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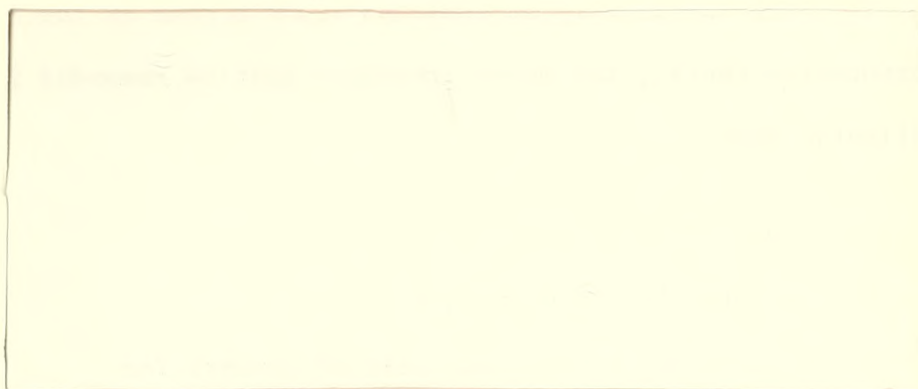
WORKING PAPER
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Price and Non-Price Determination
in the Multiproduct Firm

227-66

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Introduction

The problem of the specification of an optimal level of price, advertising, and other non-price variables has not received the theoretical consideration it deserves. This is especially true in the case of multiproduct firms which sell interrelated products. Although these firms have become prominent in our economy, there has not been a corresponding development in theory to deal with the interdependencies inherent in price, non-price, and output determination in a multiproduct firm. There have been advances in the analysis of cost interdependencies by the use of linear and non-linear programming, but a general formulation to deal with demand and cost interactions has not been developed.¹

One special case of multiproduct price determination has been considered. Martin J. Bailey has presented an analysis of demand and cost interdependency for a monopoly firm selling two products.² Mr. Bailey has proposed a graphical analysis which is based on defining the concept of differential revenue and differential cost for each product. Differential revenue is the change in the total firm's revenues produced by adding one more unit of a product to the firm's market offering. The differential cost is similarly defined as the change in the total firm's costs as the result of producing one more unit of the product. Bailey's graphical procedure

¹ See T. E. Pfouts, "The Theory of Cost and Production in the Multiproduct Firm," Econometrica XXIX (October 1961), pp. 650-658 for a non-linear formulation.

² Martin J. Bailey, "Price and Output Determination by a Firm Selling Related Products," American Economic Review XLIV (March 1954), pp. 82-93.

locates the point where differential cost equals differential revenue for a firm producing two products in a monopoly market. This point satisfies the necessary condition for maximum profit, but consideration of sufficiency and the determination of the maximum maximorum are left to a separate analysis. In the two product case with only price and cost interaction between products, the number of points meeting the necessary conditions is not likely to be large and second order conditions will be workable. If more than two products are in the firm's product line, the graphical analysis becomes very complex and impossible for more than three products. Although restricted to the two product monopoly case, Bailey's was the first analysis which simultaneously considered the essential elements of demand and cost interactions.

Purpose

The purpose of this paper is to analyze the problem of price and non-price determination in a multiproduct firm producing "n" products ($n \geq 2$). Demand and cost interdependencies will be considered and both price and non-price levels will be included as variables. The analysis will be primarily directed toward consideration of oligopoly market structures and will treat competitive and monopoly markets as special cases. It will be assumed that the oligopolistic market is one in which firms are aware that the results of their actions may be dependent upon the reactions of other members in the oligopoly.

This purpose will be implemented by first graphically exploring the nature of intra-firm demand (price and non-price) and cost interactions. These conceptual relationships will then be utilized to mathematically formulate the profit maximization problem for the multiproduct firm. This formulation will incorporate the price and non-price strategies and counter strategies of the members of the industry. Both the feasibility of generating the input for the model and of mathematically optimizing the model will be outlined.

