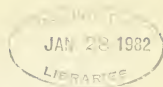WORKING PAPER
ALFRED P. SLOAN SCHOOL OF MANAGEMENT

People Drive Too Slowly

Julio J. Rotemberg

September 1981

Working Paper #1256-81

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by

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ABSTRACT

This paper studies the behavior of an optimizing driver
on a freeway and the equilibrium speed at which specified flows
of traffic move. It shows that both when passing is not allowed
(the standard car-following framework of the engineering litera-
ture) and when it is permitted, the equilibria exhibit speeds
which are below the Pareto Optimal speeds. The flat congestion
toll proposed by Oron, Pines and Sheshinski [1973] does not, by
itself, bring about Pareto Optimality.

I.   Introduction

This paper studies the behavior of a typical driver on a freeway. This
driver derives disutility both from long commutes and from unsafe ones. The
paper then studies the equilibrium at which specified flows of traffic move.
It shows that these speeds are generally inefficient and, in particular, are
lower than the Pareto optimal speeds. This inefficiency, which arises only when
traffic is congested, can be explained as follows. By making all drivers ahead
of a certain car move faster, one makes the driver of that car better off since
he now faces less congested conditions. Every driver's choices affect the welfare
of the drivers behind him. Therefore, a flat congestion toll on freeways like the
one proposed by Oron, Pines and Sheshinski [1973] does not insure Pareto Optimality.
A flat toll reduces the flow of traffic, but the speed of this reduced flow may
well be too low. Moreover, if cars that are left on their own go too slowly,
speed limits become hard to defend even if they do reduce accidents.

Two scenarios are studied. In one, passing is forbidden while in the other
it is allowed. In the former, the environment of a particular car is very similar
to the environment of the "car-following" model widely used in the engineering lit-
erature (e.g. Haberman [1977], Drew [1968], Institute of Transportation Engineers
[1976]). Each driver takes the speed of the car in front of him as given. He can
only control the distance between himself and the car he follows. Under these
circumstances, I derive the speed-density relationship which is assumed in the
engineering literature. This equation asserts simply that cars will keep larger
distances between each other the faster the traffic is moving. Both this speed-
density relationship and the inefficiency of equilibria are illustrated in models
in which the individual is assumed to keep his gap constant over time. While

this constraint is common in the engineering literature (Drew [1968], Haberman [1977]), Section II shows that it is not warranted when passing is not allowed and the individuals solve their intertemporal problems correctly. However, the resulting equilibria seem untractable. Therefore, Section III studies the optimization problem of an individual who is not allowed to pass and who must keep his gap constant. Section IV derives the equilibrium traffic patterns for this case, while Section V discusses their welfare properties. Section VI describes a stylized highway in which passing is allowed. Concluding remarks are presented in Section VII.

II.   The Representative Individual's Control Problem

Throughout the paper, all drivers are assumed to have the same preferences and vehicles. The representative individual is concerned both with the amount of time he spends on the highway and with the number of accidents he causes.[1/] Long commutes and numerous accidents are both undesirable. To a certain extent, it is possible to "buy" safety by taking more time. In this section, I study these substitutions in an environment in which the individual can only choose to vary the distance he keeps between himself and the car in front of him. This environment describes road segments like bridges and tunnels in which passing is not allowed. In the engineering literature (see for instance Haberman [1977]), it is asserted that when passing is not allowed and the speed of the surrounding vehicles is constant, individuals will eventually keep a constant gap. While assumptions that guarantee this result will be made in the next section, it will typically not be to the individual's advantage to keep the gap constant over time when traffic is congested. Safety is achieved by keeping a large gap for as long as possible while time is lost only if the

gap is large at the end of the trip. Therefore, it is to the individual's advantage to maintain a large gap during the bulk of the trip and reduce the gap near his destination.

Let the individual's utility at t be given by:

$$U = - (T - t) - A(t) \qquad (1)$$

where (T-t) is the length of time left on a trip conditional on not causing any accidents[2] and A(t) is a function of the probability of causing an accident during the remainder of the trip.[3] The inclusion of A(t) is reasonable if the person who causes the accident bears the full social costs. This may or may not correspond to the United States institutional environment. While this assumption is critical for the welfare analysis of Sections V and VI, the equilibria discussed below would not be affected if one assumed, instead, that individuals bore costs from being hit while not being able to affect the probability of being hit.

