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Abstract

The velocity of second sound as a function of pressure and temperature has been measured to 0.95°K by a pulse method. The quantitative dependence of the rise in velocity upon pressure is found to support the role of the phonons in contributing to the normal fluid flow alone.

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Introduction

The properties of liquid helium II have been most satisfactorily explained by the "two-fluid" concept advanced in the theories of Tisza (1) and Landau (2). Each theory regarded liquid helium as composed of a "normal-fluid" fraction and a "superfluid" fraction, although the origin of these two fluids was assigned to different molecular mechanisms. In the temperature range just below the lambda point, Tisza, following the suggestion of London, considered the transverse excitations to be similar to those of a Bose-Einstein gas, appropriately modified by the liquid state. Experiments showing no superfluidity in He^3 have born out this view, as opposed to the roton model of Landau. On the other hand, Landau considered the longitudinal excitations, the Debye phonons, as a component of the normal fluid flow only, rather than as associated with the fluid as a whole, which was Tisza's view. The phonons are masked by the Bose-Einstein excitations at higher temperatures, but below about 1.1°K their effect is evident. The rise in second-sound velocity (3, 5, 6) at low temperature and the sustained existence of the waves without undue attenuation tends to bear out the hypothesis that the new type of excitation predominant below 1°K is associated with the normal fluid flow only, and not with the superfluid. The present experiment takes advantage of the large pressure dependence of first-sound velocity in liquid helium to investigate whether this low-temperature excitation is indeed a phonon effect.

Experiment

Second sound is a type of wave motion most easily excited by the heating of liquid helium. Pellam has developed a pulse method (4, 5) which was used by the authors to measure the velocity to below 1°K at vapor pressure (3). Peshkov has created standing waves to determine the velocity at vapor pressure down to 1° (6) and at higher pressures down to 1.3° (7).

A pulse method of exciting second sound similar to that formerly used in this work (3, 4) was employed again. The chief innovation was the installation of a delay line in the timing mechanism. This permitted a view of the pulse on a faster, continuous sweep and hence a more accurate determination of its leading edge. A DuMont 246 oscillograph was used to trigger the pulse generator and to actuate the delay line. The pulse generator excited the carbon resistor of the second-sound chamber. Another carbon resistor, acting as a resistance thermometer, received the second-sound pulse, which was amplified and fed into the vertical deflection plates of a second 246D oscillograph. This last oscillograph was triggered by the delay line. The movable marker of the receiver oscillograph could be adjusted on the sweep so that it coincided with the edge of the pulse. By reading the marker dial and by knowing the delay, one could obtain the transit time for the pulse. The capacitative pickup within the Dewar from the transmitting pulse could be easily amplified to give a sharp leading edge when viewed on the final oscillograph. In doing this, we could not detect any instrumental delay caused by the timing system.

The second-sound chamber itself was sealed with solder for immersion in a liquid helium bath. Inside the chamber a thin sleeve separated the carbon resistors and determined the fixed path of 4.23 centimeters. Four capillaries with an inner diameter of 0.020 inch led to the chamber. Two of these acted as electrical shields for the wires while the other two were pressure lines — one to condense in helium gas from a tank and the other to observe pressure equilibrium with a check gauge. Liquid helium exists under a temperature gradient in the capillaries so that the heat leak from the λ-temperature to the chamber is limited only by the inner diameter. The size of these pressure capillaries was a compromise between this heat leak and the persistent clogging of the line by frozen materials. The pressure values of the experiment were determined on the input side by an Ashcroft Laboratory Test Gauge. A Distillation Products MB200 diffusion pump with three Kinney VSD forepumps was used to remove vapor from the liquid helium bath.

