [1991], Shapiro, Singhal and Wagner [1993]). Nevertheless, Dopuch's doubts have validity for the 1960's when companies did not have complete or easily accessed information systems on which to base ABC models or optimization models for decision support. In short, the theory at that time was far ahead of the practice.

In this paper, we restrict our attention to **linear programming** (LP) and **mixed integer programming** (MIP) models because such models are the ones most frequently used to analyze business problems. Our primary interest is in the role of LP and MIP models as analytical engines in **Advanced Decision Support Systems** (DSS's). Throughout the paper, the terminology Advanced DSS will refer to a DSS based on optimization models. Due to recent developments in information technology, especially the ready availability of data and powerful desktop computers, these models have become much more relevant to decision making in the firm than they were in the past.

The motivation for this paper stems from our experiences developing ad hoc ABC models for Advanced DSS applications. Investigation of the ABC literature revealed that connections between the two disciplines are pervasive. Perhaps the most important common feature of the two methodologies is their role in facilitating better management of business processes through cross-functional **integration**.

FIGURE 1: SEMANTIC SCHEME



The connections can be found at the four levels shown in Figure 1. In this scheme, **Concepts** refer to the ideas, constructs, and algorithmic techniques used in creating, implementing and solving models of business problems. **Models** refer to specific, data substantiated mathematical equations and relationships. **Systems** refer to computer systems that acquire data, prepare these data for the purposes of modeling, generate models from prepared data, solve models using algorithmic techniques and report on the results. **Management** refers to the organization's use of data management and modeling systems for the purposes of studying business problems and achieving better planning and control.

We believe that ABC methods and optimization methods have a great deal in common. In fact, it is difficult to identify elements of either methodology that is applicable only to it. Nevertheless, we believe that optimization models provide a deeper and more comprehensive analysis of managerial decision problems than ABC models, although the latter models can provide critical inputs to the former. At the system level, we believe that ABC systems and Advanced DSS's can and should be merged into integrated systems. At the management level, similar organizational barriers inhibit the realization of better planning and control through the use of ABC systems and Advanced DSS's. We will argue that the two disciplines can more effectively promote their individual technologies by exploiting the synergies between them.

This paper is devoted to an elaboration of these points. In the following section, we extend a numerical example from the ABC literature to an optimization model. In the five sections after that, we examine the connections between ABC models and optimization models illustrated in part by the example. After that, we discuss system implementation of ABC and optimization models. Then we discuss issues of organizational acceptance of and adaptation to ABC systems and Advanced DSS's. The final section is devoted to a brief statement of conclusions and areas of future research.

2. An Example

We present a concrete example taken from Christensen and Sharp [1993] who in turn drew on material in Horngren and Foster [1991]. They used ABC methods to evaluate the following situation (Christensen and Sharp [1993; p.39]):

> "The Wichita Machine Shop produces three types of precisely engineered components for aircraft engines - Model 1000, Model 2000, and Model 3000. Model 1000 is the high volume basic component. Models 2000 and 3000 are progressively more

sophisticated, with higher- quality and a greater number of parts involved in their manufacture. Budgeted production data for these products for 1993 are presented in Table 1.

TABLE 1: WICHITA MACHINE SHOP
BUDGETED PRODUCTION DATA FOR THE YEAR 1994

	Model 1000	Model 2000	Model 3000
Unit production	10,000	5,000	800
Direct materials cost per unit	$80	$50	$110
Number of parts per unit	30	50	120
Direct labor hours per unit	2	5	12
Machine hours per unit	7	7	15
Production orders	300	70	200
Production setups	100	50	50
Orders Shipped	1,000	2,000	800

The company has just implemented an activity-based costing system. Its single manufacturing department has been divided into six activity areas, appropriate cost drivers have been selected, and costing rates for the year have been established (see Table 2)."

TABLE 2: BUDGETED ACTIVITY-BASED CONVERSION COSTS BASED ON HISTORICAL DATA FOR 1993

Activity Area	Cost Driver	Costs	Cost Driver Total	Costing Rate
Material handling	# of parts	$258,400	646,000	$.40
Production scheduling	prod. orders	114,000	570	200.00
Setup labor	prod. setups	616,300	200	3,081.50
Automated machinery	mach. hours	3,053,700	117,000	26.10
Finishing	direct labor hours	1,092,000	54,600	20.00
Packaging and shipping	orders	190,000	3,800	50.00

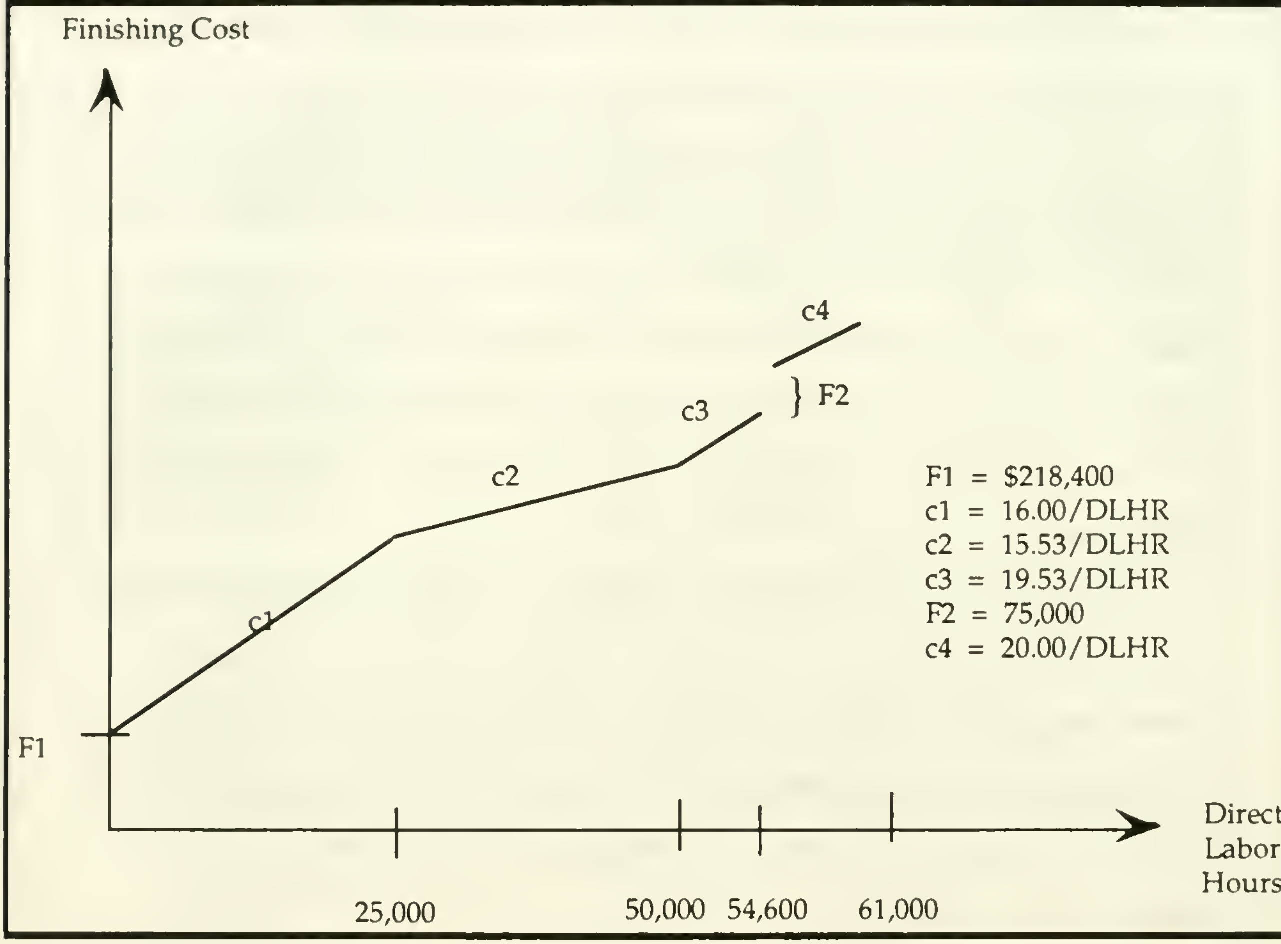
Wichita senior management is concerned with determining a strategy for product pricing, product mix and resource allocation that maximizes net revenues for the coming year, 1994. They seek an optimization model to help them formulate such a strategy. Although the information in Tables 1 and 2 indicates the types of costs and relationships that should be considered, the production levels for Wichita's three products have been pre-determined. As we indicated, one of management's concerns is to better understand how the mix among the three products should be varied. Furthermore, the ABC costing rates in Table 2 are based on fixed and known historical levels for the cost drivers. Since these levels will change next year, the information must be extended to describe how costs vary as functions of them.