The individual is, at time t, distance D(t) away from his destination. He finds himself in a stream of traffic moving at speed S(t) in which passing is not permitted. He is separated from the vehcile in front of him by the gap g(t). The instantaneous probability of causing an accident between τ and $\tau + d\tau$, given that no accident has been caused until τ, is given by $a[S(\tau) + \dot{g}(\tau), g(\tau)]$, where a dot represents a time derivative. Clearly these instantaneous probabilities increase as the car moves faster and as the distance between cars gets smaller. Moreover, I assume that the efficiency of increases in the gap as accident-preventors decreases as the gap increases. Also, as people drive faster, an extra mile per hour of speed tends to increase accidents more. Finally, an increase in the speed makes an increase in the gap more effective. In other words:

$$a_1 \geq 0 \qquad a_2 \leq 0 \qquad a_{22} \geq 0 \qquad a_{11} \leq 0 \qquad a_{12} \leq 0$$

where the subscripts denote partial derivatives.

A standard formula (see Parzen [1973], p. 168) establishes that the probability of having an accident between t and T, conditional on not having had an accident at t, is given by $p(t)$:

$$p(t) = 1 - \exp \{ - \int_t^T a[S(\tau) + \dot{g}(\tau), g(\tau)] \, d\tau \} \qquad (2)$$

Let the disutility from incurring the possibility of having an accident be given by:

$$A(t) = - k \ln [1 - p(t)]$$

where k is an arbitrary constant. Then the objective function of the individual at t can be rewritten as:

$$U_t = \int_t^T \{ - 1 - ka [S(\tau) + \dot{g}(\tau), g(\tau)] \} \, d\tau \qquad (3)$$

This objective function is subject to the following constraints:

$$D(T) = 0 \qquad \text{and} \qquad \frac{dD(\tau)}{d\tau} = S(\tau) + \dot{g}(\tau) \qquad (4)$$

which embody the notions that T is the time of arrival and that progress is made at the rate $S(\tau) + \dot{g}(\tau)$. The individual has one control variable, namely $\dot{g}$, which he uses to optimally trade off safety for speed. The Hamiltonian for this problem is given by:

$$H(\tau) = - 1 - ka \{ [S(\tau) + \dot{g}(\tau), g(\tau) \} + \lambda(\tau)[S(\tau) + \dot{g}(\tau)] + \mu(\tau)\dot{g}(\tau)$$

where λ and μ are multipliers associated with the two states, the distance away from the terminal position and the gap at τ.

The necessary conditions for an optimum are therefore:

$$\frac{dH}{d\dot{g}} = k\, a_1\, [S(\tau) + \dot{g}\,(t),\, g(\tau)] + \lambda(\tau) + \mu(\tau) = 0 \tag{5}$$

$$\frac{dH}{dD_\tau} = 0 = -\dot{\lambda}\,(\tau) \tag{6}$$

$$\frac{dH}{dg_\tau} = ka_2\, [S\,(\tau^{'}) + g\,(\tau),\, g(\tau)] = -\dot{\mu}\,(\tau) \tag{7}$$

$$H(T) = 0 \tag{8}$$

$$\mu(T) = 0 \tag{8}$$

Even when $S(\tau)$ is constant, the optimum gap is not typically constant. Consider the implications of having $\dot{g}(\tau)$ be equal to zero. By (5) this requires μ to be constant since λ is constant by (6). Therefore, the marginal effect of a change in the gap on accidents must be zero according to (7). Furthermore, the speed $S(\tau)$ must be such that the marginal effect on accidents of an increase in the speed is equal to $-\lambda$ where λ is chosen to satisfy:

$$- 1 - ka\,(S,\, g) + \lambda S = 0$$

which is implied by (8).

These implications of a constant gap can be summarized by saying that the traffic must be moving at "free flow" in this case. That is, cars must be sufficiently far apart and move quickly enough so that they do not notice each other. Then, identical cars will indeed move at a constant speed and keep a constant distance. It turns out, however, that these are precisely the conditions under

the study of traffic flows is not very interesting. Instead, when the number of cars on the road is large enough so that people can not help noticing each other, congestion, with its implied direct effects on traffic and its indirect effects on land rents (see Solow [1973], Arnott [1979] and Arnott and MacKinnon [1978]), become important.

In general, the gap must satisfy the following differential equation which is obtained by combining (5), (6) and (7).

$$a_{11} (\cdot) (\ddot{g} + \dot{s}) + a_{12} (\cdot) \dot{g} - a_2 (\cdot) = 0 \qquad (9)$$

The gap will change over time so that it is large during the bulk of the trip thus providing safety, and is small at the end of the trip thus saving time.

The speed at which traffic moves in the equilibrium which arises when each individual is following a path given by (9) and when the flow of cars is exogenous, is beyond the scope of this paper. Instead, I examine the behavior of slightly less rational individuals. These individuals pick only one value for their gap and maintain that gap during their entire trip. This simplifies the computational burden faced by individuals, and the equilibrium speed can be characterized in more detail.

