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AT MICROWAVE FREQUENCIES

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Abstract

The effect of magnetic field on the high-frequency breakdown of gases has been studied. The presence of energy resonance and the modification of diffusion are shown experimentally and explained theoretically. An application is made of both the average electron theory and the Boltzmann theory, and the correspondence between these two theories is discussed.
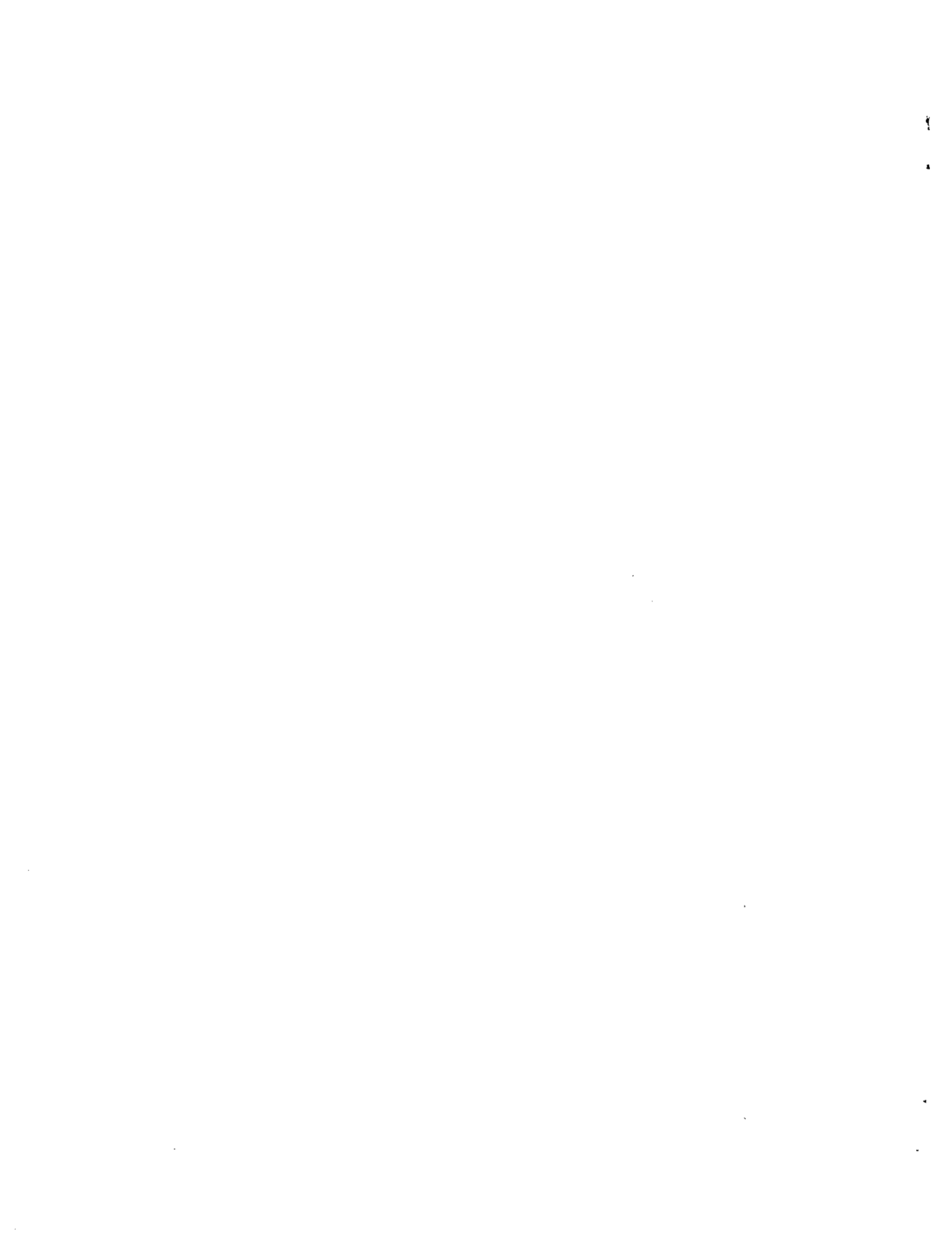
THE EFFECT OF MAGNETIC FIELD ON THE BREAKDOWN OF GASES
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The breakdown of gases by high frequency electric fields in the presence of a constant magnetic field has been studied by Townsend and Gill (1) and by A. E. Brown (2). It is the purpose of this paper to carry the analysis further, including the effect of the magnetic field on both the random diffusion of the electrons and their directed mobility.

