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AMPLITUDE MODULATION OF SYNCHRONIZED MICROWAVE OSCILLATORS

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Abstract

It can be shown that two push-pull frequency-modulated oscillators, both synchronized by the same r-f signal, can be arranged so that their combined output contains the side bands of ordinary amplitude modulation. Linearity of the scheme is excellent even for high modulation indices. The bandwidth is somewhat less than the AM bandwidth of the oscillators themselves; it is about 3 Mc/sec at X-band. Experimental work with the system indicates that it has characteristics suitable for microwave relay and possibly low-power broadcast applications.

AMPLITUDE MODULATION OF SYNCHRONIZED MICROWAVE OSCILLATORS

I. Introduction

Although microwaves, i.e., frequencies above 1000 Mc/sec, were first extensively used in detection systems (radar in particular), they have become extremely important in the fields of communication and radio broadcasting*. Their importance in these fields is partly due to the shortage of available spectra at lower frequencies. The use of microwave frequencies in these systems permits the transmission of relatively wide signal spectra.

At the present time three types of tubes are utilized for microwave oscillators, namely the microwave triode, the magnetron and the klystron. These oscillators have been extensively studied and good progress has been made in increasing their operating frequencies and output powers. Also, these oscillators (particularly magnetrons) may be readily pulse-modulated by various well-developed techniques. However, "continuous" modulation (AM, FM) of these microwave generators presents a different picture.

Methods of accomplishing frequency modulation depend to a large extent on the oscillator characteristics. In particular, klystron oscillators can be frequency modulated by simply varying the potential on one or more of the oscillator electrodes. The operating frequency of magnetron oscillators can be changed with the aid of a variable reactance element which is electronically controlled and arranged to form a part of the oscillator resonant circuit. At the present time, the frequency deviation is limited to small values, but there is reason to believe that satisfactory frequency modulation will be attained. Triode oscillators can be frequency modulated using a variable reactance element or modulating the triode electrodes. In general, frequency modulation of microwave oscillators is still in the early stages of development.

Amplitude modulation of relatively low-output-power systems employing triode oscillators and amplifiers can be accomplished using the various low-frequency techniques. That is, modulation can be performed at the final stage amplifier. However in the case of magnetron and high-power-level klystron oscillators, the available amplitude modulation techniques are poor and far from being satisfactory (1, 2, 3)**. This report

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* The term radio broadcasting is used here in the broad sense and covers the various fields which deal with the transmission of intelligence by the use of r-f energy.

** At present, RCA Laboratories are investigating an AM system for magnetron oscillators. An electronically-tuned magnetron is frequency stabilized and the power output is varied by modulating the magnetron anode potential. The accompanying frequency "pushing" is reduced by a feedback system. Therefore, the modulation bandwidth is determined by the pass-band characteristics of the feedback system. At present, modulation is limited to frequencies up to approximately 30 Kc/sec. The depth of modulation is determined by the maximum output power at which the magnetron begins to "mode" and the minimum output power at which the magnetron will oscillate. This system was discussed by Dr. J. S. Donal of RCA in a paper presented at the Spring, 1950, New England IRE meeting and shows promise of being a practical AM system for microwave oscillators.

is the investigation of a method of amplitude modulating these. The possibility of this technique was first reported in Technical Report No. 100.

II. Description of System

It can be shown that if a signal of proper amplitude and frequency is injected into the output circuit of an oscillator, the nonlinear characteristics of the oscillator cause it to operate at the frequency of the injected signal. This synchronization phenomenon, particularly in microwave (4, 5) oscillators, has been reported in Technical Reports No. 63 and No. 100. In these investigations it is shown that under synchronized operation, there exists a unique phase relationship between the injected signal and oscillator signal. Under the conditions that the injected signal and oscillator signal are steady sine waves, this phase relationship is given by

$$\theta = \sin^{-1}\left[\frac{Q_{ext}(\omega_1 - \omega)}{|p|\,\omega_o}\right] \tag{1}$$

where

θ = the phase difference between the locking signal and the oscillator signal.

$|p|$ = the apparent reflection coefficient; actually the square root of the ratio of the locking signal power directed toward the oscillator to the output power of the oscillator.

ω_1 = the angular frequency of the locking signal and hence the angular frequency of the oscillator when synchronized.

ω = the angular frequency at which the oscillator would operate in the absence of the locking signal.

ω_o = the natural resonant frequency of the oscillator circuit.

Q_{ext} = a coupling factor.

It will be noted from Eq. 1 that if the frequency and amplitude of the injected locking signal are constant, the phase is a function of the free-running frequency (ω) of the oscillator only.

The significance of this relationship is shown by considering the following example. An oscillator is synchronized by an external signal of frequency ω_1. The oscillator is adjusted so that the free-running frequency is equal to the locking signal frequency. Also, by some method, the free-running frequency of the oscillator is varied sinusoidally about ω_1. That is $\omega = \omega_1(1 - a \sin\omega_m t)$. The average operating frequency of the synchronized oscillator is constant and equal to ω_1, however the phase angle of the output signal with respect to some fixed reference will change according to Eq. 1. This change is given by Eq. 2.

$$\theta = \sin^{-1}\left[\frac{Q_{ext}}{|p|\,\omega_o}\left\{\omega_1 - \omega_1(1 - a\sin\omega_m t)\right\}\right] \quad \text{(* see footnote)} \qquad (2)$$

or

$$\sin\theta = K\sin\omega_m t \qquad (2a)$$

where

$$K = \frac{\omega_1 a\,Q_{ext}}{|p|\,\omega_o} \quad . \qquad (3)$$

Three important points should be noted in the above example. One, the sine of the phase angle rather than the phase angle is proportional to the modulation function. Two, since $-1 \leqslant \sin\theta \leqslant 1$, the maximum free-running frequency deviation for which synchronism is maintained is limited by the circuit parameters and the amplitude of the injected signal. In the above example,

$$\omega_1 a \leqslant \frac{|p|\,\omega_o}{Q_{ext}} \quad .^{**}$$

Three, the change in phase is obtained by changing the free-running frequency of the oscillator. This change in free-running frequency amounts to frequency modulation of the oscillator and hence any of the usual methods of accomplishing frequency modulation can be used to obtain this phase variation.

As noted in Technical Report No. 100 (5), this phase relationship may be used to produce an amplitude modulated wave. Consider two independent oscillators operating at the same frequency ω_1. Both oscillators are synchronized or locked to the same external signal and the output of each oscillator is connected to a common load such that the phase angle between the two oscillator signals at the load is 180°. If the oscillators are frequency modulated*** in push-pull, the resulting phase modulation will produce a wave of varying amplitude at the load. Specifically, before modulation, the voltage wave at the load from oscillator 1 may be expressed as $e_1 = V_1 \sin\omega_1 t$; the wave from oscillator 2 as $e_2 = V_2 \sin(\omega_1 t + \pi)$. Actually an arbitrary phase angle should be added to e_1 and e_2; however, at the load or common reference the phase angles would be equal and there is no loss in generality by assuming this angle equal to zero. When the oscillators are push-pull modulated the voltage waves may be written as:

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* This expression is only approximately correct and hence subject to certain restrictions. However, for simplicity the restrictions will be omitted at this time. The approximations which must be satisfied will be discussed in the following sections.