Graphical Analysis of Intra-Firm Interactions

It is instructive to graphically show the effects of interactions. The analytic value of the graphics is small, but the use of simplified graphs makes clear the nature of the interactions and demonstrates the complexity that results from simultaneous consideration of several types of interdependencies. The graphs presented here are simplified for pedagogic reasons by considering a linear demand function, constant marginal costs and constant returns to non-price factors. This implies a positive unitary elasticity for non-price factors. Advertising, sales efforts, and promotions are grouped as one non-price variable "A". With these conventions an average revenue surface "abo" can be shown as in Figure 1.

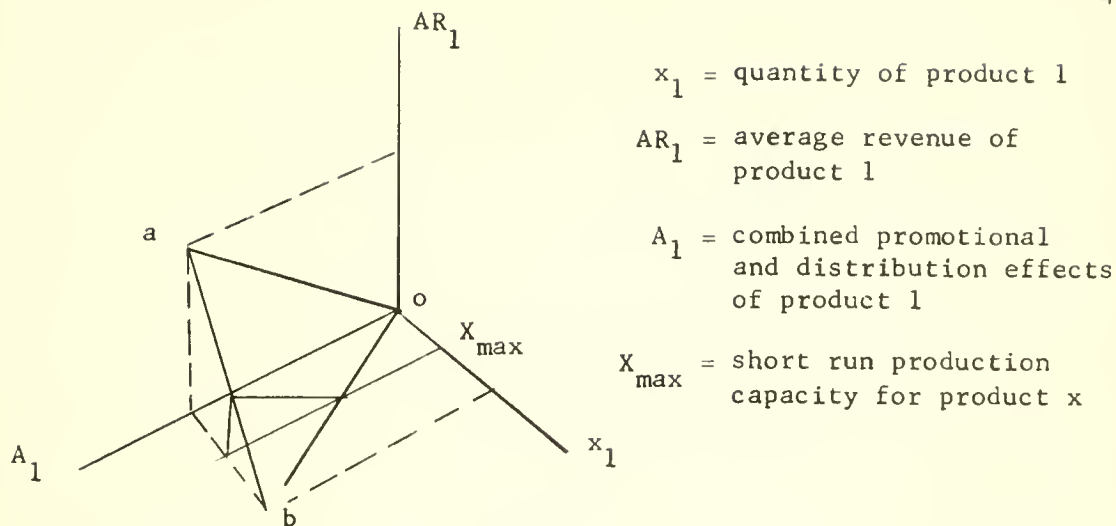


Figure 1

Price and Non-price Average Revenue Surface

This can be reduced to a marginal revenue surface by bisecting the "x" coordinate of the surface "abo" and is shown by surface "a'b'o" in Figure 2.

A marginal cost surface "cdef" can now be added (shown in Figure 2 as constant). The intersection of these two surfaces is a line "MM" along which marginal cost equals marginal revenue. Along the line of intersection "MM" total profit will vary and the optimum, given independence of products, is where profit is largest. See Figure 2. Assuming production of product one is limited in the short run by plant capacity, the maximum profit is shown as point PRF* and the associated non-price expenditure (A^*), price (P^*), and output (x^*) could be determined by returning to the appropriate axis. In this case PRF* lies at the maximum level of production, because profit is increasing under the assumptions postulated.