Two approaches are available to such kinetic problems and, as there are advantages to each, both will be used. In Part I, the average electron theory will be given. In this method the orbit of a free electron in the assumed fields is computed first, and from this one computes the displacement and the energy gain in the time $\tau = t - t_o$ elapsed since a collision. These quantities are then averaged over the phase of the a-c field at the time t_o of the last collision, over the direction in space of the velocity after the collision, and over the free time up to the next collision. The result is the mean square displacement and energy gain of the average electron between collisions. One can then discuss an average electron from its initial low energy until it ionizes a gas atom or diffuses out of the tube. The condition for breakdown is that these two final achievements shall be equally probable. This method has the advantage that each step in the analysis has a direct physical meaning.

In Part II the Boltzmann transport equation is expanded in spherical harmonics in space, and in Fourier series in time. There results a differential equation for the distribution function which is integrated. Most of the properties of a discharge follow directly from a knowledge of the distribution function.

I. Average Electron Theory

Velocity between Collisions

Consider the motion of an electron between collisions under the influence of an electric field along the x-axis, $E = E_p \exp(j\omega t)$, and a constant magnetic field B along the z-axis. The equation of motion is then

$$\vec{F} = -e\vec{E} - e\vec{v} \times \vec{B} = m\dot{\vec{v}}. \tag{1}$$

The solution of this equation is the sum of a general and a particular integral, which correspond to the superposition of a circular helical motion and a plane elliptical motion. For the helical motion whose axis is along the magnetic field, the velocity is

$$v_{1x} = (a + jb)\exp(j\omega_b\tau)$$

$$v_{1y} = (b - ja)\exp(j\omega_b\tau) \Bigg\} \quad (2)$$

$$v_{1z} = c$$

and oscillates at the cyclotron frequency $\omega_b = eB/m$. Because the helical motion contains the three arbitrary constants, the energy of this motion is constant and is given in electron-volts by

$$u_1 = \frac{mv_1^2}{2e} = \frac{m}{e}\frac{a^2 + b^2 + c^2}{2} . \quad (3)$$

For the elliptical motion, the velocity is

$$v_{2x} = \frac{eE_p}{m}\frac{j\omega}{\omega^2 - \omega_b^2}\exp(j\omega t)$$

$$v_{2y} = \frac{eE_p}{m}\frac{\omega_b}{\omega^2 - \omega_b^2}\exp(j\omega t) \Bigg\} \quad (4)$$

and oscillates at the frequency of the applied field. The kinetic energy of this motion is uniquely determined by the magnitude and frequency of the applied field and is given by

$$u_2 = \frac{eE_p^2}{8m}\left[\frac{1}{(\omega - \omega_b)^2} + \frac{1}{(\omega + \omega_b)^2} + \frac{2\cos 2\omega t}{\omega_b^2 - \omega^2}\right] . \quad (5)$$

The total energy $u = \frac{m}{2e}\left|(\vec{v}_1 + \vec{v}_2)\right|^2$ will contain cross-product terms u_{12} which are important but rather lengthy to write down. Their average value will be given later. The three constants, a, b, and c, of the helical motion are determined by the velocity $\vec{v}_o = \vec{v}_1(\tau = 0) + \vec{v}_2(t = t_o)$ immediately after a collision. As the time τ has been used in Eqs. 2 one has simply

$$a = v_{ox} - \frac{eE_p}{m}\frac{j\omega}{\omega^2 - \omega_b^2}\exp(j\omega t_o)$$

$$b = v_{oy} - \frac{eE_p}{m}\frac{\omega_b}{\omega^2 - \omega_b^2}\exp(j\omega t_o) \Bigg\} \quad . \quad (6)$$

$$c = v_{oz}$$

It is noted that the elliptical motion exhibits a resonance at frequencies near the cyclotron frequency. Exactly at this frequency Eqs. 4 no longer hold and the solution corresponds to a spiral, but as collisions interrupt the motion it will not be necessary to use this singular solution.

