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This paper is part of a larger study attempting to reconcile disparate results in the KLEM literature, headed by David O. Wood at the Energy Laboratory at MIT. I thank Professors Robert Pindyck, Tom Stoker, Ernie Berndt, and David Wood for comments on an earlier draft, and also Professor Joanne Yates for editorial assistance.

Abstract

This paper presents a model of aggregate production in the conjugate dual, cost function framework. The model provides a technique to infer full equilibrium properties of a technology from the observed partial equilibrium data. In particular, we use a three-factor (KLE) model, assuming capital to be quasi-fixed in the short run, and therefore in disequilibrium, while assuming that labor and energy adjust rapidly to equilibrium values.

The partial equilibrium model is estimated using two sets of data: time series data on U.S. manufacturing (Berndt-Wood) and pooled data on OECD countries (Pindyck). The results are used to infer full equilibrium properties. These equilibrium levels of capital are compared with the observed disequilibrium data. Results indicate divergences between actual and full equilibrium levels of capital. This casts serious doubt on the validity of the equilibrium specification used by previous authors. Furthermore, substitution elasticities between the factors were computed for the partial-equilibrium (short-run) and full-equilibrium (long-run) situations. These values are then compared and contrasted with the original work of Pindyck, Berndt-Wood, and others which assumes all factors to be in equilibrium. We conclude that under an equilibrium framework, time series data give elasticity estimates that are close to short-run values obtained via the disequilibrium model, while pooled data give values close to long-run values.

This analysis does not resolve the E-K substitutability/complementarity controversy. However, we are able to eliminate the short-run versus long-run nature of the results as a cause of the controversy. An extension of this analysis, where the data construction and specification differences are taken into account, is suggested as a method to resolve this issue.

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1. Introduction

In recent years economists have shown a renewed interest in the structure of industrial production, and in particular the substitution possibilities between input factor prices. The sudden price increases of input factors, specifically those of energy, were largely instrumental in motivating these studies. But the major analytical tool that made these studies possible was the introduction of the flexible functional forms by Diewert (1971) and Christenson, Jorgenson, and Lau (1971). The ability of these functional forms to specify and estimate the technology without placing *a priori* restrictions on substitution possibilities made them attractive for impirical work. The numerous empirical studies that followed, while providing valuable information regarding the relationship among input factors, have also created conflicting evidence. From a policy point of view the most disturbing difference relates to the degree of substitutability between capital and energy.[1]

A reconciliation of these estimates requires a careful look at the procedures and the data used by the different studies. Most of these estimates were based on the assumption that observed factor demands were in equilibrium. In this study we use a partial equilibrium factor demand model of the technology and thereby test the validity of the equilibrium assumption. The results are also used to reconcile the question of whether earlier results pertain to the short run or the long run.

A full equilibrium static model assumes that firms adjust *all* inputs to their long-run cost-minimizing value within one time period. However, in the real world there is little reason to believe that factors adjust that rapidly in response to exogenous changes in price or output. In fact, for sticky and irreversible factors such as capital, the observed

utilization level will most certainly depart from the equilibrium levels, especially during periods of rapid factor price changes. One way to remedy this is to use dynamic models where the factor adjustment processes are made explicit. These models, however, suffer from the ad hoc nature of the hypothesized adjustment process. For example, Berndt, Fuss and Waverman (1977, 1979) model the intertemporal adjustment so as to minimize the cost of adjustment. In their framework the only cause of disequilibrium is assumed to be the costs of adjustment. They neglect many other sources that could create stickiness in factor adjustments, and therefore such models could lead to incorrect results. The model used in this paper does not explicitly specify the adjustment process. Instead, we arrive at the familiar dichotomy between the short run (where some of the factors are in disequilibrium) and the long run (where all factors are in full equilibrium). A partial equilibrium model shall therefore provide an accurate and convenient way to model the production process.

We use this model to perform empirical tests on two different data sets: time series data on U.S. manufacturing from 1947 to 1971 and a set of pooled time series-cross section data for OECD countries. The technology is modeled via a three-factor--capital (K), labor (L), and energy (E)--translog cost function where capital is assumed to be quasi-fixed. The results indicate very small departures between observed and equilibrium levels of capital for the time series data, but larger differences in the pooled data sample. The results also provide some support of the conventional claim that time series data reflect short-run behavior while cross-section data reflect long-run behavior.[2]

The paper proceeds as follows. In Section 2, a verbal description of the model and its uses is presented as motivation for the more formal development that follows later on in the section. Section 3 presents an outline of a framework for using the partial equilibrium analysis towards the reconciliation of disparate results in the KLEM literature. In Section 4 we present and interpret results from applications to Berndt-Wood time series data and the Pindyck pooled data. Finally, in Section 5, we make some concluding remarks.

2. The Model and Its Uses

Our study differs from previous industrial factor demand studies in its partial equilibrium specification of the technology via a restricted cost function. In Section 2.1, we describe the assumptions underlying the model and its mechanics to prepare the reader for the more formal development of the model in Section 2.2. Finally, in Section 2.3, we consider a special case, a KLE translog technology where capital is assumed to be quasi-fixed, which will later be used in empirical work.

2.1 Motivation and Description of the Model

Traditionally, economists have modeled the production technology of firms in a static full-equilibrium framework. In the event of changes in factor prices or the level of output, these models assume instantaneous adjustment of all input factors such that total costs are minimized. However, in the real world where factor adjustment is slow and costly, it is very unlikely that such a model would be an adequate description of technology.

To remedy this, more recently, some researchers have used dynamic models, where the factor adjustment processes are explicitly specified. The exact nature of the intertemporal adjustment mechanism, of course, depends on the condition that is assumed to cause disequilibrium. Two such models can be found in Berndt, Fuss and Waverman (BFW) (1977, 1979). The first method assumes lags in adjustment to be the cause of disequilibrium and therefore the process is modeled by a Koyck lag distribution. In the more sophisticated second method, they postulate adjustment costs as the cause of slow adjustment. Then the adjustment takes place along a path that minimizes the costs of adjustment. Although conceptually appealing, both versions suffer from the ad hoc choices of the lag structure and adjustment costs. Even more importantly, other considerations stemming from government regulation, the differential treatment of investment and costing decisions within organizations, and the physical inability to change inputs (due to irreversibility or lags) are neglected in their analysis.[3] Therefore such dynamic models by not providing a complete treatment of the causes of disequilibrium can lead to misleading results.

An alternative approach is to consider a dichotomous situation between the firm's technology in the short run (the immediate time period) and the long run (some unspecified time after the immediate time period). This partial equilibrium approach is more general than the extremely restrictive full equilibrium approach, but does not provide a detailed description of the intertemporal behavior as given by the dynamic models. However, unlike the dynamic models described earlier, the adjustment process in partial equilibrium models can accommodate any or all of the causes generating the short-run disequilibrium.[4]

Therefore, we feel such a model would provide a simple but realistic framework for aggregate production analysis.

The theory used in these partial equilibrium models is well developed in the traditional microeconomic literature where costs are broken down into fixed and variable parts. In a recent study Brown and Christenson (1980) use such a model to study the substitution possibilities among factor inputs in U.S. agriculture. Their model of the farm sector assumes self-employed labor and farm land to be quasi-fixed. The results indicate that the observed levels of these quasi-fixed factors differ from the full equilibrium values predicted by the model. They also obtain short-run elasticities that are significantly different from the long-run values, thus indicating slow adjustment of these factors. These results, by exhibiting the power of the partial equilibrium technique, provided the initial motivation for this paper.

In this paper, we will be more precise about the mechanics bringing about the partial equilibrium model. The firm's input choice decision consists of two stages. In the first stage, firms adjust the variable factors to minimize variable costs. This minimization takes place within the time period in which changes in factor prices or output levels occur. The other factors need not be rigidly fixed but they do not adjust to the long-run cost-minimizing equilibrium levels. Hence we call them quasi-fixed factors. The resulting technology is in a temporary (short-run) equilibrium, which is described by the restricted (short-run) cost function.

In the second stage, firms adjust quasi-fixed factors until the <u>total</u> cost is minimized. This adjustment takes place after the end of the initial time period. The resulting cost function characterizes the

long-run (full-equilibrium) technology. Hence, we arrive at the dichotomy between the short run and the long run.

This two-stage optimization model leads to several interesting features. If the observed levels of the quasi-fixed factors are equal to the constructed full equilibrium levels, then a full equilibrium model would adequately describe the technology. However, in all other situations, the full equilibrium specification will result in an inaccurate description. Hence, by using a partial equilibrium specification, we will do at least as well as the full equilibrium specification.

Even though we can differentiate between short-run and long-run properties, this model is essentially in a static framework. The second-stage optimization does not feed into later results. The long-run cost function is only a theoretical construct used to describe the situation where all factors are utilized in the most efficient manner.

2.2 A Formal Development of the Model

The theoretical basis of restricted cost- and profit functions were developed by McFadden (1978) and Lau (1976) in their seminal work on the uses of duality theory in production economics. Here I only summarize the relevant results that are to be used in later applications.

Consider a n+m factor production technology where in the short run only n factors are variable. The other m factors are quasi-fixed in the short run and these will be called the quasi-fixed inputs. Then the observed technology will be in partial equilibrium. This short-run technology can be characterized by solving the following cost minimization problem: $CR = \text{Min. } P \cdot X \text{ s.t. } Y = F(X, Z)$ where X and P

are vectors of the levels and prices of the variable factors, Z is a m vector of quasi-fixed factor levels, Y is the level of output, and F(.) is the production function. When minimized over X, the restricted cost function can be written as: $CR = CR(P, Z, Y)$.

However, in the long run, the firms can adjust the quasi-fixed factors and thereby reach the full equilibrium which is characterized by the following total cost minimization problem; $C = \text{Min } CT(P, Q, Z, Y) = \text{Min } CT(P, Z, Y) + Q \cdot Z$, where Q is the vector of the quasi-fixed factor prices.

The first-order condition of this optimization problem is

$$(1) \qquad \frac{\partial CR}{\partial Z} + Q = 0$$

Eq. (4) can be solved to obtain the full equilibrium values of the quasi-fixed factors, Z*. Then the long-run cost function can be written in terms of the restricted cost function and the cost of the quasi-fixed factors as:

$$(2) \qquad C = CR(P, Z^*, Y) + Q \cdot Z^*$$

where Z* will be a function of P, Q, and Y.

We now have complete characterizations of the technology in both the short run and the long run. These cost functions can be used to evaluate expressions for the substitution elasticities between the input factors. The short-run Allen elasticity of substitution between factors i and j is given by:

$$(3) \qquad \sigma^s_{ij} = \frac{CR\ CR_{ij}}{CR_i\ CR_j}$$

where the subscripts of the cost function denote its partial derivatives with respect to the factor prices. Similarly, the long-run Allen

elasticity of substitution between factors i and j can be written as:

$$(4) \qquad \sigma^L_{ij} = \frac{C\ C_{ij}}{C_i\ C_j}$$

The most interesting feature of this model is its ability to represent long-run properties via the short-run (restricted) cost function which can be estimated using observable data. In other words, C, C_i and C_{ij} can be written in terms of CR, its derivatives, and Z*. These results are given in Brown and Christenson (1980) and we summarize them below.

$$(5) \qquad C_i = \frac{\partial C}{\partial P_i} = \frac{\partial CR}{\partial P_i} + \left\{ \sum_k \frac{\partial CR}{\partial Z_k} \cdot \frac{\partial Z_k}{\partial P_i} + \sum_k P_k \frac{\partial Z_k}{\partial P_i} \right\} + b_i Z_i^*$$

where $b_i = 0$ if $i \in V$

$= 1$ if $i \in F$

and we define the set F such that $i \in F$ if X_i is a fixed factor and the set V such that $i \in V$ if X_i is a variable factor.

$$(6) \qquad C_{ij} = \frac{\partial C}{\partial P_i \partial P_j} = \frac{\partial^2 CR}{\partial P_i \partial P_j} + \sum_k \frac{\partial Z_k^*}{\partial P_i \partial Z^*_k} + \sum_k \frac{\partial Z_k^*}{\partial P} \left(\frac{\partial^2 CR}{\partial Z^* \partial P} + \frac{\partial^2 CR}{\partial Z_k{}^* \partial Z_l^*} \frac{\partial Z^*}{\partial P_j} \right)$$

$$+ b_i \frac{\partial Z_i^*}{\partial P_j} + b_j \frac{\partial Z_j^*}{\partial P_i} + \left\{ \sum_k \frac{\partial CR}{\partial Z_k^*} \frac{\partial^2 Z_k^*}{\partial P_i \partial P_j} + \sum_k P_k \frac{\partial^2 Z_k^*}{\partial P_i \partial P_j} \right\}$$

At the full equilibrium the terms in the curly brackets in equation (6) will go to zero since $\frac{\partial CR}{\partial Z^*} + P_k = 0$ (from the first-order condition in equation (1)) for all $k \in F$.

To complete the computation of σ^L_{ij} we need to evaluate $\frac{\partial Z^*}{\partial P_j}$, A general method of evaluating these values is given in Brown and Christenson, but for the special case considered in this study we were able to derive a simpler method. The rest of this paper will focus on this special form of the problem which still captures the gist of the disequilibrium

concepts and also provides a set of interesting results when applied to particular data sets.

2.3 A Special Case

To proceed any further with this analysis it is necessary to characterize the restricted cost function by a particular functional form. In this study we deal with a three-factor technology where labor (L) and energy (E) and short-run variables whicle capital is assumed to be quasi-fixed. By modeling the industrial production technology by only these three factors we are making an implicit assumption about the separability of K, L, and E from non-energy materials and other inputs. As a suitable functional form for the restricted cost function, we choose the translog form (see equation (7)) due to its ability to make a good second-order approximation while maintaining flexibility of substitutability between factors.

$$(7) \qquad \ln CR = \alpha_0 + \alpha_Y \ln Y + \sum_{i=L,E} \alpha_i \ln P_i + \beta_K \ln K + \frac{1}{2}\gamma_{YY}(\ln Y)^2$$
$$+ \frac{1}{2}\sum_{i=L,E}\sum_{j=L,E} \gamma_{ij} \ln P_i \ln P_j + \frac{1}{2}\delta_{KK}(\ln K)^2$$
$$+ \sum_{i=L,E} \rho_{Yi} \ln Y \ln P_i + \sum_{i=L,E} \rho_{iK} \ln P_i \ln K + \pi \ln Y \ln K$$

For purposes of this study, we introduce a number of constraints on the cost function. In addition to the usual neoclassical properties of homogeneity of degree one in factor prices and output, symmetry of the second order cross terms, we also impose the further simplifying assumption of long-run constant returns to scale. Under these assumptions the cost function can be written as (see Appendix 1 for derivation):

$$(8) \qquad \ln CR = \alpha_0 + \alpha_Y \ln(Y/K) + \alpha_L \ln(P_L/P_E) - \frac{1}{2}\pi(\ln(Y/K))^2 + \frac{1}{2}\gamma_{LL}(\ln(P_L/P_E))^2 + \rho_{YL} \ln(P_L/P_E) \ln(Y/K) + \ln P_E + \ln K$$

An application of the Shephard's Lemma yields the variable factor share equations:

$$(9) \qquad S_L = \alpha_L - \gamma_{LL} \ln(P_L/P_E) + \rho_{YL} \ln(Y/K)$$

$$(10) \qquad S_E = 1 - \alpha_L - \gamma_{LL} \ln(P_L/P_E) - \rho_{YL} \ln(Y/K)$$

where $S_i = \dfrac{P_i X_i}{\sum_{i \varepsilon V} P_i X_i} = \dfrac{P_i X_i}{CR}$

The full equilibrium cost function for this special case simplifies to:

$$(11) \qquad C = CR(P_L, P_E, Y, K^*) + P_K K^*$$

and the first order condition of cost minimizing becomes

$$(12) \qquad \frac{\partial CR}{\partial K^*} + P_k = 0$$

where K* is the equilibrium level of the capital input.

