A Quantitative Approach to

New Product Decision Making

215-66

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ABSTRACT

The decision to either add, to reject, or to investigate more fully a new product proposal is one of the most important problems faced by businessmen. The factors surrounding the decision can be mathematically considered by four sub-models in the areas of demand, cost, profit, and uncertainty. The demand model is structured to consider life cycle, industry, competitive, and product interdependency effects and will admit non-linear and discontinuous functions. A cost minimization model is joined to the demand model to formulate a constrained profit maximization problem. The optimization is accomplished by the use of dynamic programming. The final decision is based on the businessman's criterion in combining uncertainty and the rate of return on investment.

Mathematical models and quantitative techniques have found an increasing number of applications as tools for management decision making. They are most useful to management in areas where a high degree of complexity forces an almost complete reliance upon subjective reasoning. One of the most difficult and complex decisions businessmen face is the new product decision. At some stage in a new product's development the executive must decide if the product is to be introduced, if it is to be rejected, or if more study is needed before a decision can be reached. A nebuli of complex factors relating to profit, investment, technical feasibility, and uncertainty surround the decision. This paper develops a mathematical model which considers the significant factors surrounding the new product decision and then recommends the adoption, the rejection, or further investigation of the product. The total model is based on four sub-models in the areas of demand, cost, profit, and uncertainty. After the models have been developed and the decision environment has been accurately described, dynamic programming is utilized as the basis of the solution method. The central emphasis in the model building is the creation of a realistic model that can be used by businessmen as a tool for new product decision making.

MODELING THE DEMAND FOR A NEW PRODUCT

The modeling of the demand for the new product can be begun by considering the estimated quantity to be sold in each year of some time period. This estimate of the quantity to be sold in each year is called

the life cycle of the product. The estimated life cycle is dependent upon a number of marketing factors. The estimate would be different if different prices were established. In fact, the life cycle estimate is supplied with a complete marketing program of price level, advertising expenditure, and distribution effort in mind. This basic program is called the reference marketing program and the corresponding estimate of the quantities to be sold over time is called the reference life cycle. Given these estimates, a curve can be mathematically fitted to the data. For example a Gompertz function may be a reasonable approximation of the life cycle. If the "S" shape is not a good approximation to the life cycle, other functional equations could be tried to get the best estimate of the input data.

If the reference price level of the new product were changed, the estimate of the quantity to be sold would change. These changes could be noted by a term of the form:[1]

$$k\bar{X}_{1t}(P_1^{EP})$$

$\bar{X}_{1t}$ = reference life cycle estimate for product one in year "t"

P_1 = price of product one

EP = price elasticity

k = scale constant

This form requires that the price elasticity be constant. Allowing the elasticity to be a function of price is a tempting alternative, but it results in an inconsistant representation of the demand.[2] Using the exponential form and requiring the elasticity to be constant has economic

implications which may not be reasonble in actual practice. A general form which considers non-linear and discontinuous price-quantity relationships can be formulated. This is based on the concept of a "response function." The response function measures the proportionate changes in the level of the reference estimate as a result of an absolute change in the price level. For example, if the price-quantity relationship is:

$$X_{1t} = a - bP_{1t}^2 + cP_{1t}^3$$

X_{1t} = quantity sold of product one in year "t"

a,b,c = constants in year "t"

P_{1t} = price of product one in year "t"

the price response function for product one in year "t" is:

$$PR_{1t} = a/\bar{X}_{1t} - bP_{1t}^2/\bar{X}_{1t} + cP_{1t}^3/\bar{X}_{1t}$$

$\bar{X}_{1t}$ = reference life cycle for product one in year "t"

The response function always equals one when the price equals the reference price level. The quantity sold in any year is

$$X_{1t} = \bar{X}_{1t}PR_{1t}$$

Although this relationship appears to be self-evident, the strength of the formulation lies in the fact that it can be extended to include advertising and distribution responses. For example:

$$X_{1t} = \bar{X}_{1t} PR_{1t} AR_{1t} DR_{1t}$$

$\bar{X}_{1t}$ = reference quantity of product one in year "t"

PR_{1t} = price response function for product one in year "t"

AR_{1t} = advertising response function for product one in year "t"

DR_{1t} = distribution response function for product one in year "t"

This equation reflects the changes in the quantity sold as a result of changes in the price, advertising, or distribution level, but does not require these relationships to be defined by any particular mathematical form.

The level of the variables is important, but the total response to a price change in a given year will depend on the level and sequence prices in the previous years. To account for the effects of various sequences, lagged response functions can be added to the equation. The lagged response functions measure the proportionate changes in the reference quantity sold in a year as a result of the absolute level of the price in previous years.

In addition to the dynamic effects of sequences of variables, another cumulative effect may be a consideration in new product marketing. This is the effect of introductory campaigns. These initial spurts of promotion are designed to increase the rate of diffusion of the new product innovation. This can be considered in the demand equation by specifying a shift in the reference life cycle. For example, in the simple Gompertz case, the dynamics could be incorporated by the equation:

$$\bar{X}_{1t} = G^{B^{t+t_s}}$$

t_s is the shift in the life cycle and it is a function of the size of the initial promotional campaign.

The aggregate demand for the new product can now be described as:

$$X_{1t} = \bar{X}_{1t} PR_{1t} LPR_{1t} LLPR_{1t} AR_{1t} LAR_{1t} LLAR_{1t} DR_{1t} LDR_{1t} LLDR_{1t}$$

PR_{1t} = industry price response function for product one in year "t"

LPR_{1t} = one year lagged price response function for product one in year "t"

$LLPR_{1t}$ = two year lagged price response function for product one in year "t"

AR_{1t} = advertising response function for product one in year "t"

LAR_{1t} = one year lagged advertising response function for product one in year "t"

$LLAR_{1t}$ = two year lagged advertising response function for product one in year "t"

DR_{1t} = distribution response function for product one in year "t"

LDR_{1t} = one year lagged distribution response function for product one in year "t"

$LLDR_{1t}$ = two year lagged distribution response function for product one in year "t"

Additional factors can be added to this chain after they have been described by response functions. This form is very flexible and allows the consideration of non-linear and discontinuous input relationships.