III. A Simplified Problem for the Individual

In this section, every driver has two options. He can either drive at the same speed as the preceding car (S), or he can drive slower. If he chooses the former alternative, he must maintain a constant gap. This gap is picked at a certain distance from his destination which is common to all drivers. His utility at this point is given by:

$$U(S, G) = - [\frac{1}{S} + \frac{G - G_o}{S}] - A(S, G) \qquad (10)$$

where G is the gap he chooses at this point, G_o is the gap with which he starts, and S is the speed measured in units away from the destination per hour. Note that the probability of having an accident now depends on the gap both because reductions in the gap and small gaps are dangerous. Once again I assume that:

$$A_1 \geq 0 \qquad A_2 \leq 0 \qquad A_{22}, \; A_{11} \geq 0 \qquad A_{12} \leq 0$$

which were justified above.

A rational individual chooses a gap such that the marginal loss in utility from a bigger gap in terms of time spent on the road just equals the marginal gain from accident prevention. At the preferred gap G, the marginal utility of an increase in the gap, MU_G, must be equal to zero.[4]

$$MU_G \; = \; -\frac{1}{S} - A_2(S, \, G) \; = \; 0 \tag{11}$$

Individuals change their gap when the speed of the car in front of them changes. Totally differentiating (11) and rearranging, we get:

$$\frac{dG}{dS} \; = \; \frac{s^2/1 \; - \; A_{12}}{A_{22}} \; > \; 0 \tag{12}$$

An increase in speed leads to an increase in the gap between cars. This fact forms the basis of much traffic flow theory even though, to my knowledge, it has never been derived from the assumption of utility maximization.[5] Intuitively, a reduction in the gap by one meter involves, as S goes up, lower time losses and more effective accident prevention. For both reasons, an increase in G is worthwhile when S rises.

Condition (10) is not sufficient to guarantee that a certain speed-gap combination will be an equilibrium since people have the option of traveling at

a speed below S. Therefore, for a speed-gap combination to be an equilibrium, it must be the case that no unilateral reduction in speed increases utility. Such a reduction would be accompanied by ever increasing gaps. Therefore, it would be safer than (and preferred to) the same reduction in speed combined with an unchanging gap. Hence:

$$\frac{dU}{dS} = MU_S = \frac{1}{S^2} - A_1 \geq 0 \tag{13}$$

Suppose the individual were free to choose both S and G. Then (11) and (13) would hold with equality. Assume that this occurs as long as the speed is the free-flow speed, S^*, and the gap is bigger than or equal to G^*. If cars are sufficiently far apart, they will simply travel at a constant speed and will not be affected by the presence of other cars. In Section II, such a situation prevailed when a_1 was equal to λ while a_2 was equal to zero.

A certain "monotonicity" condition is also required. This condition is that points with $S = S^*$ and $G \geq G^*$ are the only critical points of the function U. In other words, individuals are locally satisfied only when traffic is moving freely. When traffic is congested, the speed is less than S^*, and MU_S is strictly greater than zero. This last condition can be thought of as a definition of congestion since it states that the car ahead of the individual is locally affecting the individual's welfare by going slowly.

The analysis of this section can be concluded by drawing, in Figure 1, some indifference curves which satisfy the assumptions I have made on individual preferences. Indifference curve 2 corresponds to freely flowing traffic, and utility declines as the individual moves from 2 to 1 to 0. This occurs as the individual travels either faster (less safely), slower, or at more dangerous gaps than the free-flow speed and gaps.
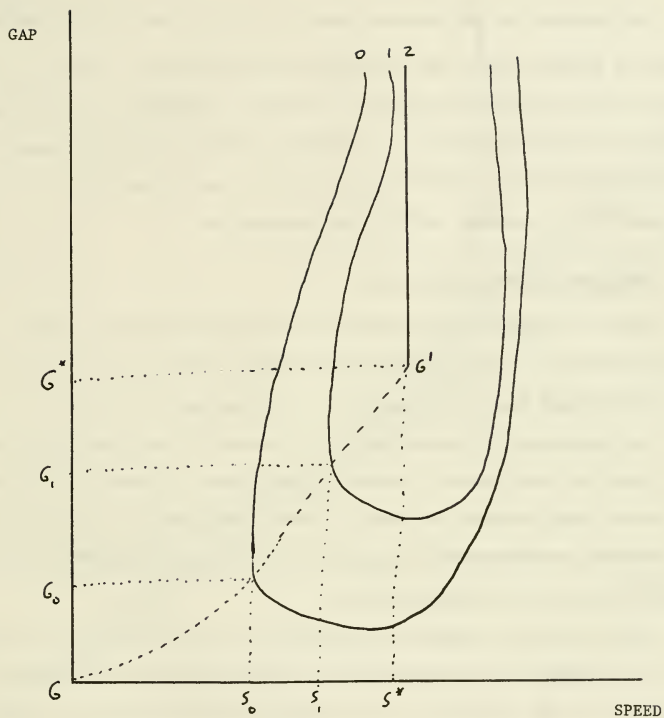
Figure 1

The slope of the indifference curves is $(\frac{1}{S^2} - A_1)/(A_2 + \frac{1}{S})$. They are vertical at points at which the individual maximizes utility -- like (S_o, G_o) and (S_1, G_1). At these points an increase in the speed accompanied by an unchanging gap makes the individual better off. The locus of the points which maximize utility is given by the line GG' together with the indifference curve 2. No speed above S* is consistent with utility maximization.