Evaluating the partial derivative, $\frac{\partial CR}{\partial K^*}$ for the translog function in equation (7) and substitution in equation (12) gives:

$$(13) \qquad CR(P, K^*, Y). \ (A_1 - \pi \ln K^*) + P_K K^* = 0$$

where $A_1 = 1 - \alpha_Y + \pi \ln Y - P_{YL} \ln(P_L/P_E)$.

Equation (13), a non-linear equation in K* can be solved to obtain the equilibrium values K*. For this particular functional form (or for many other commonly used cost functions) it is not possible to obtain a closed form solution to (13). Therefore, numerical simulation techniques are used in solving for K*.

As seen in Section 2.3, the next step in computing σ^L_{ij} is to obtain the partial derivatives of $\frac{\partial K^*}{\partial P_i}$. For the single fixed factor case the problem is greatly simplified from the general case mentioned earlier. By implicitly partial differentiating equation (13) with respect to the factor prices and solving for $\frac{\partial K^*}{\partial P_i}$ we get:

$$(14) \qquad \frac{\partial K^*}{\partial P_i} = \frac{-K^*}{P_i} \; \frac{(S_i \; N + V_i P_{YL})}{N^2 - \pi - N}, \qquad \text{for } i = L, E$$

where $N = A_1 - \pi \ln K^*$

and $V_i = \begin{cases} -1 & \text{if } i = L \\ +1 & \text{if } i = E \end{cases}$

and[5]

$$(15) \qquad \frac{\partial K^*}{\partial P_K} = \frac{K^*}{P_K} \; \frac{N}{(N^2 - \pi - N)}$$

Once the equilibrium capital level is known these values can be evaluated.

Remaining terms in the expression of σ^L_{ij}, are summarized below:[6]

$$(16) \qquad \left.\frac{\partial CR}{\partial P_i}\right|_{K=K^*} = \frac{CR}{P_i} S_i \qquad \text{if } i \epsilon V$$

$$= 0 \qquad \text{if } i \epsilon F$$

$$(17) \qquad \frac{\partial CR}{\partial K^*} = \frac{CR}{K^*} \; N$$

$$(18) \qquad \frac{\partial^2 CR}{\partial P_i \partial P_j} = \frac{CR}{P_i P_j} (S_i S_j + u_{ij} \gamma_{LL} - w_{ij} S_i) \qquad \text{if } i, j \; \epsilon \; V$$

$$= 0 \qquad \text{if } i \text{ or } j \; \epsilon \; F$$

where $w_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$

and $u_{ij} = \begin{cases} 1 & \text{if } i = j \\ -1 & \text{if } i \neq j \end{cases}$

$$(19) \quad \frac{\partial^2 CR}{\partial K^* \partial P_i} = \frac{CR}{P_i K^*} (S_i N + V_i P_{YL}) \qquad \text{if } i \in V$$

$$= 0 \qquad \text{if } i \in$$

Observe now the following two properties (partially differentiating (14) with respect to K^* and P_i):

$$(20) \quad \frac{\partial^2 CR}{\partial K^{*2}} \frac{\partial K^*}{\partial P_K} + 1 = 0$$

$$(21) \quad \frac{\partial^2 CR}{\partial K \, \partial P_i} + \frac{\partial^2 CR}{\partial K^{*2}} \frac{\partial K}{\partial P_i} = 0$$

Substituting these in equation (6) yields:

$$(22) \quad C_{ij} = \frac{\partial^2 CR}{\partial P_i \partial P_j} + \frac{\partial^2 CR}{\partial P_i \partial K^*} \frac{\partial K^*}{\partial P_j} \qquad \text{if } i, j \in V$$

$$= \frac{\partial^2 CR}{\partial P_i \partial K^*} \frac{\partial K^*}{\partial P_K} \qquad \text{if } i \in V \quad j \in F$$

$$= K^* \qquad \text{if } i, j \in F$$

Similarly from (5)

$$(23) \quad C_i = \frac{CR}{P_i} S_i \qquad \text{if } i \in V$$

$$= K^* \qquad \text{if } i \in F$$

We can summarize the computations as follows: First, solve (13) to obtain K^*. Then compute $\frac{\partial P^*}{\partial P_i}$ and the partial derivatives of CR from equations (14) - (19). Compute C_i, C_{ij} using equations (22) and (23). Finally, substitute these in (3) and [7) to get the elaticities.

3. Framework to Reconcile Disparate KLE Results

One of the most interesting uses of this partial equilibrium approach stems from its ability to distinguish between the short-run and the long-run characteristics of this technology. This provides us with a

framework to address some of the issues which are thought to generate disparities in aggregate factor demand studies. Here, we do not attempt to review the literature on industrial factor demand studies.[7] Instead in this section we develop a framework by focusing on two aspects which are likely to contribute towards the disparities. The first concerns specification of an equilibrium cost function, common to all studies generating the initial E-K substititability/complementarity debate. The second focuses on the short-run versus long-run nature of the results obtained by previous researchers.

3.1.A Specification Test

The validity of the equilibrium specification can be tested by comparing the actual level of capital, K, with the full equilibrium level, K*, obtained by solving eq. 13. If departures between the two paths occur, then that would invalidate an equilibrium specification, thus casting doubt on the results stemming from such a model. A formal test of the hypothesis K = K* requires information on the distribution of K*. But since eq. 13 does not have a closed-form solution for K*, and can only be solved via numerical methods, we are unable to compute standard errors of the estimates for K*.[8]

The results of thi specification test are, however, still conditional on the assumption that some of the factors reach long-run equilibrium instantaneously. For example, in the KLE framework, if we allow only capital to be quasi-fixed while labor and energy are at their full equilibrium levels, then the results are conditional on this assumption. Therefore, finding that the full-equilibrium level of capital is not significantly different from the actual observed level may

not be sufficient to conclude on the validity of the full-equilibrium specification. However, if we do find significant disparities between the actual and equilibrium values under this model, conditional on the assumption of observed equilibrium labor and energy levels, violation of this assumption will only enhance the conclusion. In other words, we can use this test to reject the hypothesis but not to accept it.[9]

3.2 Short-Run/Long-Run Controversy

The conventional wisdom among applied econometricians has been to interpret elasticity estimates based on cross-section data as capturing long-run effects, while those estimates based on time series data as capturing short-run effects. Pindyck (1977) and Griffin and Gregory (1976) trace the origins of this claim to a paper by Houthakker (1965). Stapleton (1980) makes references to a paper by Kuh (1959) where he points out that, if adjustment to long-run equilibrium is slow, estimates based on cross-section demand-function estimates are likely to reflect long-run elasticities while time series estimates reflect only partial adjustments. Evidence from pooled cross section-time series data is used by both Kuh and Houthakker in showing the considerable differences in coefficient values obtained from a single cross section as compared with those based on time averages. Pindyck (1977), using pooled cross section-time series data from ten OECD countries, tests this hypothesis informally, and finds the results to be inconclusive.

The adjustment cost literature mentioned earlier addresses this issue by explicitly modeling the adjustment path. But, as pointed out earlier, these studies assume rather restrictive conditions about the causes and nature of the disequilibrium.

In this paper, we only consider the polar cases: a short run which is within one time interval, and a long run which is at some undefined time after the end of the period. To address the reconciliation issue via these estimates requires careful consideration of the data and the validity of the original estimates which assumed all factors to be in equilibrium. If the previous analysis does not indicate any significant differences between the actual and equilibrium levels of the short-run fixed factors, thereby justifying the specifications used by previous studies, then we can make direct comparison between the corresponding elasticity estimates obtained via these two specifications. For example, we should check the following relationships:

$$\sigma_{ij}^{TS} \simeq \sigma_{ij}^{S} < \sigma_{ij}^{L}$$

$$\sigma_{ij}^{XS} \simeq \sigma_{ij} > \sigma_{ij}^{S}$$

where σ_{ij}'s are the elasticities of substitution between factors i and j. The superscripts TS, XS, S, and L denote times series, cross-sectional, short-run (restricted), and long-run (full-equilibrium) cases, respectively. Since these estimates are random variables, the above relationships should be checked via significance tests. However, as mentioned earlier, the procedure used in this paper does not provide a facility to readily compute standard errors of the elasticities.

However, if the discrepancies from the equilibrium are very large and significant, then the elasticity estimates of previous models (hereafter I will refer to them as "equilibrium" models or "original" models) difficult to interpret. In such a case, it is necessary to evaluate the elasticities on different data sets (time series and pooled data) on the

same or at least similar technologies, if one is to to resolve the long-run versus short-run debate.

4. Empirical Application

The methodology developed thus far conveniently lends itself to empirical applications using previously established data on aggregate industrial production. In this section we use the restricted cost function to estimate the short-run technology for two data sets, one time series and the other pooled cross section-time series, to estimate. The procedure described in Section 2.3 is then used to compute the full equilibrium levels of the quasi-fixed factors and elasticities of substitution in both the short run and the long run.

4.1 Data

Because this paper is part of a larger reconciliation study, the most logical data to be used here are those that created the initial controversy. Convenience as well as some characterizing features (explained later) led to the choice of Bernt-Wood (B-W) (1975)[10] time series data on U.S. industries for 1947 to 1971 and Pindyck (1977) pooled cross section-time series data on 10 OECD countries for 1963-1973. Both sets of data are explained in considerable detail in their respective original papers. Further details on the Pindyck data can be found in Carson (1978). A critical analysis of these two and several other data sets, paying much needed attention to the data construction methods, can be found in an unpublished memo by Hirsch (1980).

Several caveats with special relevance to this particular technique need mentioning. The first concerns the data on level of output. In the orginal B-W study the cost function specification did not include cross terms in factor prices and output. Therefore, the output term did not enter into the estimation of the factor share equations. Pindyck incuded such cross terms in his cost function and used the "net value of output" for Y in the share equations. This amounts to using real (deflated) values as instrumental variables for the level of output. In this study we estimate the entire short-run cost function and thus need a measure of the level of output. We obtain this by constructing output price as a divisia index using arithmetic value weights and price of the input factors. Diewert (1977) has shown that the divisia index forms and exact index in the case of a translog technology. Therefore we have an exact index of the 'output' of the KLE input. The input prices and the divisia index for B-W data are plotted in Figures 1 and 2 respectively. The divisia indexes for each of the 10 countries in the Pindyck data are reported in Table 1. The level of output is obtained by dividing the value of output by this divisia index.

Second, it should be kept in mind that the data is at a high level of aggregation. The conditions for the existence of aggregate production functions are extremely stringent and it is very unlikely that these conditions would be met by the data used here.[11] Although most studies have this problem, researchers find that the 'reasonable' empirical results obtained by such aggregate analysis provide some justification for using such data.

Thirdly, the price of capital P_k is assumed to be the same in the short run and in the long run. In this analysis it is crucial to note that this price is a rental price of capital and not an asset price.

Figure 1: B-W Prices for K, L and E.

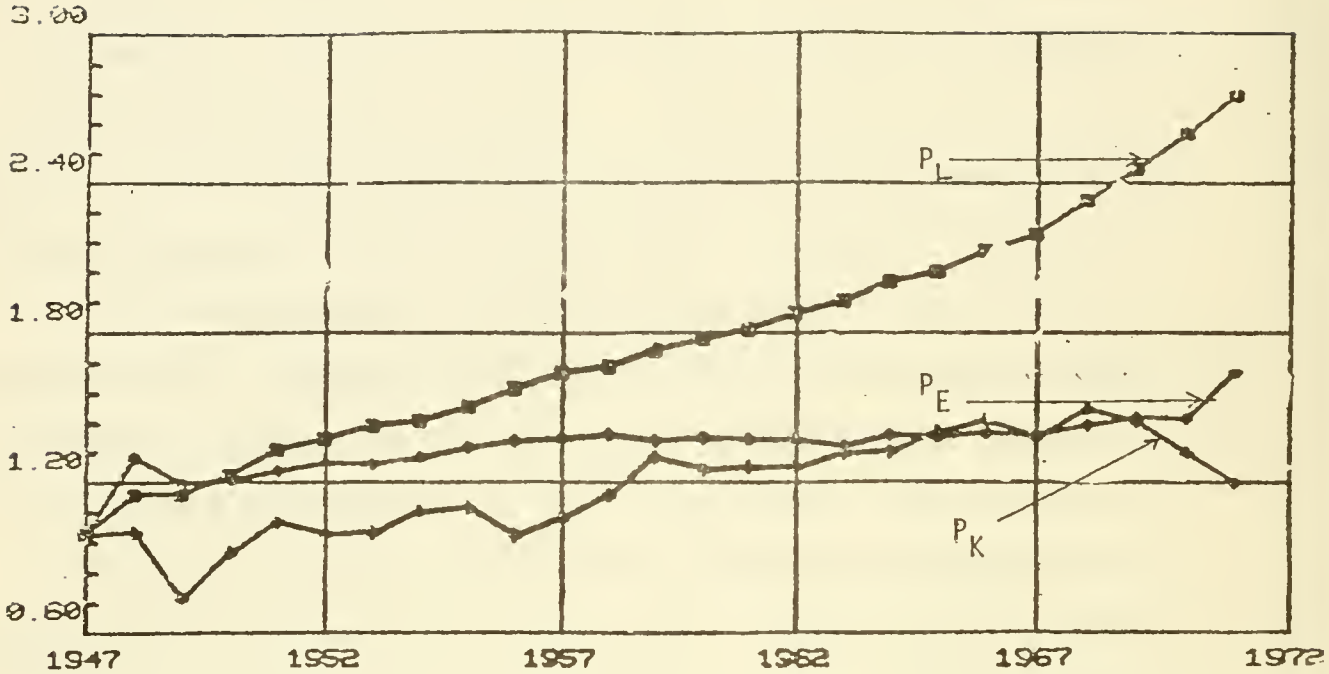


Figure 2: Divisia index of KLE prices: (B-W data)