The total industry sales described above are divided among the companies in the industry on the basis of the competitive behavior of the firms. It seems reasonable to assume that the market share on the basis of the relative marketing effectiveness of each firm in the industry. If all firms entered at the same time, the market share for firm one is:[3]

$$MS_{ilt} = \frac{PR_{1lt} AR_{1lt} DR_{1lt}}{\Sigma_{i=1}^{m} PR_{ilt} AR_{ilt} DR_{ilt}}$$

MS_{ilt} = market share for firm "i" in product market one in year "t"

PR_{ilt} = price response function for firm "i" and product one in year "t"

AR_{ilt} = advertising response function for firm "i" and product one in year "t"

DR_{ilt} = distribution response function for firm "i" and product one in year "t"

m = number of firms in the industry

If competitors enter at the different times this equation would not be reasonable because it would not account for any competitive lead the introductory firm may have developed. To account for the competitive advantages gained by early entry, the market share can be expressed as:

$$MS_{ilt} = \frac{\Sigma_{t=1}^{c} e_{1t} PR_{1lt} AR_{1lt} DR_{1lt}}{\Sigma_{t=1}^{c} \Sigma_{i=1}^{m} e_{it} PR_{ilt} AR_{ilt} DR_{ilt}}$$

e_{it} = efficiency of firm "i" in year "t"

c = number of years which are cumulated

The summation is over some period of years and e_{it} reflects the efficiency of each firm's marketing effort in a given year. This equation indicates that the introductory firm has a time lead and that if the competitor matches his marketing program, he will not receive a full proportion of the market until he has achieved full efficiency. The market share a firm receives also depends upon its competitive strategy. The introductory firm may have a non-adaptive strategy as in the case of price leader or it may follow an adaptive strategy based on sales, market share, or profits. These alternate strategies and counterstrategies can be tested and a matrix of rewards could be generated so that game theory could be utilized to select the best strategy.[4]

The sales of the new product will be affected by interaction between competitors, but the new product may also be affected by other products offered in the firm's product line. These demand interdependencies may be significant. The new product may reduce the sales of other products or it may increase the demand for other products. The interaction effects may be based on price, advertising, or sales effort interdependencies. These can be incorporated into the model by again utilizing the concept of response functions, but now "cross response functions" could be utilized. These measure the proportionate change in the reference quantity of one product as a result of an absolute change in the level of a parameter of another product. In this way the cross response relationships

can be added to the chain of response functions to specify the demand for the new product. The complete equation for the new product is:

$$X_{ijt} = \bar{X}_{jt}[PR_{jt}LPR_{jt}LLPR_{jt}AR_{jt}LAR_{jt}LLAR_{jt} \cdot DR_{jt}LDR_{jt}LLDR_{jt}] \cdot$$

$$\left[\frac{\Sigma_{t=1}^{c} PR_{ijt}AR_{ijt}DR_{ijt}}{\Sigma_{t=1}^{c}\Sigma_{i=1}^{m} e_{ijt}PR_{ijt}AR_{ijt}DR_{ijt}} \right] \cdot$$

$$[CPR_{i1kt}CAR_{i1kt}CDR_{i1kt} \cdots CPR_{i(j-1)kt}CAR_{i(j-1)kt}CDR_{i(j-1)kt} \cdot$$

$$CPR_{i(j+1)kt}CAR_{i(j+1)kt}CDR_{i(j+1)kt} \cdots \cdot CPR_{inkt}CAR_{inkt}CDR_{inkt}]$$

x_{ijt} = quantity of good "j" sold by firm "i" in period "t"

$\bar{X}_{jt}$ = reference level of industry sales for product "j" in year "t"

PR_{jt} = industry price response function for product "j" in year "t"

LPR_{jt} = one year lagged price response function for product "j" in year "t"

$LLPR_{jt}$ = two year lagged price response function for product "j" in year "t"

AR_{jt} = industry advertising response function for product "j" in year "t"

LAR_{jt} = one year lagged advertising response function for product "j" in year "t"

$LLAR_{jt}$ = two year lagged advertising response function for product "j" in year "t"

DR_{jt} = industry distribution response function for product "j" in year "t"

LDR_{jt} = one year lagged distribution response function for product "j" in year "t"

$LLDR_{jt}$ = two year lagged distribution response function for product "j" in year "t"

PR_{ijt} = price response function for firm "i" on good "j" at time "t"

AR_{ijt} = advertising response function for firm "i" on good "j" at time "t"

DR_{ijt} = distribution response function for firm "i" on good "j" at time "t"

e_{ijt} = efficiency of firm "i's" marketing program for product "j" in year "t"

CPR_{ijkt} = cross price response of product "k's" price on product "j" in firm "i" in period "t"

CDR_{ijkt} = cross distribution response of product "k's" price on product "j" in firm "i" in period "t"

CAR_{ijkt} = cross advertising response of product "k's" on product "j" in firm "i" in period "t"

Similar equations could be specified for the other products in the firm's product line. When the total profit generated by these products is calculated, the new line profit is specified. If the profits of the product line without the new product are estimated and deducted from the

new line profits, the change in total line profits is generated. This change is called the "differential profit" and it is a measure of the profits generated by adding the new product when demand interdependencies are considered.