IV.   Nash Equilibrium Traffic Flow

Section III shows that for every speed the individual picks a gap. A Nash equilibrium is a situation in which, given the other agent's choice of a gap, the individual finds it optimal to pick the same gap.

Two things could be exogenous to this system. In the first place, the flow of vehicles at all points along the highway could be assumed to be a constant which simply reflects the number of people who commute on any given day. This assumption is best suited to highway sections with no entrances, and will be the main focus of the analysis below.

Second, one could assume that the density of vehicles on a road is exogenous. This describes situations with many entrance points in which people simply decide to get on the road. The equilibrium speed can simply be read off the line GG in Figure 2 which corresponds to line GG' and indifference curve 2 in Figure 1. The density of cars is simply equal to the inverse of the gap plus the length of each car. Unless the cars move at the speed given by GG, individual cars will not find it optimal to choose the gap which supports the given density of automobiles.

When the flow F is given exogenous, S and G must obey:

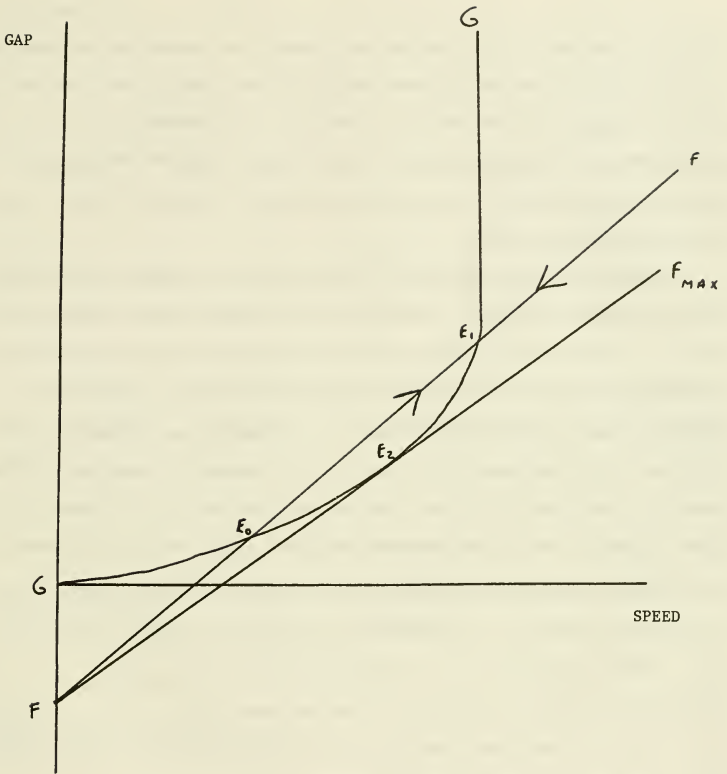$$F = \frac{S}{G + L} \tag{14}$$

Figure 2

where L is the length of the typical car. Here, F is measured in cars per hour, S in units of distance from the destination per hour, while G and L are measured in units of distance from the destination per car. This identity is intuitive since an increase in the speed, keeping the gap constant, increases the number of cars per hour that go through a point, while the reverse occurs if the speed is constant and the gap increases.

For any particular flow, say $\bar{F}$, one can draw the speed-gap combinations which are consistent with it. The locus, FF, represents such combinations in Figure 2. Points like E_0 and E_1 represent Nash equilibria for a given flow. At these points, (11) and (14) hold simultaneously. The gap chosen by each individual is such that the speed from (14) renders that gap optimal for each individual. This Nash equilibrium involves an externality. When an individual changes his gap, this affects the speed of the cars behind him through (14) — a fact ignored by the drivers concerned only with their own time and safety.

Points with slightly higher speed and gap for all drivers than an equilibrium are always feasible, i.e. they involve the same flow. Moreover, at any equilibrium, changes in the gap do not affect the driver's welfare, while increases in the speed make them strictly happier unless the speed is S*. Therefore, unless the flow of cars is so small that people drive at the free-flow speed, people drive too slowly. The Pareto optimal speed is discussed in Section V.