Temperature measurements were made with a McLeod gauge through a tube in the pumping line above the Dewar arrangement as described formerly (3). The equivalence of temperatures obtained from the McLeod gauge and those inside the chamber was established by constructing a dummy chamber of the same dimensions as the real one but filled with iron ammonium alum. Liquid helium under pressure was supplied to this chamber. A mutual inductance bridge was used to measure the relative paramagnetic

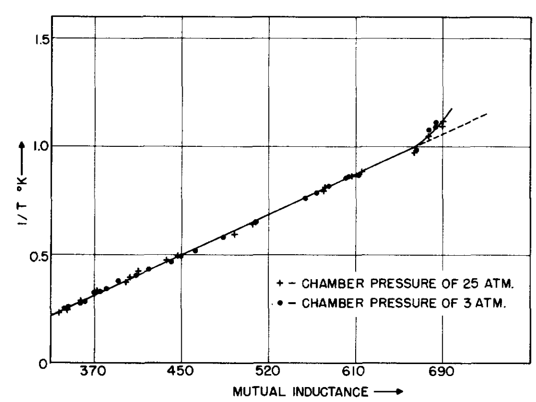

Fig. 1

Plot of the mutual inductance of the salt in the dummy chamber vs. the reciprocal of the temperature as obtained from McLeod gauge readings. Values were taken with the liquid helium in the chamber at two different pressures. The extrapolated broken line gives the true temperature in terms of that obtained from McLeod gauge readings.

susceptibility of the salt at different McLeod gauge values and at two different chamber pressures. The susceptibility of the salt, and hence the mutual inductance, varies as $1/T$. Figure 1 shows that the mutual inductance gives a straight line, except at low temperatures, when plotted against this variable obtained from the McLeod values. The deviation below 1° is attributed to the pumping pressure drop from the bath as observed previously (3) and is not due to any radical change in heat conductivity of the helium in the capillaries. The data below 1° were corrected from this curve.

Results and Conclusions

The data for second-sound velocity as a function of temperature and pressure are shown in Fig. 2. They agree, to within experimental error, with the measurements down to 1.3°K by Peshkov (7). The velocity of second sound along the vapor-pressure line as previously reported (3) was remeasured with the present apparatus and is also shown. The maximum deviation of the points is ± 1 percent from the curves drawn through them.