MODELING THE COST STRUCTURE FOR A NEW PRODUCT

If a new product is produced and distributed in a system independent of other products, its cost function may be directly specified in an equation. When the product shares common production or distribution facilities with other products in the line, the cost structure is more complex. When cost interdependencies are present the problem is to minimize the cost of producing a specified product line. Given production requirements for each product in the line, the problem is to minimize:

$$\Sigma_{j=1}^{q} c_j I_j$$

subject to

$$\Sigma_{j=1}^{q} a_{ij} I_j \geq b_i \qquad \text{and} \qquad I_j \geq 0$$

where

c_j = cost per unit of input factor "j"

I_j = amount of input factor "j" utilized

b_i = constraint on input values and quantities of goods produced

a_{ij} = technical production relationships

q = number of input factors

This can be more fully described in the dyadic form in figure one. This is the usual programming format. If the unit input costs are constant,

FIGURE 1.

Dyadic form of cost minimization model

	INPUT FACTORS				
	I_1	$I_2 \ldots I_p$	$I_{p+1} \ldots I_j \ldots I_n$		
Product one	a_{11}	$a_{12} \ldots a_{1p}$	$a_{1\,p+1} \ldots a_{1j} \ldots a_{1n}$	$\geq x_1$	Production Requirements
Product two	a_{21}	$a_{22} \ldots a_{2p}$	$a_{2\,p+1} \ldots a_{2j} \ldots a_{2n}$	$\geq x_2$	
.	.	. . .	. . .	.	
.	.	. . .	. . .	.	
.	.	. . .	. . .	.	
Product "m"	a_{m1}	$a_{m2} \ldots\ldots\ldots$	$\ldots\ldots\ldots a_{mn}$	$\geq x_m$	
Machine one	1	0....0	0......0....0	$\leq q_1$	Input Limitations
Machine two	0	1....0	0......0....0	$\leq q_2$	
.	.	. . .	. . .	.	
.	.	. . .	. . .	.	
.	.	. . .	. . .	.	
Machine "p"	0	0....1	0......0....0	$\leq q_p$	
Salesman one	0	0....0	1......0....0	$\leq q_{p+1}$	
.	.	. . .	. . .	.	
.	.	. . .	. . .	.	
.	.	. . .	. . .	.	
Salesman "n"	0	0....0	0......0....1	$\leq q_n$	
	c_1	$c_2 \quad c_p$	$c_{p+1} \ldots c_j \ldots c_n$		
	Costs per unit of input				

linear programming computational routines can be used to solve the problem. If the unit input costs are not constant, piecewise linear programming can be used to solve the non-linear problem.

By specifying various production requirements in terms of the minimum amounts of the new and old product to be produced, the program will calculate a minimum variable cost matrix. The program will yield the minimum total variable cost for producing given amounts of products. Successive runs will produce a total variable cost function. The total cost function would be:

$$TC = TVC + TFC + \Sigma_{j=1}^{P} A_j$$

TVC = variable cost function generated by linear programming routine

TFC = total fixed costs

A_j = advertising expenditure on product "j"

p = number of products in the firm's product line

MODELING THE PROFIT FOR THE NEW PRODUCT

The demand model and cost model can be combined to specify the differential profit. Assuming that profit maximization is the objective of the firm in introducing this product, the problem is to maximize the differential profits generated by the new product subject to the constraints on the product and the firm. Constraints on the profit maximization will exist in each year. The productive plant capacity, the size of the sales force, the advertising budget, or the number of trained personnel may be some of the limitations in each year of the planning period.

The maximization of the total differential profit over the planning period can be visualized as a discrete multistage decision process. In each year product parameters are given, and based on these parameters a differential profit for that year can be specified by the combination of the demand and cost models. The method of combination is shown in Figure Two. The total revenue and total costs for the new line are calculated given the product parameters. After the variables have been tested to see that the constraints are satisfied, the old line profits are deducted from the new line profits to determine the differential profits. The differential profit for each year is discounted at the corporation rate of return and the total differential profits is gained by summing the yearly rewards.

To find the optimum price level and sequence of prices for the new product over the planning period the problem is to:

MAX: $TDDP = \Sigma_{t=1}^{pp} DDP_t$

where $DDP_t = f(S_t, d_t)$

and

$$S_1 = 0$$

$$S_2 = P_1$$

$$S_3 = P_1, P_2$$

$$S_4 = P_2, P_3$$

$$S_t = P_{t-2}, P_{t-1}$$

pp = number of time periods in the planning period

TDDP = total discounted differential profit

DDP_t = discounted differential profit in year "t"