Figure 1 indicates that equilibria with high speeds are preferable to equilibria with lower speeds. The question then becomes whether the most desirable equilibrium consistent with a given flow is, in any sense, the most likely to prevail. A complete answer to this question is impossible without a dynamic analysis of the form used in Section II. However, it is possible to look at the stability of equilibria by assuming that they result from a tatônnement process. Suppose the auctioneer sets the gap for each individual. Then, equation

(14) determines the speed at which the traffic flows. The auctioneer observes people on the FF line but not necessarily on the GG line. He might be given the following rule: attempt to lower the gap by increasing everyone's speed when $MU_G < 0$, lower everyone's speed when $MU_G > 0$, and maintain the status quo when $MU_G = 0$. This, however, raises the gap when $MU_G < 0$ and viceversa when $MU_G > 0$, as illustrated by the arrows of Figure 2. Then, equilibria at which the GG curve is steeper than the FF curve are stable.[6/] If there is a vertical indifference curve that represents blissful free flow, high speed equilibria tend to be stable. However, low speed equilibria are unstable and this instability may be the root of Stop-Go traffic.

As usual, the issue of stability is linked to questions of comparative statics. It is widely assumed (see Solow [1973]; Oron, Pines and Sheshinsky [1973]; Arnott and MacKinnon [1978]; Arnott [1979]) that increases in the flow have a negative effect on speed and hence on welfare. This turns out to be true if the equilibrium is stable. Stability also insures that a reduction in the size of the cars leads to an increase in the speed.[7/] These facts are easily established algebraically. Differentiating (14), one obtains:

$$(G + L)dF + FdL + FdG = \bar{d}S \tag{15}$$

which coupled with (12) leads to:

$$dS = \frac{(G + L)dF + FdL}{1 - kF}$$

where

$$k = (1/S^2 - A_{12}) / A_{22}$$

Speed and welfare decrease in response to an increase in either F or L only when kF is larger than one, or, in other words, when the GG curve is steeper than 1/F. As mentioned above, this condition is likely to be met at high speeds.

V.   Social Optima

The social optimum is computed by maximizing the utility of the representative driver subject to the constraint that the flow be equal to the exogenously given F. This computation relies on the assumption that individuals pay the social costs of their accidents. If they did not, accidents would be more important in the social welfare function than in the utility function.

Using (14) to substitute for S in (10), the first order condition for this maximum is:

$$\frac{dU}{dG} = \frac{F}{S^2} - FA_1 - \frac{1}{S} - A_2 = 0$$

or

$$FMU_S + MU_G = 0 \tag{16}$$

The marginal utility of increasing each driver's gap must be equal to the loss in utility that results from the required increase in speed.

When the equilibrium involves the free flow of traffic, then $MU_G - MU_S = 0$; the equilibrium and the optimum clearly coincide. However, when congestion be-

comes important, $MU_S > 0$ at the equilibrium. Therefore, welfare increases along the FF curve to the right. To insure that a bounded local maximum of welfare exists at a speed higher than the equilibrium, one more assumption is needed: as long as the gap exceeds some upper bound, G^u, and the speed exceeds some upper bound, S^u, the MU_G is nonpositive and the MU_S is negative. When the car in front is beyond the horizon, only the speed should affect the welfare of a driver, and, if this speed is sufficiently large, decreases in the speed should increase welfare.

This assumption ensures that for speed-gap conditions far to the right of the equilibrium but consistent with the exogenous flow, $FMU_S + MU_G < 0$. At an equilibrium, with $S < S^*$, then $FMU_S + MU_G > 0$. Therefore, by the continuity of the first derivatives of (10), a maximum of utility exists which has a higher speed than the equilibrium.

Figure 3 illustrates these points graphically. At point E, there is an equilibrium whose speed is below S^*. The individual would like to go faster. From society's point of view, it is possible to make all individuals go faster as long as the gap increases. Since, locally, changes in the gap have no effect on utility, the movement towards point 0 increases welfare. At point 0, naturally, the indifference curves are tangent to the society's constraint which is given by the FF locus.

VI.   Multilane Highways

So far, only one lane roads were considered. This is in the spirit of the car-following models studied in the engineering literature. However, the
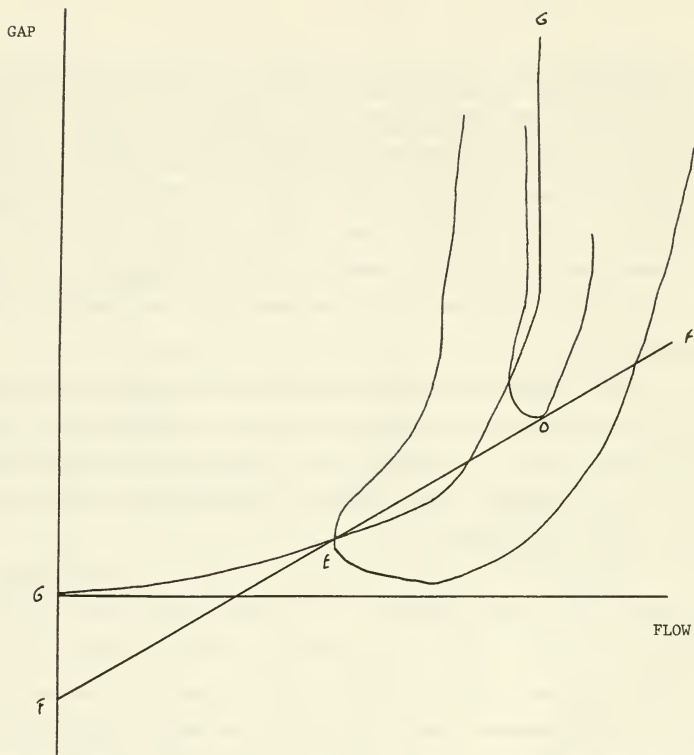
Figure 3

