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PRICING DECISIONS\*

David B. Montgomery\*\* and Glen L. Urban\*\*

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by

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## CHAPTER SIX

### PRICING DECISIONS

Price determination has long been a primary concern of economists. At the micro-economic level their analysis has centered around the use of price to achieve profit maximization under various market structures. In most economic formulations price is considered the only variable affecting demand. In contrast to this classical approach, this chapter will treat price as one of several demand determinants. First, price policies in business practice will be outlined and then univariate models which consider price as the only demand determinant will be discussed. The univariate approach will then be generalized to include non-price demand determinants such as advertising and other merchandising efforts. After this discussion of price as an element in the marketing mix and some comments on price interactions in multiproduct firms, competitive pricing situations will be discussed. Then questions in the empirical measurement of demand relationships will be considered. Finally, heuristic models of price determination will be explored.

### PRICE GOALS AND POLICIES

Before turning to a consideration of pricing models and measurements, a brief sketch of some common pricing goals and policies seems useful as a prelude to the more formal management science approaches. This discussion is not intended to be exhaustive, but rather seeks to remind the reader of the varied nature of pricing goals and policies in practice.



Some of the more common pricing objectives and policies in practice are:

1. Target Rate of Return.

Price is set so as to yield a target return on investment when the standard volume is sold. The standard volume is generally taken as the long-run average of plant utilization. Standard volume is used in order to prevent short run changes in volume or product mix from having an undue effect on price. Firms which adopt this objective are generally market leaders in relatively protected markets. Alcoa, du Pont, General Electric, and General Motors are firms which have a target return as their primary pricing objective. <sup>1</sup>

2. Maintain or Improve Market Share.

This pricing objective has been pursued by such firms as A&P, Swift, and Sears Roebuck. For General Motors it is a collateral pricing goal to the primary objective a a target return.

3. Stabilization of Price and Margin.

Kemmeerth Copper has emphasized the stabilization of price, but it is an important collateral pricing goal for such firms as Alcoa, American Can, And General Electric! Stabilization of percentage of margin is a cost-plus pricing policy, which is common in research and development contracting.

4. Pricing to Meet Competition

This is a reactive or defensive price policy where competitive price moves are countered. In many cases it is





intended as a permanent threat to price cutters. Gulf Oil, Kroezer, and National Steel are examples of firms which have followed this policy!

These are some of the principle pricing objectives and policies pursued by large American companies. Most firms explicitly or implicitly attend to all four of the above objectives. The interfirm variation in price policy comes in the relative emphasis given to each goal. Thus in actual practice firms utilize several operational criteria in established prices. The use of these rules of thumb or heuristics will be discussed in greater detail later in this chapter. Until then, the models that are considered will take profit maximization as the goal. From a normative viewpoint, this is the appropriate goal in all cases.

In contrasting business practice and theoretical considerations, Green<sup>2</sup> has noted that:

1. Businessmen rely on costs more than demand in setting price since the former are easier to estimate.
2. Businessmen generally use full historical costs rather than the incremental, future costs of economic theory.
3. Competition is more likely to be considered defensively than offensively with respect to price.
4. Safeguarding a "normal" profit is generally more important to businessmen than taking greater risk so as to maximize profits.



## UNIVARIATE MODELS FOR PRICE DETERMINATION

Classical Economic Model

The classical economic model of price determination is a univariate model linking price and quantity sold. Under certain assumptions, the classical model may be solved to yield the price which will maximize profits to the firm. Suppose that

(6-1)	$q = F(p)$	where $q$ = quantity sold
		$p$ = price per unit
(6-2)	$TC = C(q)$	$TC$ = total variable cost as a function of $q$
(6-3)	$TR = p \cdot q$	$TR$ = total revenue from the sale for $q$ units at a of $p$ per unit
	$FC = \text{fixed costs}$	

where it is assumed that the functions  $F(p)$  and  $C(q)$  are at least twice differentiable with respect to  $p$  and  $q$ , respectively. If (6-1) can be solved for  $p$  in terms of  $q$ , then this inverse function of  $F(p)$  will be denoted by

$$(6-4) \quad p = f(q).$$

Again assume that  $f(q)$  is at least twice differentiable with respect to  $q$ . Then if the product is offered at price  $p$ , an amount  $q = F(p)$  will be sold. Conversely, one may use (6-4) to determine the price at which  $q$  units of product will sell. For estimation purposes in an empirical situation, (6-1) and (6-4) are not strictly interchangeable, as will be discussed later in this chapter.

The criterion for setting price in this classical model is to maximize profits. The profit function is

$$(6-5) \quad \begin{aligned} \text{Pr} &= TR - TC = p \cdot q - C(q) - FC = [f(q)]q - C(q) - FC \\ &= R(q) - C(q) - FC \end{aligned}$$

where  $R(q) = [f(q)]q = pq = TR = \text{total revenue as a function of } q$ .



The first order optimality condition then is

$$(6-6) \quad \frac{dPr}{dq} = \frac{dR(q)}{dq} - \frac{dC(q)}{dq} = 0$$

where  $\frac{dR(q)}{dq}$  is the marginal revenue generated by the  $q^{\text{th}}$  unit of sales and  $\frac{dC(q)}{dq}$  is the marginal cost of the  $q^{\text{th}}$  unit. That the well known marginal revenue equals marginal cost level of  $q$  for profit maximization is clear from (6-6). Denote the profit maximizing  $q$  by  $q^*$ . The optimum price,  $p^*$ , is then found from  $p^* = f(q^*)$ . The second order of optimality condition is that

$$(6-7) \quad \frac{d^2R(q)}{dq^2} - \frac{d^2C(q)}{dq^2} < 0$$

at  $q = q^*$ . Recall that the functions have been assumed to be at least twice differentiable with respect to  $q$ . Graphically (see Figure 1), the second order condition requires that the slope of the marginal cost function. The intersection of the marginal cost and marginal revenue curves determines the optimum quantity  $q^*$ . The optimum price corresponding to  $q^*$  is found by the average revenue relationship [ $p^* = f(q^*)$ ].