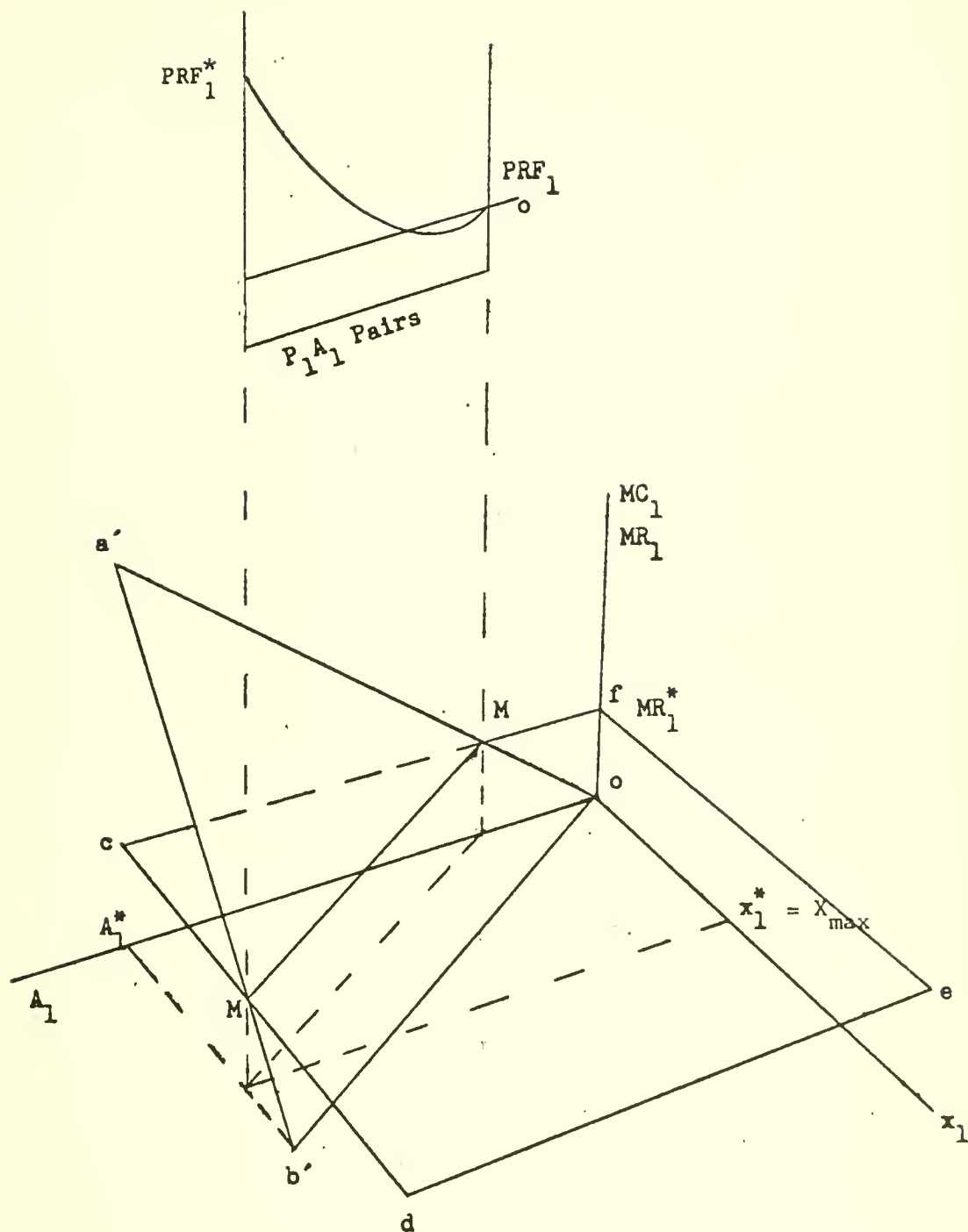


Figure 2. Optimum Profit Curve for Product x

The introduction of interdependency can be shown by describing changes in this product's marginal revenue surface as a result of an alternation of another product's parameters. If the demand of product one is affected by the price of product two, this can be expressed by the price cross elasticity:

$$CP_{12} = \frac{\partial x_1/x_1}{\partial P_2/P_2}$$

x_1 = quantity of product 1
 P_2 = price of product 2

In the constant cross price elasticity case the marginal revenue surface will undergo a parallel shift with changes in P_2 and a new intersection where marginal cost equals marginal revenue will be determined. For continuous changes in the price of product two a profit surface will be generated. See Figure 3. The optimum pricing is where the sum of the profit of product one and two is greatest while constraints upon the firm are met.

The interaction effects of the non-price expenditures of product two on product one can be expressed by the non-price cross elasticity:

$$CA_{12} = \frac{\partial x_1/x_1}{\partial A_2/A_2}$$

x_1 = quantity of product 1
 A_2 = non-price level for product 2

In the simple constant cross non-price elasticity case, the marginal revenue surface will rotate about the origin with changes in A_2 and again will generate a new profit surface.