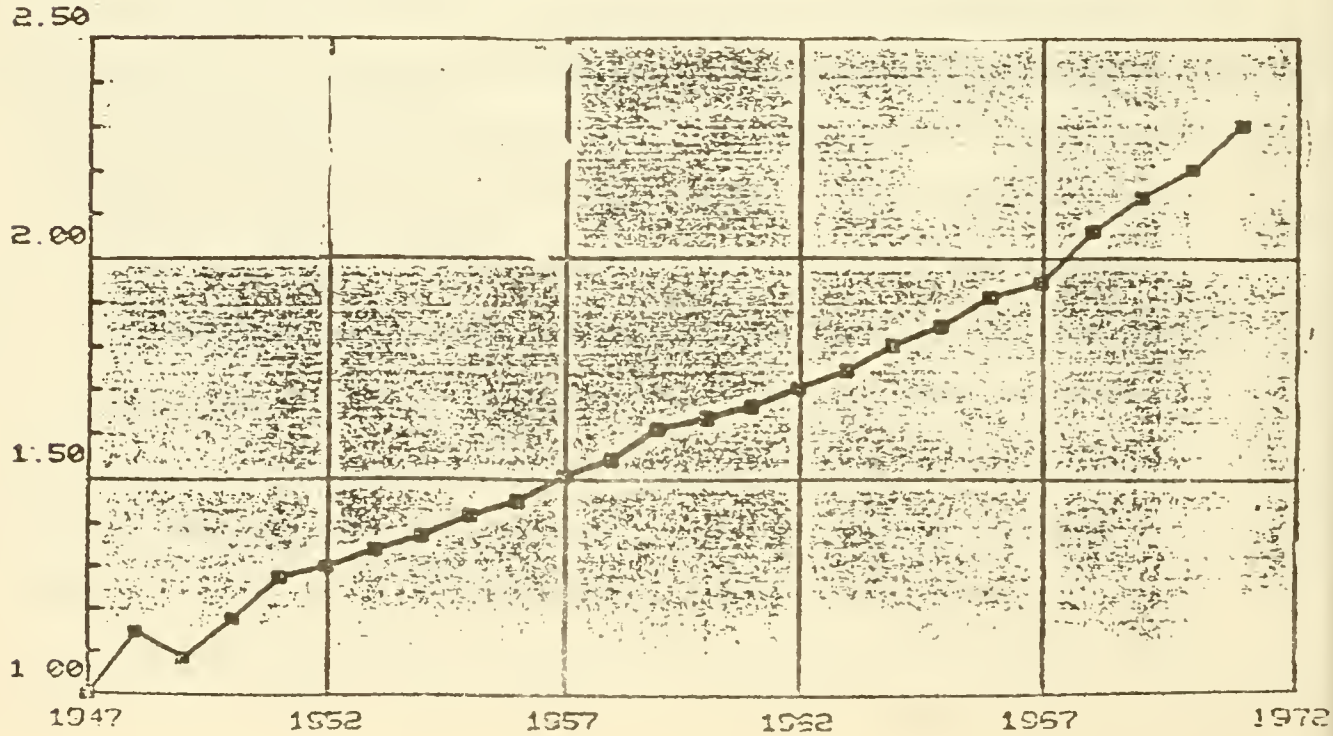


Table 1. Divisia index of input factor prices: Pindyck data.

	BELGIUM	FRANCE	ITALY	JAPAN	W.GERMANY
1963	1.	1.	1.	1.	1.
1964	0.988821	0.977626	0.980355	1.07118	1.06326
1965	0.938036	1.04301	0.970367	0.933087	1.09161
1966	1.03808	1.05248	1.04652	1.02745	1.3726
1967	1.1031	1.08051	1.06728	1.06525	0.983316
1968	1.21395	1.19249	1.11868	1.07704	1.08385
1969	1.24517	1.10082	1.12806	1.04766	1.17356
1970	1.16052	1.17267	1.17092	1.15749	0.993156
1971	1.24037	1.23282	1.33686	1.20515	1.19[illegible]
1972	1.26217	1.33613	1.38332	1.09412	1.40893
1973	1.31017	1.33933	1.34344	1.04232	1.46[illegible]

	NETHERLAND	NORWAY	SWEDEN	U.K.	U.S.A.
1963	1.	1.	1.	1.	1.
1964	1.04257	1.03051	1.23281	0.969299	1.00182
1965	1.04923	0.94409	1.21693	0.994501	0.994622
1966	1.13424	0.948092	1.28507	1.06367	1.00469
1967	1.12964	1.1139	1.32992	1.01067	0.989237
1968	1.25354	1.09363	1.29001	1.031	1.0223
1969	1.25202	1.01301	1.47451	1.08494	1.07986
1970	1.27112	1.18307	1.42597	1.12467	1.02436
1971	1.19798	1.23157	1.47522	1.17454	1.0992
1972	1.33677	1.26611	1.53131	1.20155	1.15277
1973	1.51426	1.32215	1.5514	1.22278	1.18676

4.2 Estimation of the restricted cost function

Due to the full equiibrium nature of the specification used in the original papers by B-W and Pindyck, it was sufficient to estimate the factor share equations to obtain the relevant substituion properties. However, as seen in Section 2.3, in this partial equilibrium set-up we need to estimate the entire restricted cost function to obtain the long-run values of the quasi fixed factors. Although there does not exist simulataneity between the cost function and the share equations, the error terms of these equations are correlated. To account for the contemporaneous correlation of the errors, we use Zellner's seemingly unrelated regression (SURR) to obtain estimates of the short-run parameters. Details of the error structure and the estimation technique are given in Appendix 2. The estimated coefficients for both data sets are reported in Table 2.

In the case of B-W data, the translog form described in eqs. 15 and 16 provide a good fit of the short-run technology. However, for the Pindyck data, the fit was not that good (see column 2, Table 2) This coud be attributed to a number of reasons. First, as Pindyck notes in his paper, the assumption of a homogenous production function across the 10 countries in the sample, is not justified. To remedy this, he allows the intercept terms in the share equations to vary across countries. Fuerthermore, he finds that a better specification is obtained if U.S. and Canada are pooled separately from Japan and Europe. On this study allowing such a specification would create significant computational difficulties. A more sohisticated computational procedure that is currently being developed, would enable us to carry out more realistic specifications in the next stage of the study. (I hope to have these results very soon.)

Table 2

SURR Estimation Results:

(1) Restricted cost function: eq. (9)

(2) Labor share equation: eq. (10)

		B-W data 1947-1971	Pindyck: Ten countries pooled 1963-1973 unconstrained	constraining $\pi=0$
α_0		-0.2333 (-8.3432)	-0.5466 (-2.3233)	-0.6064 (-6.7579)
α_Y		0.8815 (27.7364)	1.6406 (3.6660)	1.76048 (18.132)
α_L		0.76822 (52.333)	0.9124 (124.365)	0.9126 (124.422)
π		-0.1642 (-9.0981)	-0.1104 (-0.2736)	0 -
γ_{LL}		0.0729 (17.4902)	0.0207 (7.7562)	0.0209 (7.8275)
ρ_{YL}		0.0458 (5.4567)	0.0105 (1.3299)	0.0106 (1.3416)
$\bar{R}^2$	(1)	0.99	0.36	0.36
	(2)	0.88	0.81	0.81

*t-statistics are shown in parenthesis.

Second, even within the specification used here, we notice that the coefficient π, has a low t-statistic, indicating its insignificant contribution towards explaining the dependent variable (see column 2, Table 2). An alternative specification, where the cross terms between K and Y are omitted (i.e. with $\pi = 0$), was estimated. These results are in column 3, Table 2. To test the null hypothesis of $\pi = 0$, we construct a likelihood ratio test as follows; The likelihood ratio test statistic, can be written as:

$$-2 \ln \quad = N \ln \left(\frac{r}{r}\right)$$

where and are the determinants of the estimated error covariance matrices for the restricted and unrestricted models respectively, is the likelihood ratio and N the number of observations. Under the null hypothesis, this statistic will be distributed as chi-squared with degrees of freedom equal to the number of restrictions. The computed test statistic of 0.0936 is lower than the X_1^2 value at 75 percent of 0.1015. Thus the null hypothesis cannot be rejected. In what follows we use this constrained specification with all 10 countries pooled together.

4.3 Equilibrium levels of capital, K*

Once the parameters of the short-run cost function is estimated, we use eq. 13 to solve for K*, the full equilibrium values of the quasi fixed capital. Values for K and K* are compared in Figure 3 (for B-W data) and Figures 4a-4j (for Pindyck data).

Before commenting on these results, we define "over capitalization"

Figure 3: Actual and Equlibrium Levels of Capital: B-W data.

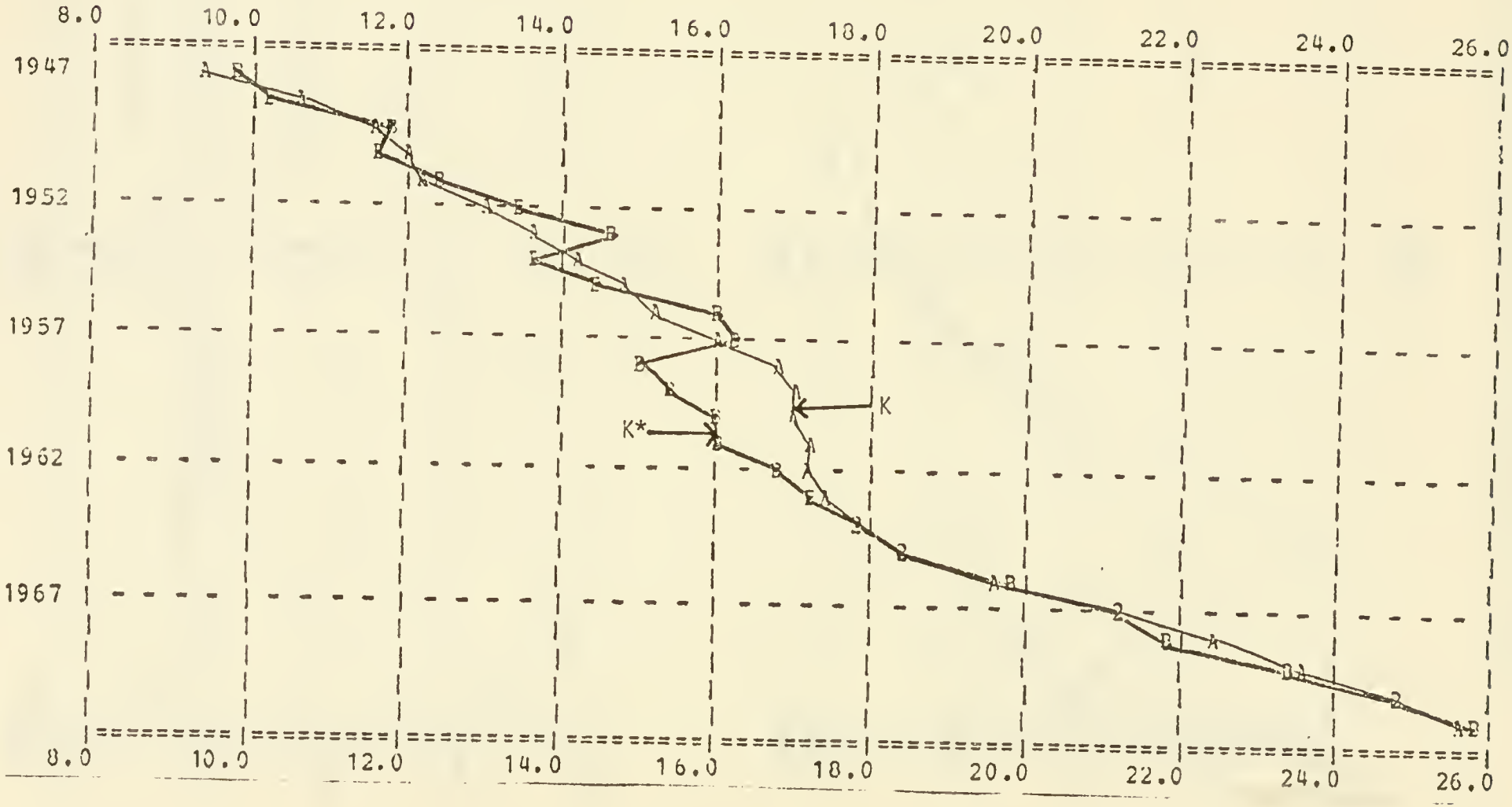
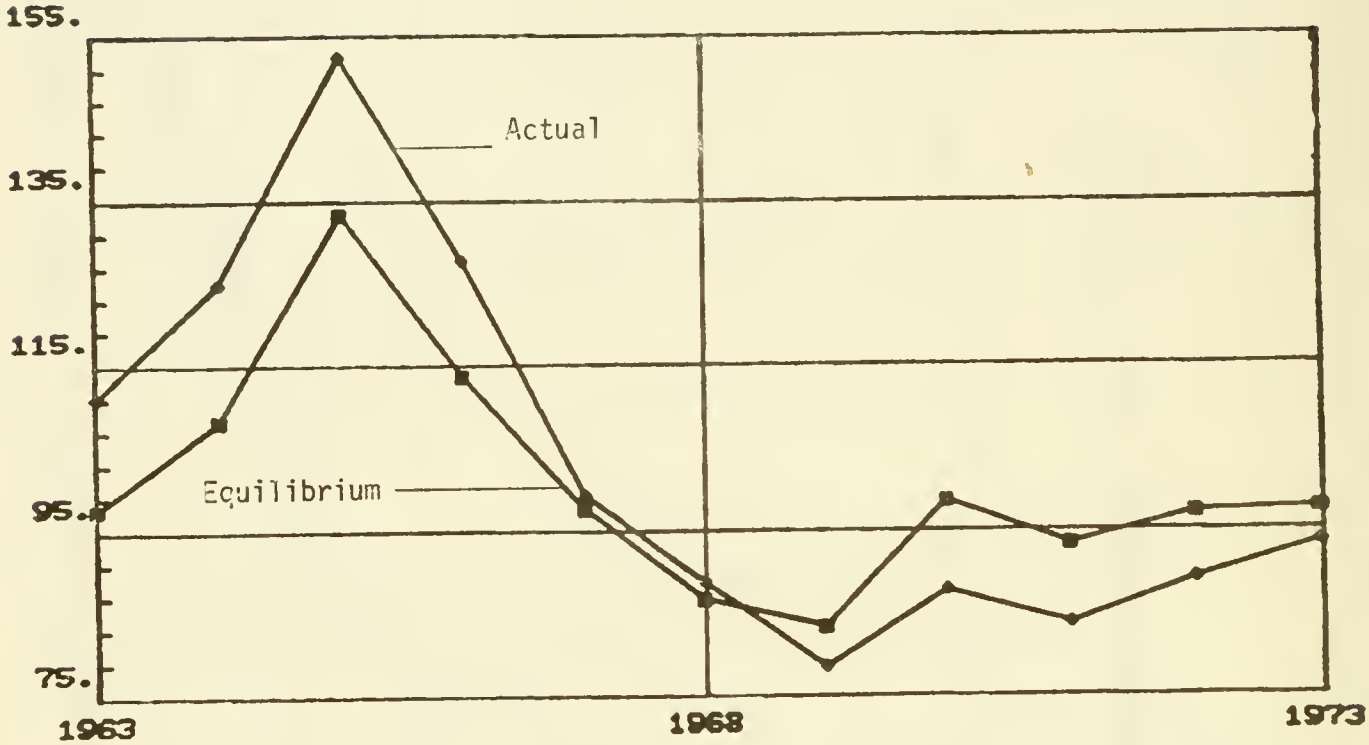