Diffusion

From the velocities one obtains the displacements x_1, y_1, z_1, x_2, y_2 by integration. From these one can calculate the mean displacements $\bar{x}_1$, etc. but these all vanish on averaging over orientations of v_o, which is assumed isotropic, and over collision times t_o. In the average, an electron stays where it is in a high-frequency discharge.

We are interested in the mean square displacements $\overline{x_1^2}$, $\overline{y_1^2}$, $\overline{z_1^2}$ because these lead (3) to the diffusion coefficient. One finds that the cross-product terms such as $\overline{x_1 y_1}$ all vanish when averaged.

$$x_1^2 + y_1^2 = 2 \frac{a^2 + b^2}{\omega_b^2}(1 - \cos \omega_b \tau)$$

$$z_1^2 = c^2 \tau^2 . \tag{7}$$

Averaging over orientations and times t_o, one finds that

$$\overline{a^2} = \overline{b^2} = \overline{c^2} = \frac{v_1^2}{3} ; \quad \overline{ab} = 0 . \tag{8}$$

The cross terms between the helical and elliptical motions also vanish but terms $\overline{x_2^2}$ and $\overline{y_2^2}$ do not vanish. However, these latter terms represent the mean square displacements due to mobility in the applied a-c field and are not wanted in calculating the diffusion.

The average of a quantity X over the free times $\tau_c = 1/v_c$ between collisions is, by definition

$$\bar{X} = \int_0^\infty X \exp(-v_c \tau) v_c d\tau . \tag{9}$$

Applying this to the quantities in Eqs. 7 and defining the diffusion coefficients in terms of the mean square displacements, we obtain

$$D_{yy} = D_{xx} = \frac{\overline{x_1^2}}{2\tau_c} = \frac{v_1^2 v_c}{3(\omega_b^2 + v_c^2)}$$

$$D_{zz} = \frac{v_1^2}{3v_c} . \tag{10}$$

This definition of the diffusion tensor is of necessity symmetric. We shall see later that there are skew-symmetric terms which the random-walk definition cannot give. Diffusion along the z-axis is not altered by the magnetic field, but in the plane at right angles to the field it is reduced in the ratio $v_c^2/(\omega_b^2 + v_c^2)$. For a given collision frequency, the diffusion coefficient is proportional to the energy of the electrons in their helical motion.

Energy Gain

The mean energy gain between collisions is best obtained by considering the power input to an electron $P = -e\vec{E} \cdot \vec{v}$. As the velocity v_{2x} is out of phase with the field, the corresponding power P_2 into this motion is zero in the average and only the power P_1 need be considered where

$$P_1 = -eE_p \cos \omega t(a \cos \omega_b \tau - b \sin \omega_b \tau) \tag{11}$$

and the constants a and b are given by Eqs. 6. Averaging over orientations of the initial velocity the terms in v_o drop out. Averaging over t_o also we obtain

$$\bar{P} = \frac{e^2 E_p^2}{4m} \left[\frac{\sin(\omega + \omega_b)\tau}{\omega + \omega_b} + \frac{\sin(\omega - \omega_b)\tau}{\omega - \omega_b} \right] \tag{12}$$

from which the average energy is obtained by integrating with respect to τ from 0 to τ.

$$\overline{u_{12}} = \frac{eE_p^2}{4m} \left[\frac{1 - \cos(\omega + \omega_b)\tau}{(\omega + \omega_b)^2} + \frac{1 - \cos(\omega - \omega_b)\tau}{(\omega - \omega_b)^2} \right]. \tag{13}$$

Averaging this quantity over collision times gives the mean energy gain between collisions

$$u_c = \frac{eE_p^2}{4m} \left[\frac{1}{(\omega + \omega_b)^2 + v_c^2} + \frac{1}{(\omega - \omega_b)^2 + v_c^2} \right] \equiv \frac{eE_e^2}{mv_c^2} . \tag{14}$$