** This also defines the range of frequencies at which the oscillator can be synchronized by the injected signal.

***It should be remembered that under locked operation the average output frequency is constant. The term frequency modulation used here and in subsequent discussions shall refer to the changes in frequency that would occur in the absence of the locking signal. In any case the meaning will be clear from the related discussion.

$$e_1 = V_1 \sin\left[\omega_1 t + \theta(t)\right] \tag{4}$$

and

$$e_2 = V_2 \sin\left[\omega_1 t + \pi - \theta(t)\right] \quad . \tag{4a}$$

The total voltage at the load (e_r) is

$$e_r = e_1 + e_2 = V_1 \sin\left[\omega_1 t + \theta(t)\right] + V_2 \sin\left[\omega_1 t + \pi - \theta(t)\right] \quad . \tag{5}$$

Assuming $V_1 = V_2$ and expanding the trigonometric expression, the equation becomes

$$e_r = 2V_1 \sin\theta(t) \cos\omega_1 t \quad . \tag{5a}$$

If the mean free-running frequency of the oscillators is equal to the frequency of the locking signal and if the frequency modulation is sinusoidal at a frequency ω_m, the phase change will be given by Eq. 2 which is

$$\theta(t) = \sin^{-1}(K \sin\omega_m t)$$

or

$$\sin\theta(t) = K \sin\omega_m t \quad .$$

Therefore

$$e_r = 2V_1 K \sin\omega_m t \cos\omega_1 t \quad . \tag{6}$$

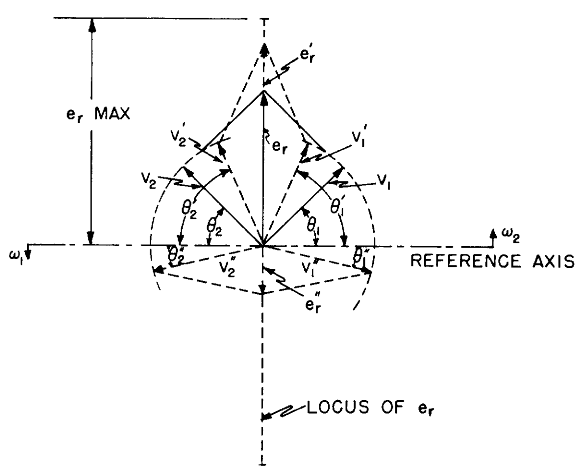

Fig. 1

Vector diagram of output voltage of a locked-oscillator system composed of two independent oscillators locked to the same external signal. Note that vectors are rotating at the reference angular frequency, ω_1.

A vector representation of the above output is shown in Fig. 1.

It will be noted that Eq. 6 is the expression of the side-band frequencies of an amplitude-modulated voltage wave. That is, the equation is the expression of a voltage wave containing the frequencies $(f_1 - f_m)$ and $(f_1 + f_m)$.

From the preceding discussion, it is seen that amplitude modulation of microwave oscillators may be accomplished using a system composed of two independent oscillators locked to the same external signal; furthermore, this amplitude modulation can be accomplished using the usual frequency modulation techniques. The output signal is that obtained from a balanced-modulator system and as such is composed of the side bands only, the carrier being suppressed. However, for systems in which a carrier is desired, the carrier can be supplied by the locking signal source.

III. Secondary Effects in the System

In the foregoing analysis a number of important assumptions were made that must be investigated in order to determine the actual system characteristics. It was assumed that the voltage at the load was simply the sum of the output voltage of the individual oscillators. This assumption implies that the external locking signal is coupled to the oscillators and yet not coupled to the load. With reference to Fig. 2 which shows a

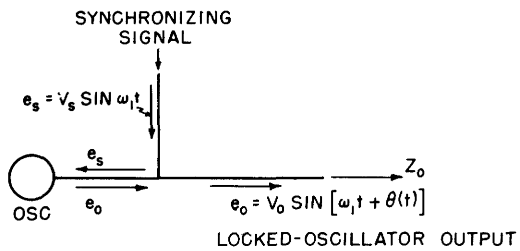

Fig. 2

Typical microwave locked-oscillator system. Ideal injection of locking signal.

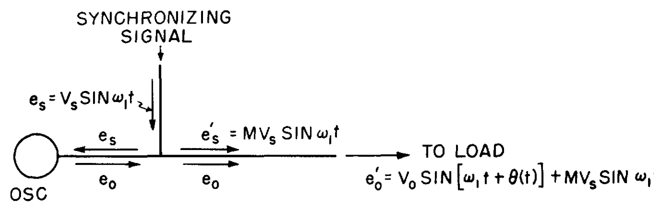

Fig. 3

The output of a locked-oscillator system in in which a portion of the synchronizing signal is coupled to the load.

typical locked-oscillator system, this means that the synchronizing signal is assumed to propagate only toward the oscillator as indicated. In a practical system, perfect directional coupling is impossible and therefore a portion of the locking signal will be propagated toward the load. Hence, the voltage output will be modified as indicated in Fig. 3. In this case, Eqs. 4 and 4a must be written as

$$e_1 = V_1 \sin\left[\omega_1 t + \theta(t)\right] + N_1 V_s \sin\omega_1 t \tag{7}$$

and

$$e_2 = V_2 \sin\left[\omega_1 t + \pi - \theta(t)\right] + N_2 V_s \sin(\omega_1 t + \pi) \tag{7a}$$

where N_1 and N_2 are constants which depend upon the degree of coupling between the injected signal and the load; the phase angle π in the expression $N_2 V_s \sin(\omega_1 t + \pi)$ arises from the fact that the oscillator voltages at the load are 180° out of phase. The combined output will be

$$e_r = 2V_1 K \sin\omega_m t \cos\omega_1 t + (N_1 - N_2)\, V_s \sin\omega_1 t \quad . \tag{8}$$

It is noted that the output contains a carrier term in addition to the original side-band term. The relative phase of the added carrier is shown vectorially in Fig. 4. This carrier term being 90° out of phase with the appropriate carrier for AM will introduce amplitude and phase distortion in the final modulated output. The amount of distortion introduced depends upon the difference $N_1 - N_2$; therefore if the coupling between the locking signal and the load in each oscillator circuit are made equal or nearly equal, the deviation from the "ideal" locking will have negligible effect.

It should be noted also that if a mismatch exists between the oscillator and the output circuit (Fig. 2), a portion of the locking signal will be reflected back to the load. The effect of this coupling to the load will be similar to that just described. However, due to the arbitrary phase angles associated with the reflected waves, the second term of Eq. 8 will contain an arbitrary phase angle.

In arriving at the conclusions contained in Eq. 6, it was also assumed that there existed no coupling between the two oscillators. If coupling exists, the effective synchronizing voltage to which either oscillator is locked will be the sum of the voltages

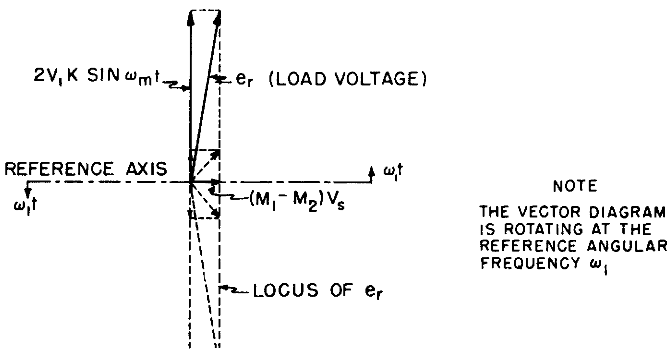

Fig. 4 Vector diagram of side-band output plus an out-of-phase carrier.

produced by the reference oscillator and by the other oscillator. When the oscillators are modulated, this effective synchronizing voltage will be phase modulated. Hence, since θ is the phase difference between the oscillator signal and the locking signal, an additional phase modulation will be introduced. Also, the magnitude of the effective locking signal will vary with time. This variation will produce a change in $|\rho|$ and hence in K of Eq. 3. If NV_2 is the amplitude of the voltage coupled to oscillator 1 from oscillator 2 and V_s is the amplitude of the external locking voltage, then the phase angle of the effective synchronizing voltage will be

$$\beta = \tan^{-1}\left[\frac{NV_2 \sin\epsilon(t)}{V_s + V_2 N \cos\epsilon(t)}\right] \tag{9}$$

where $\epsilon(t)$ is the phase angle with respect to the reference voltage of the voltage fed from oscillator 2 and is a function of time when oscillator 2 is modulated. The phase between the effective locking voltage and the oscillator voltage is given by Eq. 2. Since the phase of the voltage from the reference oscillator is fixed, the phases of all other voltages may be expressed with reference to it. Therefore the phase θ_1' of the voltage output from oscillator 1 will be

$$\theta_1' = \theta + \beta \quad . \tag{10}$$

From Eqs. 2 and 9 this becomes

$$\theta_1' = \sin^{-1}\left[K \sin\omega_m t\right] + \tan^{-1}\left[\frac{NV_2 \sin\epsilon(t)}{V_s + NV_2 \cos\epsilon(t)}\right] . \tag{10a}$$