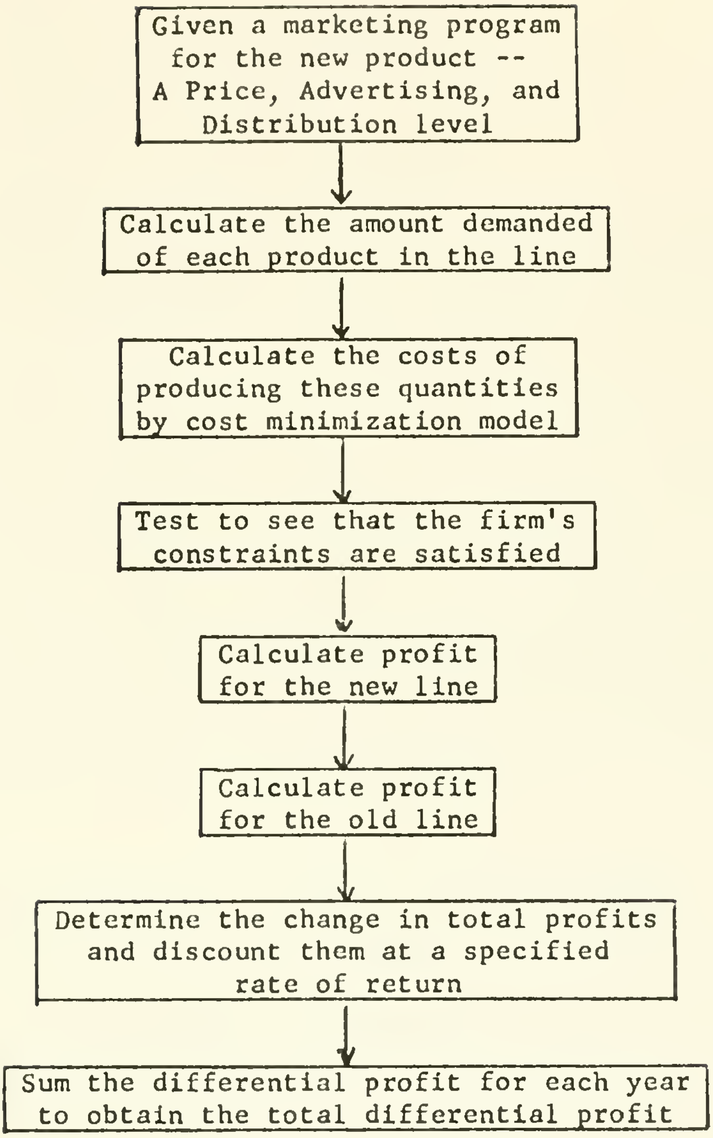


Figure 2

PROFIT MODEL

s_t = state of system at period "t"

d_t = decision at period "t"

P_t = price established in year "t"

This deterministic process can be solved by the upstream algorithm of dynamic programming. The backward induction procedure could then be repeated for various levels of advertising and distribution to specify the optimum value of the parameters of the new product.[5]

The output of the search program is the optimum levels and sequences for the new product parameters of price, advertising, and distribution. The optimization can be re-run with various levels of production constraints and alternate competitive strategies to determine the opportunity costs of specified policies and constraints.

MODELING THE UNCERTAINTY ASSOCIATED WITH A NEW PRODUCT

The maximum differential profit of new products is an important parameter in the new product decision, but it must be balanced against the uncertainty associated with the product proposal. Since the new product probably will amplify or compensate for profit fluctuations in the existing products offered by the firm, the uncertainty interdependencies should be considered in the decision process. This interdependency can be approached by considering the "differential uncertainty" connected with the new product. The differential uncertainty is the change in the total line uncertainty. Using the variance of the new and old line profits as surrogates for uncertainty, the differential uncertainty is:

$$DU^2 = V' + V - 2\ COV(Pr, Pr')$$

DU = differential uncertainty

V' = variance of new line profits

V = variance of old line profits

COV(Pr,Pr') = covariance of new and old line profits

$= E[(Pr-E(Pr)) \cdot (Pr'-E(Pr'))]$

Pr = old line profits

Pr' = new line profits

E = expected value operator

The covariance term will be significant since the new line includes all or some of the old line products. The total variance (V) of a group of items can be shown by a variance-covariance matrix:

$$V = \begin{vmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \cdots\cdots \sigma_{1m} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \cdots\cdots \sigma_{2m} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \cdots\cdots \sigma_{3m} \\ \vdots & \vdots & \vdots \qquad\quad \vdots \\ \sigma_{n1} & \sigma_{n2} & \sigma_{n3} \cdots\cdots \sigma_{nm} \end{vmatrix}$$

The total variance is:

$$V = \Sigma_{i=1}^{n} \Sigma_{j=1}^{m} a_j\, a_i\, \sigma_{ij}$$

a_i, a_j = proportional commitment to "i" and "j", $\Sigma a=1$

(The proportional commitments are the proportion of profits contributed by each product.)

σ_{ij} = covariance of "i" and "j" $= E[(y_i - \mu_i) \cdot (y_j - \mu_j)]$

If each product's profit is normally distributed, the variance can be expressed as:[6]

$$V = \sum_{i=1}^{n} a_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} a_i a_j \sigma_{ij}$$

The determination of the direct variance of profits (σ_i^2) can be analytically determined by combining the uncertainty of the quantity and cost estimate. The variance of the distribution of the profit is the joint distribution of the expression:

PROFIT = P·X - C·X

P = price

C = cost

x = quantity

The variance of this joint profit distribution is:

$\sigma^2_{Profit} = \sigma^2_{Px} + \sigma^2_{xC} - 2\,COV(Px,xC)$

σ^2_{Px} = variance of joint distribution of price times quantity

σ^2_{xC} = variance of joint distribution of cost times quantity

COV(Px,xC) = covariance of the two joint distributions of price times quantity and cost times quantity.

$COV(Px,xC) = E[(Px - E(Px)) \cdot (xC - E(xC))]$

The mean of the cost distribution is the expected value of the joint distribution.

$E(xC) = E(x)E(C) + COV(x,C)$

E = expected value operator

COV(x,C) = the covariance of x and C, which is $E[(x - E(x)) \cdot (C - E(C)]$

If the quantity estimates and cost estimates are independent, the covariance term is zero and the mean of the joint distribution is simply

the product of the individual means. The mean of the total revenue distribution (Px) necessary for the calculation of the covariance is simply $P \cdot E(x)$, since price is specified and treated as certain.

The variance of the joint cost distribution is:[7]

$$\sigma^2_{xC} = \sigma^2_x \sigma^2_C + [E(x)]^2 \sigma^2_C + [E(C)]^2 \sigma^2_x$$

Substituting price for cost in this formula and remembering that price is considered certain (i.e. $\sigma^2_p = 0$), the variance for the total revenue distribution is: $\sigma^2_{Px} = p^2 \cdot \sigma^2_x$. These variances can now be substituted into the joint profit equation to calculate the profit variance in a given time period. Once the means and variances of profit are determined for each year, they must be combined to yield an overall mean and variance of the total profit for the period under consideration. The sum of the means of each year when discounted will reflect the best estimate of total profit. In dealing with the variances in the demand model, complications are introduced by the fact that the entrance of competition is distributed along the time dimension. The combined variance must be calculated for each possible competitive entrance time. These combined variances when weighted by the probability of competition entering at each specific time will give the aggregate total variance of profit. The combined variance, given a specific entrance time for competition and assuming independence of variances, is the sum of the individual yearly variances weighted by the fraction of the discounted profit contributed in that year.