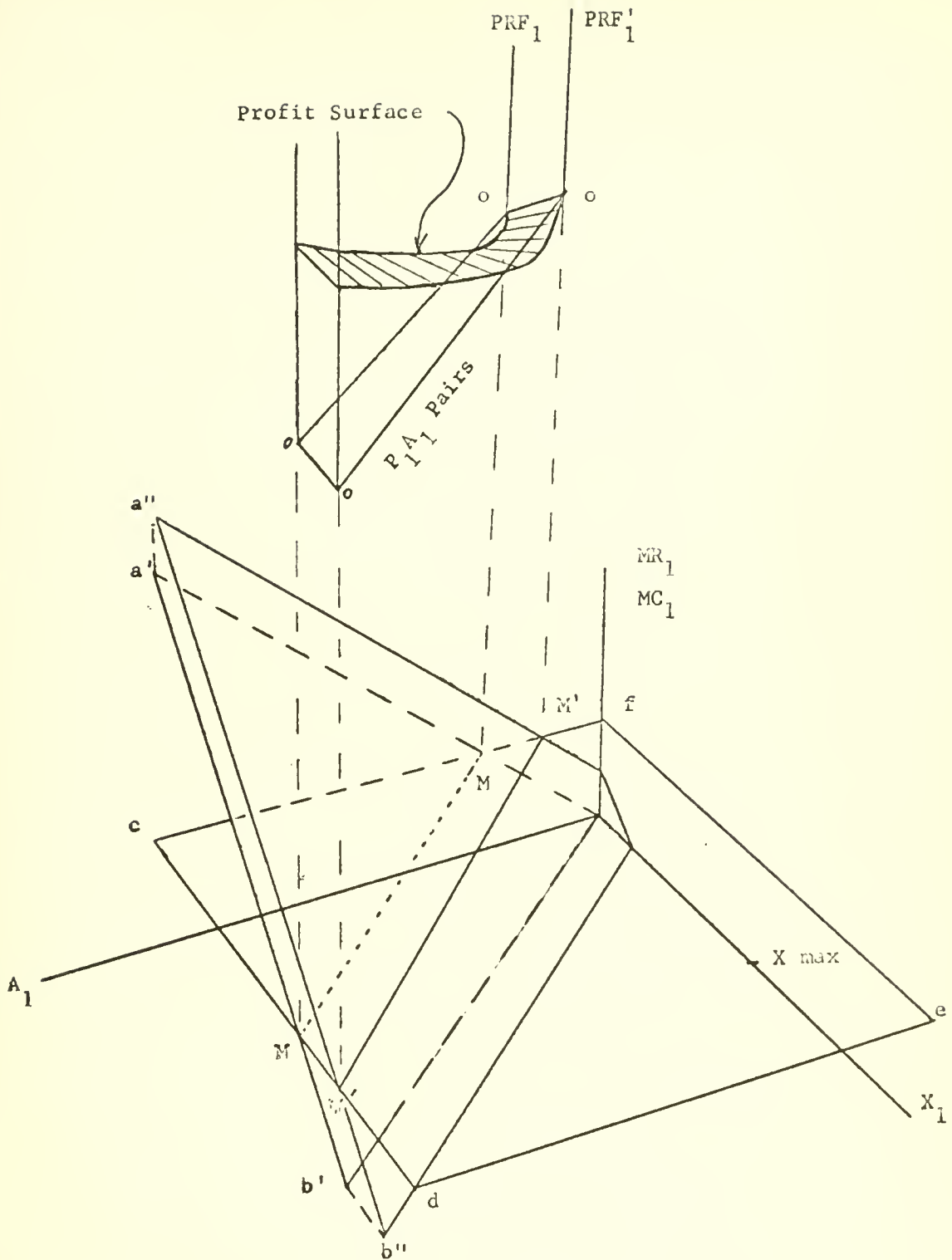


Figure 3. Profit Surface for Product x Generated
by Price Interdependency

Cost interactions can be expressed by the cross cost elasticity:

$$CC_{12} = \frac{\partial MC_1 / MC_1}{\partial x_2 / x_2}$$

x_2 = quantity of product 2
 MC_1 = marginal cost of product 1

In the simple constant cross elasticity case a parallel shift in the marginal cost plane of product one occurs as quantities of product two are changed. This again will generate a profit surface. When cost, price, and demand interactions are allowed, a complex set of profit surfaces is generated. The profit optimizing situation is the point at which the sum of the profit of product one and two is a maximum. This point cannot be determined graphically, even with the simplified assumptions of the presentation here. The total profit function is very complex and becomes more complex when marginal costs are not constant, demand is not linear, and the elasticity of non-price expenditure is not positive and unitary. A large number of points will meet the necessary conditions even in this two product case.