In developing these functions, it is important to distinguish between those for which the cost driver is a resource that may be scarce and therefore may constrain the optimal strategy, and those for which the cost driver is merely an accounting device and not a resource that will constrain the strategy. We refer to the former functions as **cost/resource functions**, and to the latter as **cost/accounting functions**.

In addition, as Christensen and Sharp point out, costs may be short-term variable, short-term fixed and committed fixed. Construction of the cost/resource and cost/accounting functions should incorporate these characteristics. In the context of the concerns of Wichita's management, short-term refers to controllable within the coming year whereas committed refers to unavoidable within the coming year unless Wichita decides to liquidate the associated resource.

As an example of a cost/resource function, further analysis of Wichita's finishing activities as a function of direct labor hours reveals the relationship shown in Figure 2. We consider the fixed cost of $218,400 to be committed and

FIGURE 2: COST/RESOURCE FUNCTION FOR FINISHING ACTIVITIES



unavoidable; that is, Wichita clearly does not have the option of liquidating the resource associated with the cost driver. The variable cost of direct labor decreases after the hours reach 25,000 for the year reflecting an economy of scale in Wichita's operations. The variable cost increases after 50,000 reflecting significant overtime. To acquire direct labor hours beyond 54,600 next year, Witchita will need to add extra shifts, with an additional indirect fixed cost of $75,000 for supervisory personnel, insurance, energy, etc. Moreover, the variable cost for overtime is $20. per direct labor hour. For further discussion of this type of nonlinear cost function, see Roth and Borthick [1991].

Similar analysis of the other cost driver relationships at Witchita produces the data shown in Table 3.

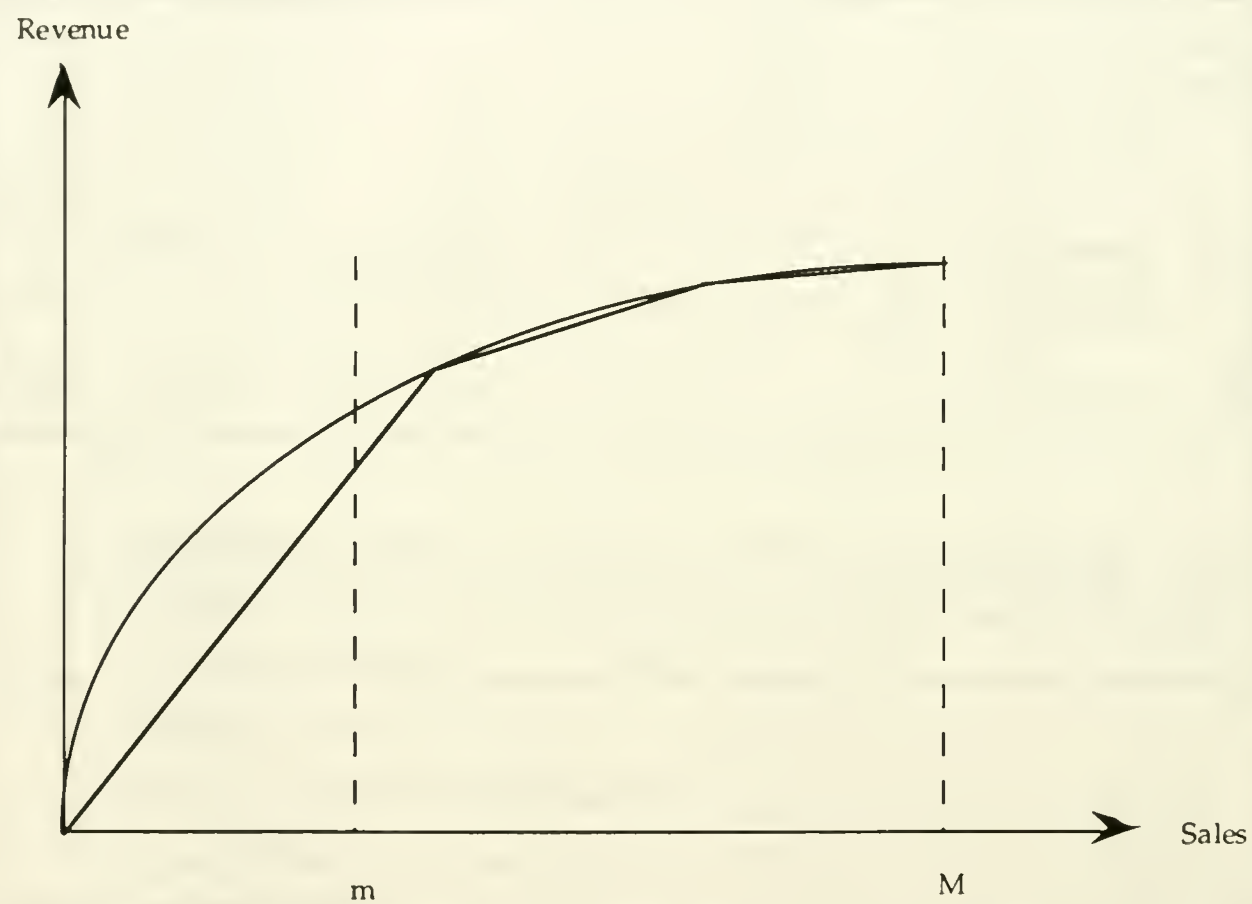
TABLE 3: DATA FOR COST/RESOURCE AND COST/ACCOUNTING FUNCTIONS											
	F1	c1	B1	c2	B2	c3	M1	F2	c4	M2	Cost Driver
Material Handling	51680	.32	646000	.36			846000				# of Parts
Production Scheduling	11400	180					700				Production Orders
Setups	319410	1484.45					250				Production Setups
Automated Machinery	2137590	7.83					117000	125,000	9.00	150000	Machine Hours
Finishing	218400	16.00	25000	15.35	5000	19.53	54600	75,000	20	61000	DL Hours
Packaging Shipping	38,000	40					3800	5,000	45	4200	Orders Shipped

Implicit in the data in Table 3 is management's judgment that the first three cost drivers - number of parts handled, production orders scheduled, and production set-ups - are not scarce resources that may require allocation to production. In other words, the upper limits on the values of these drivers are greater than any levels that might be achieved next year. By contrast, the last three cost drivers – machine hours, direct labor hours and orders shipped – are viewed as potentially scarce resources that may require allocation. Moreover, decisions to expand these capacities should also be considered. Expansion will involve fixed as well as variable costs.