The covariances (σ_{ij}) are equally as important as the variances. These can be determined by using the procedure suggested by Harry Markowitz or by other subjective methods.[8] After the specification of the variances and covariances has been accomplished, the differential uncertainty can be calculated as suggested above when given normal or lognormal distributions for all parameters. If the normal or lognormal distributions are not reasonable approximations of the input distributions, a Monte Carlo analysis could be carred out to determine the distribution of differential profits about the mean estimate of differential profit.

MODELING THE DECISION FOR THE NEW PRODUCT

The differential profit and differential uncertainty must be combined to indicate whether the new product should be introduced (GO decision), should be rejected (NO decision), or should be investigated more fully (ON decision). The risk and return plane must be divided into GO, ON, and NO areas. The GO, ON,and NO areas can be defined by two methods:

(1) Define the total risk-return utility preference map and then by specifying a minimum utility for GO and maximum for NO divide the map into three areas.

(2) Define constraints on the decision process that can be represented on the risk-return plane to divide the areas. These constraints need not be in terms of utility, but some other measure (e.g. profits).

The first approach is very difficult to carry out in practice, since determining a utility map for an individual is difficult and almost impossible for a corporation. There could be a question as to whether a corporation utility function actually exists. The second approach has been formalized by A. Charnes and others.[9] They propose two constraints to divide the GO, ON, and NO areas. The constraints are based on a probability of the investment making a specified payback and a minimum dollar profit. These constraints can be adapted and utilized for the model proposed in the previous sections.

The constraints chosen to divide GO, ON, and NO areas for this model are:

(1) For a GO decision the probability of obtaining a given discounted rate of return must be greater than a specified level.

(2) For a NO decision the probability of obtaining a given discounted rate of return must be less than a specified level.

These constraints can be derived in terms of the differential profit and differential uncertainty.[10] For the GO decision the constraint is:

$$P(\frac{TDDP}{I} \geq 1) \geq A_G$$

A_G = minimum probability for a GO decision

P = probability operator

I = total investment in new product

TDDP = total discounted differential profit

or $P(TDDP \geq I) \geq A_G$. This can be expressed as

$$P\left(\frac{TDDP - E(TDDP)}{DU} \geq \frac{I - E(TDDP)}{DU}\right) \geq A_G,$$

DU = differential uncertainty.

Since (TDDP - E(TDDP))/DU is normally distributed with a mean of zero and a variance of one, the equation can be restated in an equivalent form as $[(I - E(TDDP))/DU] \leq t_{GO}$, where t_{GO} is the fractile of (TDDP - E(TDDP))/DU associated with A_G.

In Figure 3, the shaded area represents the probability required for a GO decision. If $A_G > .5$, then $t_{GO} < 0$, so let $t_{GO} = -|t_{GO}|$, then

$$E(TDDP) \geq |t_{GO}| \, DU + I$$

is the equation for the GO constraint level of probability of achieving the specified rate of return.

Similarly for a NO decision the constraint is:

$$E(TDDP) \leq |t_{NO}| \, DU + I, \text{ if } A_N > .5$$

A_N = maximum probability for NO decision

t_{NO} = fractile corresponding to A_N in N(0,1)

In Figure 4, the shaded area represents the probability that must not be exceeded for a NO decision.

If $A_N < .5$, $t_{NO} > 0$, the equation for the NO decision is:

$$E(TDDP) \leq - t_{NO} + I$$

These constraints can be plotted as straight lines on the certainty equivalence plane and the decision areas can be specified. (See Figure 5).

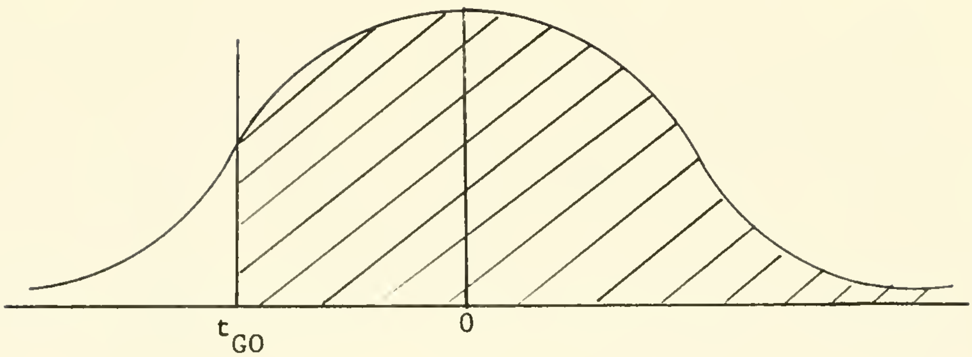


FIGURE 3.

GO Decision Fractile

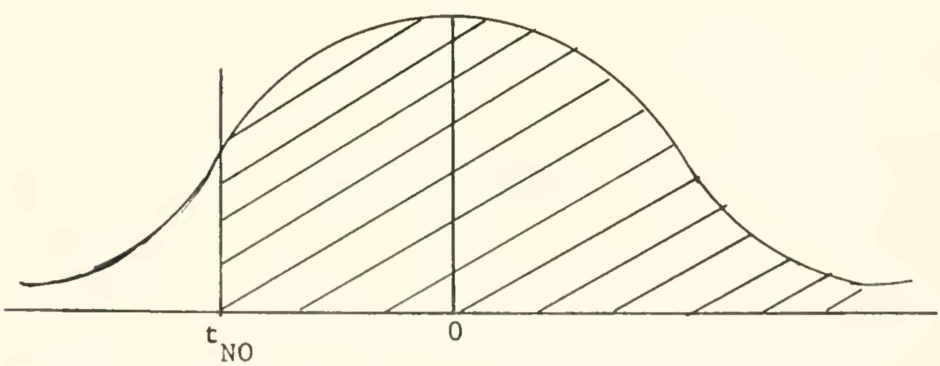


FIGURE 4.