In addition to these internal complexities of product interdependency, the situation is further complicated by the fact that the monopoly curves indicated in this section would probably not be the ones appearing to an oligopoly firm. In the oligopoly case the curves would be dependent upon the price and non-price policies of the other firms in the industry. The demand surfaces may have "bends" that reflect the adaptive strategies of competitors. Bends could be expected when competitors follow price decreases and non-price increases, but remain

stationary in terms of price increases and non-price decreases. Many other strategies are possible. Competitors may be very sensitive to price changes, but be indifferent to changes in the firm's non-price variables. Some firms may not react to price or non-price changes, but may instead react to changes in the levels of their profits, sales, or market share. The choice of strategy is an important adjunct to the price and non-price decision for an oligopoly firm.

Mathematical Formulation of a Multiproduct Firm

Although the price, non-price, and output determination problem cannot be solved graphically, the cross elasticity concepts can be utilized in a mathematical formulation of the multiproduct firm's decision.

The starting point for the mathematical model is the representation of the demand for one product offered by the firm. In general, let the demand for product "j" be as follows:

$$x_j = f(\text{reference sales forecast, industry price and non-price effects, oligopoly effects, interaction effects})$$

The reference sales forecast represents the estimate of the demand for the next time period given some level of price and non-price variables by the firm and competitors. This estimate reflects estimated aggregate economic and monetary conditions, as well as specific industry and company predictions.

The combined effects of price and non-price activity of all firms in an industry may cause a shift in the reference sales forecast. This may be formulated as follows:³

$$a P_{jI}^{EPI} A_{jI}^{EAI}$$

a = scale constant

P_{jI} = industry price level for product j

A_{jI} = industry non-price level for product j

EPI = industry price elasticity for product j

EAI = industry non-price elasticity for product j

The proportion of the total industry sales a member of the oligopolistic industry achieves can, as suggested by Philip Kotler, be represented by the ratio of the firm's price and non-price effectiveness to the total industry effectiveness.⁴

$$\text{oligopoly effect for product } j \text{ in firm one} = \frac{P_{1j}^{EPI} A_{1j}^{EAI}}{\sum_i P_{ij}^{EPI} A_{ij}^{EAI}}$$

³ This is similar in form to the Cobb-Douglas production function. See Paul H. Douglas, Theory of Wages (New York: Macmillan 1934). See Philip Kotler, "Competitive Strategies for New Product Marketing Over the Life Cycle," Management Science XII (December 1965) B-104-B-119.

⁴ Kotler, *ibid.*, p. B-107.

P_{ij} = price of product j by firm i

A_{ij} = non-price level for product j by firm i

E_{Pi} = price elasticity for firm i and product j

E_{Ai} = non-price elasticity for firm i and product j

In this expression the elasticities are subscripted to allow the possibility of differentiated products. Given this expression the effects of various strategies and counter-strategies can be related to the share of market a firm will receive. For example, if firm one is the price leader for a homogeneous product market, a lowering of price by the firm would be followed by other firms with no change resulting in the share of market. Industry effects as described in the previous section may be produced however. Given a strategy and set of counter-strategies, this expression will predict market share changes for changes in price and non-price variables.

Demand interdependencies may originate because of substitution or complementarity effects of one product with other products offered by the firm under consideration, or with products offered by other firms. The interdependencies of product M relative to product j can be expressed by:

$$b P_{iM}^{CPjM} A_{iM}^{CAjM}$$

b = scale constant

P_{iM} = price of product M by firm i

A_{iM} = non-price level for product M by firm i

$CPjM$ = cross price elasticity for product j and M

$CAjM$ = cross non-price elasticity for product j and M

This expression estimates the changes in the sales of one product as the result of changes in parameters of interrelated products offered by the firm or other firms.

Combining these expressions the demand for product j is:

$$x_j = \bar{x}_j a_j^{EPI} A_j^{EAI} \left[b_j \prod_M P_{iM}^{CPjM} A_{iM}^{CAjM} \right] \left[\frac{P_{1j}^{EPI} A_{1j}^{EAI}}{\sum_i P_{ij}^{EPI} A_{ij}^{EAI}} \right]$$

\bar{x}_j = reference sales estimate

M = number of products sold by firm

other notation as previously stated.