The data just discussed describes the cost side of Wichita's operations. As we indicated, Wichita management is also concerned with pricing and product mix decisions. In terms of product mix, management is willing to consider sales variations in the ranges 9000 to 12500 for Model 1000, 4500 to 6250 for Model

2000, and 720 to 1000 for Model 3000. Within these ranges, demand is price sensitive according to the generic revenue function shown in Figure 3 where m is the minimal quantity that must be sold and M is the maximal quality that can be sold.

FIGURE 3: PRICE SENSITIVE REVENUE FUNCTION



In the decision model, these function are approximated by the three linear pieces shown in the Figure. Table 4 contains the price and range information for each of the three products which we interpret as follows. If analysis shows that Wichita wishes to sell 11000 units of Model 1000, they should set a price of \$378.82 for the year 1994 according to the calculation

$$\frac{8000 \times 382 + 2500 \times 372 + 500 \times 362}{11000} = 378.82$$

Implementation of a pricing strategy becomes the responsible of the sales manager and the sales people. They may either market and sell the product according to the standard rate per unit, or allow price negotiations with customers that average out to the standard rate. In any case, the revenue functions are simply forecasts. The implication is that Wichita should review their annual strategy on a shorter term basis and make adjustments in pricing and manufacturing plans to account for actual business conditions. This point is discussed at greater length in Section 6.

TABLE 4: PIECEWISE LINEAR APPROXIMATIONS TO REVENUE FUNCTIONS

Product	Range 1	Price 1	Range 2	Price 2	Range 3	Price 3
Model 1000	[0, 8000]	$382	[8000, 10500]	$342	[10500, 12500]	$392
Model 2000	[0, 3000]	480	[3000, 5000]	430	[5000, 6250]	470
Model 3000	[0, 400]	1100	[400, 800]	990	[800, 1000]	970

The data in Table 4 completes the specifications needed to construct an optimization model of Wichita's operations for next year. This model is given in the Appendix. Note that the model integrates cost, revenue and resource information associated with receiving, scheduling, manufacturing, shipping and sales activities. An optimal strategy computed by the model is summarized in Tables 5 and 6.

TABLE 5: OPTIMAL PRODUCT MIX AND PRICES FOR 1994

Product	Sales Volume	Price	Revenue
Model 1000	12,153	$377.21	$3,611,971
Model 2000	5,419	439.53	2,110,857
Model 3000	800	1090.00	784,000
TOTAL			$6,506,828

From Table 5, we see that none of Wichita's three products are constrained by the product mix constraints imposed by management. Thus, all three implicitly show a positive net revenue because they are above their lower bounds on sales. Gross revenues from Table 5 exceed total manufacturing costs from Table 6 by $588,226. This net quantity is the one that was maximized by the MIP model in the Appendix. Non-manufacturing costs must be subtracted to compute Wichita's projected net profit for 1994.

TABLE 6: BUDGETED ACTIVITY-BASED CONVERSION COSTS FOR OPTIMAL STRATEGY FOR 1994

Activity Area	Cost Driver	Costs	Cost Driver Total	Costing Rate
Material handling	# of parts	$289,189.	731,523	$0.395
Production scheduling	prod. orders	126,600.	640	197.81
Setup labor	prod. setups	654,896.	226	2897.77
Automated machinery	mach. hours	3,340,700.	135,000	24.75
Finishing	direct labor hours	1,294,988.	61,000	21.23
Packaging and shipping	orders	212,229.	4,183	50.74
Total Cost		$5,918,602.		

Comparison of Tables 2 and 6 shows that costs, cost driver totals and costing rates for the projected optimal strategy for 1994 in Table 6 are somewhat different than their amounts in Table 2. The differences are due to higher activity levels for the cost drivers which in turn are due to higher sales of Models 1000 and 2000 in the optimal strategy for 1994 than they were in 1993.

Comparison of Tables 3 and 6 shows that the cost driver levels for the first three drivers are below their maxima. This is as it should be since these drivers are accounting variables, not resources. On the other hand, the resources machine hours and direct labor hours are at capacity according to Table 3, and the third resource, orders, is near capacity.

An important output from an ABC analysis is the determination of unit costs for the company's products. If management does not have recourse to an optimization model, the unit costs can help them determine prices for the products. They are also useful for valuing inventories.

TABLE 7: PRODUCT UNIT COSTS FOR 1994

Product	Sales Volume	Unit Cost
Model 1000	12,153	$347.55
Model 2000	5,419	401.19
Model 3000	800	1064.71

The unit costs in Table 7 were computed from the costing rate information for the cost drivers from Table 6. We have repeated the volume information from Table 5 in Table 7 as a reminder that the unit costs determined by the optimization model are based on product mix decisions regarding volume. The unit costs would be different for different levels of volume.

An important by-product of applying an optimization algorithm to the Wichita model is the calculation of shadow prices or marginal prices on the resources. A shadow price measures the net gain in maximal net revenue if an additional unit of the resource were made available. For the Wichita model, the shadow prices are shown in Table 8.

TABLE 8: SHADOW PRICES FOR 1994	
Resource	Shadow Price
Automated machinery hours	\$20.48/hr
Direct labor finishing hours	\$0.06/hr
Packing and shipping orders	0

Note that the shadow price takes into account the marginal unit cost that Wichita must pay for the resource. Thus, the shadow price of \$20.48 per hour on automated machinery capacity would be \$29.48 per hour if the \$9. per hour variable marginal cost were not present. Similarly, the shadow price of \$0.06 per hour on direct labor finishing capacity would be \$20.06 if the \$20.00 per hour variable cost were not present. The shadow price of \$0. on order shipping capacity reflects the fact that total shipping capacity was almost but not completed consumed.

We conclude our discussion of the Wichita example by discussing briefly how the model can be extended to evaluate make-or-buy-decisions. Suppose, for example, that Wichita is considering the sub-contract of a major component of the Model 1000 for the coming year to the Moore Tooling Company. By subcontracting the component, the production data for the Model 1000 from Table 1 would change to that data in Table 9. The reduction in utilization of Wichita's automated machinery and finishing resources might allow them to profitably expand production and sales.

TABLE 9: ADJUSTED PRODUCTION DATA FOR THE YEAR 1994 WITH SUB-CONTRACTING OF MAJOR COMPONENT OF MODEL 1000

	Model 1000
Direct materials cost per unit	$60.
Number of parts per unit	20.
Direct labor hours per unit	1.5
Machine hours per unit	5

Moore offers to undertake the sub-contact according to the following terms: $100. per unit for the first 1000 units, $85. per unit for the next 4000 units, and $70. per unit for the next 7000 units. To evaluate the option, we add it to the optimization model. Wichita management also decides to add an option to expand capacity in the packing and shipping department because: (1) The buy option does not serve to reduce packaging and shipping resource utilization; and, (2) the department's capacity was near depletion in the original optimal plan. In particular, for an additional lumpy or one-time cost of $6000. plus a variable cost of $50. per order shipped, Wichita can expand their packing and shipping capacity by 600 orders for the year.

The buy option and the resource expansion option are added to the model using MIP modeling constructs similar to the ones used in the original model. Re-optimization of the model indicates that Wichita should acquire 6,080 units of the component from Moore and expand packaging and shipping order capacity to be able to handle 4312 orders. These decisions will increase net revenues for 1994 by $36,759. by selling 14000 of the Model 1000, 5480 of the Model 2000, and 720 of the Model 3000.