A similar expression may be written for oscillator 2.

$$\theta_2' = \sin^{-1}\left[K \sin\omega_m t\right] + \tan^{-1}\left[\frac{NV_1 \sin\epsilon'(t)}{V_s + NV_1 \cos\epsilon'(t)}\right] . \qquad (10b)$$

It should be noted that the angle $\epsilon(t)$ in Eq. 10a and the angle $\epsilon'(t)$ in Eq. 10b depend upon the particular system used as will be noted in Fig. 2. Referring to this figure, the angles $\epsilon(t)$ and $\epsilon'(t)$ will depend upon the electrical line length between the load and the point at which the reference voltage is injected. As was noted previously, the constant K in Eq. 2 is a function of the magnitude of the effective locking voltage. This magnitude may be expressed as

$$|V_s'| = \sqrt{(V_s + NV_2 \cos\epsilon(t))^2 + N^2V^2 \sin^2\epsilon(t)} \quad . \qquad (11)$$

From the preceding discussion, it is clear that coupling between the oscillators introduces additional phase modulation which will produce distortion in the output. However, if the coupling is small so that $NV \ll V_s$, the coupling effect will be negligible. For example, if $NV = 0.01V_s$, from Eq. 9, the maximum added phase angle will be less than 1° and from Eq. 11, the magnitude of the locking voltage will be changed by less than 1 percent.

The above analysis may be used to calculate the effects of a mismatched load. In this case N becomes the reflection coefficient of the mismatch.

In all of the preceding analysis, it has been tacitly assumed that the two oscillators have identical modulating characteristics. In general, the combined output will be as shown in Fig. 5. The output voltage may be written as

$$e_r = (V_1 \cos\theta_1 - V_2 \cos\theta_2) \sin\omega_1 t + (V_1 \sin\theta_1 + V_2 \sin\theta_2) \cos\omega_1 t \quad . \qquad (12)$$

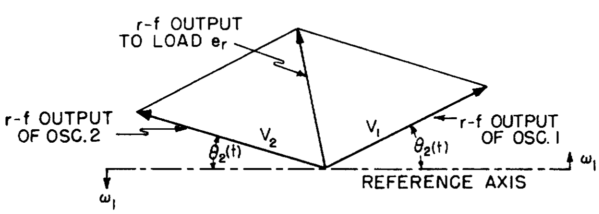

Fig. 5 Vector diagram of locked oscillator output for unsimilar oscillators.

If the amplitude and phase variation of the output voltage of the two oscillators are identical, Eq. 12 reduces to the previous expression given by Eq. 5. However, if $V_1 \neq V_2$ and $\theta_1 \neq \theta_2$, the output will contain an amplitude-modulated signal in quadrature with the intelligence-carrying side-band signals. It is shown in Appendix I that if the amplitudes and phases are expressed as $V_1 = AV_2$ and $\sin\theta_1 = A' \sin\theta_2$ where A and A' are constants, the amplitudes of the harmonics introduced depend upon $|\sin\theta_{max}|$, as well as A and A'. For the magnitude of the second harmonic introduced to be less than 1 percent of the fundamental, A and A' should differ from unity by less than 2 percent.

In addition to the above assumptions, the previous discussions make the important assumption that the phase relationship expressed by Eq. 1 is valid when the free-running oscillator frequency (ω) is varied. It should be noted that the phase angle between the locking signal and the oscillator signal will be given by Eq. 1 only if the frequencies ω_1

and ω are constant. When ω is modulated sinusoidally, the phase variation must satisfy the following differential equation (ref. 5)*.

$$\frac{d\theta}{dt} + \frac{|p|\omega_o}{Q_{ext}} \sin\theta = a\omega_1 \sin\omega_m t \quad . \tag{13}$$

This equation is nonlinear and therefore the modulation system is inherently a nonlinear one. Also, since this equation determines the output response, the modulation bandwidth of the system depends upon the solution of this nonlinear differential equation. Therefore, if the response is determined for sinusoidal modulation, this response does not establish the bandwidth characteristics of the system. By an analysis of Eq. 13 on an electronic differential analyzer (15), it was found that Eq. 13 may be satisfactorily approximated by

$$\frac{d\theta}{dt} + S\theta = k \sin\omega_m t \tag{14}$$

where $S = |p|\omega_o/Q_{ext}$, and $k = a\omega_1$. Since Eq. 14 is linear, the response given by this equation for sinusoidal modulation will establish the system bandwidth characteristics.

The expression for θ which satisfies this differential equation is

$$\theta = \frac{k}{\sqrt{S^2 + \omega_m^2}} \sin(\omega_m t + \mu) \tag{15}$$

where

$$\mu = \tan^{-1}\left(\frac{\omega_m}{-S}\right) \quad .$$

From Eq. 15, it is seen that the approximate modulation bandwidth of the system is equal to S which is a function of the Q of the locked-oscillator resonant circuit and the magnitude of the locking signal.

IV. Experimental System and Measurement Techniques

From the preliminary analysis given in section III, the proposed modulation system should have the following features: (a) two microwave oscillators with provisions for frequency modulation and similar modulating characteristics, (b) provision for locking both oscillators to the same reference oscillator, (c) small coupling between the two oscillators, (d) small coupling between the reference oscillator and the load and (e) 180° phase difference between the locked oscillator voltages at the load. For the most part, these factors determined the various circuit components used in the actual system investigated.

An X-band system was decided upon. This choice was governed by the relatively small size of 3-cm waveguide and the availability of 3-cm equipment. The exact operating frequency was 9200 Mc/sec which is about the center of the X-band. The two oscillators

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* For the derivation of this equation see Appendix I of this reference.

used were reflex klystrons (2K25). These oscillators may be readily frequency-modulated and when operated at the center of the mode, the frequency deviation is linear for relatively large changes in repeller voltage.

The klystrons were operated in the first mode – the oscillation region characterized by the largest negative repeller voltages. The characteristics of this mode of operation gave two decided advantages over the others: a smaller rate of tuning – change in frequency per volt change of repeller potential – and a greater linear frequency deviation range. The small rate of tuning had the advantage in that a relatively large modulation voltage was needed. For example, when the oscillators were operated in the third mode, the change in repeller voltage corresponding to the required change in frequency was 0.5 volt. When the mode of operation was changed to the first mode, the required voltage change increased to 2 volts. The small value of the rate of tuning reduces the modulation produced by changes in the power supply potentials. To further decrease this incidental modulation (5), only the klystron anode and heater potentials were obtained from the normal power supply; the repeller voltage was supplied by batteries.

Oscillator modulating characteristics were determined by measuring the output power and frequency for various repeller voltages. Two tubes were selected which when operated at the center of the desired mode had equal output powers and operating frequencies; with the two tubes selected, it was possible to adjust the operating potentials so that equal repeller voltage changes produced equal frequency deviations also.