policy relevance of my conclusions would be somewhat limited if they only applied to bridges and tunnels. In this section, a very stylized model in which passing is permitted is presented. Once again, I conclude that unless the equilibrium speed coincides with S*, people drive too slowly.

The analysis proceeds in two stages. First, I show that under some reasonable assumptions on the technology of passing, equilibria exist in which overtaking will not take place even though it is allowed. At these equilibria, individuals have the option of slowing down and being passed. As long as the effects of being passed are not too large, one can infer that individuals would prefer to go faster at the equilibrium gap from the fact that they refuse to go slower and be overtaken. Hence, society would be better off if people drove faster!

Individuals are in traffic which moves at the speed S, and they choose a gap at the beginning of the trip. However, the individual can overtake the car in front of him at any point.

Overtaking a car is a discrete choice. The benefits from passing increase as the distance between cars gets larger. This is so mainly because it makes passing safer. It also may increase the speed at which passing can take place. For each speed S, the benefits from passing, which will be denoted by B, are positively related to the gap of the "followed" car. Therefore, passing will take place as long as $B(S, G_F) > 0$, $B_2 > 0$, where G_F is the gap maintained by the followed car, and B is assumed to be continuously differentiable. $B(S,0)$ must be negative and $B(S,\infty)$ must be positive if $S < S^*$. Therefore, as long as $B(\cdot)$ is continuous, there exists for each speed below S*, a gap that is just small enough to deter passing. Denote this gap by $\hat{G}(S)$. It is such that:

$$B[S, \hat{G}(S)] = 0$$

where I assume, for simplicity, that when cars are indifferent between passing and not passing they do not pass.

Denote the equilibrium gaps when overtaking is impossible, by $G'(S)$. If $G'(S) \leq \hat{G}(S)$, then the equilibria studied in Section IV will persist when passing is allowed. That is, if the gap that makes passing just desirable is larger than the gap which people want when they are assured that no passing will take place, then passing will not take place.

Suppose instead that $\hat{G}(S) < G'(S)$. Then, $G'(S)$ is not an equilibrium since all drivers want to pass each other. There is a new incentive for drivers to keep near each other. By doing so, they avoid being passed. Being overtaken is costly because the passed car must slow down to reestablish the original gap. Under fairly weak assumptions on the passing technology, $\hat{G}(S)$ is an equilibrium when $\hat{G}(S) < G'(S)$. Then, MU_G is larger than zero since A_{22} is positive. Since the gap is smaller than in the no-passing equilibrium, no car wants to reduce his gap at $\hat{G}(S)$. The question is whether any car wants to increase his gap and be passed. A sufficient condition for this not to occur is that the loss from allowing oneself to be passed is larger than the gain to a single person who passes. This is intuitively appealing since the gain from passing one car is a small reduction in trip length accompanied by a larger probability of causing an accident. On the other hand, the loss from having a gap larger than $\hat{G}(S)$ involves being passed many times (and therefore considerably lengthening the trip) in exchange for only slightly more safety.

Let $\tilde{G}$ be any gap larger than $\hat{G}(S)$ where $\tilde{G}$ is the average gap of the passed car. Let $\tilde{S}[S, \tilde{G}(S)]$ be the average speed that the car which is passed can maintain when the traffic moves at S and keeps a gap of size $\tilde{G}$. $\tilde{S}_2$ is negative. A person who keeps a larger gap will be passed more often. Then, the following assumption

guarantees that $\hat{G}(S)$ will be an equilibrium:

$$\forall \; S < S*, \qquad \forall \; \tilde{G} < \hat{G}(S) \; : \; U[S,\hat{G}(S)] - U[\tilde{S}(S,\tilde{G}(S)),\tilde{G}(S)] \geq B(S,\tilde{G}) \qquad (17)$$

That is, the loss from being passed repeatedly by keeping a gap $\tilde{G}$ is no smaller than the gain to a passing car.