This is a fundamental quantity in this theory. At low pressures ($v_c \to 0$) we see that it approaches twice the mean energy $\overline{u_2}$ of the elliptical motion of an electron, and in no case does it exceed this motion. At higher pressures, such that there are many collisions per oscillation, the energy $\overline{u_2}$ loses its meaning and the collision energy becomes $eE_p^2/2mv_c^2$. One can
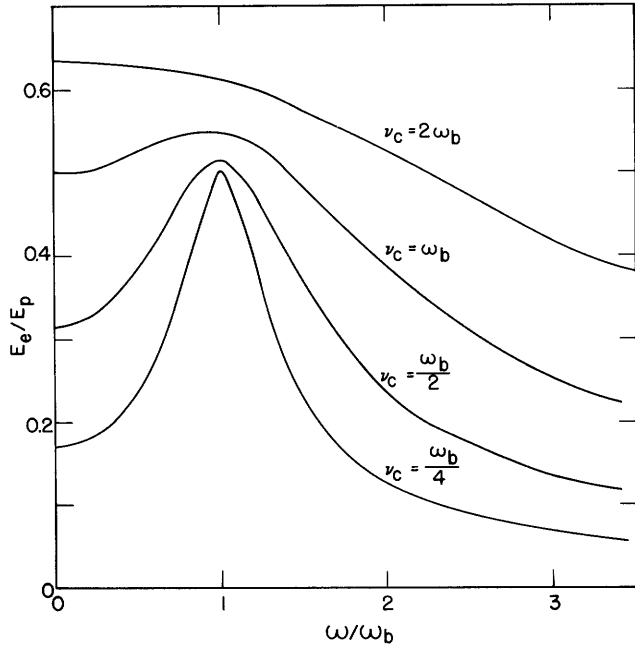
use Eq. 14 to define an effective field E_e which is the root-mean-square field at high pressure. This concept is useful when the collision frequency ν_c is independent of velocity since this single function takes into account the effects of frequency and magnetic field on the energy.

At low pressures the effective field has a maximum at resonance with the cyclotron frequency as shown in Fig. 1.

Fig. 1.
The effective field E_e as a function of the frequency showing the resonance in the presence of a magnetic field.

Breakdown

The electrons produced by ionization have initially very little energy, but this increases by steps of u_c until the energy reaches a value u_1, at which ionization occurs. This is above the ionization potential V_1 by an amount which we shall neglect. Excitations are disregarded in the following simple theory. The number N of free times to ionize is $N = u_1/u_c$ when ν_c is constant.

The electrons thus double their number by ionization every N collisions and unless some equally effective process exists which removes electrons their number will increase exponentially. In most cases diffusion to the walls of the discharge tube is the balancing process. In absence of the magnetic field, the random-walk theory (4) gives the mean square distance $\Lambda^2 = N\ell^2/3$ reached in N free paths of mean square length ℓ^2, so that if the average electron reaches the wall in a distance Λ the diffusion process will just balance ionization. This is the condition for breakdown and we can write it

$$\frac{u_c}{u_1} = \frac{\overline{\ell^2}}{3\Lambda^2} = \frac{\overline{v^2}}{3\Lambda^2 \nu_c^2} \qquad (15)$$

where Λ is now a length characteristic of the discharge tube and known as the diffusion length (5).

If there is a magnetic field u_c will be altered according to Eq. 14.

At the same time the random-walk theory must be altered to take into account the curved paths between collisions. This may be done by appropriately decreasing ℓ or increasing Λ. We shall adopt the latter and denote the new length by Λ_e. Its value will be given later.