Frequency modulation of the 2K25 klystrons was obtained by varying the repeller potentials. Push-pull modulation of the repellers was accomplished by means of a phase inverter. The modulation circuit used is shown in Fig. 6. Since the required modulating

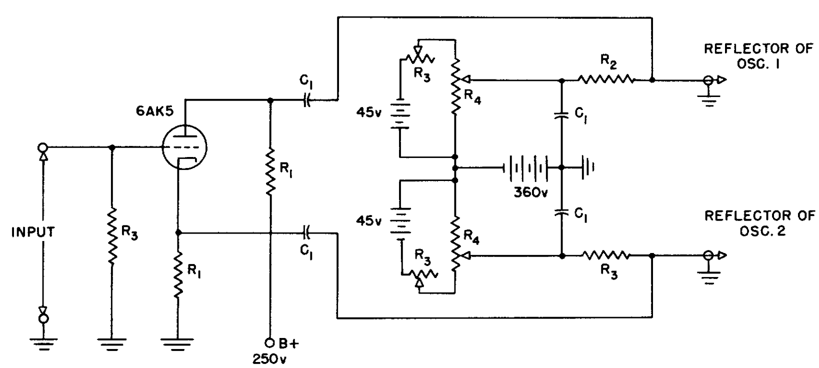


Fig. 6

Schematic diagram of circuit used to frequency-modulate the locked oscillators.

voltages were small (approximately 2 volts rms), the decrease in gain resulting from this particular method of phase inversion was tolerable.

Figure 6 shows also the circuit used to supply the repeller potentials. As was noted previously, the various operating potentials of the oscillators were adjusted to obtain equal outputs and frequency deviations. Since the required anode potentials were not the same for both oscillators, two separate power supplies were used. As will be noted from Fig. 6, the repeller potentials were supplied by the same set of batteries. However, a small adjustable potential was inserted in each repeller circuit. This adjustable potential was used to compensate for the unequal operating voltages and to provide means of accomplishing fine frequency adjustments.

A magic T was used to provide for the injection of the locking voltages. By connecting the oscillators to opposite arms of the T, the power delivered to one of the other arms was coupled to both oscillators. This method of injecting the external signal simultaneously provided the necessary features (b), (c), (d) and (e) listed above. The basic circuit arrangement used is shown in Fig. 7. If the two oscillators are matched to the guide, no power from the locking oscillator is coupled to the load; if the impedance looking toward the locking oscillator is equal to Z_0 then no coupling will exist between the two oscillators; also, if the oscillators are placed an equal distance from the T-junction, there will be a 180° phase difference between the oscillator signals due to the characteristics of the magic T.

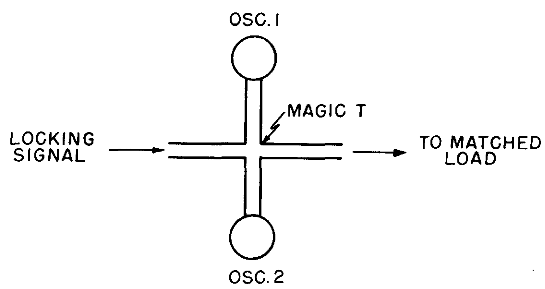


Fig. 7 Basic circuit arrangement (ideal).

It was found that with standard klystron waveguide mountings, the decoupling between the external signal and the load was at least 40 db. Therefore no special matching of the oscillators was required. In order to provide the necessary match to the oscillator output, approximately 20 db of attenuation was inserted in the load arm and in the locking oscillator arm. This produced a minimum decoupling of 40 db between the oscillators.

It was found necessary to insert a phase-shifting section between one oscillator and the T to compensate for the difference in electrical-line length between the oscillators and the T-junction. This difference was the result of unequal physical distances between junction and klystron output probes and slight differences in the klystron couplings. The arrangement of these various components are shown in Fig. 8.

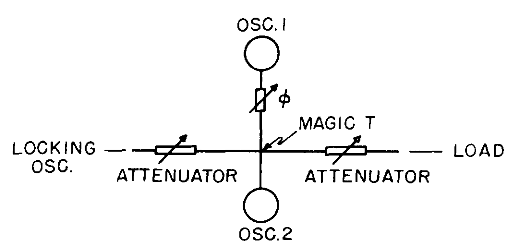


Fig. 8 Circuit diagram showing added components.

It should be noted from Fig. 8 that the output from the two oscillators will be coupled to the external source; therefore, to prevent the oscillators from "pulling" the locking frequency, the external-source output must be several db above the oscillator outputs. Obviously, the frequency stability of the system will be determined by the frequency stability of the locking source. In selecting a source to supply the locking voltage, these factors must be properly considered.

The external oscillator used was a 2K39 reflex klystron with an output power of 200 mw. The minimum attenuation between the locking source and the magic T was 10 db. Since the output power of the 2K25 klystrons was 16 mw, the power from these klystrons coupled to the external oscillator was at least 18 db below the output of 2K39 klystron.

(To reduce the output of the 2K25 klystrons and yet operate them in the first mode, the klystrons were operated at reduced potentials.) With this minimum attenuation no "pulling" of the external oscillator was observed.

A Pound system (10)* was used to frequency stabilize the locking oscillator. This provided a 9200 Mc/sec signal that varied less than 10 Kc/sec over long time periods.

A portion of the locking voltage was inserted in the system output to provide the necessary carrier. The carrier was supplied to permit detection of the modulating signal by simply using a silicon-crystal as a detector. The complete system that was used is shown in Fig. 9. The phase-shifting section in the carrier line was used to provide the proper phase relationship between the carrier and the modulated output. The magnitude of the carrier was adjusted with the aid of a variable attenuator.

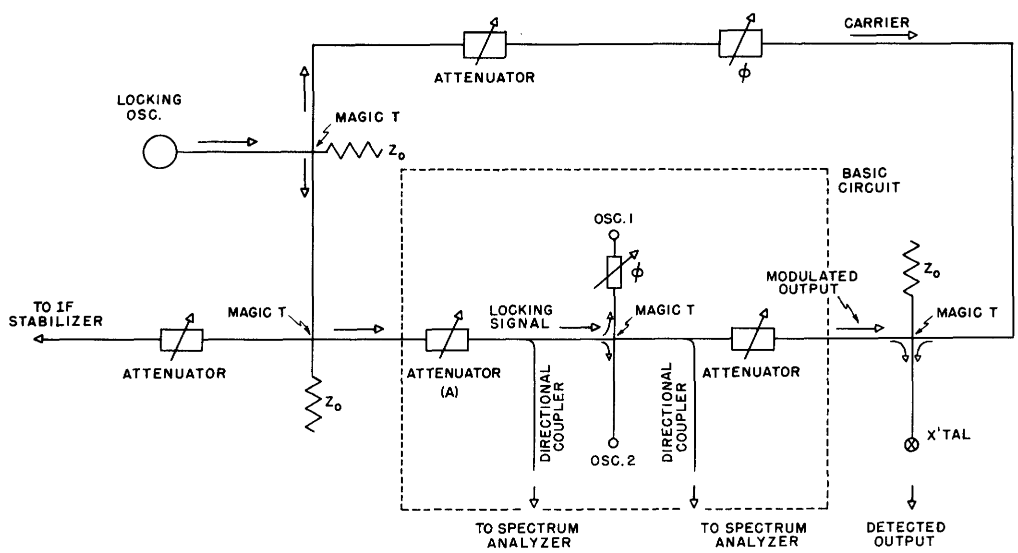


Fig. 9

The procedure used in locking the oscillators was as follows. The locking signal source was first adjusted for maximum output at 9200 Mc/sec and stabilized (11). The locking signal was monitored by a spectrum analyzer. The power to the analyzer was obtained from a directional coupler located in the locking circuit. This arrangement is shown in Fig. 9. Oscillators 1 and 2 were then adjusted. During the tuning of the oscillators, the locking signal was decoupled by inserting a minimum of 50 db of attenuation at A (Fig. 9). This decoupling was necessary to prevent the external signal from "pulling" the oscillators and thereby altering the "free-running" frequency of the oscillators. The sensitivity of the spectrum analyzer was sufficient to detect the locking voltage after attenuation; also, owing to the finite directivity of the directional coupler, a detectable amount of power from each oscillator (oscillators 1 and 2) was coupled to the analyzer. Therefore the output from the locking source,

*For further information on the Pound system and microwave-oscillator stabilization in general see references 11, 12 and 13.

oscillator 1 and oscillator 2 were viewed on the scope simultaneously (14). Oscillators 1 and 2 were tuned so that their markers coincided with that of the locking signal. Under this condition the free-running frequency of oscillators 1 and 2 were equal to the frequency of the external signal. The accuracy to which the frequencies can be made equal depends upon the resolving characteristics of the analyzer; with the analyzer used in this experiment the frequencies were within 30 Kc/sec. After the free-running frequencies were set, the attenuation at A was decreased and the oscillators were locked. The spectrum analyzer was then connected at the load to monitor the output of the locked oscillators. Since this output will be a minimum when a 180° phase difference exists between the oscillator signals, the phase-shifter in the output line of oscillator 1 was adjusted until a minimum output was indicated by the analyzer.