This example has revealed several important connections between ABC models and optimization models for decision support. These and others are discussed in the following sections.

3. Optimization Models Require ABC Inputs

To construct an optimization model for strategic or tactical planning in a company that is not using ABC, the management science practitioner must, in effect, carry out an ad hoc ABC analysis of many of the company's operations. On the other hand, if the company is already using ABC, the practitioner should find it easier to construct the optimization model because many costs and cost relationships needed by the model have already been determined. Yet, the ABC model will require extension if it is to be linked to an optimization model. As we saw in the Wichita example, the creation of an optimization model requires prescriptions of functions relating costs to the activity levels of cost drivers, rather than mere costing rates which are the standard output of an ABC model. Unlike an accounting analysis, activity levels in a decision model are variables whose values are to be determined.

Furthermore, we distinguished in the example between cost/resource functions and cost/accounting functions depending on whether or not the functions' cost drivers are potentially scarce resources. If the drivers do not correspond to resources, they are considered accounting variables with unlimited capacity. For some costs and cost drivers, the distinction between the two types of cost drivers and functions may be ambiguous, especially if the company might operate in the future in a manner that is significantly different from the past. When in doubt, the practical solution is to treat a cost driver as a resource by estimating an upper bound on its activity level beyond which management intervention will be required. For example, receiving space at a plant may not currently be a scarce resource but will become so if production increases by 50%.

We also note that the cost functions in the example were simple in that each cost was a function, albeit sometimes a nonlinear one, of a single driver.

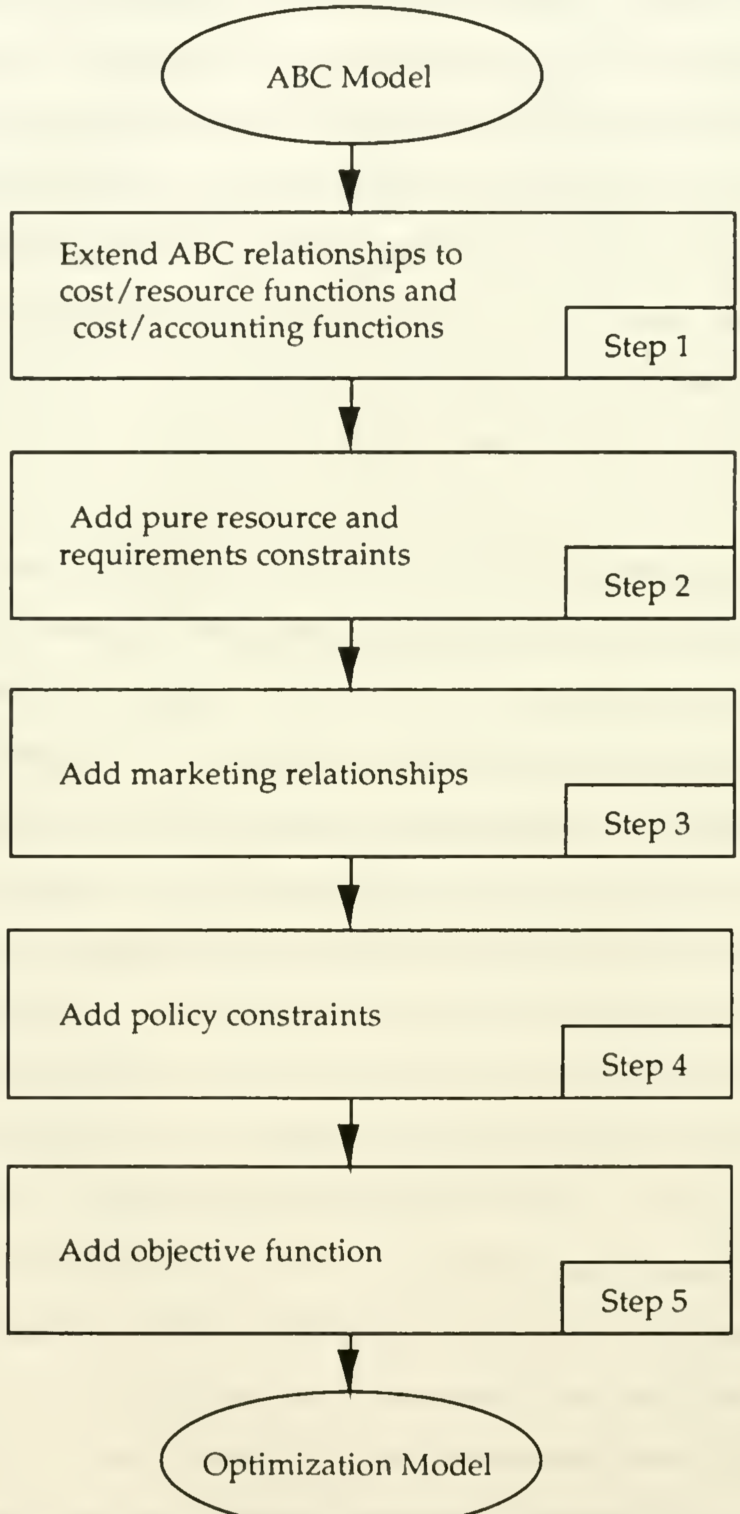
More generally, one can expect that costs may be functions of multiple drivers, some drivers may appear in more than one function, and some costs may be drivers in other functions. These complications do not substantially affect the construction and solution of an optimization model derived from an ABC model as long as the effects are separable and additive. These more complex functions can still be easily represented, at least to a good approximation, in linear and mixed integer programming models. If the functions involve nonseparable and/or non-additive terms, however, optimization model construction and solution becomes more difficult. This is the case, for example, when there are cross- products of drivers in the cost functions.

4. Optimization Models Extend ABC Models To Provide Deeper Analyses of Business Decision Problems

ABC is justifiably extolled as a tool to help managers make better decisions. As the Wichita example demonstrated, however, an optimization model offers management a deeper analysis than an ABC model. An obvious manifestation of this is the model's optimal allocation of resources to activities, a computational necessity that prompted development of linear programming and the simplex method nearly 50 years ago. In the example, the resources to be allocated were machinery hours, direct labor hours, and packing and shipping order capacity. For the optimal strategy, machinery hours, and to a far lesser extent direct labor hours, were the binding (limiting) resources.

The steps involved in extending an ABC model to an optimization model are summarized in Figure 4. In addition to cost/resource functions describing the acquisition and allocation of resources, an optimization model may incorporate pure resource constraints that do not involve costs. An example is a constraint on the total production capacity of a machine that has been paid for

FIGURE 4: EXTENDING ABC MODELS TO OPTIMIZATION MODELS



and will not be sold or replaced. The optimization model may also incorporate pure requirement constraints. An example is a lower bound constraint on the volume of sales of a particular product.

Mixed integer programming adds additional decision modeling capabilities to global optimization of a company's strategy. As we saw in the Wichita example, MIP can resolve complexities presented by economies of scale or make-or-buy. It can also resolve complexities presented by lumpy go/no-go decisions such as the opening and closing of facilities.

Of course, not all data required by an optimization model can be derived from an ABC model. Some transformation coefficients and bounds on transformation activities may be physical quantities determined by engineering specifications or studies. Similarly, transportation capacities and transportation rates may be obtained from trucking industry data bases rather than the company's internal data bases. Data describing demand for the company's products or services may be developed using a forecasting model.