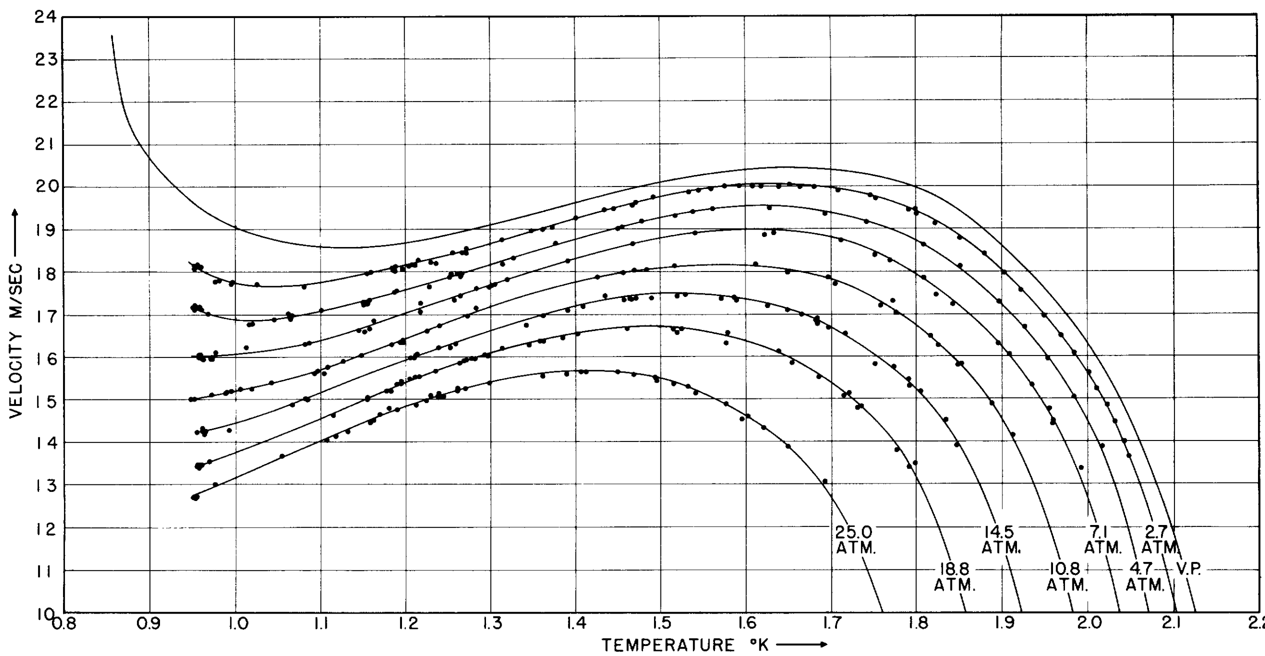

Fig. 2 The velocity of second sound as a function of temperature and pressure. Experimental points for the vapor pressure curve are not shown.

Landau's proposal that the phonons contribute to the normal fluid flow implies that at the absolute zero of temperature the second-sound velocity must approach the value $c_1/\sqrt{3}$, where c_1 is the velocity of ordinary (first) sound (2). The value of c_1 at the vapor pressure is about 250 meters per second, which is an order of magnitude larger than the velocity of second sound. The second-sound velocity must therefore rise rapidly as the temperature is lowered. The first-sound velocity increases with pressure, so that the second-sound curves at various pressures must cross somewhere in the temperature region blow 0.95°K if they are to arrive at Landau's value of velocity.

It has been found, however, that a computation based on a linear superposition of the terms giving the entropy and normal fluid fraction for the Bose-Einstein and phonon contributions to the excitations does not agree quantitatively with the observed velocity. The experimental curve shows no indication of the phonon term down to 1.2°K, but is rising rapidly at 1.0°. A phonon term simply added to the Bose-Einstein term extrapolated from higher temperature does not show an appreciable rise until about 0.6°. Landau gets the rise at 1° by using a roton expression which decreases more rapidly (exponentially instead of algebraically), but the phonon term is then too large in comparison to be absent at 1.2° as observed. The suddenness of the onset of the phonon contribution below 1.2° suggests the possibility that an interaction between the Bose-Einstein excitations and the phonons is removing the former rapidly as the latter becomes sufficiently large in comparison.

Nevertheless, the temperature at which this rise must begin corresponds to the point where the entropy of the phonons begins to be of the same order of magnitude as the entropy of the Bose-Einstein excitations as the temperature is lowered. The pressure dependence of this temperature may be computed from the known pressure dependence of the first-sound velocity, and compared with the observed pressure dependence. The phonon nature of the low-temperature contribution to the second-sound curve may be checked by this comparison. To remove the possibility of phonon Bose-Einstein interaction from affecting the result, this pressure dependence will be obtained in the limit of small phonon contribution.

An empirical value of the Bose-Einstein entropy may be obtained by extrapolation from measurements between 1.2° and the lambda point, and is given to sufficient accuracy for the present purpose by

$$S_{BE} = S_\lambda \left(\frac{T}{T_\lambda}\right)^{5.5}$$

where S_λ is the entropy at the lambda point, and T_λ is the lambda temperature. The entropy of the phonons is given by Debye's expression

$$S_{ph} = \frac{16}{45}\pi^5 \frac{k^4 T^3}{h^3 c_1^3 \rho}$$

where k is Boltzmann's constant, h is Planck's constant, and ρ is the density. The temperature at which deviation from the extrapolated second-sound velocity curve occurs may then be given approximately by

$$S_{ph} = \text{const} \times S_{BE}$$

where the value of the constant determines the amount of deviation. This expression can be expected to hold only for small deviations. Solving for temperature

$$T_d^{2.5} = \text{const} \times \frac{T_\lambda^{5.5}}{c_1^3 \rho S_\lambda}$$

where the new constant is a combination of the previous constant and Boltzmann's and Planck's constants, and T_d is the temperature at which the observed second-sound velocity deviates a certain amount from the value extrapolated from high temperature. Rather than passing to the small phonon contribution limit, the constant may be eliminated by use of the logarithmic derivative

$$\frac{1}{T_d}\frac{dT_d}{dP} = 2.2\frac{1}{T_\lambda}\frac{dT_\lambda}{dP} - 0.4\frac{1}{S_\lambda}\frac{dS_\lambda}{dP} - 1.2\frac{1}{c_1}\frac{dc_1}{dP} - 0.4\frac{1}{\rho}\frac{d\rho}{dP} \quad .$$