Figure 4. Actual and Equilibrium levels of Capital: P ndyck data

4a. CANADA



4b. FRANCE

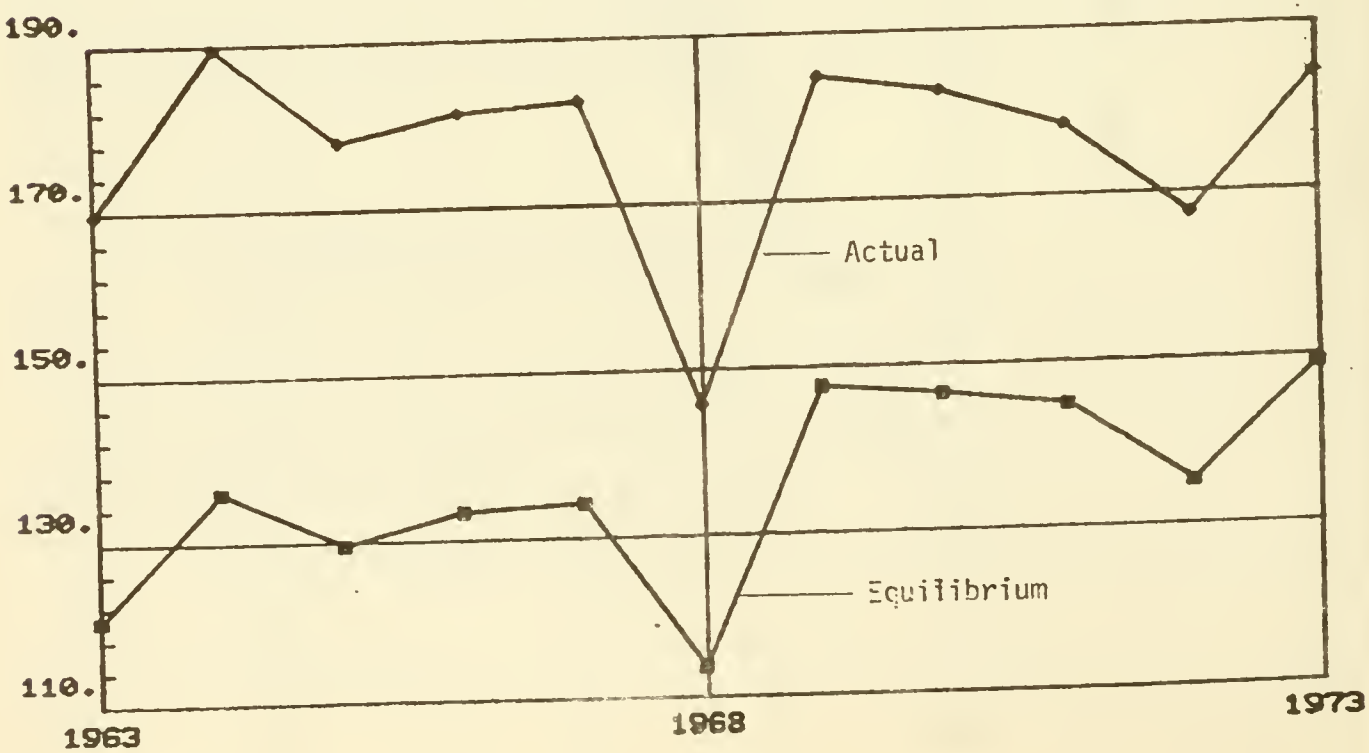
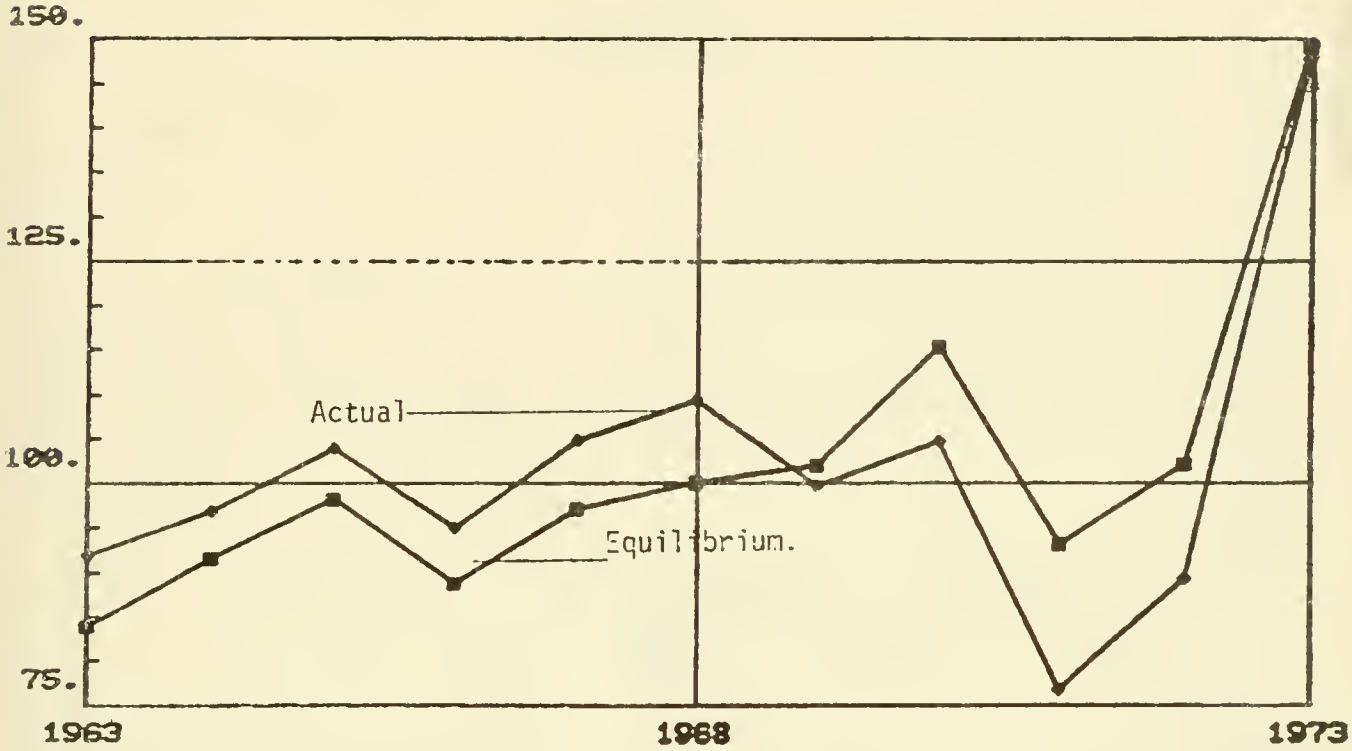


Figure 4. cont. Actual and Equilibrium levels of capital: Pindyck data.

4 c. ITALY



4d. JAPAN

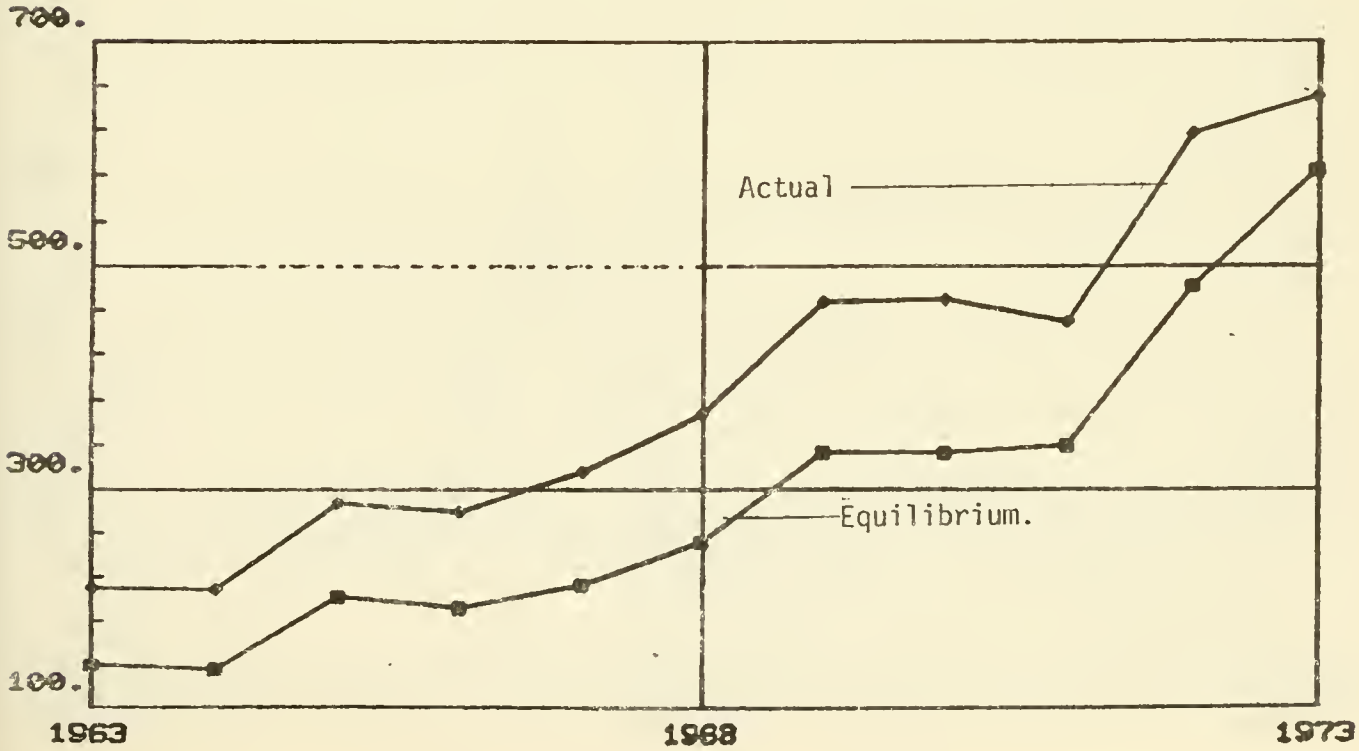
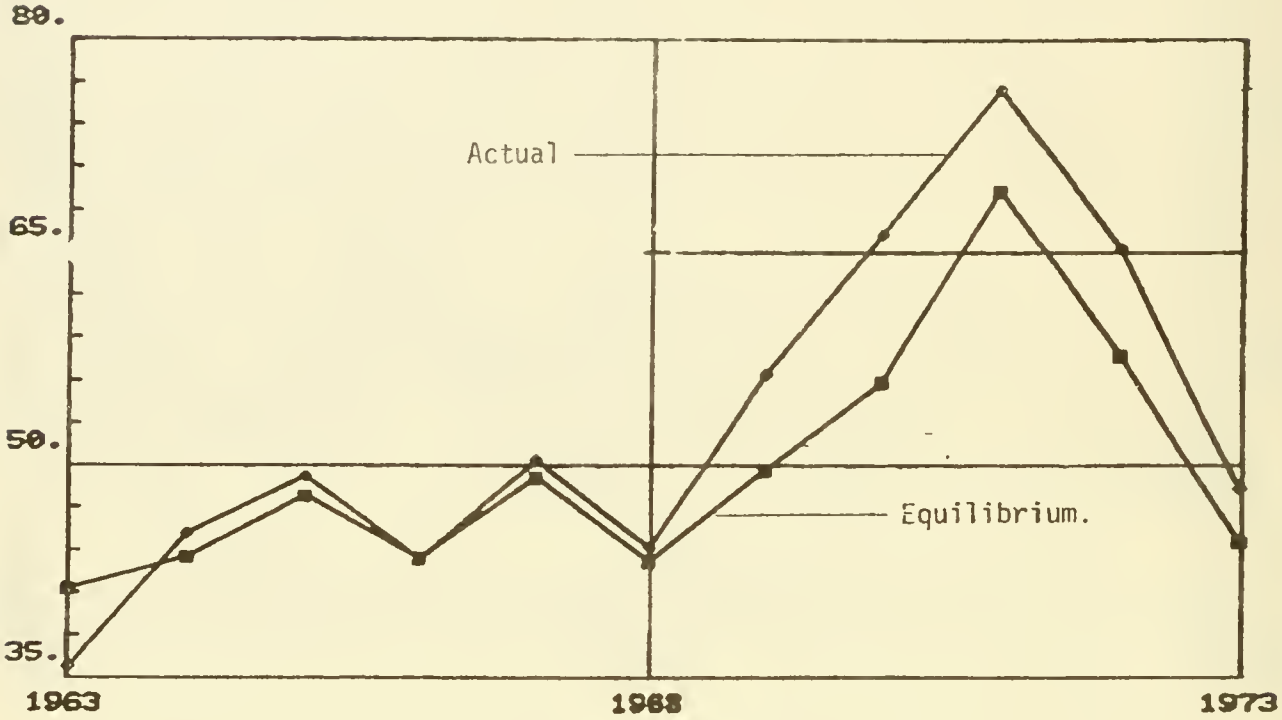


Figure 4. cont. Actual and equilibrium levels of capital: Pindyck data.

4e. NETHERLANDS.