With the system adjusted as indicated above, the oscillators were sinusoidally modulated and the resulting output detected. This detection was accomplished using the spectrum analyzer or a silicon-crystal rectifier depending upon the frequency of modulation. For modulation frequencies up to 750 Kc/sec, the crystal rectifier was used. Since linear detection was needed, the carrier was adjusted until the power incident on the crystal produced approximately 0.4 ma of rectified current. By mixing a small amount of modulated-output power with the carrier output, good linear detection was obtained.

For modulation frequencies above 750 Kc/sec, the individual side bands were detected with the spectrum analyzer.

V. Experimental Results

Measurements were made of the r-f power output and repeller potentials under static conditions. In obtaining these measurements, the circuit shown in Fig. 10 was used to change the repeller voltage of the oscillators. It will be noted from this diagram that by changing the position of the grounded resistance potentiometer, the potential of one reflector is increased while the potential of the other reflector is decreased. Since this action results from the change in current flow in the branches of a parallel circuit, the changes in reflector potential are not equal. However if the values of the resistances are properly chosen, the voltage changes will be within 1 percent over a 2 volt range. The r-f power output was measured with the spectrum analyzer.

Figure 11 shows the result of the static tests. The reflector voltage indicated is for one reflector and is measured from an arbitrary reference. Since the reflectors were at a potential of approximately 360 volts below ground, a separate battery source of approximately 360 volts was used as the reference. In this manner the voltage changes could be measured with a high degree of accuracy. The r-f power output was measured from an arbitrary reference also. The accuracy of the power measurements is $\pm$ 0.1 db. It will be noted that the results are in agreement with the expected behavior of the system and that the system has very good linearity.

Since it is impossible in any practical system to obtain two oscillators with exactly equal power outputs, complete cancellation of the output at zero modulation is impossible. It was found possible however to adjust the klystrons so that the minimum output was at least 30 db below the maximum power. Further reduction of this minimum was not attempted since it would have necessitated measuring powers with more accuracy than was possible with the equipment at hand. Also, further reduction was unwarranted in that the remaining unbalance was negligible for practical purposes.

In Fig. 12, the results of this test are interpreted in terms of side-band power and modulation voltage.

Changes in temperature of the klystrons will cause changes in the free-running frequency and hence will produce a random modulation of the output. To examine the temperature stability of the system, various points around the minimum output were measured over comparatively long periods of time. These results are shown in Fig. 13. During these tests and in all subsequent operation, the 2K25 klystrons were enclosed by a metal shield to protect the tubes from varying drafts. For the tubes used in this experiment, it was found that a warm-up period of at least 30 minutes was necessary to reach the temperature equilibrium. The results shown in Fig. 13 are for changes after this warm-up period.

A further check on the linearity of the system was made by modulating the oscillators with a sinusoidal signal and applying the detected output and modulation signal to the plates of an oscilloscope. Figure 14 shows the trace of the detected output. Figure 15 is the trace of this detected output versus the modulation signal. As will be noted, the linearity is quite good.

For all of the above tests, the locking signal power to the oscillators was adjusted to produce an effective reflection coefficient $|p|$ of 0.3. With reference to Fig. 9, this is the ratio of the magnitude of the voltage wave from the locking source propagating toward oscillator 1 (or oscillator 2) to the magnitude of the voltage output from oscillator 1 (or oscillator 2). This value of $|p|$ was the maximum value of reflection coefficient that remained reasonably constant over the modulation range (5). It will be noted from Eq. 2b, section III that the change in $\sin\theta$ for a given change in ω is inversely proportional to $|p|$. Therefore, by using a comparatively large value of $|p|$, the random modulation produced by temperature changes, changes in klystron potentials due to power supply hum, and general klystron instability are kept small. The validity of this reasoning is illustrated in Fig. 16, where the traces show the result of changing the locking signal power while the oscillators were modulated with a sinusoidal signal of constant amplitude.

The result of overmodulation is illustrated in Fig. 17. In section III, it was shown that the oscillator locking range is restricted to the values of ω for which $|\sin\theta| \leqslant 1$. When the modulation voltage applied to the klystron was such that