Moreover, an optimization model will probably contain constraints reflecting company policy toward risk, customer service, or other criteria in addition to cost. A simple example is a constraint stating that no supplier of a critical raw material is permitted to supply more than 50% of the company's total needs. In a later section, we return to a more detailed discussion of multi-objective optimization.

Finally, optimization models allow the cost information developed by ABC models to be integrated with decisions relating to other company activities. The product pricing information included in the Wichita optimization model is an example. Another example is the integration of a marketing model that relates money spent on advertising and promotion to sales with an ABC model of the costs of supply. A third example is a capital investment model that describes how a company can raise capital to expand its production and distribution capacities.

5. Optimization Models Provide New Perspectives on ABC Models and Vice Versa

One important area where optimization models provide a new perspective is in distinguishing between transactional data and data for decision support. As discussed in Shapiro [1993a], it is necessary and desirable to aggregate transactional data when constructing optimization models for tactical and strategic planning. The most important aggregation is of products into product families. Aggregations are also appropriate for the development and statistical validation of ABC models. In fact, product families can and should be defined so that they are homogeneous with respect to both the business issues being evaluated by the model and the costs and cost relationships being estimated by the ABC methods. In most cases, we would expect considerable overlap between these two criteria for aggregating products.

Other aggregations are necessary and desirable for construction of optimization models. Depending on the nature of the company business, markets can be aggregated into market zones, possibly leaving the largest customers of the company as separate entities. Similarly, suppliers may be aggregated into supplier zones for previously defined product families. ABC methods for estimating costs and cost relationships associated with in-bound and out-bound logistics can be based on aggregate, rather than transactional data.

The aggregations just discussed have the side benefit of bringing the bulk of a company's business into sharper definition. For example, suppose a product family definition for a company with 50,000 SKU's is based on 10 types of manufacturing processes, 8 types of shipping characteristics, and 5 types of customers. These numbers multiply out to 400 potential product families. Experience has shown, however, that 300 or more of these families will contain extremely low product volumes and can be ignored or lumped into one family labeled "other". The result will be an optimization model for decision support that is appropriately detailed, or aggregated, for insightful analysis and efficient computation. The same or similar aggregations would be highly appropriate for estimating and using ABC models.

A second perspective on ABC provided by optimization models is information about which of the company's resources are scarce, and their marginal values. Optimization models can also be used to develop parametric functions describing the total and marginal value of resources as they vary through ranges of values. Furthermore, these models can search through large combinations of go/no-go decisions involving fixed costs related to resource utilization, plant openings, and plant closings to find an optimal configuration.

By contrast, ABC models provide a systematic view of costs and cost relationships that is important to represent in optimization models. First, ABC addresses directly distinctions between avoidable and unavoidable costs that depend on the scope of the analysis. These should be translated into appropriate structures in an optimization model addressing the same business situation.

In addition, ABC provides classifications of activities and costs relevant to the construction of models. For example, Cooper and Kaplan [1991] propose the following four level hierarchy of activities at a plant:

1. Unit-level activities
2. Batch-level activities
3. Product-level activities

4. Facility-level activities

This activity analysis can be directly translated into a collection of interlocking variables, constraints and costs providing a complete accounting for the plant in an optimization model.

Since all companies must have accounting systems, and therefore should have ABC systems, ABC provides structure for creating optimization models for decision support in situations where models have not been traditionally used. New decision models for companies in the service industry, such as those providing financial and telecommunications services, are good examples of this synergy. The same approach can be taken to develop new models of service organizations within manufacturing firms.

6. ABC Models and Optimization Models Differ Depending on Whether the Business Decisions Problems They Are Addressing are Strategic, Tactical or Operational

Kaplan [1988] claims that many companies have difficulty developing unified ABC and other cost models because they must address three very different functions: individual product cost measurement, inventory valuation, and operational control. Individual product cost measurement is a strategic or long-term tactical exercise that incorporates cost information relating to the entire organization. ABC methods were developed in large part to meet this need. Although many data pertaining to product cost measurement can be objectively estimated from historical data, other data may need to be subjectively estimated.

Inventory valuation is a short-term tactical exercise that the firm carries out for financial and tax statements. Costing analysis for operational control is an exercise to provide feedback to managers on resources consumed and avoidable costs incurred. The information is based solely on objective data. Due to the diversity in the scope and purposes of cost systems for these needs, Kaplan [1988] concludes that "one cost system isn't enough".

Costing analysis for operational control touches on the controversy between the **theory of constraints** (TOC) and ABC. TOC was originally proposed by Goldratt [1988], in part as a reaction against the arbitrary allocation methods of cost accounting. According to Barfield, Raiborn and Dalton [1991], TOC is "a method of analyzing the bottlenecks that keep a system from achieving higher performance; states that production cannot take place faster than the slowest machine or person in the process."

A few brief comments about this debate are appropriate to our discussion here. First, the "bottlenecks" identified by TOC can be directly linked to binding constraints in optimization models of the system being studied. Since TOC addresses shorter term planning problems than ABC, it is concerned with the sequencing of activities and decisions, an orientation that tends to focus on time-dependent bottlenecks.

TOC is also concerned primarily with identifying and controlling variable costs that are avoidable in the short term. As MacArthur [1993] points out, TOC is therefore a complementary approach to ABC which generally takes a longer term view of a company's activities. Our perspective is that optimization models for effective decision support are available for planning problems of any length, including short-term production scheduling. Moreover, models are rigorous formalizations of both ABC and TOC.

Nevertheless, the application of optimization models to business decision problems faces similar difficulties of purpose and scope. And, the synergy between ABC and optimization modeling is highly relevant to methods for integrating inter-temporal cost analyses and decision support. Inter-temporal integration of models can be formally achieved by applying decomposition methods to a hierarchy of nested models (see Bitran and Tirupati [1993]). The hierarchical modeling approach is an interpretation of the concept of **hierarchical planning** espoused by Hax and Meal [1975].

For example, Graves [1982] presents a decomposition scheme for integrating a tactical model for determining production plans for families of products with detailed operational models for scheduling individual products. The link between the two models are inventory balances between sums of inventories of individual products in a family and aggregate family inventory variables. These equations are priced out using Lagrange multipliers, thereby separating the tactical planning model from individual operational scheduling models. The multipliers can be interpreted as a rewards to the tactical model for providing aggregate inventory capacity for the product families and charges to the scheduling models for consuming this capacity. The multipliers are interatively computed until globally optimal detailed schedules and a tactical plan are determined, at least to a close approximation.

Even if formal hierarchical modeling methods such as the one just described are not employed, the concept can provide useful guidelines for ad hoc coordination of optimization models and Advanced DSS's addressing business problems with different scopes. Moreover, optimization models addressing common problems over different time frames should be as consistent as possible with respect to overlapping cost and other data. ABC provides important insights for achieving consistency among the costs and cost relationships in these models, especially in the identification and treatment of costs which may be fixed in the short term but become discretionary in the longer term. Conversely, exercises directed at achieving consistency among optimization models with different planning horizons are appropriate to resolving the cost management differences reviewed at the start of this section.

The Wichita example developed at the start of the paper is a good example of the need for inter-temporal analysis. The optimization model developed in Section 2 addresses a one year snapshot of Wichita's business. The snapshot reflects and extends an underlying ABC model for determining product unit costs. For Wichita and most manufacturing companies, some of the data, especially those pertaining to product sales, will vary significantly from their forecasted averages for the year on a month by month basis. In addition to purely random variations, sales may be expected to vary due to seasonal effects or simply changes in the market. Other data may be less volatile, such as direct labor costs or machine capacities, but still subjected to unexpected changes during the year that seriously affect the profitability of the company's product line.