4f. NORWAY

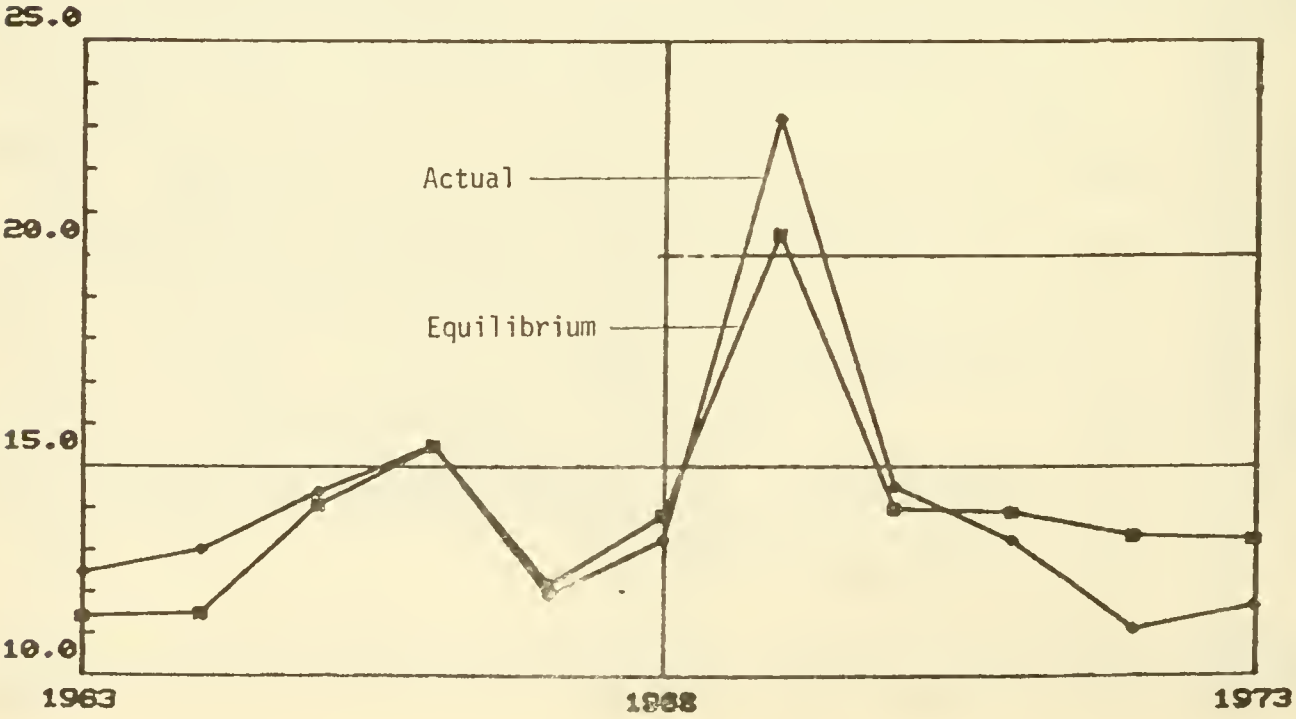
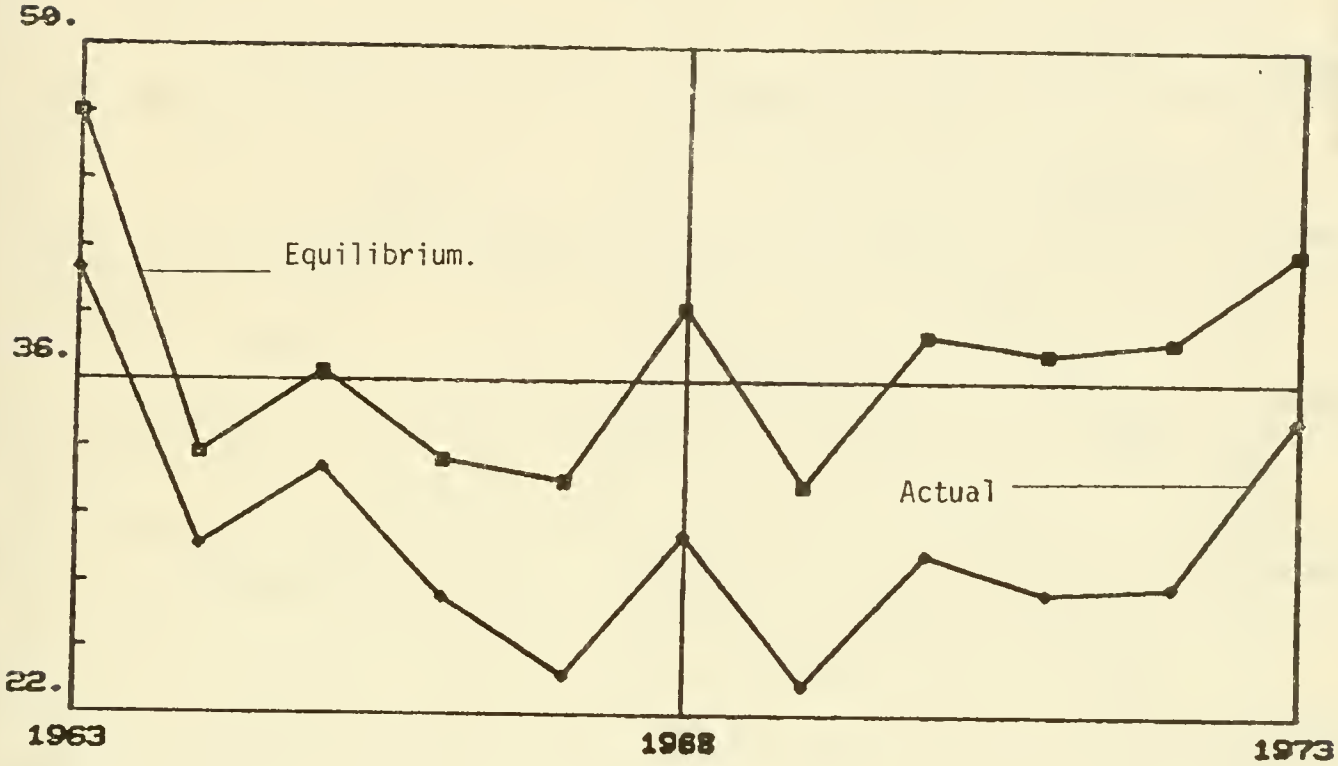


Figure 4. cont. Actual and Equilibrium levels of capital: Pindyck data.

4g. SWEDEN



4h. U.K.

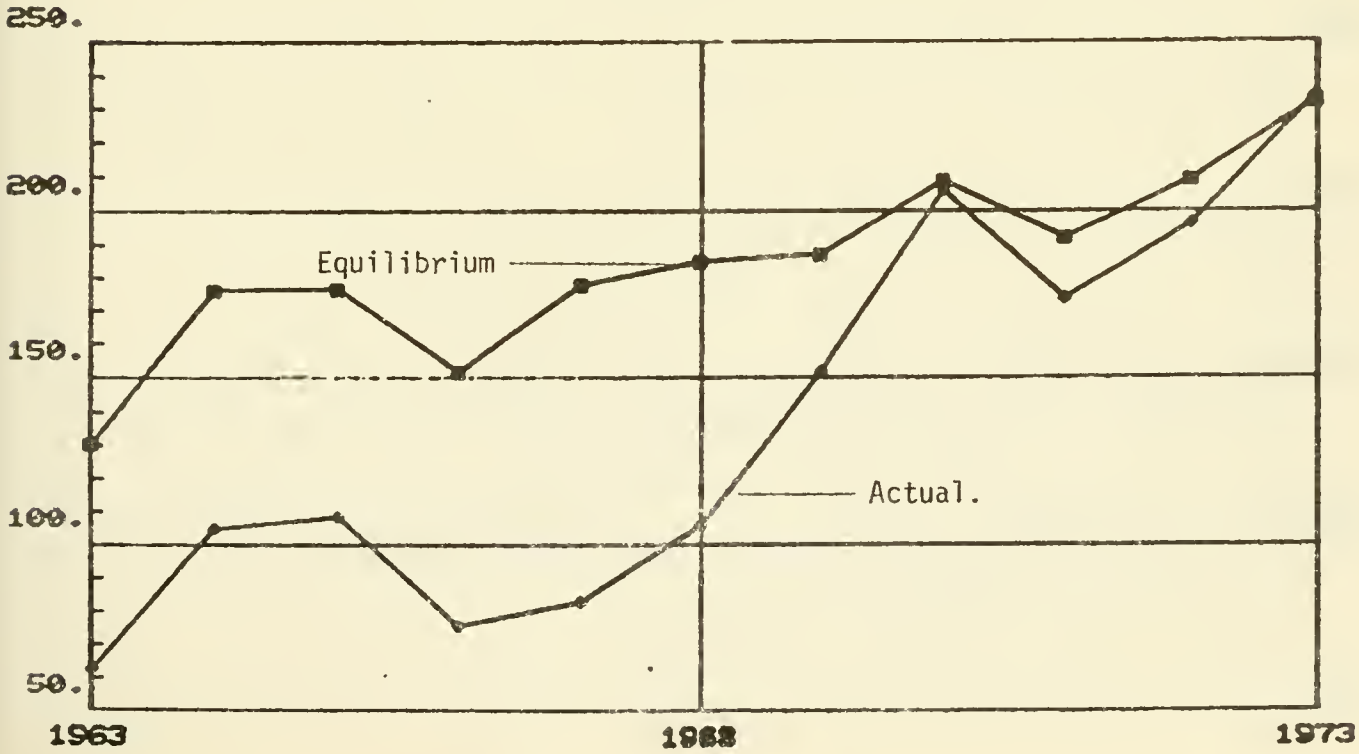
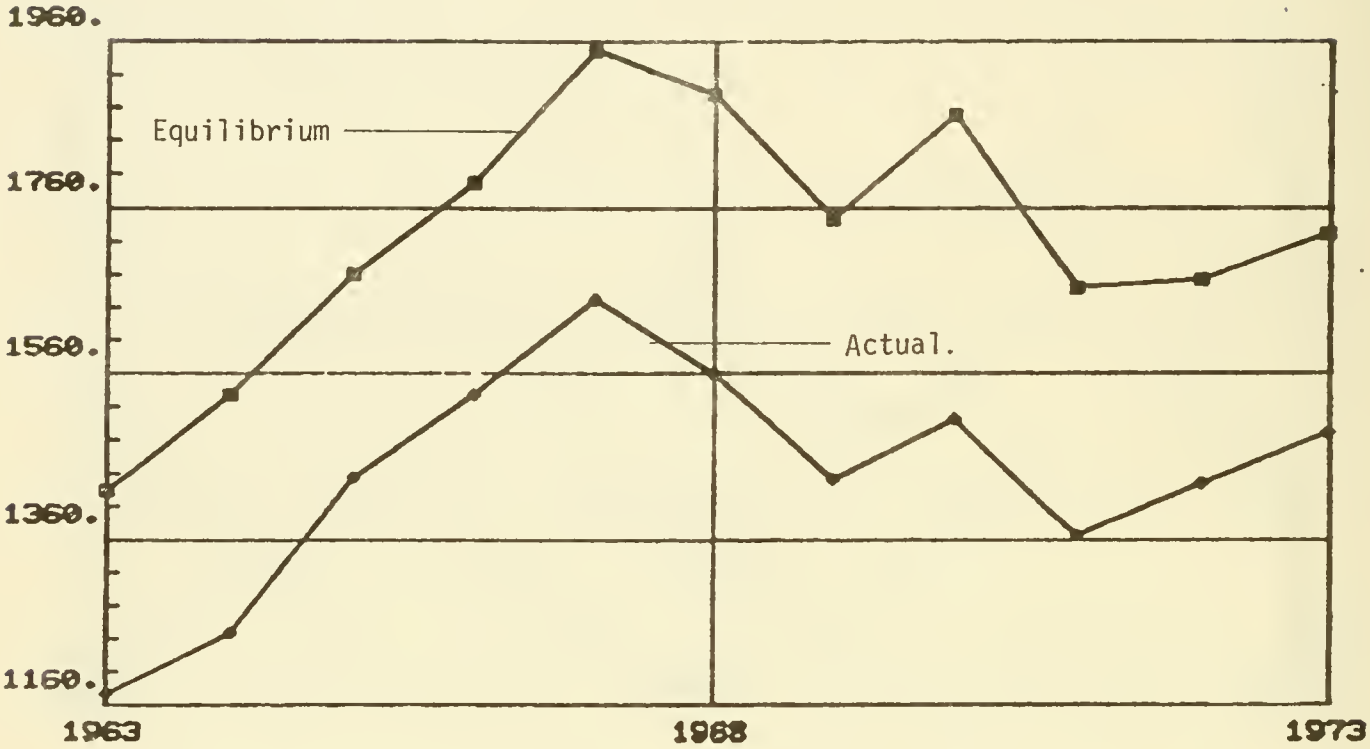
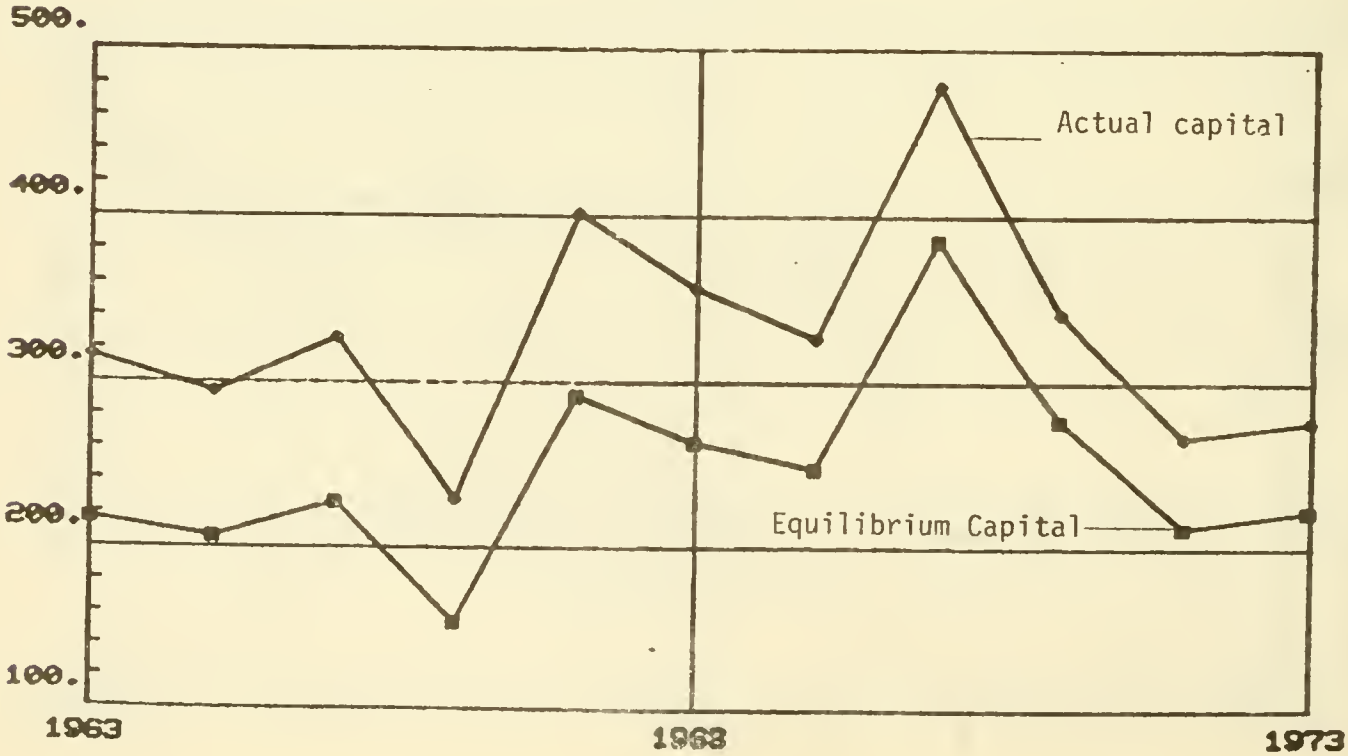


Figure 4. cont. Actual and equilibrium levels of capital: Pindyck data.