NO Decision Fractile

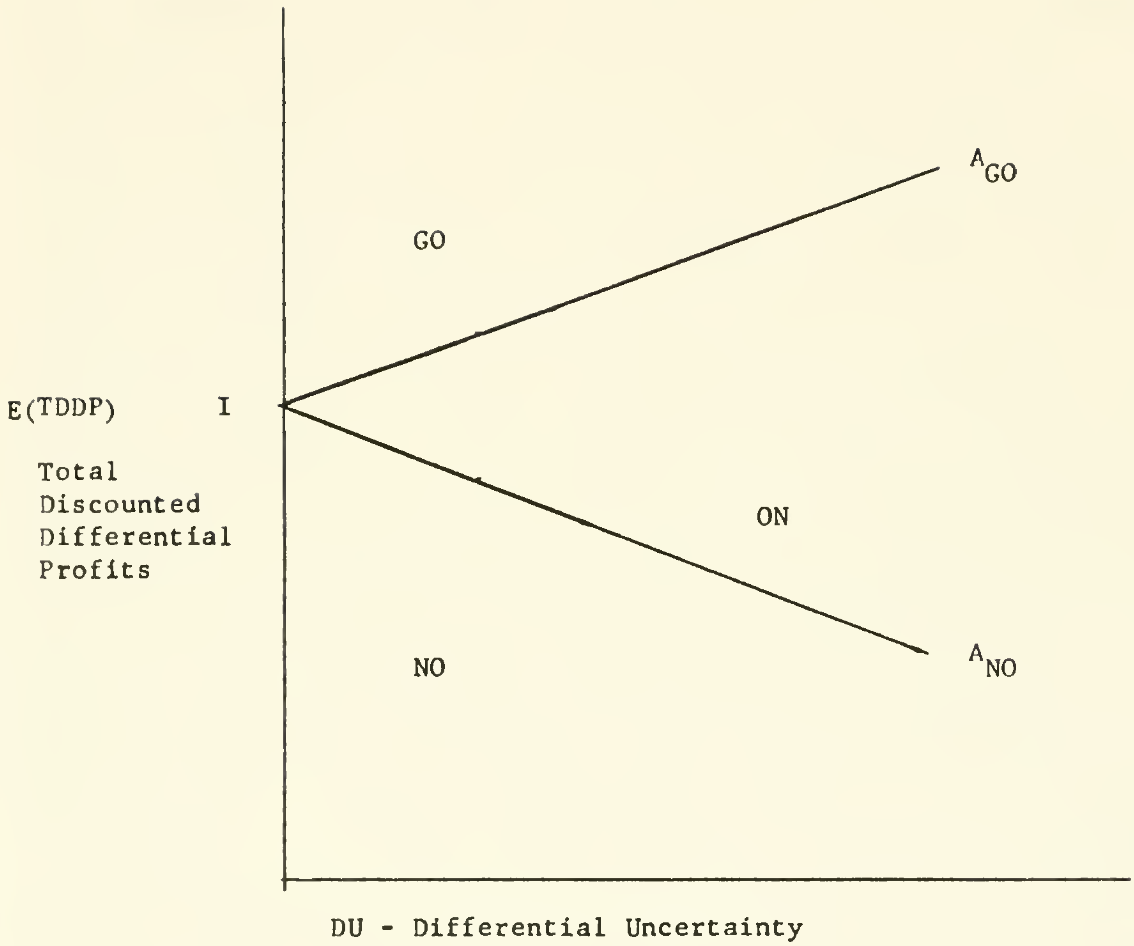
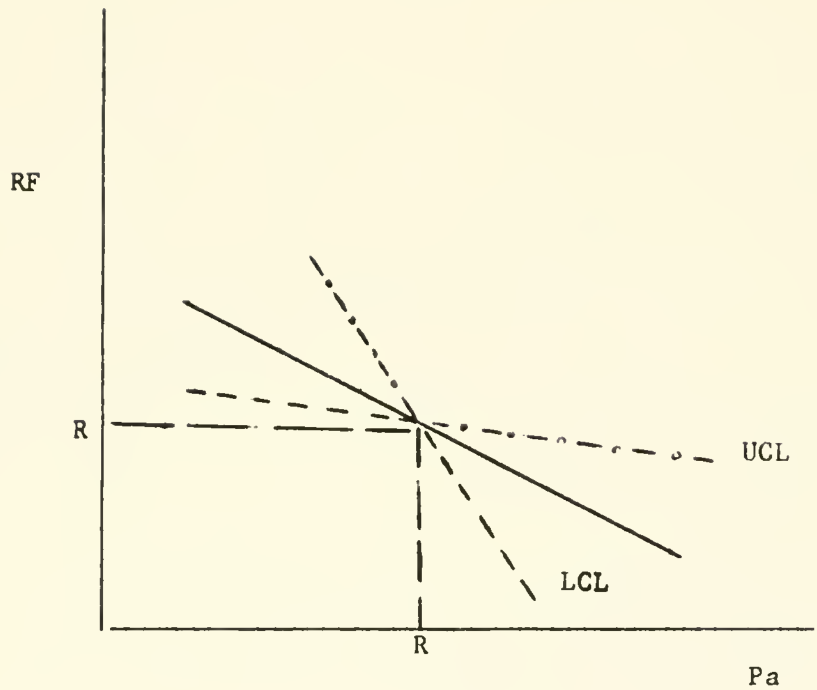


FIGURE 5.