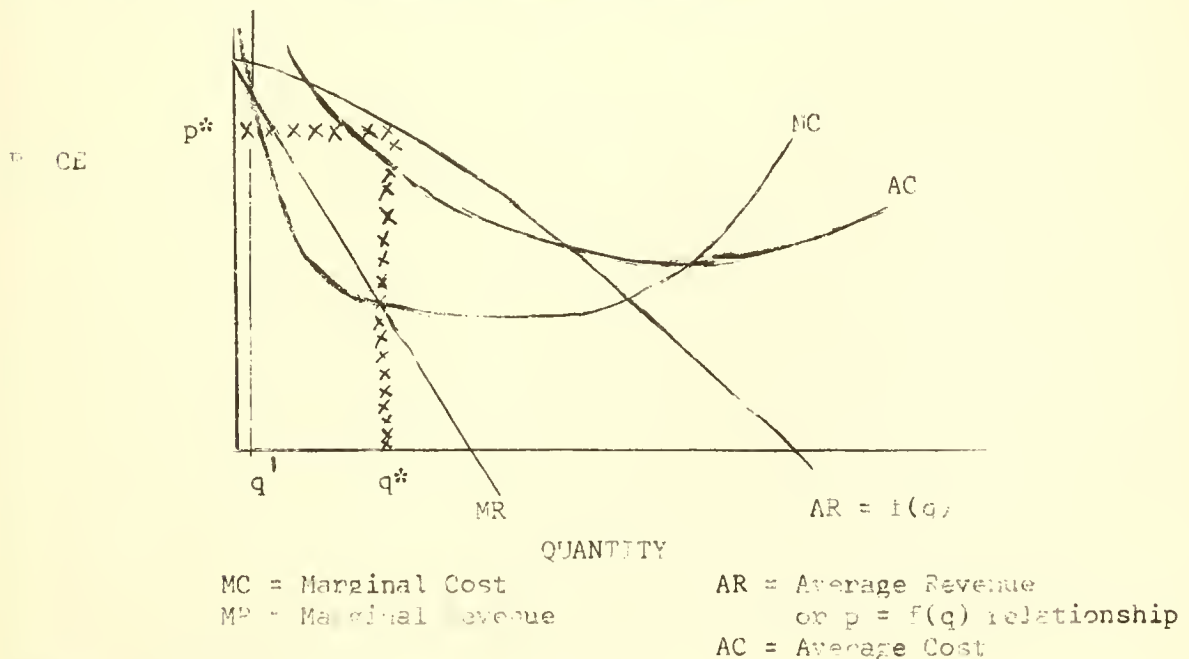


Figure 1. Monopoly Price Determination



In figure 1,  $q^*$  is chosen rather than  $q'$  since only  $q^*$  satisfies the sufficiency condition (6-7). Figure 1 is a monopoly market example. If a purely competitive market existed, the average revenue and marginal revenue curves would be horizontal and the optimum price would continue to be described by (6-6) and (6-7).<sup>3</sup>

The economic model is based on pricing to the market demand so as to maximize profit. This is in contrast to "cost plus pricing" used by many firms. It should be pointed out however, that the cost plus price may by coincidence correspond to the optimum price, even though cost plus pricing ignores the market responses to price completely.

Given demand and cost functions that are differentiable, the economic model specifies the best price if the first order conditions yield a solution and if the sufficiency can be checked.

### Breakeven Model

Another commonly used technique for price determination is breakeven analysis. In this analysis the price-volume relationship is established such that:

$$(6-8) \quad TR - TC = p \cdot q - [c \cdot q + FC] = 0$$

where  $c$  is the per unit variable cost. For a given price,  $p$ , (6-8) may be solved to find the number of units,  $q$ , which must be sold in order to breakeven at that price. The price specification is not based on a profit criterion, and it will not identify the profit maximizing point.

The cost plus and breakeven models are not as sophisticated as an economic model which yields an explicit specification of the optimum price; however, they may be useful analytical tools for exploring the implications of price policy and in many circumstances be more operational than explicit profit maximizing.





## MULTIVARIATE MODELS FOR PRICE DETERMINATION

Price as Part of the Marketing Mix

The determination of the best price should not be made without considering the other variables that might determine market success. For example, the interactions between advertising and pricing decisions is important. A high price and a large advertising expenditure may be as effective as a low price and small advertising expenditure. The determination of the optimum price and advertising combination is a problem in multivariate optimization. The profit equation now is:

$$\begin{aligned}
 (6-9) \quad Pr &= p \cdot q - TC - A - FC \\
 &= p \cdot F(p,A) - C[F(p,A)] - A - FC \\
 &= R(p,A) - \bar{C}(p,A) - A - FC
 \end{aligned}$$

where Pr = profit

q = F(p,A) = quantity sold

p == price per unit

A = advertising expenditures

TC = C(q) = total variable costs

FC = fixed costs

R(p,A) = p · F(p,A) = total revenue at price p and advertising expenditure A

and

$\bar{C}(p,A) = C[F(p,A)]$  = total variable costs at price p and advertising expenditure A

The maximum profit conditions can be specified by the application of the multivariate calculus model if the functions are differentiable.<sup>4</sup>



Differentiating (6-9) with respect to  $p$  and  $A$  yields the following two equations:

$$(6-10) \quad \frac{\partial \text{Pr}}{\partial p} = \frac{\partial R(p,A)}{\partial p} - \frac{\partial \bar{C}(p,A)}{\partial p} = 0$$

$$(6-11) \quad \frac{\partial \text{Pr}}{\partial A} = \frac{\partial R(p,A)}{\partial A} - \frac{\partial \bar{C}(p,A)}{\partial A} - 1 = 0$$

For explicit specifications of  $R(p,A)$  and  $\bar{C}(p,A)$ , equations (6-10) and (6-11), if they can be solved simultaneously, will identify one or more  $(p,A)$  combinations which may maximize profits.<sup>5</sup> In order to identify the profit maximizing  $(p,A)$  combination the second order or sufficient conditions must be checked.