Given constant direct and cross elasticities, this equation represents the demand for one product of a multiproduct firm in an oligopolistic market. It should be noted that this formulation could be extended to include more than one non-price variable by the specification of the appropriate cross elasticities.

The total revenue for the firm is:

$$TR = \sum_j P_j x_j$$

The variable cost of producing one of the firm's goods could be expressed as:

$$TVC_j = AVC_j(x_j) \prod_M (x_M)^{CCjM}$$

TVC_j = total variable cost of producing product j

AVC_j = average variable cost function for product j , if produced independently of other products

x_j = quantity of product j produced

x_M = quantity of product M produced

$CCjM$ = cross cost elasticity of product j and M ($M \neq j$)

The total cost for the firm is:

$$TC = \sum_j TVC_j - FC$$

where FC = fixed costs. The profit is obviously total revenue less total costs.

The problem for the firm in the short run is to maximize the total profit subject to existing production constraints.

Determination of Optimum Price Level and Non-Price Expenditure

The profit function formulated in this paper does not possess nice convexity or concavity properties, so the utilization of standard non-linear programming routines is impossible.⁵ Although the function could be theoretically maximized by Lagrangean analysis, the practical problem of solving the set of partial differential equations in this case eliminates it as a possible solution method. It is also true that the optimum points will satisfy the Kuhn-Tucker conditions, but the location of these points for this function is not efficient with existing gradient methods. Developments in non-linear programming are possible and probable, so it may be anticipated that this function will bend under future analytical approaches.

⁵Even if the oligopoly effects are omitted, the function is only simplified to a sum of convex and concave functions.

The powerful characteristics of analytical solutions being unavailable, simulation appears to be a likely resort. In this application the method implies a trial and error search and solution method.

A first approach would be to test an exhaustive set of discrete price and non-price levels for each product offered by the firm. With "n" products this would indicate (K^{2n}) trials for the exhaustive set, where K = number of values of each variable to test. This could be a large set even for a relatively small number of products in the firm's product offering. For example, if ten values for price and non-price variables for each of four products were to be tested in all combinations, the number of trials would be 10^8 or one hundred million trials. Preliminary tests of this model indicate that a large size computer can run two thousand trials in one minute, so that in general, running the exhaustive set is outside practical consideration with the computer of today.

Compiling an exhaustive set may not be necessary. Search routines may be instituted to reduce the number of trials. For example, a two stage search procedure which first locates the rough optimum by trials of all combinations of the quartiles in the range and then tests all the combinations of the variables in best rough quartile range could reduce the calculation burden by a factor of about one thousand. With this search procedure the four product case could be tested on a ten item range for each variable in about fifty minutes on a high speed

computer. More efficient search methods are available and new routines may be found. For example, trial problems indicate that the eight variable problem can be solved in five minutes by the use of the Best Univar search technique developed by Merrill M. Flood and Alberto Leon.⁶ The advent of higher speed computers may make this approach even more feasible. Certainly the proposed model is relevant when a reasonable ($n \leq 4$) number of products are considered.

If one is compelled to resort to a trial and error search routine, certain advantages can be gained at a small computing cost. In the case at hand the previous formulation has assumed that all direct and cross elasticities be constant. When simulation is used, this assumption can be relaxed by the use of "response functions". A response function measures the proportionate change in the demand of the product as the result of an absolute change in value of a product parameter. Response functions may be non-linear and discontinuous. Each Y^{EY} term can be replaced by a function Y^P which specifies the proportionate change in the sales of the product as a result of a change in the parameter Y . This formulation requires the response functions to be independent, but assumption of constant elasticities has been removed.

⁶ Merrill M. Flood and Alberto Leon, A Generalized Direct Search Code for Optimization, Preprint 129, Mental Research Institute (University of Michigan, June 1964). The time required depends upon the starting point of the search and the desired accuracy of the solution.

In addition to a flexibility of functional forms, simulation would also allow the possibility of testing additional constraints. The search routine could contain tests to assure that an advertising budget is not exceeded or to assure that the distribution system is not utilized beyond capacity.