When the mean free path is much smaller than Λ_e, the inter-collision energy gain u_c is correspondingly smaller than the ionization potential. From Eq. 15 we see that breakdown should occur at the same effective field if the ratio of the mean free path to the effective diffusion length is the same, that is, the effective field for breakdown is a function of $p\Lambda_e$ only.
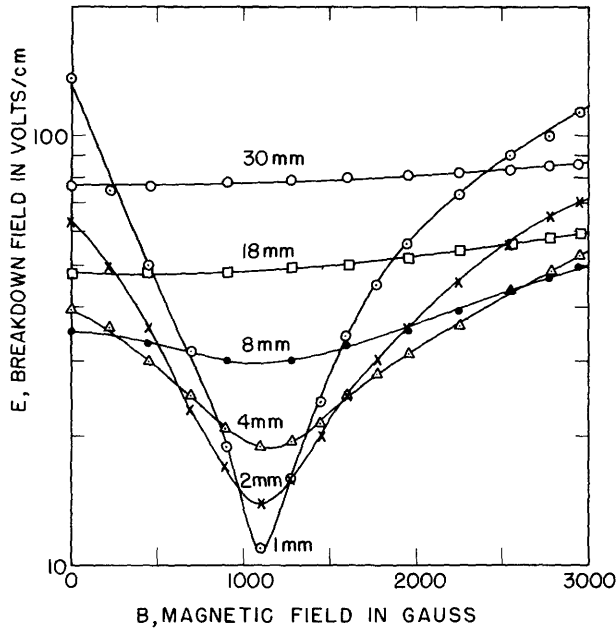

Fig. 2.
Breakdown of helium in transverse electric and magnetic fields; diameter = 7.32 cm, height = 4.60 cm.

Combining Eqs. 14 and 15 we get

$$E_e^2 = \frac{2}{3} \frac{\overline{u}u_1}{\Lambda_e^2} . \qquad (16)$$

Experimental data for breakdown in helium containing small admixtures of mercury vapor are shown in Fig. 2. The result of using the effective field with $v_c = 2.37 \times 10^9 p$ as given by Brode's (6) collision probability measurements in place of the actual field but without taking the variation of Λ_e into account is shown in Fig. 3 for the same data. This shows that the resonance effect of the magnetic field and the high frequency are removed by using the effective field.

In order to test formula 10 for the diffusion coefficient, breakdown was studied in a flat cylindrical cavity whose length was
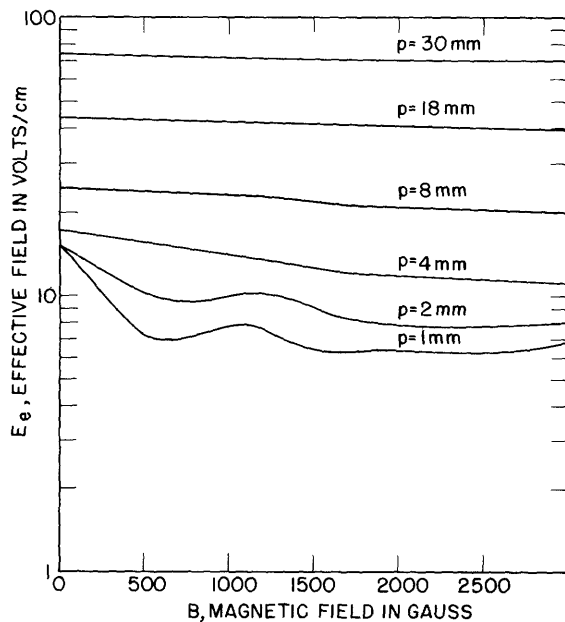