For a two variable problem the sufficiency conditions for relative maximum profit are:

$$(6-12) \quad \frac{\partial^2 \text{Pr}}{\partial p^2} + \frac{\partial^2 \text{Pr}}{\partial A^2} < 0$$

$$(6-13) \quad \begin{vmatrix} \frac{\partial^2 \text{Pr}}{\partial p^2} & \frac{\partial^2 \text{Pr}}{\partial p \partial A} \\ \frac{\partial^2 \text{Pr}}{\partial A \partial p} & \frac{\partial^2 \text{Pr}}{\partial A^2} \end{vmatrix} > 0$$

where (6-13) is a determinant and the condition can be restated as:

$$(6-14) \quad \left[ \frac{\partial^2 \text{Pr}}{\partial p^2} \cdot \frac{\partial^2 \text{Pr}}{\partial A^2} - \frac{\partial^2 \text{Pr}}{\partial A \partial p} \cdot \frac{\partial^2 \text{Pr}}{\partial p \partial A} \right] > 0$$

The points found by the solution of (6-10) and (6-11) must be substituted in (6-12) and (6-13) to see if they satisfy the sufficiency conditions. More than one point may satisfy the necessary and sufficient conditions. This reflects the occurrence of relative maxima. The greatest relative maximum



is called the maximum maximorum and may be located by substituting the relative maximum points  $(p, A)$  into the profit equation (6- ) and then selecting the point  $(p^*, A^*)$  producing the greatest profit.

This two variable maximization was outlined assuming competition to be exogeneous. The only way the model as outlined above could include competitive effects is by constraining the prices to be considered. Competitive retaliation might be considered as constraining the range of possible prices. Other constraints may be present. Government regulation or pressures may limit the freedom to establish prices. Advertising expenditure could also be limited by the financial policies of the firm. If such constraints are present, the calculus model will have to be expanded by Lagrangean analysis.<sup>6</sup> The constraints would be placed into Lagrangean forms so that the problem could then be considered an unconstrained maximization. If the price must be less than a value  $L$ ,  $p \leq L$ , the equality form of the constraint would be  $p + S^2 = L$  where  $S$  is a new unconstrained slack variable.<sup>7</sup> The Lagrangean form of the constraint is

$$(6-15) \quad p + S^2 - L = 0$$

and the function to be maximized is

$$(6-16) \quad Pr(p, A) - \lambda (p + S^2 - L),$$

where  $\lambda$  is termed the Lagrange multiplier. The unconstrained maximization of (6-16) with respect to  $p$ ,  $A$ ,  $S^2$ , and  $\lambda$  will result in the values of  $p$  and  $A$  which will yield the maximum profit subject to the price constraint. Additional constraints may be handled in a similar fashion by adding one slack variable and one Lagrange multiplier per constraint. Note that an equality constraint doesn't require the use of a slack variable.



In addition to advertising effects, price determination should also reflect price interaction with other elements of the marketing mix. For example, the channel of distribution of the product may affect the price decision. The price established for middlemen would have to reflect the functions the middleman is expected to carry out and his price policies in setting the final retail price. Price will also interact with advertising, and personal selling intensity. These additional marketing mix aspects will be considered in the product planning chapter.

#### Pricing Decisions and the Product Line

The multiproduct nature of most firms makes consideration of product interdependency an important aspect of pricing. This further complicates the pricing decision whenever complementarity or substitution effects within the firm's product line are significant. By complementarity it is meant a positive demand interaction between two products in the firm's product line. That is, as demand increases (decreases) for one product, it is likely to increase (decrease) for the other. An example would be the complementarity between Sears Roebuck's appliances and their appliance service contracts. Increased sales of an appliance are likely to be associated with increased sales of the corresponding service contracts. Substitution effects in the product line occur whenever the firm's products compete with each other. The automobile manufacturers provide a prime example here. A customer who buys a Mustang is not likely to be a good prospect for a Galaxy in the near future and visa versa. Since the firm has as its overall goal the maximization of profits across the entire product line, complementarity and substitution effects are important aspects of the pricing of individual products in the line.





A general formulation of the theory of the multiproduct firm has been given by Holdren.<sup>8</sup> The firm is assumed to have n separate products in its product line. The general formulation then is given by

$$(6-17) \quad \begin{aligned} q_1 &= f_1(p_1, p_2, \dots, p_n; a_1, a_2, \dots, a_m) \\ &\vdots \\ q_n &= f_n(p_1, p_2, \dots, p_n; a_1, a_2, \dots, a_m) \end{aligned}$$

where  $p_i$  = the unit price of product  $i$  ( $i = 1, \dots, n$ )

$a_j$  = cost of non-price offer variant  $j$  ( $j = 1, \dots, m$ )

The  $a$ 's may be such non-price items as advertising, personal selling intensity, package design, etc.

The total cost to the firm of selling  $q_1, q_2, \dots, q_n$  units of its products may be expressed in functional form as

$$(6-18) \quad C = C(q_1, q_2, \dots, q_n; a_1, a_2, \dots, a_m).$$

The firm's profit function then is

$$(6-19) \quad Pr = \sum_{i=1}^n p_i q_i - C$$

The necessary condition for maximum profits is given by

$$(6-20) \quad \begin{aligned} \frac{\partial Pr}{\partial p_1} &= q_1 + \sum_{i=1}^n \left( p_i - \frac{\partial C}{\partial q_i} \right) \frac{\partial q_i}{\partial p_1} &= 0 \\ &\vdots &\vdots \\ &\vdots &\vdots \end{aligned}$$

$$\frac{\partial Pr}{\partial p_n} = q_n + \sum_{i=1}^n \left( p_i - \frac{\partial C}{\partial q_i} \right) \frac{\partial q_i}{\partial p_n} = 0$$

$$\begin{aligned} \frac{\partial Pr}{\partial a_1} &= \sum_{i=1}^n \left( p_i - \frac{\partial C}{\partial q_i} \right) \frac{\partial q_i}{\partial a_m} - \frac{\partial C}{\partial a_1} &= 0 \\ &\vdots &\vdots \\ &\vdots &\vdots \end{aligned}$$

$$\frac{\partial Pr}{\partial a_m} = \sum_{i=1}^n \left( p_i - \frac{\partial C}{\partial q_i} \right) \frac{\partial q_i}{\partial a_m} - \frac{\partial C}{\partial a_m} = 0$$



While Holdren was able to draw a few interesting conclusions from analysis of this model, the complexity of (6-20) is such that solution for the optimal price and non-price mix for the product line will be probabilities in all but the simplest cases. Even when solutions to the necessary condition of (6-20) have been obtained, there remains the complex problem of testing for sufficiency at each of these solutions. Thus this multi-product, total marketing mix form of model would seem to have its greatest use as an analytical framework for analyzing broad market and policy implications. It does not appear to be as promising in actually determining an optimal marketing mix for a product line.