Consider a situation in which all cars ahead of a particular car are going at speed S and keeping a gap $\hat{G}(S)$. Then, if (17) holds, the particular car will not want to keep a gap larger than $\hat{G}(S)$ since for such a gap $B(S,\tilde{G})$ would be larger than zero. Therefore, $\hat{G}(S)$ will indeed be an equilibrium.

Graphically, the possibility of passing lowers the GG curve at certain speeds (those for which $\hat{G}(S) < G'(S)$). This is shown in Figure 4. It is just as if people drove more recklessly. While such a change in driving patterns tends to increase the speed of the stable equilibria, it has no obvious effect on the number of accidents. Peltzman [1975] allows drivers to choose the amount of time a trip will take and makes the probability of accidents depend on that time. Then, more reckless driving (in response to automobile safety regulations for instance) reduces the time spent on each trip and increases the likelihood of accidents. When traffic is congested, however, my paper emphasizes that it is incorrect to think that people can pick directly the time their trip will take.

Under certain conditions, people drive too slowly even when passing is allowed. The key assumption is that it is possible to increase the gap slightly from $\hat{G}(S)$ and thereby reduce the speed only slightly by being passed by a few cars. Since people do not take advantage of this opportunity to slow down even though increases in the gap are not undesirable, they would prefer to drive faster. This is, of course, socially possible as was discussed in the previous section.
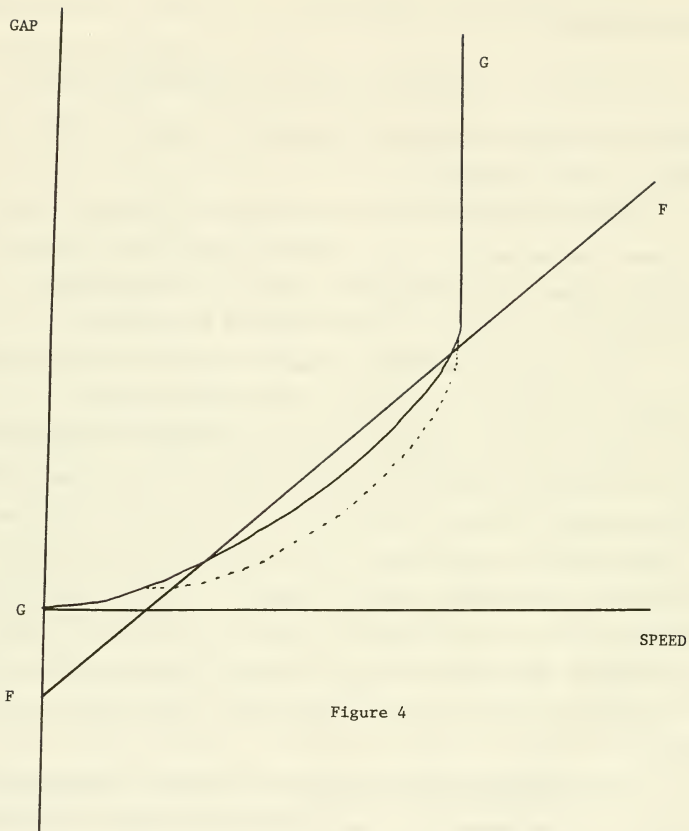
Figure 4

I therefore make the following assumption:

$$\forall \ S \leq S* \ \exists \ \tilde{G} > \hat{G}(S) : \ MU_G \ \Big|_{\hat{G}} \ [\tilde{G} - \hat{G}(S)] + MU_S \Big|_S \ [\tilde{S}(S,\tilde{G}) - S]$$

has the same sign as:

$$U[\tilde{G},\tilde{S}(\tilde{G})] - U[\hat{G}(S),S] \qquad (18)$$

In other words, a first order Taylor approximation of (18) at $[\hat{G}(S),S]$ has the same sign as (18). This condition will be met as long as there exist feasible combinations $[\tilde{S}(S,\tilde{G}), \tilde{G}]$ which are not too far from $[S,\hat{G}(S)]$.

If $G'(S) \leq \hat{G}(S)$, then the equilibrium with passing is equivalent to the equilibrium without passing. For this case, the analysis in Section IV established that $FMU_G + MU_S$ is greater than zero at the equilibrium, and that people drive too slowly. If, instead, $G'(S) > \hat{G}(S)$ then, at $\hat{G}(S)$, MU_G is strictly positive. Since $U(\tilde{G},\tilde{S}(\tilde{G})) - U(\hat{G}(S),S)$ is nonpositive at an equilibrium, (18) implies that MU_S evaluated at $(\hat{G}(S),S)$ is positive.

Since being passed involves no gain in utility in spite of increasing the gap which is desirable, it must be the case that the reduction in speed is welfare decreasing. Therefore:

$$FMU_G + MU_S > 0$$

and a simulataneous increase in the gap and the speed along the FF curve increases welfare.