$$|(\omega_1 - \omega)| \geqslant \frac{|p|\ \omega_o}{Q_{ext}} \quad ,$$

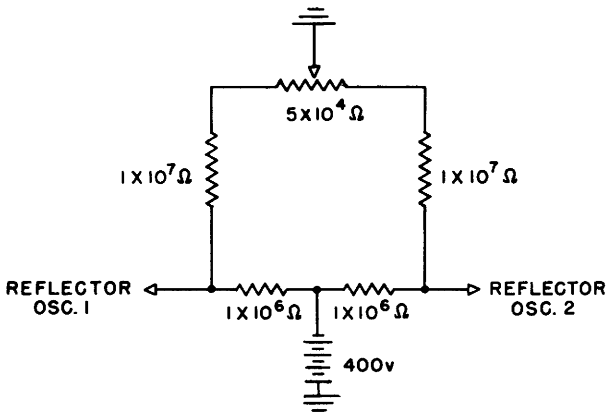


Fig. 10

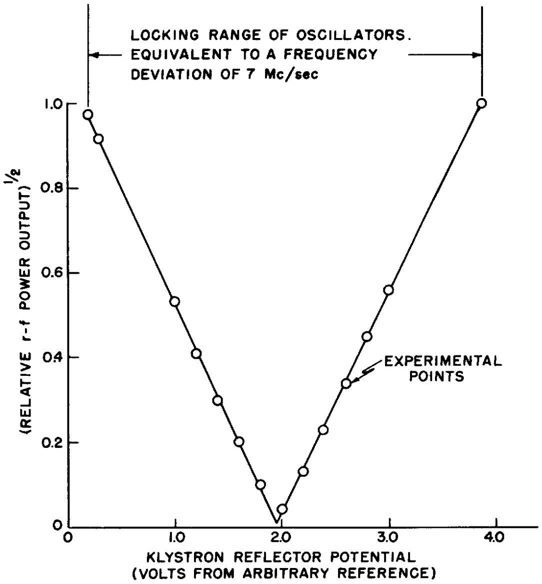


Fig. 11

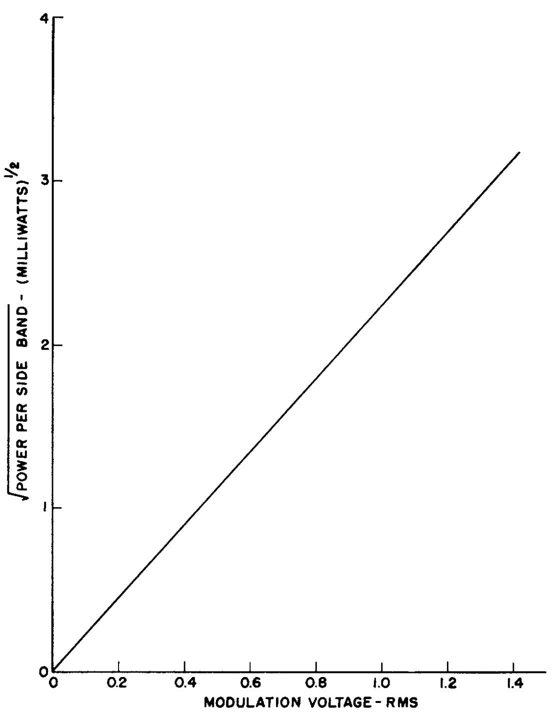


Fig. 12

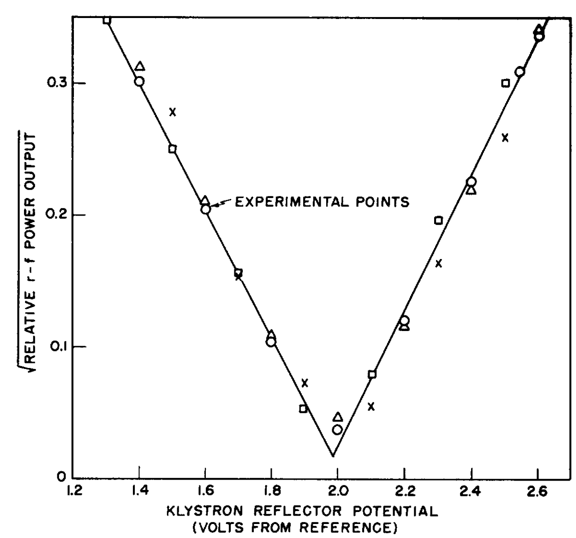


Fig. 13

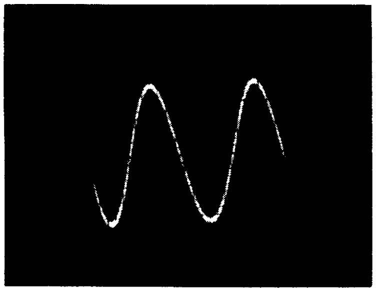
Fig. 10 Schematic diagram of circuit used to obtain a "push-pull" reflector-voltage change for static tests of the modulation system.

Fig. 11 Static test of modulation characteristics.

Fig. 12 Modulation characteristics of locked oscillator system. These results are interpreted from Fig. 11.

Fig. 13 Changes in r-f output power due to changes of free-running oscillator frequency. These frequency changes were the result of temperature changes of the klystrons.

Conditions for Figs. 14 and 15



$|p| = 0.3$

V_m = 1 volt (rms)

f_m = 2 Kc/sec

f_c = 9200 Mc/sec

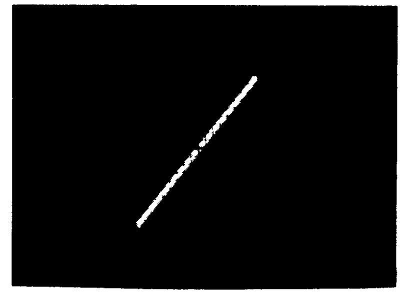
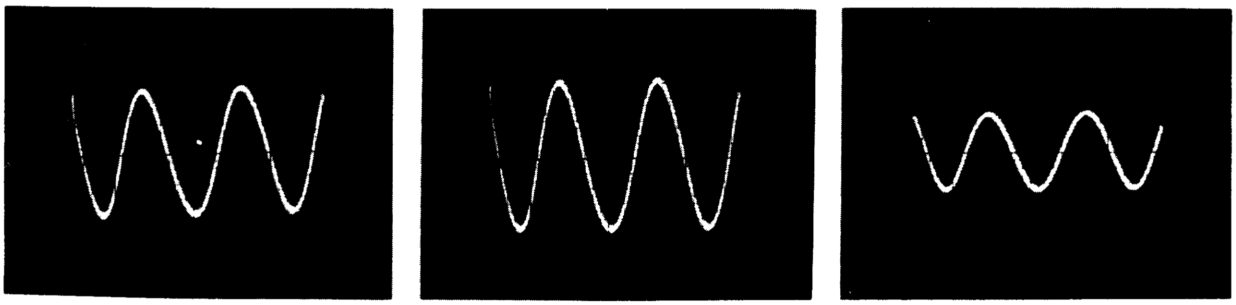


Fig. 14

Trace of the detected output.

Fig. 15

Trace of the detected output vs. modulation signal.



(a)

V_m = 1 volt (rms)

f_m = 20 Kc/sec

f_c = 9200 Mc/sec

$|p| = 0.30$

(normal locking voltage)

(b)

V_m = 1 volt (rms)

f_m = 20 Kc/sec

f_c = 9200 Mc/sec

$|p| = 0.25$

(locking voltage reduced)

(c)

V_m = 1 volt (rms)

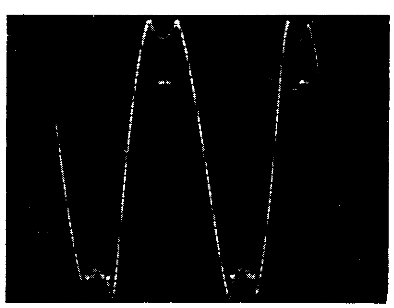
f_m = 20 Kc/sec

f_c = 9200 Mc/sec

$|p| = 0.43$

(locking voltage increased)

Fig. 16 Change in the modulated output when the amplitude of the locking voltage is changed. All other factors were kept constant.



$|p| = 0.3$

V_m = 2.5 volts (rms)

f_m = 2 Kc/sec

f_c = 9200 Mc/sec

Fig. 17

Trace of detected output showing the result of overmodulating the locked oscillators. It will be noted that at the peaks of the modulating signal the oscillators are no longer controlled by the locking voltage. The trace between the points of "unlocking" is the result of the beat frequencies between the two oscillators.

the oscillators were no longer controlled by the external signal. The shaded portion between the locking peaks shown in Fig. 15 is the result of the beat frequencies between the "unlocked" oscillators.

It will be remembered from section III that very little can be said about the exact bandwidth. However, it was shown that for signal amplitudes for which $\sin\theta$ may be replaced by θ in the equation

$$\frac{d\theta}{dt} + S \sin\theta = f(t) \tag{16}$$

the bandwidth is a function of S. In order to evaluate this constant S, consider the following equation

$$\sin\theta = \frac{Q_{ext}(\omega_1 - \omega)}{|p| \ \omega_o} \quad . \tag{17}$$

If under locked operation, ω_2 is the oscillator frequency at which "unlocking" occurs, then $\sin\theta = -1$ and Eq. 17 may be written as

$$\frac{-|p| \ \omega_o}{Q_{ext}} = (\omega_1 - \omega_2) \quad , \text{ if } \omega_2 > \omega_1 \quad . \tag{18}$$

If ω_3 is the frequency at which "unlocking" occurs then $\sin\theta = 1$ and Eq. 17 may be written as

$$\frac{|p| \ \omega_o}{Q_{ext}} = (\omega_1 - \omega_3) \quad , \quad \text{if } \omega_3 < \omega_1 \quad . \tag{19}$$

Subtracting Eq. 18 from Eq. 19, the result is

$$\frac{|p| \ \omega_o}{Q_{ext}} = \frac{(\omega_2 - \omega_3)}{2} = S \quad . \tag{20}$$

To obtain $\omega_1 - \omega_3$, the change in reflector potential between the two "unlocking" points was obtained directly from the static characteristic curves (Fig. 11). The klystrons were then operated with the locking signal removed, and the change in frequency that resulted from the above change in reflector potential was measured. This change in frequency was substituted in Eq. 20 to calculate S.

Figure 18 shows the predicted amplitude response for sinusoidal modulation using the above calculated value of S in the linear approximation of Eq. 16 together with the sinusoidal response obtained experimentally. It will be noted that the experimental results are in close agreement with the calculated response. From the experimental results, it seems safe to assume that the system bandwidth may be calculated using the linear assumption of Eq. 16. In the system under investigation, the bandwidth was approximately 3.2 Mc/sec.