Input Considerations

The feasibility of the application of this model rests upon the ability to generate meaningful input, as well as on the presence of a practical solution method. The elasticities could be estimated on a subjective basis that reflects the decision makers best judgment. This approach might be justified since the decision must be made and if the model is not used, a much simpler and perhaps less accurate decision procedure would be used. Subjective inputs, however, should be used only after all empirical information relating to the problem has been considered.

Input for the present model can be generated by statistical regressions on empirical data. The industry elasticities and cross elasticities could be estimated from a regression of industry sales on past industry price and non-price variables. The use of logarithms would make the regression linear and usual econometric procedures could be used to estimate the constants -- the elasticities and cross elasticities.⁷

⁷ See Ronald E. Frank and William F. Massy, "Short Term Price and Dealing Effects in Selected Market Segments," Journal of Marketing Research II (May 1965) 171-185 for an application of the technique to market data.

A similar procedure could not be used to determine individual firm elasticities since logarithms of the market share term would not produce a linear equation. The competitive price and non-price elasticities could be estimated by a direct search routine to minimize the associated Chi Square statistic.⁸ The competitive input is completed with a formalization of the reaction functions of the competing firms. These might be obtained by examining the past competitive responses to price and non-price changes. The proposed model could be run for various strategies and counter strategies. The resulting payoff matrix could then be analyzed by game theory techniques.⁹ Successive applications of the model for a number of future periods with the estimated reaction functions could yield valuable information concerning the existence of an equilibrium, the rate of convergence to the equilibrium, and the stability of the equilibrium, if it exists. As a theoretical aside, repeated passes through the model might be useful in evaluating current notions of oligopoly competition, collusion, and warfare.

The cost cross elasticities could be approximated by examining the cost records of the firm for various quantity mixes or by formulating a linear programming model to minimize the cost of producing specified

⁸See David B. Montgomery, "A Probability Diffusion Model of Dynamic Market Behavior," MIT Working Paper 205-66 (May 1966) 128-141, for an application of this technique.

⁹Kotler, *ibid.*

quantities of the firm's products. Successive runs of the cost minimization model and a regression procedure could yield estimates of the cost cross elasticities.

The examination of past data could be supplemented by directed studies to measure the perceived interrelationships between products. Such a procedure has been developed by Barnett and Steffle.¹⁰ If this experimental procedure or the regression approaches that have been outlined in this section are not practical in specific situations, subjective estimates might make the model workable.

A Note on the Special Cases of Monopoly and Competitive Markets

If the multiproduct firm faced a monopoly market the above formulation can be used as given except the share of market would, by definition of monopoly, be unitary. If one conceives of a monopoly as existing only when the product is an independent demand unit, the interaction terms would be neglected and the model would collapse to the classical monopoly decision case with the addition of non-price decisions.

If the firm faced a competitive market, the model presented here would add nothing to the classical analysis since in the competitive market no price or non-price decisions are made. The multiproduct

¹⁰ Norman L. Barnett and Volney J. Steffle, "An Empirical Approach to the Development of Advertising New Products," Presented at the 1966 Fall Conference of the American Marketing Association.

firm in this case must only decide what quantities to produce. Furthermore demand interdependencies in the purely competitive case are irrelevant to the output decision since by definition the firm could sell as much of each product as desired. Cost interactions may still persist, but then a linear programming model to maximize profit given prices would be a more efficient approach than the model proposed in the previous sections.

The model proposed in this paper may be of value for the multiproduct monopolist, but the multiproduct firm facing competitive markets would find the formulation largely irrelevant.

Summary

This paper has proposed a theoretical model for price and non-price determination in a multiproduct firm. After graphically describing the underlying demand and cost interdependency relationships, mathematical expressions of the interrelated demand and cost functions were developed. Approaches to the estimation of the equation parameters and competitive responses were outlined. Given the demand, cost, and competitive relationships, the model determines the profit implications of various price and non-price levels. The maximization of the firm's profits in the proposed model cannot be analytically specified, so trial and error search routines were suggested to make the optimum price, non-price, and output determination. With existing search routines and this model, meaningful multiproduct firm decision problems could be attacked.

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