4 i. U.S.A.



4j. W. GERMANY



and "under capitalization" as situations where $K > K^*$ and $K < K^*$ respectively. If $K = K^*$ then the observed technology will be in full equilibrium and the short-run and long-run properties will be identical.

In the case of B-W data, over most of the range, differences between K and K* (see Figure 3) are small. During 1958-1961 we observe over-capitalization, while in 1953-1958 and in 1968 we observe under-capitalization. Upon casual observation we feel that, over most of the range, an equilibrium specification would provide an adequate representation. To formally test the hypothesis of $K = K^*$ it is necessary to obtain the standard errors of these estimated levels of K*. Unfortunately, since closed form solutions do not exist for eq. 13, we are unable to compute the standard errors of K* without resorting to non linear numerical techniques such as monte-carlo simulations.

In the cae of Pindyck data, the differences between K and K* are considerably larger than in B-W case, for most countries in the sample. We observe over-cpaitalization for France (Figure 4b), Japan (fig 4d) and W. Germany Figure 4j); for Sweden (Figure 4g, U.K. (Figure 4h) and the U.S.A. (Figure 4i) the situation is reversed, to give under-capitalization; Canada (Figure 4a) and Italy (Figure 4c) exhibit over-capitalization in the first half of the sample period and under-capitaliztaon in the second half; the two variables are closest together and cross over several times in cases of Norway (Figure 4f) and The Netherlands (Figure 4e). These results indicate significant departures between full equilibrium and the observed data. Thus, an equilibrium specification, as the one used in the original study may lead to incorrect results. But before concluding on this issue, it is necessary to keep in mind the inadequacy of the partial equilibrium

specification which assumed a homogenous technology across all 10 countries, as a model of the true underlying technology. The errors thus introduced could have caused the observed discrepencies between K and K*. However, since the partial equilibrium model includes the possibility of full equilibrium as a special case, this method will always be superior to the full equilibrium specification.

4.4 Substitution elasticities:[12] Short run and the long run

The elasticities of substitution computed via the algorithm in Section 2.3 are given in Tables 3 and 4 f(or the short run and the long run respectively) for B-W data. A summary, giving the mean over the sample period for each of the 10 countries in the Pindyck data are reported in table 6. The inter temporal details f the Pindyck case are in Appendix 3. As with the K* values earlier, here too we are unable to compute the standard errors of the elasticity estimates.

For the B-W data, the elasticity estimates of L, E own- and cross-effects show very little difference between the short run and the long run (see Tables 3 and 4). This result is to be expected due to the closeness of K and K* values discussed in the previous section. In the short run LE show substitutability. In the long run labor is found to be substitutable with both enregy and capital. But, as with the original B-W study, we still find EK to be complements.

The long-run elasticities, σ^L_{LL}, σ^L_{LE} and σ^L_{EE} are consistently greater than the short-run values, σ^S_{LL}, σ^S_{LE} and σ^S_{EE} respectively. This difference is large only in the case of LL. Since the elasticities are dependent not only on the parameters of the static cost function but also on the data it is possible to compute estimates for each time period (see

Tables 3 and 4). There are some noticeable trends in these results. For example, all of the short-run elasticities have a decreasing trend over time (see Table 3). The long-run values in table 4, show a downward trend for LL, LE and EE, but have an upward trend for LK, EK and KK.

To test the hypothesis that "time series data captures short-run effects," we consider the following relationship described in Section 3.

$$\sigma_{ij}^{s} \simeq \sigma_{ij}^{TS} < \sigma_{ij}^{L}$$

where σ_{ij}^{TS} refers to the elasticities resutling from an equilibrium specification of a LKE model using the B-W <u>time series</u> data. These results together with a summary of the short-run and long-run values from Tables 3 and 4, are given in Table 5. It should be noted that the standard errors in columns 2 and 3, Table 5, only take into account the variation over time. The estimation error component of the variance in the elasticities are not computed due to the problems mentioned earlier. Therefore, we are unable to perform any formal tests on the above hypothesis. Hence, we form a heuristic test by considering the interval within two or three standard deviations around each variable.

To test the left hand side of the above relation, we compare columns 1 and 2 of Table 5. The hypothesis can be accepted (i.e. the intervals overlap within 2 standard errors) only in the case of σ_{EE}. For σ_{LL} and σ_{LE} the short-run values are much smaller than the time series estimates in column 1. However, the right hand inequality (compare cols. 1 and 3, Table 5) is found to be satisfied in all except σ_{EK} (where the time series values are larger than the long-run values). In the case of σ_{EL}, the inequality will be true only if we form intervals using 3 standard errors. But here again, if estimation error in the partial equilibrium

Table 3: Short-run Allen Elasticities: B-W data.

	SZLL	SZLE	SZEE
1947	-0.071907	0.417835	-2.42796
1948	-0.085281	0.453023	-2.40653
1949	-0.08558	0.45374	-2.40571
1950	-0.07771	0.433898	-2.42269
1951	-0.072615	0.419868	-2.42772
1952	-0.073856	0.423377	-2.427
1953	-0.071191	0.415763	-2.42809
1954	-0.070353	0.413309	-2.42809
1955	-0.070215	0.412899	-2.42807
1956	-0.071795	0.417513	-2.42799
1957	-0.068519	0.40783	-2.42744
1958	-0.066369	0.401221	-2.42551
1959	-0.060018	0.380411	-2.41117
1960	-0.060244	0.381187	-2.41193
1961	-0.058737	0.375965	-2.40648
1962	-0.056823	0.369153	-2.39822
1963	-0.052741	0.353912	-2.3749
1964	-0.052965	0.354777	-2.37641
1965	-0.050243	0.344078	-2.35635
1966	-0.048324	0.336239	-2.33958
1967	-0.047155	0.331339	-2.32821
1968	-0.044525	0.319953	-2.29917
1969	-0.044162	0.318343	-2.29477
1970	-0.042983	0.313032	-2.27973
1971	-0.051165	0.347757	-2.36361

Table 4: Long-run Allen Elasticities: B-W data.

	LZLL	LZLE	LZEE	LZLK	LZEK	LZKK
1947	-0.262267	0.58878	-2.92846	0.738658	-0.389258	-3.08269
1948	-0.262451	0.616278	-2.86199	0.72356	-0.38301	-3.21891
1949	-0.242138	0.618701	-2.83352	0.699473	-0.469894	-3.40604
1950	-0.24633	0.603495	-2.88227	0.714709	-0.451261	-3.27711
1951	-0.245577	0.592875	-2.90346	0.720145	-0.456776	-3.2279
1952	-0.240965	0.596087	-2.8929	0.713345	-0.476161	-3.28226
1953	-0.236854	0.590758	-2.89802	0.711924	-0.495943	-3.28949
1954	-0.240613	0.58847	-2.90524	0.717323	-0.479809	-3.24661
1955	-0.238811	0.588371	-2.90353	0.715433	-0.487854	-3.26103
1956	-0.230021	0.592704	-2.88823	0.702913	-0.527046	-3.35972
1957	-0.229171	0.585546	-2.89767	0.706218	-0.533004	-3.32968
1958	-0.232057	0.580276	-2.90592	0.712447	-0.521426	-3.27872
1959	-0.233162	0.564262	-2.91299	0.721642	-0.524079	-3.19901
1960	-0.228562	0.565481	-2.90824	0.716146	-0.544629	-3.24173
1961	-0.226489	0.56177	-2.90539	0.715651	-0.556441	-3.24346
1962	-0.222701	0.557074	-2.89938	0.713676	-0.577281	-3.25602
1963	-0.220841	0.54573	-2.88649	0.716697	-0.593743	-3.2272
1964	-0.218646	0.546683	-2.88551	0.713861	-0.603784	-3.24929
1965	-0.218927	0.538502	-2.87336	0.717641	-0.608351	-3.21654
1966	-0.21698	0.532804	-2.86027	0.717821	-0.622355	-3.21255
1967	-0.210718	0.529957	-2.84775	0.712005	-0.656135	-3.25555
1968	-0.210014	0.52142	-2.82502	0.71457	-0.666939	-3.23232
1969	-0.203982	0.521007	-2.81845	0.70784	-0.698766	-3.28326
1970	-0.193448	0.518148	-2.80272	0.69621	-0.758846	-3.3701
1971	-0.190022	0.543765	-2.8612	0.679536	-0.758479	-3.50795

Table 5: Summary of Substituion Elasticities: B-W data

	Equilibrium model: σ_{ij}^{Ts}	Partial Equilibrium Model[2] Short-run: σ_{ij}^{s}	Long-run: σ_{ij}^{L}
σ_{KK}	-2.75 (0.36)		-3.270 (0.080)
σ_{KL}	0.697 (0.074)		0.713 (0.011)
σ_{KE}	-1.09 (0.23)		-0.554 (0.101)
σ_{LL}	-0.226 (0.019)	-0.062 (0.013)	-0.228 (0.018)
σ_{LE}	0.610 (0.043)	0.384 (0.043)	0.568 (0.031)
σ_{EE}	-2.70 (0.23)	-2.389 (0.047)	-2.880 (0.033)

[1]Standard errors in parentheses: source, Table XVII, Hirsch (1980).

[2]Standard errors over time in parentheses.

model is taken into account, the results would be even stronger. In summary, we conclude that an equilibrium time series model captures some but not all of the long-run substitution effects.

For the Pindyck data, the long-run own elasticities are consistently larger than the short-run values (compare row 1 with row 7 and row 3 with row 9, Table 6). But the cross elasticity between L and E is indicating larger values for the short-run than in the long-run. The differences between σ^{s}_{LE} and σ^{L}_{LE} are still small. As with the B-W case, LE and LK show substitutability, but unlike the B-W data, we find EK also to be substitutes in the long run. Thus, the EK substitutability/ complementarity controversy remains unresolved. Nevertheless, we have found an interesting result by being able to conclude that the causality of the controversy does not stem from the time series and the cross-section nature of the data.

We now go on to use these results to test the following hypothesis; elasticity estimates derived from an equilibrium model using pooled cross-section - time series data, captures the long-run effects in the underlying technology. As described in Section 3, this test can be conducted by checking the following relationship:

$$\sigma^{s}_{ij} < \sigma^{cs}_{ij} \simeq \sigma^{L}_{ij}$$

where σ^{cs}_{ij} is obtained by estimating an equilibrium model on the Pindyck cross section - time series data. Results for σ^{cs}_{ij} obtained by Pindyck are given in Table 7. As with the B-W case, the test will be conducted in a heuristic manner.

In the short-run own E and L elasticities are much lower than those obtained via the cross-section specification used by Pindyck. Thus,

Table 6

Elasticities of Substitution for Capital, Labor and Energy: via the partial equilibrium specification: Pindyck data.

	Canada	France	Italy	Japan	Netherlands	Norway	Sweden	U. K.	U.S.A.	W.Germany
Short-run										
σ^S_{LL}	-0.07	-0.094	-0.096	-0.104	-0.111	-0.073	-0.082	-0.098	-0.067	-0.1
σ^S_{LE}	0.737	0.784	0.786	0.796	0.804	0.744	0.763	0.789	0.731	0.792
σ^S_{EE}	-7.821	-6.524	-6.456	-6.136	-5.875	-7.629	-7.128	-6.373	-7.978	-6.273
Long-run										
σ^L_{KK}	-1.279	-1.231	-1.256	-1.224	-1.311	-1.265	-1.274	-1.255	-1.299	-1.239
σ^L_{KL}	0.933	0.937	0.935	0.938	0.93	0.934	0.933	0.935	0.931	0.936
σ^L_{KE}	0.761	0.801	0.798	0.811	0.8	0.769	0.779	0.8	0.753	0.805
σ^L_{LL}	-0.8	-0.878	-0.862	-0.901	-0.849	-0.815	-0.824	-0.866	-0.782	-0.883
σ^L_{LE}	0.71	0.761	0.768	0.776	0.798	0.717	0.742	0.771	0.706	0.773
σ^L_{EE}	-13.891	-11.932	-11.69	-11.299	-10.462	-13.642	-12.743	-11.557	-14.042	-11.464

Table 7

- Elasticities of Substitution for Capital, Labor, and Energy

Ten countries pooled

	CANADA	FRANCE	ITALY	JAPAN	NETH	NOR	SWED	UK	USA	WGER
σ_{KK}	-0.99 (0.12)	-0.91 (0.11)	-1.14 (0.14)	-0.68 (0.08)	-1.35 (0.18)	-1.12 (0.14)	-1.39 (0.18)	-2.18 (0.37)	-0.95 (0.11)	-0.57 (0.07)
σ_{KL}	0.82 (0.10)	0.82 (0.10)	0.81 (0.11)	0.81 (0.11)	0.81 (0.11)	0.82 (0.10)	0.80 (0.11)	0.77 (0.13)	0.82 (0.10)	0.81 (0.11)
σ_{KE}	0.76 (0.13)	0.76 (0.13)	0.82 (0.09)	0.86 (0.07)	0.77 (0.12)	0.78 (0.12)	0.80 (0.11)	0.66 (0.18)	0.61 (0.21)	0.82 (0.10)
σ_{LL}	-0.84 (0.09)	-0.90 (0.09)	-0.79 (0.09)	-1.32 (0.15)	-0.65 (0.08)	-0.76 (0.08)	-0.65 (0.07)	-0.38 (0.05)	-0.81 (0.08)	-1.42 (0.16)
σ_{LE}	0.96 (0.14)	0.95 (0.15)	0.97 (0.09)	0.96 (0.13)	0.97 (0.09)	0.96 (0.11)	0.96 (0.08)	0.97 (0.08)	0.93 (0.23)	1.94 (0.19)
σ_{EE}	-15.86 (1.07)	-16.20 (1.12)	-10.96 (0.50)	-11.17 (0.51)	-12.25 (0.61)	-13.75 (0.78)	-10.77 (0.47)	-13.10 (0.70)	-24.21 (2.93)	-15.82 (1.06)

source; Pindyck (1977)

we have evidence supporting the left hand inequality in the above relationship. The LE values for most countries lie within 2 standard deviations of σ_{LE}^{cs}, but since σ_{LE}^{s} σ_{LE}^{L}, it is difficult to interpret these results. In the long run, own elasticities are much closer to the cross-section results than with the short-run cases. Values for most countries, fall within 2 or 3 standard deviations of each other. The long-run cross elasticities are even better agreement. They fall within one standard deviation for many of the countries. Therefore, on this heuristic level, the evidence indicates convincing support for accepting the hypothesis.