As a further check on the bandwidth, the oscillators were modulated with a pulse signal and the resultant output waveform was observed. The waveform of a 2 microsecond

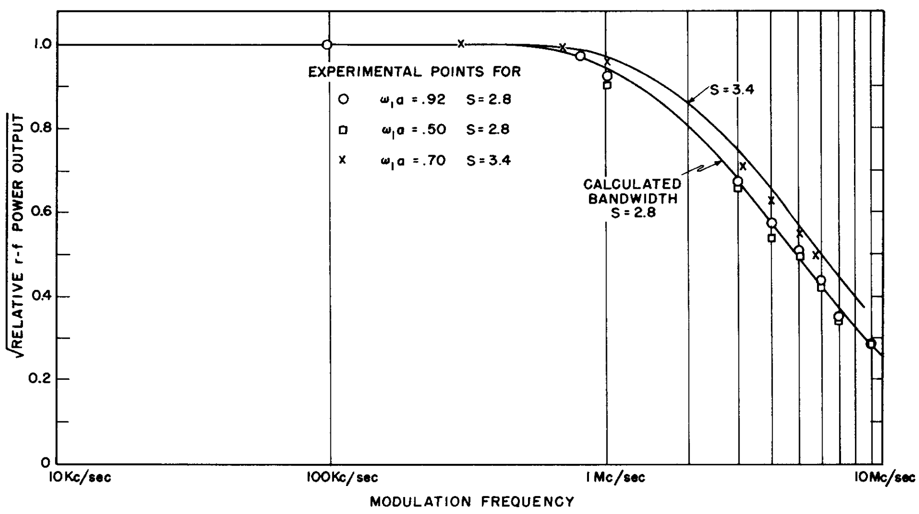


Fig. 18 Amplitude response for sinusoidal modulation.

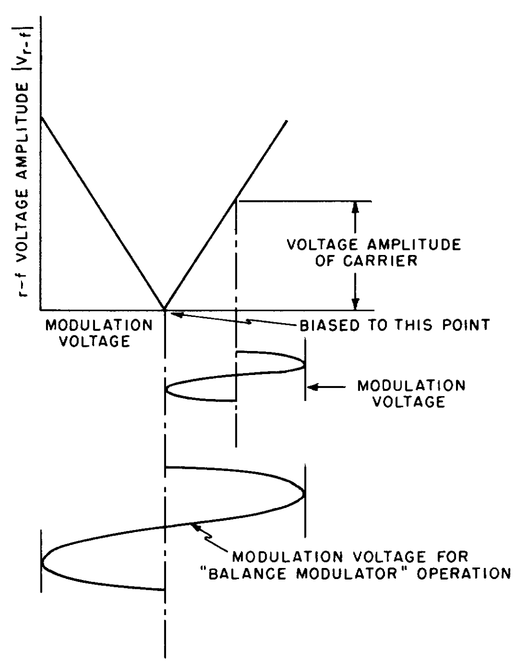


Fig. 19 A method of obtaining amplitude-modulated output composed of a carrier and side bands. Note the decrease in available side-band power when the system is used to supply the carrier.

pulse remained essentially unchanged. No accurate measurements of the rise time of the leading edge of the pulse could be made since the resultant trace on the synchroscope was small. It is of interest to note that the modulating voltage waveform had a spike of approximately 0.2 microsecond width immediately preceding the main pulse and that this spike could be distinguished in the output waveform.

In order to obtain a qualitative check on the over-all characteristics, the system was modulated with the audio output of a commercial receiver. The modulated output was then detected with a silicon-crystal rectifier and the audio signals were applied to a speaker. In the course of this operation, both speech and music were detected and no unusual distortion was noted. The performance was noted for several hours of continuous operation and no adjustments were made during this time.

In all previous tests, the system was operated as a balanced modulator system and the carrier when needed was supplied by the locking generator. However an examination of the results shown in Fig. 12 indicates that a complete (side bands plus carrier) amplitude-modulated output may be obtained from the locked oscillators. This may be accomplished by biasing the reflectors in such a manner that for zero modulation voltage the r-f output is given by a point on one of the operating lines shown in Fig. 12. If a modulation voltage is now applied, the r-f output will be amplitude modulated about this point. This is shown schematically in Fig. 19. The amplitude of the modulating voltage must be limited so that the r-f output varies between the minimum value and the value at which the oscillators unlock. A test was made of this operation and the results were as predicted. It will be noted that the side-band power is reduced to half that attained with the balanced-modulation type of operation.

VI. Conclusions

From the experimental investigation, it is clearly seen that the basic formulation of this modulation system is correct. The practicability of the system depends to a great extent on the power level at which it is to be used. It is seen that the maximum r-f efficiency that can be attained with the system investigated is 50 percent. That is, under maximum modulation, half of the total output of the locked oscillators is absorbed by the locking source; in effect, it is an absorption modulator. This poor r-f efficiency however is somewhat compensated by the extreme linearity of the system and the low modulation-signal voltage and power requirement.

The system has a further disadvantage in that two oscillators are used in a push-pull arrangement. As with all such systems, this requires the use of two oscillator tubes with similar characteristics. However, the only critical characteristic is the frequency variation of the oscillators. That is, the frequency modulation must be linear. If the frequency variations are linear, the difference in output power, required modulation voltage and locking characteristics may be readily compensated.

It is believed that the over-all characteristics of this modulation technique are such that the modulation method would be advantageous in many microwave systems. This is particularly true in low-power-level systems where r-f efficiency is a secondary consideration. Therefore a further and more complete investigation of the bandwidth of this system is warranted. Also, in order to obtain the desired modulated output, it is essential that the output voltages of the locked oscillators have the proper phase relationship (180° phase difference at the load) at zero modulation voltage. A change in the free-running oscillator frequency as the result of temperature changes of the oscillator will alter this phase relationship. Therefore, consideration should be given to the problem of stabilizing the locked oscillators with respect to these frequency drifts. Since these frequency drifts are random and of very long periods, it should be possible to design a low-pass stabilizing circuit with a cut-off frequency well below the essential modulation frequencies.

As the system now stands a locking signal must be supplied. With the present available methods of injecting this locking signal, the external source output power must be several orders of magnitude greater than the oscillators being locked. This disadvantage may be eliminated if the two oscillators could be locked to each other and then modulated in push-pull to produce the amplitude-modulated output. This possibility was checked very briefly and no definite results were obtained. However, the brief results observed seem to warrant further investigation of this system.

Appendix I

If two oscillators are locked to an external signal of fixed frequency, the voltage output of each oscillator may be expressed with reference to the locking signal as

$$e_1 = V_1 \sin(\omega_1 t + \theta_1) \tag{A.1}$$

and

$$e_2 = V_2 \sin(\omega_1 t + \theta_2) \tag{A.1a}$$

where ω_1 = frequency of external signal.