Thus it is evident that the simultaneous consideration of marketing mix and product line effects is very difficult. The addition of competitive effects to these two factors imposes an even higher order of complexity upon the analysis. These topics will receive further analytic consideration later in this book, but it should not be surprising to find that heuristic procedures have been developed to make pricing decisions. Examples of heuristic procedures are given in the concluding section of this chapter and in the Howard and Morgeroth reading which follows the chapter.

### Competitive Models

One of the disadvantages of the classical economic model is that in oligopolistic situations the demand function is usually not differentiable in which case the marginal revenue curve is discontinuous. Although the necessary conditions can be checked at the discontinuity, the calculus model does not yield satisfactory results in this interdependent bargaining situation since the uncertain nature of competitive reactions is not explicitly considered.



Several approaches can be taken to this problem. One approach is to attach subjective probabilities to each possible competitive reaction. This Bayesian approach has been presented by Green.<sup>9</sup> In this approach, the possible competitive reactions to various price levels are defined and a probability of occurrence is associated with each of them. When these probabilities are multiplied by the profit payoffs of establishing the respective price levels, the expected value of the payoff is generated. The selected price is the one that yields the greatest expected profit.<sup>10</sup> This procedure of treating the distribution of potential results as known is an analysis of the risk aspects of the problem.

Bayesian models are not the only method of approaching the risk aspects of competition. In pricing situations where competitors submit sealed bids such as in construction and aerospace marketing, other probability models can be developed. The profit generated by the bid depends upon the bid price and the costs of fulfilling the bid. Given that the objective of the firm is to maximize profits, Churchmen, Ackoff, and Arnoff have developed a model to specify bids so that the expected value of profits is maximized.<sup>11</sup> The expected value of profit is:

$$(6-21) \quad E(\text{Pr}) = [\text{PROB}(p)] \cdot (p - C)$$

$p$  = bid a price  
 $E(\text{Pr})$  = expected value of profit  
 $\text{PROB}(p)$  = probability of winning the contract at bid (or price)  $p$   
 $C$  = estimated cost of fulfilling contract

If the probability of winning the contract,  $\text{PROB}(p)$ , could be determined for each possible bid ( $p$ ), the price or bid corresponding to the maximum expected value of profit could be found. The probability of winning the bid is the probability of submitting a bid lower than all other competitors. If contractors form their bids independently, the probability of being lower than all bids is the product of the probabilities of



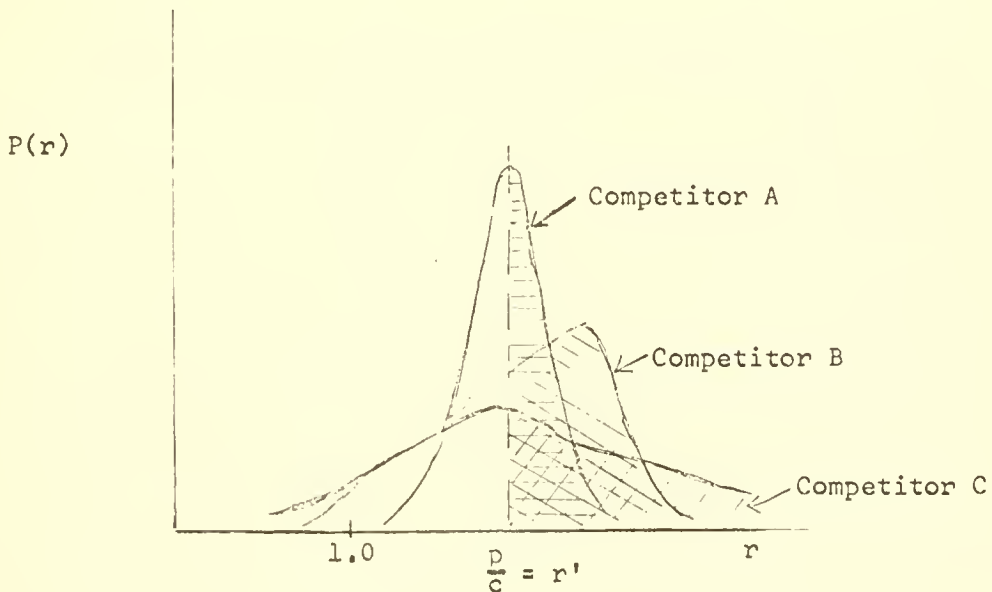
being lower than each of them:

$$(6-22) \quad \text{PROB}(p) = \text{PROB}(p)_1 \cdot \text{PROB}(p)_2 \cdot \dots \cdot \text{PROB}(p)_j \cdot \dots \cdot \text{PROB}(p)_n$$

where  $\text{PROB}(p)_j$  = probability of submitting a bid lower than competitor  $j$  with a bid of  $p$ . The probability of submitting a bid lower than competitor  $j$  may be determined by an analysis of competitor  $j$ 's past bidding behavior if he is expected to continue to behave in this manner.

It might be noted that the distribution may change to reflect the past success of the competitor in bidding. For example, if he has been successful and is reaching capacity, higher prices might then be bid. If he has been unsuccessful in the past, lower prices might reflect a need to maintain at least a minimum level of production. In the absence of useful past data or if the changes just discussed would make use of his past bidding behavior suspect, subjective distributions may be utilized.

In any case, the distributions for competitors might appear as in Figure 6-2.



where  $r$  = ratio of bid to cost estimate

$P(r)$  = probability that the bid to cost estimate ration will lie between  $r$  and  $r + \partial r$ .