5. Concluding Remarks

In this paper we modeled the industrial production process in a partial equilibrium framework where some of the factor inputs are assumed to be quasi-fixed in the short-run. By allowing the quasi fixed factors to vary in the long-run, we can make inferecnes about the full equilibrium (long-run) behavior. In particular, we consider a 3-factor KLE production function where capital is assumed to be quasi-fixed in the observed data.

A restricted cost function is used to estimate the short-run technology, in two empirical applications. One of the most interesting results stem from the differences between actual and equilibrium levels of capital. For the B-W time series data, these differences are found to be small. But for the pooled Pindyck data, we notice large departures between actual and equilibrium capital levels. Although the tests used here are not formal, we employ heuristic methods that indicate these differences to be significant enough to invalidate the equilibrium

specification used by previous researchers. Furthermore, since the partial equilibrium model contains the full equilibrium model as a special case, it is always safe to use such a partial equilibrium specification in modeling a technology.

The fact that actual capital departed significantly from equiibrium levels also has interesting policy implications. For example, the "under capitalization" observed in certain countries could be remedied by policies which would increase incentives for higher capital investment. Increasing investment tax credits or permitting firms to depreciate capital costs more rapidly (e.g., to expense all capital costs with one year lag, as suggested in the U.K.) could provide such incentives. On the other hand, when "over capitalization" is present the policies should aim at achieving the opposite effect. However, specific policy recommendations require the use of more sophisticated models. These models show disaggregate to identify particular sectors where the effects are strongest.

Substitution elasticities between the input factors provided another host of intersting results. First we observe that the long-run elasticities are almost always larger than the short-run values (i.e., no over-shooting). This can be explained by a slow adjustment process. Second, the time series models such as B-W, seem to underestimate the long-run elasticities but overestimate slightly the short-run values. The pooled data gave full equilibrium elasticities that were close to the corresponding values obtained by Pindyck. It is hard to interpret these results in a straightforward way, because of the large departures from the equilibrium. However, we can say this: the long-run estimates obtained via ful equilibrium models, are closer to short-run values when

using time series data but give estimates closer to long-run values when using pooled data. This can be thought of as the appropriate modification to the claim made by Pindyck (1977) and Griffin-Gregory (1976).

This analysis was not able to resolve the E-K complementarity/substitutability debate. Even under the partial equilibrium assumptions, E and K were found to be substitutes when using Pindyck data but complements when using B-W data. However, we can now eliminate the short-run/long-run nature of the result as the cause of the disparity. Thus, data construction and coverage issues may be at the heart of the controversy. An extension of this study where the data would be adjusted to acccommodate accounting and other differences would serve as a convenient vehicle to address the above issue.

Footnotes

[1]Similar disparities were observed in the literature on Capital-Labor production function studies. Berndt (1976), in an attempt to reconcile between alternative estimates of elasticity of substitution, found that the results were very sensitive to data construction techniques.

[2]Griffin-Gregory (1976) and Pindyck (1977) make this claim.

[3]See Berndt, Morrison and Watkins (1980) for a comparative assessment of dynamic energy models.

[4]Although we refer to it as a disequilibrium, this can also be thought of as a temporary equilibrium. This is made clear in the description of the model which follows.

[5]Here we do not attempt to review the literature on industrial factor demand studies.

[6]Computing such values necessitates the use of Monte Carlo simulations, which were too costly to be carried out here. However, we can make subjective statements by comparing the time paths of K and K*.

[7]Although I do not have a rigorous proof, the following heuristic reasoning might be helpful in providing some insight. Suppose, contrary to assumption, observed labor is not at the equilibrium level. Then, solving the long-run total cost minimization problem we would have extra degrees of freedom over which the optimization can be done. In other words, capital quantities will have to adjust for changes in labor levels as well as their own changes.

[8]Note that here we assume separabiity of KLE from M and therefore the value of output will refer to the sum of the KLE inputs in this value added framework.

[9]Hirsch (1980) has taken a closer look at the validity of some of the assumptions behind the existence of aggregate production functions in the context of the KLEM literature. He finds the aggregation problems to be extremely serious but reasonable empirical results are considered as providing some justification.

[10]It is straightforward to compute the usual own and cross price elasticities of demand from the Allen elasticities:

$$\eta_{ii} = \sigma_{ii} S_i \quad \text{and} \quad \eta_{ij} = \sigma_{ij} S_j .$$

In this paper we only deal with the Allen elasticities which should be interpreted as indicating substitutability if positive and complementarity if negative.

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APPENDIX I

Imposing restrictions on KLE translog function with one fixed input (k):

Consider the following translog restricted cost-function.

$$\text{(A.1)} \quad \ln CR = \alpha_0 + \alpha_Y \ln Y + \sum_{i=L,E} \alpha_j \ln P_j + \beta_K \ln K + \frac{1}{2}\gamma_{YY}(\ln Y)^2 + \frac{1}{2}\sum_{i=L,E}\sum_{j=L,E} \gamma_{ij} \ln P_i \ln P_j + \frac{1}{2}\delta_{KK}(\ln K)^2 + \sum_{i=L,E} \rho_{yi} \ln Y \ln P_i + \sum_{i=L,E} \rho_{iK} \ln P_i \ln K + \pi \ln \gamma \ln K$$

If this equation is directly estimated, there are 16 free parameters that have to be estimated. However, imposing theoretical restrictions of homogeneity of degree one in input prices and the symmetry conditions gives the following linear restrictions:

(A.2) symmetry $\gamma_{ij} = \gamma_{ji} \quad \forall\, i, j$

(A.3) homogeneity of degree one,

$$\sum_i \gamma_{ij} = \sum_j \gamma_{ij} = 0 \qquad i, j = L,E$$

$$\sum_i \rho_{Yi} = \sum_j \rho_{iK} = 0$$

and $\sum_i \alpha_i = 1$

For the cost function in equation (1), these constraints can be explicitly written as:

(A.4) $\gamma_{LE} = \gamma_{EL}$

$$\gamma_{LE} + \gamma_{EE} = \gamma_{EL} + \gamma_{LL}$$

Thus in compact notation

$$\gamma_{LL} = \gamma_{EE} = -\gamma_{EL} = -\gamma_{LE}$$

(A.5) $\rho_{YL} = -\rho_{YE}$

(A.6) $\rho_{LK} = -\rho_{EK}$

(A.7) $\alpha_L + \alpha_E = 1$

Imposing restrictions (A.4) - (A.8) in the cost function we get[1]

(A.8) $$\ln CR = \alpha_0 + \alpha_y \ln Y + \ln P_E + \alpha_L \ln(P_L/P_E) + \beta_K \ln K + \frac{1}{2}\gamma_{KK}(\ln Y)^2 + \frac{1}{2}\gamma_{LL}(\ln(P_L/P_E))^2 + \frac{1}{2}\delta_{KK}(\ln K)^2 + \rho_{YL} \ln Y \ln(P_L/P_E) + \rho_{LK} \ln K \ln(P_L/P_E) + \pi \ln Y \ln K.$$

Now the number of free parameters is 10.

As a further simplification, we can impose constant returns to scale on the technology by introducing the following linear restrictions:

(A.9) $$\alpha_Y + \beta_K = 1$$
$$\rho_{YL} + \rho_{LK} = \rho_{YE} + \rho_{EK} = 0$$
$$\gamma_{YY} + \pi = 0$$
$$\pi + \delta_{KK} = 0$$

Rewriting equation (A.9) we get

(A.10) $$\beta_K = 1 - \alpha_Y$$
$$\rho_{LK} = -\rho_{YL}$$
$$\gamma_{YY} = \delta_{KK} = -\pi$$

The resulting cost function (now with only 6 free parameters) can be written as

(A.11) $$(\ln CR - \ln P_E - \ln K) = \alpha_0 + \alpha_Y \ln(Y/K) + \alpha_L \ln(P_L/P_E) - \frac{1}{2}\pi(\ln(Y/K))^2 + \frac{1}{2}\gamma_{LL}(\ln(P_L/P_E))^2 + \rho_{YL} \ln(P_L/P_E) \ln(Y/K).$$

---

[1]Note the following property, $(\ln(\frac{x}{y}))^2 = (\ln x)^2 - 2 \ln x \ln y + (\ln(y))^2$.

Appendix II

Estimating the Restricted Cost Function

The estimation method chosen here involves simultaneous estimation of the restricted cost function, CR, and one of the two share equations. Due to the adding-up of property of the share equations, one of the share equations can be left out while still extracting all the information. In general, the system of equations to be estimated can be written as follows,

$$\text{(B.1)} \qquad \ln CR = \alpha_0 + \alpha_Y \ln Y + \sum_{i\varepsilon V} \beta_i \ln Z_i + \sum_{i\varepsilon V} \alpha_i \ln p_i$$

$$+ \frac{1}{2}\sum_i\sum_j \gamma_{ij} \ln P_i \ln P_i + \frac{1}{2}\sum_i\sum_j \rho_{ij} \ln \rho_i \ln Z_j + \frac{1}{2}\sum_i\sum_j \delta_{ij} \ln Z_i \ln Z_j$$

$$+ \frac{1}{2} \gamma_{YY} (\ln Y)^2 + \sum_i \rho_{Yi} \ln Y \ln P_i + \sum_i \pi_i \ln Y \ln Z_i + \varepsilon_C$$

$$\text{(B.2)} \qquad S_i = \alpha_i + \rho_{Yi} \ln Y + \sum_j \gamma_{ij} \ln P_j + \sum_j \rho_{ij} \ln P_j + \varepsilon_i$$

for all except one $i\varepsilon V$.

The estimation method should be one that is capable of incorporating both the cross equation parameter restrictions and the specific structural relationships of the error terms. Before pursuing these any further, I will present the simple single fixed factor KLE model considered in this study, thereby, the analysis and description will be much simplified while maintaining the concepts.

$$\text{(B.3)} \qquad (\ln CR - \ln K - \ln P_E) = \alpha_0 + \alpha_Y \ln(Y/K) + \alpha_L \ln(P_L/P_E) - \frac{1}{2} \pi(\ln Y/K)^2$$

$$+ \frac{1}{2} \gamma_{LL} (\ln P_L/P_E) + \rho_{YL} \ln (Y/K) + \tilde{\varepsilon}_L$$

Since the share equation is derived by applying Shepard's Lemma to the cost function, the error terms of the two equations will be related as follows:

$$S_L = \frac{\partial \ln CR}{\partial \ln P_L} \text{ from Shepard's Lemma}$$

By integrating equation (B.4) with respect to $\ln P_L$, we get

$$\text{(B.5)} \quad \ln CR = \int S_L \, d(\ln P_L) = \int (\alpha_L + \gamma_{LL} \ln(P_L/P_E) + \tilde{\varepsilon}_{YL} \ln (Y/K) + \tilde{\varepsilon}_L) d(\ln P_L)$$

$$= \alpha_L \ln P_L + \frac{1}{2} \gamma_{LL} \ln (P_L/P_E) \ln P_L + \rho_{YL} \ln (Y/K) \ln P_L$$

$$+ \tilde{\varepsilon}_L \ln P_L + \text{function } (P_E, Y, K)$$

Similarly, by integrating S_E with respect to $\ln P_E$, we get

$$\text{(B.6)} \quad \ln CR = (1 - \alpha_L) \ln P_E - \frac{\gamma_{LL}}{2} \ln (P_L/P_E) \ln P_E - \rho_{YL} \ln (Y/K) \ln P_E$$

$$- \tilde{\varepsilon}_L \ln P_E + \text{function } (P_L, Y, K)$$

For consistency of equations (B.5), (B.6) with (B.3),

$$\text{(B.7)} \quad \tilde{\varepsilon}_C = \tilde{\varepsilon}_L \ln (P_L/P_E + \tilde{\eta}_C = 0$$

where $\text{plim } (\tilde{\varepsilon}_L \cdot \tilde{\eta}_C) = 0$ and $\text{plim } \tilde{\eta}_C = 0$

The most efficient estimator of the system of equations (B.3) and (B.4) should take into account this explicit relationship between the error terms as well as the cross equation restrictions.

Furthermore, if the error terms are assumed to be serially uncorrelated, then by using iterative Zellner estimation, we take into account the cross equation restrictions and the contemporaenous correlation between the error terms. The only loss of efficiency of using Zellner as opposed to FIML is its inability to incorporate the specific error structure.