VII. Conclusions

The results of this paper have a variety of policy implications. First and foremost, they point to the folly of speed limits. A speed limit that

reduces the speed from its equilibrium value decreases the gap if it is to be consistent with the exogenously given flow. Therefore, a speed limit reduces welfare since individuals opt, at an equilibrium, not to decrease their speeds even when they think this leads to an increase in their gap. Furthermore, from a theoretical viewpoint, speed limits are not necessarily effective at reducing accidents since they lead to lower gaps.

Instead, this paper indicates the advantages of minimum speed laws since it proves that under reasonable conditions people drive too slowly. For enforcement purposes, it would probably be necessary to impose minimum gaps as well. This is so because it would be difficult to blame an individual for going too slowly when traffic is very dense. These minimum speeds and gaps should depend on the flow of traffic on the highway and should therefore vary over time. In particular, when cars are sufficiently dispersed and traveling at the speed they would use in the absence of any other cars, government intervention is unnecessary.

When the flow is higher, cars become more bunched together and the equilibrium speed falls. Congestion creates an externality; the presence of one more car on the road affects the tradeoff between accidents and length of the trip faced by all other drivers. This paper emphasizes that there is a further externality which results from the actual choice of gap the typical driver makes. The larger the gap, the faster a given flow must move. When traffic is congested, it is possible to take advantage of this second externality by forcing the traffic to move faster and mitigate the negative effect on welfare of congestion. Note that when there is a congestion externality, or, in other words, when the addition of one agent to an environment affects the other agents, there often is a further externality: the behavior

of any agent in the congested state affects the welfare of others. In a congested museum, the proximity of any person to a work of art reduces the consumption opportunities of the other people. Therefore, Pareto optimality can typically not be achieved by taxing only congestion per se. Instead, the behavior of people in the congested state must be optimally regulated. It is not enough to tax vehicles which enter New York City one must also levy fines on those which cause gridlock. In the context of this paper, Pareto optimality can be achieved as follows: for each flow of traffic, minimum speeds and gaps must be imposed so that traffic patterns correspond to those derived in Section V. Then, a change in the flow still affects the welfare of drivers. Therefore, congestion tolls of the form of those proposed by Oron, Pines and Sheshinski [1973] must be levied so that each individual pays the social costs of his commute.

Highways are often designed to ensure a certain level of "capacity". In the engineering literature (Institute of Transportation Engineers [1976]), capacity is defined as the maximum flow that a highway can accommodate. This actually refers to the maximum flow consistent with equilibrium on a given highway.[8/] Methods of regulating the number of cars that enter a highway so as to keep the flow near the capacity have been proposed by Wattleworth (cited in Drew [1968], p. 429, and in Gazis [1974], p. 231). The analysis of this paper casts some doubts on the usefulness of the capacity measures. First, when the traffic flow is equal to the capacity, the typical individual may be worse off than if he had to wait in line to enter a highway which allowed him to drive at the speed corresponding to free flow. Second, in the presence of minimum speed limits which are effective, there are many flows which are higher than the "capacity" that are feasible. Furthermore, these higher flows may

well be the most effective way to transport people to and from work.

The results of this paper seem to depend heavily on the assumption that individuals must pick a gap at the beginning of their trip and maintain it forever. This assumption was made because the equilibrium in which people change their gap over time, according to the analysis of Section II, would be described by a complicated differential equation. However, I think that the conclusion that people drive too slowly would still emerge from a model in which people changed their gaps optimally. In such a model, people would still never drive at speeds such that lower speeds, accompanied with the same gap, were utility increasing since safer alternatives would be available to the individual driver. Therefore, small increases in the speed could never reduce welfare.

Footnotes

1/   The monetary costs of this trip are assumed to be independent of his
      actions during the trip.

2/   It must be noted that putting (T-t) in the utility function is equivalent
      to putting T in since there is nothing the individual can do to affect the
      time that has already elapsed. This utility function is linear in time.
      In this context, it is not clear whether it should be concave to reflect
      a declining marginal utility of leisure, or convex since, in the presence
      of discounting, arriving earlier makes leisure more valuable.

3/   The utility function in (1) is very similar to the utility function con-
      sidered in Peltzman [1973].

4/   For this condition to reflect a maximum of utility, it must be true that
      $-A_{22}$ is negative. This holds by assumption.

5/   Drew [1978] simply assumes that individuals adjust their gap towards a
      "safe headway" which is postulated in such a way as to ensure this result.

6/   If, instead, when $MU_G < 0$ the auctioneer lowers the gap and the speed, then
      these equilibria are unstable. However, it seems reasonable to think that
      when the gap is "too small", the cars accelerate and the speed increases.

7/   Honda commercials imply that this is always true.

8/   This flow is given by FF_{MAX} in Figure 2.

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