Decision Quadrant

A decision is specified when the total discounted differential profit generated by the dynamic programming routine and the differential uncertainty are plotted on the certainty equivalence plane. This decision format assumes that the project has a single measure of uncertainty. The project may not have the same uncertainty at different commitments when the uncertainty is measured by the variance of the estimated discounted differential profit. As different prices are established, the profit variance will change even if the quantity variance is constant. In fact, the estimates of quantity variance may be different for different levels of price. At the reference quantity all uncertainty is reflected in the distribution about the life cycle estimate, but the price-quantity relationships may be subject to additional estimation uncertainty. This is because the reference estimate is to be the decision maker's best estimate. This may be based on a market test or on past studies relating to the response relationship. If there is additional uncertainty connected with values other than the reference value, this would cause the variance of the differential profit to vary as different price levels are established. For example, the confidence limits may be as in Figure 6. The fact that the uncertainty will vary with different prices, advertising, and distribution poses a problem for the decision model, since now multiple points will be plotted rather than one TDDP-DU point. The points will represent different combinations of mean estimates of discounted differential profit and variance based on a different set of trial values of the input variables. See Figure Seven.



RF = response function

Pa = parameter

LCL = lower confidence limit = ----

UCL = upper confidence limit = -.-.-

R = reference level

FIGURE 6.

Confidence in Response Function

FIGURE 7.

Decision Quadrant

This complication can be handled in a number of ways. First, it is presumed that the GO area is preferred to the ON area and the ON area is preferred to the NO area whenever possible. This means if any points fall in GO, the decision will be GO and the remaining question is which point in the GO area is to be chosen. If no points are in the GO area and some fall in the ON area, the decision is ON, and the selection of the exact point can be deferred until after the next "best" study. If no points fall in the GO or ON area, a NO decision is reached. The most difficult problem is the choice of the best point if any points lie in the GO area.

If only one point lies in the GO area the problem will not appear. When more than one point is in the GO area, the selection of the "optimum" point is important since each point represents a different level of commitment to the product and a different marketing mix for the product. This problem can be approached in several ways. The most obvious is a preference approach. The executive could specify the points in increasing order of preference and choose the most preferred as the optimum. This could be a lengthy process if many points were present, but it would be possible.

Another solution is a chance constrained programming approach. Chance constrained programming attempts to solve the problem:

optimize: $f(c,x)$

subject to: $P(Ax \geq b) \geq \alpha$

where A, b, c are random variables and $P(B) \geq \alpha$ indicates that the probability of B occurring must be greater than α.[11]

Although the analytic algorithm of chance constrained programming can not be applied to the new product model proposed here because of the nature of the objective function, it is useful conceptually. If f(c,x) is defined as discounted differential profit (DDP) and if the constraint represents the probability of making the minimum rate of return required at the GO level, the logic of the chance constrained approach is applicable. In this case the problem is:

maximize: E(TDDP)

subject to: $P(\frac{TDDP}{I} \geq 1) \geq A_G$

A_G = minimum GO probability

TDDP = discounted differential profit

I = total investment

E = expected value operation

P = probability operator

This formulation is called the "E" model. The "E" model can be solved from the plot of points on the E(TDDP)- DU quadrant. For example, if the points are plotted as in Figure 7, the point "A" would be the solution to the single stage chance constrained "E" model. Point "A" has the greatest expected profit level in the GO area. The use of expected value of profit is only one choice of several objective functions. The decision maker may wish to minimize risk. Then the problem is:

minimize: DU

subject to: $P(\frac{TDDP}{I} \geq 1) \geq A_G$

DU = differential uncertainty

TDDP = discounted differential profit

I = total investment

A_G = minimum GO probability

This is called the "V" model or more fittingly here the "DU" model and the solution would be point "C" for this example. See Figure 7. Point "C" has the lowest value of the differential uncertainty in the GO area. If the businessman was interested in a "satisficing" solution, the problem would be to maximize the probability of achieving the minimum rate of return. For the GO decision criterion this would be:

maximize: $P(\frac{TDDP}{I} \geq 1)$

subject to: $P(\frac{TDDP}{I} \geq 1) \geq A_G$

P = probability operator

TDDP = discounted differential profit

I = total investment

A_G = minimum probability for a GO decision

This is called the "P" model and the solution to this model in this example would be point "B". See Figure 7. Point "B" is the farthest radical distance from the probability constraint line and therefore is associated with the highest probability of any of the points in this example.

The solution of the decision model depends upon the criterion the businessman chooses to use to determine the "optimum". Perhaps the profit maximization model would be the one most commonly used. When

this is true, the choice of the points in the GO area is made on the basis of the plotting of the maximum total discounted differential profit (as generated by the dynamic program routine) and the differential uncertainty associated with this program. If the decision maker does not choose profit maximization as the criterion for "optimum", the use of the preference approach or the "P" or "V" chance constrained models would be appropriate and many trial value points would be plotted.

If a GO decision is reached, a commitment to market the product is made. If a NO decision is reached, the product is rejected. If an ON decision is specified, an information gathering study is carried out. The decision maker might find it instructive to look at the plot of DDP-DU and see how much improvement must be made before a GO decision can be reached. If he can see no possible way of achieving the information necessary to reach the GO area, or if he can not justify the funds for an additional study, he may feel a pre-emptive NO decision is in order. Perhaps he would withhold further consideration of the project for a time.

The decision approach outlined in this paper is analogous to the sequential procedures prepared by A. Wald.[12] He suggested that information be compiled bit by bit and that a decision be made as soon as the sumulative evidence was sufficient. Much of this analysis deals with specific distributions with known means or variances, but this proof of optimality for sequential testing is general.