Appendix 3

Details of Pindyck Results

Table A1 . Restricted Allen Elasticities: (short run)

(σ^{S}_{LL})

	BELGIUM	FRANCE	ITALY	JAPAN	W.GERMANY
1963	-0.076151	-0.0984	-0.106538	-0.112938	-0.108202
1964	-0.074878	-0.098531	-0.103888	-0.109817	-0.106204
1965	-0.075607	-0.09641	-0.102882	-0.113103	-0.105417
1966	-0.072035	-0.09562	-0.101021	-0.110058	-0.101405
1967	-0.070158	-0.095183	-0.099558	-0.107406	-0.103636
1968	-0.068578	-0.094115	-0.098004	-0.104519	-0.103252
1969	-0.067104	-0.094507	-0.09612	-0.102453	-0.099415
1970	-0.068001	-0.093037	-0.09266	-0.097944	-0.098976
1971	-0.06625	-0.092249	-0.087815	-0.095145	-0.094487
1972	-0.063517	-0.090309	-0.085188	-0.094458	-0.091668
1973	-0.063172	-0.088313	-0.083257	-0.092769	-0.089592

	NETHERLAND	NORWAY	SWEDEN	U.K.	U.S.A.
1963	-0.125467	-0.076904	-0.09615	-0.102265	-0.071564
1964	-0.121259	-0.07563	-0.090505	-0.10265	-0.070261
1965	-0.119087	-0.075266	-0.088332	-0.101729	-0.069641
1966	-0.11753	-0.075303	-0.085884	-0.099185	-0.068299
1967	-0.115226	-0.073372	-0.084195	-0.100871	-0.067918
1968	-0.111559	-0.071576	-0.082585	-0.098846	-0.066154
1969	-0.108084	-0.072328	-0.078105	-0.096081	-0.064252
1970	-0.10498	-0.0707	-0.07697	-0.094478	-0.064629
1971	-0.103593	-0.070385	-0.075219	-0.094619	-0.064837
1972	-0.098825	-0.070564	-0.073571	-0.093433	-0.064649
1973	-0.092738	-0.067924	-0.070911	-0.090934	-0.064749

Table A2. Restricted Allen Elasticities: (short run)

(σ^{s}_{LE})

	BELGIUM	FRANCE	ITALY	JAPAN	W.GERMANY
1963	0.752186	0.789964	0.800441	0.807767	0.802417
1964	0.749484	0.790145	0.797183	0.804286	0.800037
1965	0.751042	0.787179	0.795909	0.807948	0.799079
1966	0.743184	0.786045	0.793494	0.804561	0.793999
1967	0.738807	0.785411	0.791545	0.801479	0.796866
1968	0.734978	0.783843	0.789419	0.797971	0.796379
1969	0.731279	0.784421	0.786765	0.795359	0.791351
1970	0.733544	0.782229	0.781658	0.789335	0.790755
1971	0.729079	0.781027	0.773949	0.785356	0.784393
1972	0.721734	0.778001	0.769471	0.78435	0.780443
1973	0.720771	0.774772	0.766034	0.781822	0.776856

	NETHERLAND	NORWAY	SWEDEN	U.K.	U.S.A.
1963	0.82022	0.753543	0.786808	0.795115	0.742102
1964	0.816289	0.751089	0.778312	0.795611	0.739051
1965	0.814162	0.750318	0.774806	0.79442	0.737568
1966	0.812595	0.750395	0.770679	0.791038	0.734287
1967	0.810214	0.746196	0.76772	0.793296	0.733084
1968	0.806246	0.742132	0.764805	0.790576	0.728827
1969	0.802275	0.743852	0.756195	0.786709	0.723755
1970	0.798543	0.74009	0.753887	0.784378	0.72478
1971	0.796811	0.739347	0.750216	0.784586	0.725339
1972	0.790547	0.739768	0.746636	0.782824	0.724833
1973	0.781774	0.733352	0.740584	0.778988	0.725102

Table A3 .Restricted Allen Elasticities: (short run)

(σ^{s}_{EE})

	BELGIUM	FRANCE	ITALY	JAPAN	W.GERMANY
1963	-7.42979	-6.34195	-6.01387	-5.77742	-5.95067
1964	-7.50198	-6.33641	-6.11718	-5.89051	-6.02673
1965	-7.46047	-6.42725	-6.15726	-5.77155	-6.0572
1966	-7.66746	-6.46168	-6.23273	-5.88165	-6.21703
1967	-7.78006	-6.48088	-6.29324	-5.98075	-6.12718
1968	-7.87703	-6.52828	-6.35875	-6.09228	-6.14247
1969	-7.96923	-6.51083	-6.43984	-6.1745	-6.29923
1970	-7.91293	-6.57676	-6.59385	-6.36126	-6.31764
1971	-8.02347	-6.61263	-6.82115	-6.48258	-6.51171
1972	-8.20094	-6.7024	-6.95034	-6.513	-6.63009
1973	-8.2236	-6.79714	-7.04815	-6.58893	-6.73615

	NETHERLAND	NORWAY	SWEDEN	U.K.	U.S.A.
1963	-5.36205	-7.39327	-6.43853	-6.1821	-7.69548
1964	-5.49507	-7.45918	-6.69323	-6.16658	-7.77385
1965	-5.56622	-7.47981	-6.79618	-6.20383	-7.8116
1966	-5.61626	-7.47774	-6.91569	-6.30885	-7.89433
1967	-5.69706	-7.58886	-7.00032	-6.23886	-7.9244
1968	-5.82685	-7.69475	-7.08276	-6.32309	-8.02965
1969	-5.95507	-7.65012	-7.3213	-6.44155	-8.15265
1970	-6.07418	-7.74728	-7.38396	-6.51212	-8.12799
1971	-6.12986	-7.7663	-7.48252	-6.50587	-8.1145
1972	-6.32391	-7.75553	-7.57729	-6.55889	-8.12673
1973	-6.59026	-7.91774	-7.73461	-6.67324	-8.12024

Table A4. Full Equilibrium Allen ELASticities: (long run)

(σ^{L}_{LL})

	BELGIUM	FRANCE	ITALY	JAPAN	W.GERMANY
1963	-0.80572	-0.891402	-0.888817	-0.944837	-0.900917
1964	-0.802161	-0.886332	-0.876985	-0.941823	-0.90042
1965	-0.793472	-0.884348	-0.869925	-0.922507	-0.896517
1966	-0.799761	-0.882071	-0.878541	-0.925248	-0.910676
1967	-0.801219	-0.880435	-0.871961	-0.919837	-0.874706
1968	-0.807592	-0.884297	-0.869889	-0.907657	-0.880037
1969	-0.805909	-0.868235	-0.862716	-0.889946	-0.879704
1970	-0.791818	-0.868588	-0.84835	-0.885633	-0.853539
1971	-0.794062	-0.869076	-0.851307	-0.876678	-0.864467
1972	-0.791469	-0.871757	-0.842243	-0.854518	-0.874031
1973	-0.79614	-0.86359	-0.818235	-0.834615	-0.868601

	NETHERLAND	NORWAY	SWEDEN	U.K.	U.S.A.
1963	-0.880565	-0.836546	-0.824336	-0.894393	-0.790736
1964	-0.87205	-0.837712	-0.846492	-0.871813	-0.786259
1965	-0.86292	-0.819296	-0.837897	-0.871332	-0.784156
1966	-0.86602	-0.813260	-0.83984	-0.875804	-0.782006
1967	-0.852549	-0.826889	-0.835347	-0.862371	-0.777516
1968	-0.858144	-0.810632	-0.817816	-0.861847	-0.776345
1969	-0.845443	-0.790517	-0.827897	-0.865493	-0.779344
1970	-0.831978	-0.81089	-0.615212	-0.859236	-0.77107
1971	-0.813277	-0.806965	-0.811313	-0.860873	-0.77744
1972	-0.819089	-0.804159	-0.805809	-0.85333	-0.780721
1973	-0.825428	-0.803346	-0.80017	-0.847254	-0.782518

Table A5. Full Equilibrium Allen Elasticities: (long run)

(σ^{L}_{LE})

	BELGIUM	FRANCE	ITALY	JAPAN	W.GERMANY
1963	0.73087	0.767879	0.784349	0.785306	0.785095
1964	0.727866	0.769168	0.781662	0.780351	0.781528
1965	0.732125	0.765069	0.781103	0.789675	0.780802
1966	0.719503	0.763817	0.775781	0.783872	0.770247
1967	0.712941	0.763194	0.77414	0.780076	0.781621
1968	0.705913	0.760033	0.771357	0.776963	0.779855
1969	0.70108	0.764202	0.768829	0.776386	0.772312
1970	0.707855	0.760847	0.76419	0.768079	0.776608
1971	0.700999	0.758948	0.752185	0.763877	0.76493
1972	0.691339	0.753856	0.747565	0.766904	0.757038
1973	0.688751	0.750772	0.747857	0.767272	0.752811

	NETHERLAND	NORWAY	SWEDEN	U.K.	U.S.A.
1963	0.816066	0.725594	0.776643	0.775146	0.720124
1964	0.811531	0.721747	0.759644	0.780287	0.716982
1965	0.809899	0.724994	0.756317	0.778589	0.715422
1966	0.806984	0.726534	0.749859	0.772613	0.711360
1967	0.805803	0.717239	0.746518	0.778655	0.710818
1968	0.79885	0.715353	0.746181	0.774693	0.705188
1969	0.795274	0.722689	0.731458	0.768181	0.69735
1970	0.792302	0.712375	0.731081	0.76598	0.700932
1971	0.793343	0.712285	0.726738	0.765954	0.700052
1972	0.783131	0.713578	0.722945	0.764885	0.698494
1973	0.769106	0.704673	0.715718	0.760478	0.698403

Table A6. Full Equilibrium Allen Elasticities: (long run)

(σ^{L}_{EE})

	BELGIUM	FRANCE	ITALY	JAPAN	W.GERMANY
1963	-13.1903	-11.6633	-10.9911	-10.8514	-10.9389
1964	-13.3015	-11.6193	-11.1232	-11.0654	-11.0886
1965	-13.1556	-11.7875	-11.1582	-10.7069	-11.1255
1966	-13.5997	-11.841	-11.3626	-10.9471	-11.5364
1967	-13.8268	-11.8684	-11.4403	-11.1154	-11.1287
1968	-14.064	-11.9886	-11.5566	-11.2655	-11.1926
1969	-14.2276	-11.8466	-11.6697	-11.3191	-11.5022
1970	-14.0053	-11.9794	-11.8764	-11.6643	-11.3693
1971	-14.2346	-12.0536	-12.3421	-11.8468	-11.8231
1972	-14.552	-12.2498	-12.5311	-11.7593	-12.1219
1973	-14.635	-12.3807	-12.5488	-11.774	-12.2949

	NETHERLAND	NORWAY	SWEDEN	U.K.	U.S.A.
1963	-9.64462	-13.3536	-11.4139	-11.3625	-13.5838
1964	-9.8609	-13.491	-12.0582	-11.1885	-13.6959
1965	-9.9495	-13.3902	-12.1998	-11.2592	-13.7508
1966	-10.0699	-13.34	-12.4464	-11.4961	-13.8906
1967	-10.1466	-13.66	-12.5793	-11.2713	-13.9112
1968	-10.4343	-13.7375	-12.6126	-11.4339	-14.1008
1969	-10.6087	-13.4941	-13.1485	-11.6914	-14.3586
1970	-10.7575	-13.8407	-13.1741	-11.789	-14.2431
1971	-10.7461	-13.8461	-13.3343	-11.7876	-14.2705
1972	-11.1584	-13.8031	-13.474	-11.8414	-14.321
1973	-11.7145	-14.1079	-13.7314	-12.0243	-14.3236

Table A7. Full Equilibrium Allen Elasticities: (long run)

(σ^{L}_{EK})

	BELGIUM	FRANCE	ITALY	JAPAN	W.GERMANY
1963	0.769568	0.806571	0.809149	0.823907	0.812602
1964	0.767372	0.805399	0.805284	0.822324	0.811803
1965	0.765341	0.803951	0.803128	0.819069	0.810598
1966	0.763991	0.803018	0.804539	0.818771	0.812388
1967	0.762575	0.802404	0.80228	0.816694	0.804621
1968	0.762887	0.802833	0.801073	0.812888	0.805799
1969	0.760764	0.799003	0.798371	0.807908	0.804153
1970	0.757342	0.798349	0.792867	0.804967	0.797286
1971	0.756087	0.798062	0.790926	0.801448	0.798028
1972	0.751978	0.797695	0.786834	0.795426	0.799119
1973	0.753043	0.794455	0.778774	0.789196	0.796487

	NETHERLAND	NORWAY	SWEDEN	U.K.	U.S.A.
1963	0.812134	0.779097	0.788036	0.8089	0.760773
1964	0.809158	0.778441	0.791198	0.803512	0.757969
1965	0.806346	0.772842	0.787603	0.803023	0.756649
1966	0.806758	0.771095	0.786622	0.803082	0.754513
1967	0.802686	0.773361	0.784275	0.800391	0.752511
1968	0.803102	0.766864	0.77818	0.799391	0.750223
1969	0.798653	0.761398	0.777704	0.799065	0.748929
1970	0.793907	0.766077	0.773095	0.796669	0.74663
1971	0.788109	0.764568	0.770439	0.797164	0.749016
1972	0.787782	0.763891	0.767296	0.794592	0.749871
1973	0.786613	0.760875	0.76301	0.791641	0.750583

Table A8. Full Equilibrium Allen Elasticities: (long run)

(σ^{L}_{KK})

	BELGIUM	FRANCE	ITALY	JAPAN	W.GERMANY
1963	-1.28507	-1.22164	-1.24383	-1.18794	-1.23192
1964	-1.28703	-1.22865	-1.25334	-1.18483	-1.22801
1965	-1.30155	-1.22636	-1.26046	-1.21557	-1.23124
1966	-1.28337	-1.22749	-1.24457	-1.20543	-1.20432
1967	-1.27661	-1.22861	-1.2499	-1.20626	-1.2558
1968	-1.2637	-1.22119	-1.24901	-1.21502	-1.24779
1969	-1.26244	-1.24303	-1.2542	-1.23295	-1.23932
1970	-1.28475	-1.23914	-1.26552	-1.22821	-1.27356
1971	-1.27712	-1.23666	-1.24993	-1.23341	-1.24301
1972	-1.27403	-1.22867	-1.25597	-1.26137	-1.2293
1973	-1.26652	-1.23473	-1.2948	-1.28496	-1.23113

	NETHERLAND	NORWAY	SWEDEN	U.K.	U.S.A.
1963	-1.29993	-1.24372	-1.30817	-1.22679	-1.29502
1964	-1.30171	-1.23937	-1.26289	-1.25736	-1.29859
1965	-1.3094	-1.26367	-1.26953	-1.25584	-1.30012
1966	-1.30114	-1.27222	-1.26093	-1.24372	-1.29986
1967	-1.31481	-1.24863	-1.26305	-1.26595	-1.30532
1968	-1.2977	-1.26678	-1.28373	-1.26184	-1.30279
1969	-1.30732	-1.29757	-1.25863	-1.25038	-1.29348
1970	-1.31921	-1.26427	-1.27357	-1.25501	-1.30676
1971	-1.34377	-1.26901	-1.27477	-1.25314	-1.29779
1972	-1.32269	-1.27343	-1.27853	-1.26055	-1.29246
1973	-1.29805	-1.26807	-1.27998	-1.26288	-1.29008

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