Figure 2. Competitor Bidding Distributions





In Figure 6-2 the competitor bidding distributions have been developed for the ratio of the competitor's bid to the cost estimate,  $c$ , made by the firm analyzing this competitive situation. The use of  $c$  normalizes the distribution whatever the actual  $p$  and  $c$ . If the firm makes the bid  $p = R'c$  (as noted in the figure), the probability of winning the bid from any given competitor is just the area in the upper tail ( $r > r'$ ) of the appropriate distribution in the figure. These probabilities are then inserted into (6-22) in order to determine the probability that the firm will win the contract with a bid of  $p = r'c$ . This probability is then used in (6-21) in order to determine the expected profit from a bid of  $p$ . The decision rule then is to choose that bid  $p$  which maximizes (6-21).

If the number of bidders ( $n$ ) in (6-22) is not known, the probability of being lower than bidder  $j$  is conditional upon the probability that  $j$  will bid. In this case, the probability of being lower than bidder  $j$  is:

$$(6-23) \quad \text{PROB}(p)_j = 1 - \text{PROB}(j) \cdot \text{PROB}(p|j)$$

$$\begin{aligned} \text{PROB}(j) &= \text{probability competitor } j \text{ will bid} \\ \text{PROB}(p|j) &= \text{probability of winning contract at bid price } p, \\ &\quad \text{if competitor } j \text{ bids} \end{aligned}$$

The probabilities of (6-23) when placed in (6-22) define the probability of winning the contract which in turn is used in (6-21) to define the expected profit. If specific distributions of the probability of being lower than a competitor [ $\text{PROB}(p)_j$ ] and the probability of a number of bidders could be determined, explicit expressions of the expected profit can be specified.<sup>12</sup> Trial and error search procedures could be used to determine the optimum price to bid so as to maximize the expected profit whenever the resulting expressions are analytically tractable.



In some pricing situations, the distributions and expected results cannot be formulated. The state of ignorance may be such that meaningful subjective estimates cannot be made, in which case the risk situation is replaced by one of uncertainty. The competitive situation under uncertainty can be approached by game theory as was noted in use of game theory for advertising decisions in Chapter 4.

In pricing decisions the payoff may often be characterized as a non-zero sum game. For example, if all firms lower prices the total rewards to all competitors may decrease. A good example of this phenomenon is presented in the gasoline industry. The payoff to any one firm obtained by reducing prices is very large, but it is almost certain to be followed by competitors. Successive price cuts can lead the industry to a very low price and profit level.<sup>13</sup> Figure 6-3 is a hypothetical matrix that could explain the self destructive rivalry of two firms.

		Firm Two's Price				
		1	2	3	4	5
Firm One's Price	1	2 / 2	7 / 1	10 / 1	13 / 1	10 / 1
	2	1 / 7	4 / 4	12 / 1	14 / 1	11 / 1
	3	1 / 10	1 / 12	6 / 6	15 / 1	12 / 1
	4	1 / 13	1 / 14	1 / 15	8 / 8	13 / 1
	5	1 / 10	1 / 11	1 / 12	1 / 13	7 / 7

Figure 3. Self-Destructive Game



If firm one and two currently are both charging a price of four and firm one lowers his price to three, he would get a reward of 15. This is a maximax strategy. Firm two would have his payoff reduced to one so he would certainly follow the reduction and may even reduce his price to two and obtain a reward of 11. This process could continue until both firms are charging a price of one. There is not incentive to stop the price spiral until both firms realize the destructive nature of the process. If the firms colluded, they probably would establish a price of four, since there the total rewards of the game are a maximum.

Tacit collusion resulting from the realization of the nature of the game might also lead to price stability. Formulating the game payoff table might be just as effective in producing this realization as an actual war and therefore serve a useful function. The usual zero-sum strategies are not reasonable for this example.<sup>14</sup> For example, if each player followed the maximax strategy, the firms would be led to a price of one and a payoff of two, as discussed in the previous paragraph. If both players used the most conservative strategy -- the maximin strategy -- the game would also be played at a price of one and payoff of two. This example dramatizes the dangers of applying zero-sum game strategies to non-zero sum games. Not all non-zero sum games will produce such perverse results. Some non-zero games will yield equilibrium maximin solutions.

Some pricing games may be zero-sum games. For example, if two retailers are competing for a fixed number of customers, they may play a zero sum game in selecting a loss leader. A loss leader is a product selected for a very large price reduction even to the extent of a loss on that product. The reason for using a loss leader is to attract people to the store. If it is assumed that the profit per consumer is constant,



the profit payoffs will be directly proportional to the number of customers attracted to the store. If the number of people to visit the two stores is fixed (e.g., by geographical considerations) and the cost of loss leading each item is the same, the game is a two person zero sum game. A hypothetical example is given in Figure 6-4. Retailer one can loss lead either chicken or coffee and retailer two can lead steak or butter in this example.

		RETAILER TWO'S LEADER	
		1 - Steak	2 - Butter
RETAILER ONE'S LEADER	Payoff to 1 \ Payoff to 2		
	1 - Chicken	500	400
	2 - Coffee	400	500
		300	600
		600	300

Figure 4. Two Person Zero Sum Pricing Game

The payoffs are the number of people shopping at the store in the time period of interest. The differences in the payoffs reflect the different relative preferences of consumers with respect to the item used as a loss leader.

If firm one is conservative and attempts to obtain the greatest payoff assuming the strongest play is made by his opponent, the maximin strategy, he would use chicken as a loss leader. If firm two also used the maximin criterion, he would use chicken as a loss leader. The resulting pair would not be an equilibrium pair since player two could see a way of improving his position assuming his competitor did not change his strategy. Firm two could increase his payoff by one hundred by changing to steak





assuming firm one remained with chicken. This force destroys the possibility of a maximin equilibrium for this game if competitors must always play the same alternative or, in other words, display a pure strategy.