A better optimization model for annual planning would be a multi-period model where the periods might be, for example, 3 one-month periods followed by 3 one-quarter periods. The periods would be linked by ending inventories for each period that become beginning inventories for the next period. The periods might also be linked by other decisions such as the option to commit to a short-term contract with an outside supplier of raw materials or parts.

A multi-period optimization model of this type is best utilized on a rolling basis. Just before the start of each month, the data for the model would be updated and the model would be re-optimized to determine a revised plan for the coming year and a specific plan for the coming month. The revised plan would include guidelines for production and product pricing for the month. Smoothing constraints could be imposed on the model to limit the extent of the planning changes from month to month.

Model data, which would be derived from an ABC system and from other sources, would be divided into three classes: volatile, semi-volatile, and stable.

Volatile data, such as sales forecasts, would require re-estimation each month. Semi- volatile data, such as available machine capacity, would need to be checked on a monthly or fairly short-term basis. Stable data, such as the cost of material handling, would be checked once or twice a year.

Two critical conditions must be satisfied for a company to exploit a dynamic decision support tool such as the one just described. First, the company must have flexible and efficient software systems for data acquisition, management and modeling. As we discuss in a later section, such software is available or can be efficiently developed. The second critical condition is that the company must be organized, or more likely must be re- organized, to effectuate integrated planning on a regular basis. This means that managers with coordinating responsibilities must participate in the planning process and be willing and able to implement the integrating plans once they have been determined. We return to a discussion of this point in Section 9.

Organizing a company for enhanced integrated planning is much more difficult to achieve than developing the models and the computer systems to support it. Nevertheless, the effort to create a more disciplined and timely process of data acquisition, data analysis and decision support within a company can yield huge benefits.

7. ABC Models and Optimization Models Are Appropriate for Studying Customer Service, Quality, and Timing Factors as Well as Cost

ABC has been criticized because it focuses too heavily on costs and revenues. Some managers complain that service and quality factors, and the timing of their decisions, are equally or more important to the success of their business than controlling costs. Not coincidentally, the application of optimization models has also been subject to the same criticism.

We believe these criticism are unfounded because, in the final analysis, all companies seek to maximize net revenues. Superior service and quality can often be attained only at higher costs which should be measured and included in the decision making process. Kaplan [1992] presents a number of convincing arguments in defense of ABC in the face of these criticisms, many of which are relevant to the use of optimization models.

Moreover, ABC models and optimization models can and should explicitly address the non-cost factors affecting a company's competitive position. For example, many companies now select suppliers on the basis of the total cost of acquiring parts, not simply the prices they must pay the suppliers. In addition to direct acquisition costs, these companies include costs associated with quality assurance, production delays due to late deliveries, and rework costs due to faulty parts. ABC models can include drivers relating to these other cost factors as well as those relating to direct acquisition costs.

A number of other examples could be cited if space permitted. We present instead an example based on the Wichita case discussed above. Suppose the production manager at Wichita faces a scheduling problem for next month that has excess demand. In particular, even with overtime shifts, Wichita will not be able to ship all orders on time; that is, during the week that the order was promised to be shipped. The overload of orders is a temporary phenomenon that is expected to subside in six to eight weeks. Nevertheless, some decision must be made about what to do during the next month.

From the perspective of the production manager, the overload can most easily be handled by delaying order shipments until normal supply exceeds demand. From the perspective of the sales manager, however, such a strategy would raise havoc with Wichita's customer service reputation. She insists that no more than 10 percent of the orders be shipped one week late and no more than

2 percent of the orders be shipped two weeks late. Order shipments more than two weeks late are not allowed. The production manager counters by suggesting that the sales manager's requirements are too stringent and will cost Wichita too dearly in avoidable overtime costs.

The conflict can be evaluated and resolved by using a multi-objective optimization model and method to perform a systematic analysis of the tradeoffs between cost and order completion. The idea behind the method is to use Lagrange multipliers to price out order completion constraints [Shapiro [1993b]]. For each setting of these multipliers, an MIP model is solved to determine a production schedule that is optimal (minimal cost) <u>for the order completion levels it provides</u>. The multipliers are then updated based on the differences between these levels and the target levels specified by the sales manager. The targets may be changed if they appear too easy or too difficult to achieve. The MIP model is then re- optimized.

The production manager and the sales manager can try to resolve their conflict by selecting a mutually acceptable schedule from among those generated by the method just described. If necessary, Wichita's CEO can arbitrate the dispute, based on objective results from the multi-objective optimization analysis.

In summary, to the extent that they can be quantified, performance criteria other than cost can be explicitly incorporated in ABC models and optimization models. Moreover, we contend that it is important to quantify service, quality and time factors, at least in part, in any business planning situation, even if models are not part of the planning process. Quantification imposes rational thinking about the objectives of the company. ABC models can be used to measure the costs directly associated with service, quality and time. Optimization techniques such as the one just described are available for combining total costs with measurements of these other criteria to systematically identify effective strategies that display the tradeoffs among such criteria.

8. ABC Systems and Advanced DSS's

The development of systems for creating and using ABC models and optimization models is the raison d'etre of much of our previous discussion. In this section, we briefly discuss major points of similarities and differences between ABC systems and Advanced DSS's. The practical integration of such systems is an area of current study.

ABC systems are new in concept as well as in their implementations. By contrast, successful Advanced DSS's running on mainframes began to appear in the 1970's, and some even before then. Their portage to pc's only became possible in the late 1980's with the advent of pc's with large memories and fast computing speeds. Because Advanced DSS's running on pc's are flexible, easy-to-use, and provide a self-contained environment for decision support, the interest in using them more pervasively has been greatly enhanced.

Scope of the Models

ABC systems such as EasyABC, NetProphet and ProfitManager provide general ABC modeling capabilities that can be applied to a wide range of costing problems in manufacturing and service companies. The systems contain routines to facilitate the development of an ABC model from raw cost data. The selection of cost drivers and the cost relationships for a model are based on the subjective judgment of the analyst rather than statistical regression methods.

Advanced DSS's, such as SLIM (Strategic Logistics Integrative Modeling system; Shapiro, Singhal and Wagner [1993]), or PIMS (Process Industry Modeling System; Bechtel Briefs [1986]) are built around model generators and optimizers that are specialized to a limited application domain. In other words, Advanced DSS's are developed for vertical applications. Moreover, the model generators are not accessible to the user. This means that the user cannot modify the models to fit the idiosyncrasies of his/her decision problems.

General purpose optimization model generation software, such as GAMS (Brooke, Kendrick and Meeraus [1992]) and AMPL (Fourer, Gay and Kernighan [1993]), will allow, in principle, a "non-specialist" to construct an optimization model for his or her application. In reality, the non-specialist must be able to express the model in mathematical form to use these packages, a skill that is beyond the capabilities of the vast majority of potential model users. Moreover, these general purpose packages do not perform well when faced with the data and modeling complexities of "industrial strength" optimization models.

Scope of Analysis and Associated Data and Data Management Programs

The scope of analysis with an ABC model is generally that which a single analyst can perform in a few days or at most a few weeks. This conclusion is substantiated in part by the sizes of memory and free hard disk space required by off-the-shelf ABC systems. Furthermore, these systems do not provide built-in facilities for down loading large data sets from corporate data bases and manipulating them prior to building an ABC model.

The scope of analysis with an optimization model might be quite broad and involve integration of decision options that cuts across several functional lines. Accordingly, the model and its data bases can be much larger than those associated with ABC models. This suggests that a large scale, integrative optimization model might be derived, in part, from several ABC models.