The phase angle between the oscillator voltage and the external signal voltage is given by the expression

$$\sin\theta = m(\omega_1 - \omega) \tag{A.2}$$

where ω is the free-running frequency of the locked oscillator and m is a constant depending upon the magnitude of the locking voltage and the circuit parameters. If the free-running frequency of each oscillator is made equal to ω_1 and the voltage output from each oscillator is fed to a load such that the voltages have a 180° phase difference, the load voltage will be

$$e_r = e_1 - e_2 = V_1 \sin\omega_1 t - V_2 \sin\omega_1 t \quad . \tag{A.3}$$

When the free-running frequencies of the locked oscillators are modulated in push-pull, Eq. 3 becomes

$$e_r = V_1 \sin(\omega_1 t + \theta_1) - V_2 \sin(\omega_1 t - \theta_2)$$

$$= (V_1 \cos\theta_1 - V_2 \cos\theta_2) \sin\omega_1 t + (V_1 \sin\theta_1 + V_2 \sin\theta_2) \cos\omega_1 t \quad . \qquad (A.4)$$

If the frequency deviation of each oscillator is linear, then for sinusoidal modulation of the free-running frequency of each oscillator

$$\sin\theta_1 = M \sin\omega_m t \quad , \quad M \leqslant 1$$

and

$$\sin\theta_2 = M_2 \sin\omega_m t \quad , \quad M_2 \leqslant 1 \quad .$$

Since there is no loss in generality by assuming $M \geqslant M_2$, M_2 may be expressed as kM, $k \leqslant 1$. Therefore

$$\sin\theta_1 = M \sin\omega_m t \qquad (A.5)$$

and

$$\sin\theta_2 = kM \sin\omega_m t \quad . \qquad (A.5a)$$

Writing $V_2 = hV_1$ and substituting Eqs. A.5 and A.5a into Eq. A.3, e_r becomes

$$e_r = V_1 \left[(\cos\theta_1 - h \cos\theta_2) \sin\omega_1 t + (M \sin\omega_m t + khM \sin\omega_m t) \cos\omega_1 t\right]$$

$$= V_1 \left[(\cos\theta_1 - h \cos\theta_2) \sin\omega_1 t + M(1 + kh) \sin\omega_m t \cos\omega_1 t\right] \quad . \qquad (A.6)$$

In Eq. A.6, the second term is the side-band expression of the amplitude-modulated wave and the effect of unequal oscillator voltages and frequency deviations is a modification of the peak amplitude of this expression. The first term however represents distortion. This distortion may be clearly seen by expanding this expression in terms of $\sin\omega_m t$ as follows.

$$\cos\theta_1 = \sqrt{1 - M^2 \sin^2\omega_m t} = 1 - \tfrac{1}{2} M^2 \sin^2\omega_m t - \tfrac{1}{8} M^4 \sin^4\omega_m t - \ldots \quad M < 1. \qquad (A.7)$$

By substituting kM in Eq. A.7, the expression for $\cos\theta_2$ is obtained.

$$\cos\theta_2 = 1 - \tfrac{1}{2} (kM)^2 \sin^2\omega_m t - \tfrac{1}{8} (kM)^4 \sin^4\omega_m t - \ldots \qquad (A.8)$$

Therefore

$$\cos\theta_1 - h \cos\theta_2 = (1-h) - \frac{M^2}{2}(1-hk^2) \sin^2\omega_m t - \frac{M^4}{8} (1-hk^4) \sin^4\omega_m t - \text{higher order terms}\ldots \qquad (A.9)$$

By expressing $\sin^n\omega_m t$ in terms of multiple angles, it will be noted that Eq. A.9 may be expressed in terms of the even harmonics of $\sin\omega_m t$ plus a constant. Since Eq. A.9 is composed of even powers of $\sin\omega_m t$, all the terms will contribute to the constant term and the second harmonic term. It should be further noted that Eq. A.9 is valid only for $M < 1$. Therefore for modulation factors (M) near unity, a large number of terms of Eq. A.9 must be used before the contribution to the lower harmonics become negligible. However by using a few terms of Eq. A.9 to obtain the amplitudes of the various harmonics, a minimum amplitude of these harmonics for large modulation factors will be obtained. Using the first three terms of Eq. A.9 the d-c term is given as:

$$1 - h - \frac{M^2}{4}(1 - hk^2) - \frac{3}{64} M^4(1 - hk^4) - \frac{5}{256} M^6(1 - hk^6) - \ldots \qquad (A.10)$$

The amplitude of the second harmonic is given as

$$\frac{M^2}{4}(1 - hk^2) + \frac{M^4}{16}(1 - hk^4) + \frac{15}{512} M^6(1 - hk^6) + \ldots \qquad (A.11)$$

The amplitude of the fourth harmonic is given as

$$-\frac{M^4}{64}(1 - hk^4) - \frac{3M^6}{256}(1 - hk^6) - \ldots \qquad (A.12)$$

As was noted above, these expressions are valid only for $M < 1$ and as M approaches unity the above expressions become inaccurate. However, the amplitudes of the harmonics will always be greater than the values calculated from the above equations.

For values of $M = 1$, the amplitude of the harmonics may be calculated by considering the variation of $\cos\theta$, as the phase angle θ is modulated between $\pi/2$ and $-\pi/2$. When $\sin\theta$ is modulated sinusoidally such that θ varies from $\pi/2$ to $-\pi/2$. When $\sin\theta$ is modulated sinusoidally such that θ varies from $\pi/2$ to $-\pi/2$ ($M = 1$), then

$$\cos\theta = |\sin(\omega_m t + 90)| = |\cos\omega_m t| \quad . \qquad (A.13)$$

Expanding $\cos\theta$ as given in Eq. A.13 in a Fourier series, it becomes

$$\cos\theta = \frac{2}{\pi} + \frac{4}{3\pi} \cos 2\omega_m t - \frac{4}{\pi 15} \cos 4\omega_m t + \ldots \quad M = 1 \quad . \qquad (A.14)$$

Now the above expression gives the maximum amplitude of the harmonics, since for M less than 1, the harmonic amplitudes are reduced as shown by Eq. A.8. Therefore for $M = 1$, $\cos\theta_1$ will be given by Eq. A.14. However, since $\sin\theta_2 = kM \sin\omega_m t$ $k < 1$, $\cos\theta_2$ must be expressed by Eq. A.8. For k near unity, Eq. A.8 converges very slowly and therefore, to be accurate a large number of terms must be used. To overcome this difficulty, the harmonic amplitudes can be nested between two expressions. If $\cos\theta_2$ is approximated by $h \cos\theta_1$, where $\cos\theta_1$ has the value given by Eq. A.14, the approximation will be greater than the actual value of $\cos\theta_2$. On the other hand if a few terms of Eq. A.8 are used to approximate $\cos\theta_2$, the approximation will be less than the actual value. From the preceding discussion, it is seen that in general Eq. A.4 will be expressed as

$$e_r = V_1A_o \sin\omega_1 t + V_1A_2 \cos 2\omega_m t \sin\omega_1 t + V_1A_4 \cos 4\omega_m t \sin\omega_1 t + \ldots$$
$$+ V_1A_1 \sin\omega_m t \cos\omega_1 t \quad , \qquad (A.15)$$

where A_m is the amplitude of the harmonics. It is seen that unequal oscillator output voltages and frequency deviations add harmonics of the modulating frequency to the modulated wave. The amplitudes of the added harmonics are best expressed as a ratio to the fundamental. Thus for $M << 1$,

$$\frac{A_o}{A_1} \cong \frac{1 - h - \frac{M^2}{4}(1 - hk^2) - \frac{3M^4}{64}(1 - hk^4)}{M(1 + hk)}$$

$$\frac{A_2}{A_1} \cong \frac{M}{4}\left[\frac{(1 - hk^2) + \frac{M^2}{4}(1 - hk^4) + \frac{15M^4}{128}(1 - hk^6)}{(1 + hk)}\right]$$

$$\frac{A_4}{A_1} \cong -\frac{M^3}{64}\ \frac{\left[(1 - hk^4) + \frac{3M^2}{4}(1 - hk^6)\right]}{(1 + hk)} \quad .$$

Finally for

$$M = 1 \qquad k = 1$$

$$\frac{A_o}{A_1} \cong \frac{2}{\pi}\ \frac{(1 - hk)}{1 + hk}$$

$$\frac{A_2}{A_1} \cong \frac{4}{3\pi}\ \frac{(1 - hk)}{1 + hk}$$

$$\frac{A_4}{A_1} \cong \frac{4}{15\pi}\ \frac{(1 - hk)}{1 + hk}$$

From the preceding analysis it is seen that the harmonic distortion increases with the modulation factor. Also the difference in frequency deviation between the two oscillators (this is expressed by the amount k differs from unity) has a greater effect on the distortion added than the difference in oscillator output voltages. In general, for good linearity the oscillators should have nearly identical operating characteristics.

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