Although pure strategies will not yield an equilibrium, a strategy based on randomizing the item to be led each time period will produce an equilibrium. If firm one plays a mixed strategy against firm two's steak, the expected payoff would be:

$$(6-24) \quad V_{11} = P_1(400) + (1-P_1)(600)$$

$P_1$  = proportion of time strategy one is played by firm one  
 $V_{11}$  = payoff of firm one using a mixed strategy against his competitor's pure strategy one

The expected payoff against firm two's strategy of butter is:

$$(6-25) \quad V_{12} = P_1(500) + (1-P_1)(300)$$

This payoff set is graphically shown in Figure 6-5

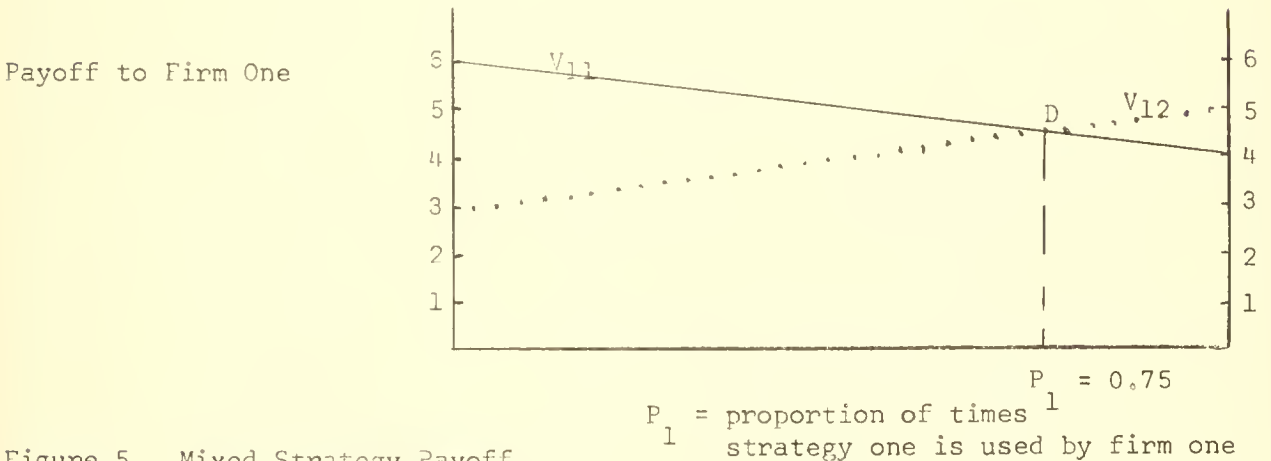


Figure 5. Mixed Strategy Payoff.

The maximin strategy is to find the point where the minimum payoff is a maximum. The dotted line in Figure 4 shows the minimum payoffs against the competitor's best decision. The highest minimum payoff is at point D.



Here  $P_1^* = .75$  and the expected maximin payoff is 450. If firm two carries out a similar analysis he will also find a best mixed strategy with a payoff of 450 in this case. The mixed strategy pairs will be an equilibrium. If there are more than two alternatives, the problem of finding the mixed equilibrium strategy is more difficult and linear programming routines must be utilized. But a two person zero-sum game will always have an equilibrium maximin strategy pair.<sup>15</sup>

The limitations of this game formulation stem from the fact that it allows only two competitors and requires the total rewards received by the firms to be constant. If the game is not a zero-sum game or if there are more than two firms competing, the analysis may not yield a maximin equilibrium strategy.

Although game theory and risk analysis outlined in this section are useful in determining prices, the difficulty in formulating competitive probabilities and the non-zero sum multi-firm nature of the environment has led firms to approach the price determination problem by developing heuristic strategies. These heuristic developments will be discussed in the last section of this chapter and additional management science approaches will be considered in the product planning chapter.



Estimating Demand Relationships

In order to apply the models discussed in this chapter, the firm generally must have knowledge of demand relationships in its market environment. For purposes of discussion consider the deceptively simple price-quantity relationship. This relationship, also known as a demand schedule, represents the quantity of a product which would be demanded at various price levels. At any point in time, however, the firm generally only knows the quantity which is demanded at their present price. A single data point, of course, is not sufficient to determine even the simplest demand schedule. Subjective estimates of the demand equation  $q = F(p)$  may be made, but managers often prefer to have empirical market data to integrate with their subjective judgments prior to reaching a price decision.

What methods are available for obtaining empirical information on the price-quantity relationship? Three basic approaches are available: questionnaires, regression analysis, and experimentation. These approaches as well as their limitations are discussed below.<sup>16</sup>

Questionnaire Methods. Various approaches have been used here. Customers may simply be asked how much they would purchase of a particular product (or brand) at a number of alternative prices. In the case of a new product customers may be given a choice between the new product and some amount of cash. The amount of cash being varied between customers in order to estimate the price sensitivity of the new product. Somewhat more subtle approaches are available. For example, the interviewer may ask the consumer about the price difference between competing products and brands. If many consumers are aware of the difference, relatively higher price sensitivity may be presumed than if few consumers are aware of the difference.



The questionnaire method has serious limitations. At least somewhat heroic assumptions must be made in this approach:

1. Consumers can perceive how they would react to different price changes.
2. Consumers will honestly and accurately report these perceptions.
3. Consumers' perceptions in the interview situation are a reliable prediction of their future market behavior.

Clearly, all three assumptions are suspect and any given questionnaire procedure should attempt to minimize the incidence of violation of these assumptions.

Regression Analysis. If data on past market response to price are available, the firm may attempt to measure the sensitivity of market and its prices. Suppose for the moment that demand for the firm's product can be specified as

$$(6-26) \quad q_i = a p_i^b p_j^c z^d$$

where

- $q_i$  = demand for firm i's product
- $p_i$  = price of firm i's product
- $p_j$  = price of firm j's product
- $z$  = disposable income
- $a$  = scaling factor
- $b$  = price elasticity for firm i
- $c$  = cross price elasticity of firm i's demand with firm j's price
- $d$  = income elasticity of firm i's demand

Now if natural logarithms are taken on both sides of (6-26), a standard regression format is obtained as

$$(6-27) \quad \ln(q_i) = a + b \cdot \ln(p_i) + c \cdot \ln(p_j) + d \cdot \ln(z).$$

The coefficients in (6-26) would generally be expected to have the following regions

$$(6-28) \quad \begin{array}{l} b < 0 \\ c > 0 \\ d > 0 \end{array}$$





Since  $b$  is the price elasticity of firm  $i$ 's demand, it represents the proportionate change in demand for its product which may be expected from a change in its price. Its expected negative sign represents the fact that demand changes and price changes will tend to move in opposite directions. Similar arguments apply to "c" and "d."