User Interfaces

Both types of systems prefer the flexibility of a Windows environment for viewing and manipulating data. Both support linkages to spreadsheets for developing input data and viewing reports. In fact, some companies develop hand-crafted ABC systems using the capabilities of today's spreadsheet packages. Similarly, software packages are available that provide optimization modeling capabilities directly from spreadsheets. However, these capabilities are often insufficient for serious applications.

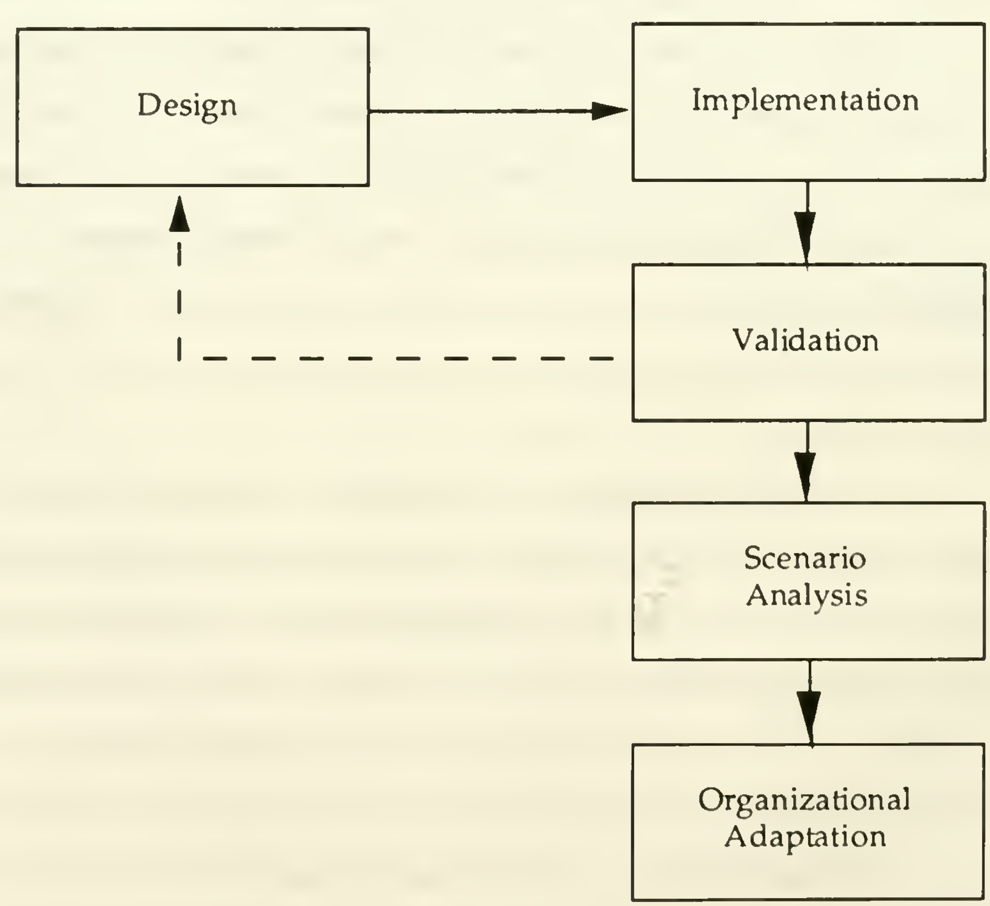
9. Organizational Adaptation to ABC Systems and Advanced DSS's

The advent of powerful and flexible pc's and other desktop computers has been a major catalyst for wider applications of ABC systems, Advanced DSS's, and other modeling systems. Since the technology is very new, most companies are only in the early stages of adapting their organizational structure to exploit the insights such systems can provide. The adaptations are part of the broad trend of **business process re-engineering** (Hammer and Champy [1993], Davenport [1992]).

Figure 5 depicts the evolutionary phases of strategic and tactical modeling systems based on ABC and optimization models. Strategic and tactical business planning problems are those that the company wishes to evaluate on a yearly, quarterly, monthly, and possibly even a weekly basis. Our primary interest in this section is to discuss organizational adaptation to ABC systems and Advanced DSS's for analyzing such problems. By contrast, organizational adaptation to systems for operational control problems, which are those requiring analysis on a daily or more frequent basis, is a separate topic that we will not address here. The reader is referred to Section 6 for an earlier discussion of modeling systems for operational control.

Implementation in Figure 5 refers to construction of both an ABC system or an Advanced DSS and the data acquisition, reporting and management programs needed to link the systems to corporate data bases. For some data, new sources and acquisition programs will need to be devised. Validation refers to the process of comparing results produced by an ABC model or an optimization model derived from historical data against historical results. The

FIGURE 5: EVOLUTION OF MODELING SYSTEMS



validation run can be used as the standard against which to measure improvements. Often, modifications to the design of the models and the systems are uncovered during validation exercises; hence, the feedback in the Figure from validation to design.

Once validated, the systems can be used to analyze scenarios of the company's future. These might be scenarios describing demand for the company's products, make-or-buy options, inventory build-up requirements due to seasonal factors, and so on. Scenarios are evaluated by relaxing historical restrictions in the validation model, adding decision options, and using data describing projections or forecasts of the company's future. For each scenario, management is concerned with identifying optimal, or at least demonstrably good, plans for product pricing, product mix, sourcing, manufacturing and distribution. Managerial judgment must then be employed to select the specific plan to set in motion from among the many plans identified by the scenario analysis.

Although it may yield important benefits to the company, scenario analysis is often not extended to a mainstream activity. Rather, it is performed by a project team that is temporarily assembled to study pressing strategic and tactical planning issues, and then disbanded. The analytical tools are not formally integrated into the planning processes of the company, and organizational adaptations to their repetitive use are not considered, much less achieved.

Cooper et al [1992] begin their discussion of this situation by posing the question on p. 54.

> "Does activity-based management automatically follow from an activity-based costing project?"

Their paper reports on experiences in eight companies that had successfully carried out ABC projects. They found that (Cooper et al [1992; p. 57]) "(m)any companies do not have an explicit game plan for making the transition from generating information in the ABC (scenario) analysis stage to having line managers make decisions in an action stage."

The barriers to ongoing and integrated use of ABC systems and Advanced DSS's after completing successful projects with them is strictly organizational. The technical efforts to create and validate them and their data bases have already been accomplished. Moreover, because scenario analysis has demonstrated the benefits to be realized, the companies should have clear incentives to seek their ongoing application. Despite the incentives, many companies are slow in moving to more pervasive management processes involving modeling systems largely because they are unaccustomed to integrating cross-functional decisions based on the rational analysis of data. Nevertheless, those that can quickly overcome the organizational barriers will realize great competitive advantage.

We conclude by noting that the benefits to the organization from ABC systems and Advanced DSS's are consistent with and supportive of three of the new rules associated with re- engineering discussed by Hammer and Champy [1993]. These are

- "Businesses can simultaneously reap the benefits of centralization and decentralization." (p. 93)

ABC systems and Advanced DSS's enable management to identify strategies that integrate and optimize cross-functional activities. These centralized strategies can be translated into guidelines for individual managers responsible for executing the various parts. The tools can be used on a regular basis to update the rules.

- "Decision making is part of everyone's job." (p. 95)

Hammer and Champy are explicit about the relevance of systems in this regard. On p. 96, they state

> "Modern database technology allows information previously available only to management to be made widely accessible. When data is combined with easy-to-use analysis and modeling tools, frontline workers – when properly trained – suddenly have sophisticated decision-making capabilities. Decisions can be made more quickly and problems resolved as soon as they crop up."