The first two terms arise from the Bose-Einstein contribution to the entropy, and their sum has the numerical value of -0.008. The second two terms arise from the phonon entropy, and their sum has the numerical value of -0.029. The total pressure dependence of T_d is therefore

$$\frac{1}{T_d}\frac{dT_d}{dP} = -0.037 \text{ deg/atm-deg}$$

of which about 80 percent comes from the phonon term in the entropy. This large fraction is due to the large pressure dependence of first-sound velocity in liquid helium.

The observed pressure dependence of T_d may be obtained from the curves of Fig. 3. If the phonon term were not present in the expression for second-sound velocity, the velocity would go to zero with the square root of the temperature (1, 2). The ratio $c_2/\sqrt{T}$ therefore exhibits a horizontal straight line at higher temperature, but somewhat below the lambda point, which may be easily extrapolated to low temperature. The deviation from the extrapolated line is then taken from the rising part of the curve at low temperature. Lines of constant deviation are shown for various values of the deviation. The corresponding values of dT_d/dP are plotted against the amount of deviation in Fig. 4. The values of dT_d/dP and $(1/T_d)\, dT_d/dP$ are nearly the same because T_d is near 1°K. The extrapolated value at zero deviation is in good agreement with the value computed above.

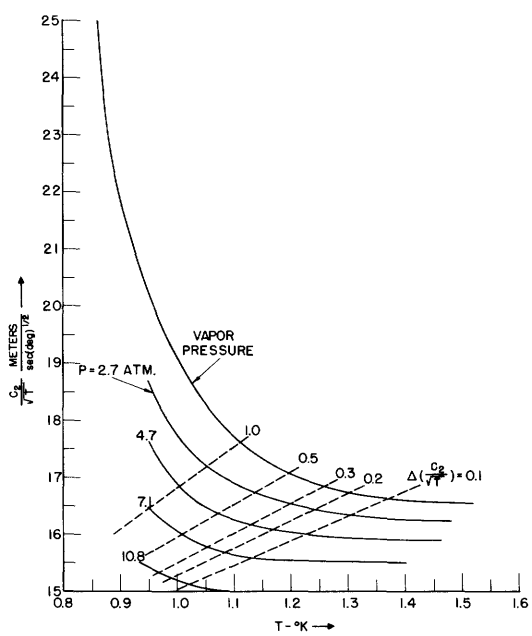

Fig. 3

Plot of $c_2/\sqrt{T}$ vs. T for various pressures. The high temperature portions of the curves extrapolate to low temperatures as horizontal lines. Lines of constant deviation from the extrapolated horizontal line, $\Delta(c_2/\sqrt{T})$, are shown for various values of the deviation. The temperatures of constant deviation, T_d, are given by the intersections of the P = const. and $\Delta(c_2/\sqrt{T})$ = const. curves.

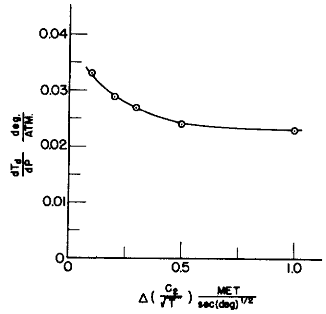

Fig. 4

Pressure derivative of the temperature of constant deviation, (dT_d/dP), as a function of the amount of deviation, $\Delta(c_2/\sqrt{T})$, taken from Fig. 3. The value expected from the phonon contribution to the normal fluid flow at small deviations is about 0.037.

This agreement supports the hypothesis that the phonons contribute to the normal fraction of fluid only, and are responsible for the rise of second-sound velocity at low temperature. The phonons do not, however, combine linearly with the Bose-Einstein excitations, unless the Bose-Einstein spectrum is greatly different from what has been assumed heretofore.

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