A. Wald showed that the minimization of risk is achieved by a sequential testing procedure and that it produces a smaller expected

number of trials than any other method.[13] This means that if costs of studies are greater than zero, the cost of a sequential procedure is less than any other testing method.

To apply Wald's proof to the new product decision model proposed here, one more factor must be considered. The statistical test Wald proposes assumes homogeneous tests at each decision. In fact, however, the model presented here assumes the "best" test will be carried out at each ON step. This further strengthens the optimality characteristics. Based on Wald's proofs, it can be reasonably concluded that the decision model proposed here for new product decisions will produce the minimum number of studies on the average for new product decisions. Since the studies are undertaken in order of decreasing desirability, i.e. the best test first, the return on research funds will be maximized. This implies that the optimum use of research funds will be made by application of the proposed decision model.

SUMMARY

The proposed new product decision model explicitly analyzes demand, cost, allocation, and uncertainty interactions and determines if a new product should be added (GO decision), should be rejected (NO decision), or should be investigated further (ON decision). The model is capable of analyzing complex input functions that represent non-constant direct and cross elasticities. Competitive strategies and cumulative competitive effects can be specified and analyzed in the model. The dynamic effects of diffusion of the new product innovation and price sequencing can be comprehended. Input distributions

can be non-normal and different estimates of differential uncertainty are allowed at various levels of commitment to the project. The marketing mix effects are functionally considered, so that a maximizing combination of market parameters will be generated for the new product.

The use of the decision model tells the decision maker when to leave the information network, and if the "best" study is chosen at each ON step, the procedure results in the optimum allocation of research funds in the long run. The output of the model in the GO state is the optimum price, advertising, and distribution marketing mix over the life cycle of the new product and the evaluation of changes in old line parameters which will help increase the differential line profits. The proposed new product decision model is an integrated formulation capable of encompassing the significant decision factors.

FOOTNOTES

[1] Philip Kotler, "Competitive Strategies for New Product Marketing Over the Life Cycle," Management Science, XII (December 1965), B-106.

[2] Given: $X_1 = \bar{X}_{1t} (P_1^{EP})$

Taking logs:

$$\ln X_1 = EP(\ln P_1) + \bar{X}_{1t}$$

Taking the total differential:

$$dX_1/X_1 = (EP)dP_1/P_1 + (\ln P_1)d(EP)$$

It is now evident that the $(\ln P_1)d(EP)$ term does not represent proportionate changes in X. The expression

$$(\ln P_1)d[(dX_1/X_1)/(dP_1/P_1)] \neq \cdot dX_1/X_1$$

The $(\ln P_1)d(EP)$ term results in an inconsistent representation of demand. This is analogously true for all X^{EX} forms when EX is allowed to vary.

[3] Kotler, B-107.

[4] Kotler, B-104 to B-119.

[5] The complete search program is called SPRINTER: Specification of PRofits with INteraction under Trial and Error Response. This program systematically re-runs the dynamic programming optimization over a range of input marketing programs. In a case study SPRINTER evaluated a range of two million programs.

[6] Alexander M. Mood and Franklin A. Graybill, Introduction to the Theory of Statistics, 2nd Edition (New York: McGraw-Hill, 1963) 211.

[7] This is derived from the basic computational formula for variance. The variance of the distribution of the cost times the quantity sold is noted as σ_{xC}.

$$\sigma^2_{xC} = E(xC)^2 - [E(xC)]^2 \quad , E = \text{expected value operator}$$

since $[E(xC)] = E(x)E(C)$ if x and C are independent (i.e. $COV(x,C)=0$)

$$\sigma^2_{xC} = E(x^2C^2) - [E(x)E(C)]^2$$

since $E(x^2C^2) = E(x^2)\ E(C^2)$ if x^2 and C^2 are independent,

$$\sigma^2_{xC} = E(x^2)\ E(C^2) - [E(x)\ E(C)]^2$$

But $E(x^2) = \sigma^2_x + E(x)^2$ and $E(C^2) = \sigma^2_C + E(C)^2$,

so $\sigma^2_{xC} = (\sigma^2_x + E(x)^2)(\sigma^2_C + E(C)^2) - E(x)^2\ E(C)^2$

or $\sigma^2_{xC} = \sigma^2_x\sigma^2_C + E(x)^2\sigma^2_C + E(C)^2\sigma^2_x + E(x)^2E(C)^2 - E(C)^2E(x)^2$

or finally the variance of the total cost distribution is:

$$\sigma^2_{xC} = \sigma^2_x\sigma^2_C + E(x)^2\sigma^2_C + E(C)^2\sigma^2_x$$

[8] Harry M. Markowitz, Portfolio Selection (New York: John Wiley, 1959), pp. 96-101.

[9]A.Charnes, W.W. Cooper, J.K. DeVoe, and D. S. Learner, DEMON: Mark II Extremal Equations Approach to New Product Marketing (Systems Research Memorandum No. 110, The Technological Institute, Northwestern University, 1964) pp. 10-11.

[10] These proofs are not identical to, but are based on, proofs by A. Charnes et. al. DEMON. The proofs presented here differ in three respects. First they are related to a profit-risk plane of total discounted differential profit -- variance of differential profit rather than cash flow profits -- variance of quantity sold. Second, this proof is for the normal rather than the lognormal distribution. Third, the constraint is based on a probability of making a specified rate of return rather than on a payback requirement.

[11] For an explanation of chance constrained programming, see A. Charnes, "Deterministic Equivalents for Optimizing and Satisficing Under Chance Constraints," Operations Research, XI (January-February 1963), pp. 18-39.

[12] Wald, Sequential Analysis (New York: John Wiley, 1947).

[13] A. Wald and J. Wolfowitz, "Optimum Character of the Sequential Probability Ratio Test," The Annals of Mathematical Statistics, XX (September 1948), pp. 326-339.

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