An interesting measurement of price and demand response in retail markets has been reported by Massy and Frank.<sup>17</sup> Using consumer panel data, they examined price and demand effects in the sales of a frequently purchased consumer product. Their analysis included lagged price and demand variables that reflected the dynamic effects of the sales response as well as a term to reflect the expected market share of the brand. Their results and model are reported in their paper at the end of this chapter.

There are pitfalls in the regression approach. For example, if an important demand determinant has been left out of the model, errors will be introduced into the estimates of the coefficients in the regression. In (6-26) this will cause the elasticity estimates to be in error. If both supply and demand are changing in time, the manager is faced with a simultaneous equations problem which leads to more complex estimation procedures. It may, of course, no longer even be possible to identify the demand relationship. (That is, the demand relationship may be confounded with the supply relationship.) Many other problems such as heteroscedasticity, autocorrelation, multi-collinearity, and errors in variables must be considered in order to make proper application of regression. While a discussion of these problems is beyond the scope of the present book, there is need to proceed with caution in order to avoid inappropriate application of the regression model. This caution is especially important in view of the ready availability of regression programs and their deceptively



simple underlying models.

Experimentation. Experimental approaches are finding increasing application in marketing. In the area of price policy, both laboratory and field experimentation have been used. The principle advocate of the experimental approach to determining a demand schedule has been Pessemier.<sup>19</sup> He has used simulated shopping trips where prices are varied as the basis for estimating the demand schedule. While this approach is subject to the usual criticisms which can be made of laboratory experiments in terms of the relation to behavior in the outside world, Pessemier's approach is an interesting one which should be developed further.

Field pricing experiments have also found increasing use, particularly in supermarkets and department stores where experimentation may be relatively easy. The use of sophisticated methods such as confounded factorial designs and covariance analysis has greatly enhanced the utility and accuracy of experimental results.<sup>19</sup> There are three principle problems in price experimentation:

1. Cost. The method is generally expensive.

2. Competitive retaliation. Competitors will attempt to disrupt an experiment if they learn of it. Increased promotion or a special sale on their part may greatly disrupt an experimental program.

3. Governmental constraints. Federal legislation limits the ability of a firm to vary its price in different areas and to different classes of customers, even on an experimental basis.<sup>20</sup>

In spite of these limitations the experimental approach may be expected to find increasing use in the future.



Heuristic Approaches to the Pricing Decision

The overwhelming complexity of the pricing decision faced by most businessmen has forced them to develop useful and satisfactory rules of thumb or heuristics for determining prices. The most commonly used heuristic is to price at some percent above cost. Other more elaborate heuristics are in use and some of them encompass the concepts outlined in the previous sections. Management scientists have attempted to identify the heuristics used by executives. The identification is based upon constructing a descriptive model of the decision procedure. If the model is accurate in replicating and predicting the executive's price decisions, the model is assumed to be a valid representation of the decisions maker's pricing heuristics. Although descriptive models would seem to be a useful and necessary first step, ultimately attention should be given to normative procedures which build upon these descriptive models.

The pioneering management science work in identifying heuristic pricing procedures was carried out by Cyert and March.<sup>21</sup> They developed a descriptive model of pricing behavior in a department store. The heuristic procedure followed by the store in determining prices was based on two goals. The first was a sales volume goal and the second was a mark up objective. These are not compatible goals. A high mark up may mean less sales volume and higher sales may be obtained by a lower mark up. These goals were not aimed at the normative criteria of profit maximization, but are rather heuristic goals that experience had indicated would yield satisfactory results.



The pricing in the department store studied was carried out in three stages: (1) normal pricing, (2) sale pricing, and (3) mark down pricing. The regular prices were determined by applying a standard industry margin or by pricing at the manufacturer's suggested price levels. The normal pricing heuristic reflected a tacit agreement between competitors to establish similar initial prices.

Sale pricing was used when the sales volume goal was not being achieved. Five heuristic rules were used in sale pricing:

1. If the normal price falls at one of the levels listed below, establish the indicated sale price.

normal price	sale price
1.00	.85
1.95	1.65
2.50	2.10
2.95	2.45
3.50	2.90
3.95	3.30
4.95	3.90
5.00	3.90

2. Reduce normal prices not encompassed by rule one by at least fifteen percent if the normal price is less than or equal to three dollars and by at least  $16\frac{2}{3}$  percent if the normal price is greater than three dollars.
3. All sale prices must end in a 0 or 5.
4. No sale price can fall in a normal price line value.
5. Always choose .90 over .85 in the cents part of the price.

With these rules a logical flow diagram and mathematical model were used to describe the complete heuristic routine. The flow diagram used by Cyert, March, and Moore is shown in Figure 6.