We would add that ABC models and optimization models are <u>necessary</u> as well as desirable tools for analyzing the many business problems involving large sets of numerical data.

- "Plans get revised instantaneously." (p. 99)

The word "instantaneously" in this rule is too extreme. We would argue instead that plans need to be revised on a frequent, timely and competitive basis. Of course, models can and should play a central role in identifying effective revised plans. A key issue in determining organizational adaptation to repetitive use of modeling systems is the cycle time for updating the data bases and exercising the systems to develop the revised plans.

10. Conclusions

We have argued in this paper that the theory and practice of ABC and optimization modeling have considerable overlap. Moreover, the extension of ABC models to optimization models for decision support is an intuitive process that should appeal to many managers. The practical synthesis of the two modeling methodologies at the systems level is an area of current research.

ABC systems and Advanced DSS's based on optimization models have central roles to play in business re-engineering exercises that are changing organizational structures in thousands of companies. The implanting of such systems to achieve more effective integrated planning requires major modifications to decision making processes within the firm. Nevertheless, companies that learn how to exploit the new decision support technologies will realize enormous competitive advantage.

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Appendix – Wichita Annual Planning Optimization Model

The 1994 annual planning model to be optimized by Wichita involves production variables, driver variables and fixed cost variables. It also involves one zero-one variable that serves the technical purpose of modeling returns to scale of one of the cost/resource functions. It is assumed that all production is sold. That is, there is no change in inventories.

<u>Indices</u>

i = 1 (Model 1000), 2 (Model 2000), 3 (Model 3000)

j = 1, 2, 3, 4, 5, 6 (Cost drivers)

h = 1,..,Nj (Ranges for cost driver j – see Table 3)

k = 1, 2, 3 (Linear pieces of revenue curve – see Figure 2)

<u>Decision Variables</u>

Pik = Production of product i in linear revenue piece k

Fj = Fixed cost decision associated with cost driver j
(1=Incur fixed cost to gain additional resource;
0=Do not incur fixed cost)

Djh = Driver j activity level in range h

I51 = 1 if D51=25000; 0 if D51<25000 (Technical variable)

With this background, we can state Wichita's 1994 annual decision planning problem.

Objective Function

$$\begin{aligned}
\text{MAX} \quad & 302\,P11 + 292\,P12 + 282\,P13 \\
& + 400\,P21 + 380\,P22 + 360\,P23 \\
& + 1020\,P31 + 940\,P32 + 860\,P33 \\
& - 0.32\,D11 - 0.36\,D12 - 180\,D2 - 1484\,D3 \\
& - 7.83\,D41 - 125000\,F4 - 9\,D42 \\
& - 16\,D51 - 15.35\,D52 - 19.53\,D53 - 75000\,F5 - 20\,D54 \\
& - 40\,D61 - 5000F6 - 45\,D62
\end{aligned}$$

The first three rows of the objective function contain terms from Table 4 corresponding to gross revenues of the three products Model 1000, Model 2000 and Model 3000. Note that the direct material costs from Table 1 have been subtracted from the prices given in Table 4. The remaining terms compute manufacturing costs based on the drivers and the functions in Table 3.

In the following constraints, the coefficients are taken from Tables 1 and 3.

1. Material Handling Cost/Accounting Function

Driver = # of parts

$$\begin{aligned}
D11 + D12 = \; & 30\,P11 + 30\,P12 + 30\,P13 + 50\,P21 + 50\,P22 + 50\,P23 \\
& + 120\,P31 + 120\,P32 + 120\,P33
\end{aligned}$$

$$D11 \leq 646000$$

$$D12 \leq 200000$$

2. Production Scheduling Cost/Accounting Function

Driver = production orders

$$D2 = .03\,P11 + .03\,P12 + .03\,P13 + .014\,P21 + .014\,P22 + .014\,P23 + .25\,P31 + .25\,P32 + .25\,P33$$

$$D2 \leq 700$$

3. Setup Cost/Accounting Function

Driver = production setup

$$D3 = .01P11 + .01P12 + .01P13 + .01P21 + .01P22 + .01P23 + .0625\,P31 + .0625\,P32 + .0625\,P33$$

$$D3 \leq 250$$

4. Machining Cost/Resource Function

Driver = machine hours

$$D41 + D42 = 7\,P11 + 7\,P12 + 7\,P13 + 7\,P21 + 7\,P22 + 7\,P23 + 15\,P31 + 15\,P32 + 15\,P33$$

$$D41 \leq 117000$$

$$D42 - 18000F4 \leq 0$$

This is the first function based on a driver, machine hours, that is a resource. The nominal level of this resource is 117000 hours. The second inequality constraint introduces the decision option to expand available machine hours for 1994 by 18000 hours at a fixed cost (that is, a lumpy or one time cost) of $125000.

5. Finishing Cost/Resource Function

Driver = direct labor hours

$$D51 + D52 + D53 + D54 = 2\,P11 + 2\,P12 + 2\,P13 + 5\,P21 + 5\,P22 + 5\,P23 + 12\,P31 + 12\,P32 + 12\,P33$$

$$\begin{aligned} D51 &\leq 25000 \\ 25000\,I51 - D51 &\leq 0 \\ -25000\,I51 + D52 &\leq 0 \\ D53 &\leq 4600 \\ D54 - 6400\,F5 &\leq 0 \end{aligned}$$

This is the most complex of the cost/resource functions. Since there is an economy of scale after 25000 direct labor hours, we must introduce a zero-one variable I51 that forces the driver level to 25000 before the economy of scale is realized. This variable is used in the second and third inequalities. If I51 = 1, then by the second inequality D51 ≥ 25000 implying D51 = 25000 because of the first inequality constraint, and D52 ≤ 25000 by the third inequality constraint. That is, 25000 hours are available at the reduced rate. On the other hand, if I51 = 0, the second inequality merely states that D51 ≥ 0 which is a redundant constraint since all decision variables are constrained to be non-negative, and D52 ≤ 0 by the third inequality which implies D52 = 0.

Since this cost/resource function experiences a diseconomy of scale after 50000 direct labor hours and up to 54600 hours, which is the nominal capacity for the year, the mathematical trick just explained in the previous paragraph is not needed since the optimizer will only use these hours once D52 = 25000. Finally, the last inequality is similar to the one in the previous cost/resource function; namely, it forces a fixed or lumpy cost to be incurred if this resource is expanded by 6400 hours.

6. Packaging/Shipping Cost/Resource Function

Driver = orders shipped

$$D61 + D62 = 0.1\,P11 + 0.1\,P12 + 0.1\,P13 + 0.4\,P21 + 0.4\,P22 + 0.4\,P23 + P31 + P32 + P33$$

$$D61 \leq 3800$$

$$D62 - 400\,F6 \leq 0$$

7. Product Mix Policy Constraints

These are constraints based on management's judgment about the most effective ranges for sales of the three products.

$$P11 \leq 8000$$

$$P12 \leq 2500$$

$$P13 \leq 2000$$

$$P11 + P12 + P13 \geq 9000$$

$$P21 \leq 3000$$

$$P22 \leq 2000$$

$$P23 \leq 1250$$

$$P21 + P22 + P23 \geq 4500$$

$$P31 \leq 400$$

$$P32 \leq 400$$

$$P33 \leq 200$$

$$P31 + P32 + P33 \geq 720$$

The optimal solution to this MIP is given in Tables 5 and 6.