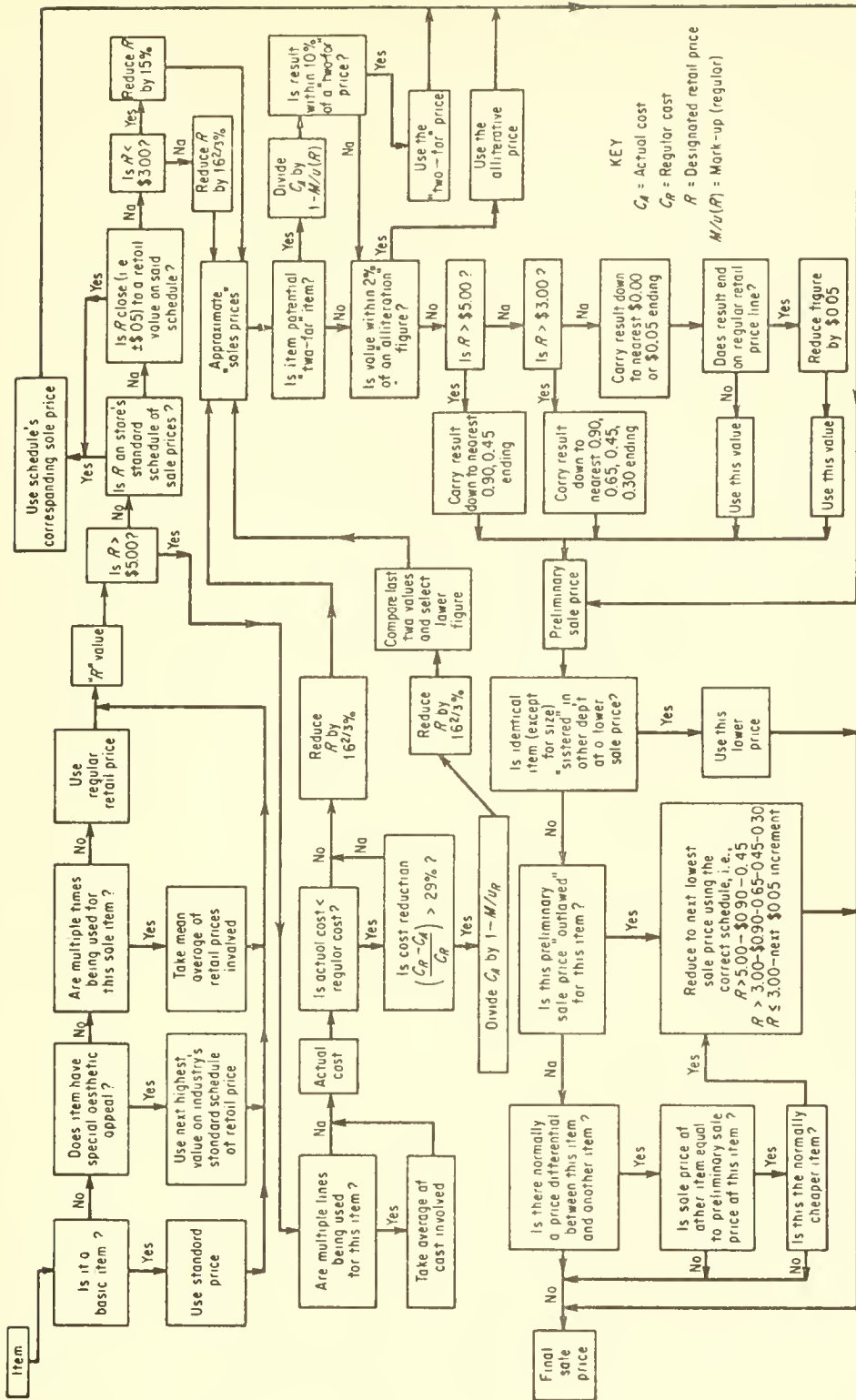


The procedure reflected heuristic rules for handling some of the complexities of pricing discussed in earlier sections. The considerations of competitive effects are handled by using standard prices (see box one) and suggested retail prices (see box two). The product line interdependencies are considered in boxes three and four by the indicated pricing rules.

If sales pricing was not successfully achieving the sales volume goal, mark down pricing was indicated. The general pricing rule was to reduce the price by at least  $1/3$  and end the price with \$.85. This basic rule and its exceptions were described in a flow diagram and a mathematical model.

The hypothesized models were tested for validity. The models correctly predicted to the penny 188 out of 197 of the normal prices, 36 out of 58 sale prices, and 140 out of 159 mark down prices during the test. These test results indicated that the model was a good description of the actual pricing procedure used in the particular department store studied. The procedure was not a normative one of profit maximization, but rather a procedure made up of a number of heuristic rules that had produced satisfactory results in the past.





Flow chart for sale pricing decision.



Another study of a heuristic pricing procedure is included in the readings that follow this chapter. This study by Howard and Morgewroth describes a rather simple pricing procedure followed by a large company facing an oligopoly market structure. The model is simple although executives of the firm felt it was a complex unprogrammable decision. The basic heuristic rule of the model is to follow price increases by competitors if the district sales office agrees, and to follow competitive price decreases if sales decrease. The general rule is tempered by holding periods where decisions are delayed while additional information is gained. The competitive nature of the oligopoly has led this firm to play the role of the imitator.

Tests of the model showed that its structure and decision output clearly corresponded with 31 actual decisions. Howard and Morgewroth's model appears to be a reasonable description of the heuristic procedure used by the company.

These models indicated that in the real world the complexity of pricing decisions is encompassed in heuristic procedures rather than the optimal approaches suggested earlier in this chapter. This indicates an area of potential for management scientists. Efforts could first be concentrated on developing descriptive models of the existing pricing procedures and then interjecting more powerful optimality characteristics into the models. This should be a healthy approach since it will allow better communication between the management scientist and the decision maker. With this working relationship an evolutionary process of upgrading the existing pricing procedures will lead to improved and more nearly optimal pricing decisions.



15. R. Duncan Luce and Howard Raiffa, Games and Decisions (New York: John Wiley, 1964), pp. 71-73.

16. This discussion draws upon W. Aldeison and P. Green, Planning and Problem Solving in Marketing (Homewood, Ill.: Irwin), 1964, pp. 247-252 and W. Baumol, Economic Theory and Operations Analysis, (Second Edition), (Englewood Cliffs, N.J.: Prentice-Hall), 1965.

17. William F. Massy and Ronald E. Frank, "Short Term Price and Dealing Effects in Selected Market Segments," Journal of Marketing Research II (May 1965), pp. 171-85. This article is reprinted in the selected readings following this chapter.

18. Edgar A. Pessemier, "An Experimental Method for Estimating Demand" in Frank, Kuehn, and Massy, Quantitative Techniques in Marketing Analysis, Homewood, Ill.: Irwin), 1962, pp. 153-165.

19. Ibid.

20. S. Banks, Experimentation in Marketing

and Howard, Legal Aspects of Marketing

21. Richard M Cyert and James G. March, A Behavioral Theory of the Firm (Englewood Cliffs: Prentice Hall, New Jersey, 1963), pp. 128-148.

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