Fig. 3.
Representation of experimental data of Fig. 2 in terms of the effective field.
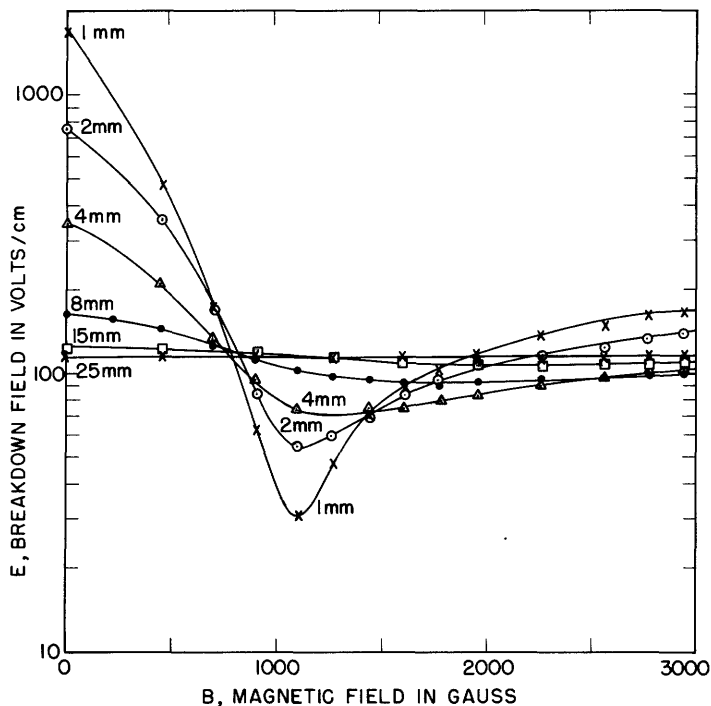
Fig. 4.
Breakdown of helium in transverse electric and magnetic fields; diameter = 7.32 cm, height = 0.318 cm.

very short compared to the radius. With the magnetic field placed transverse to the axis most of the diffusion has to take place perpendicular to the magnetic field and hence will show the full reduction. By Eq. 10 the mean square of the distance travelled by an electron is proportional to the diffusion coefficient D, and therefore the effective diffusion length Λ_e appropriate to infinite parallel plates is
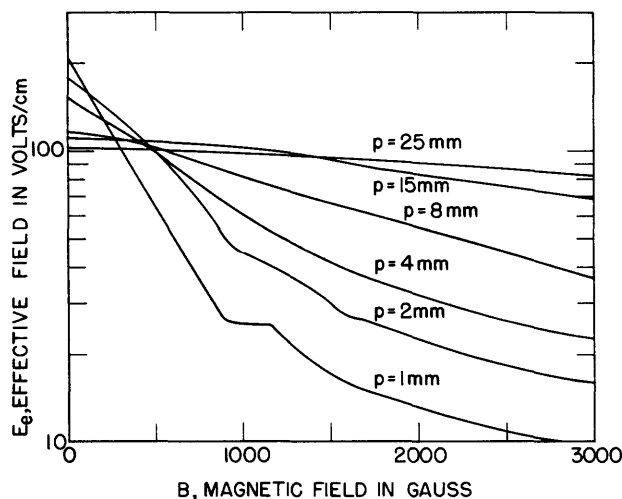
Fig. 5.
Representation of experimental data of Fig. 4 in terms of the effective field.

$$\Lambda_e^2 = \frac{\omega_b^2 + v_c^2}{v_c^2} \Lambda^2 . \qquad (17)$$

The effect of a magnetic field is to make the dimensions of the cavity at right angles to the field appear larger to an electron. By Eq. 16 this should reduce the effective field for breakdown in the same proportion. Figure 4 shows a set of breakdown curves for helium in a cavity 2 7/8 inches in diameter and 1/8 inch high. In Fig. 5 the effective field for the same data is plotted. In this figure the resonance peak has been removed as in

Fig. 3, but now the curves are not horizontal because the magnetic field is increasing the effective cavity size.

However, Eq. 16 does not correspond with experiment except when used for comparative purposes. Equation 14 and the random-walk theory give the number of free times for the average electron to ionize and reach the wall, respectively. But in a discharge it is the "faster-than-average" electron which ionizes and the "more-mobile-than-average" electron which leaves the tube and these are not the same electron. The mean-free-path method must therefore fail in predicting quantitative breakdown, and the failure should be worst when there are many collisions and therefore the greatest deviations from the mean.

II. Boltzmann Theory

Spherical and Fourier Expansions

The Boltzmann transport equation is given by

$$C = \frac{\partial F}{\partial t} + \nabla \cdot (\vec{v}F) - \nabla_v \cdot \frac{e\vec{E}F}{m} - \nabla_v \cdot \frac{e}{m} \vec{v} \times \vec{B}F \qquad (18)$$

where C is the production rate due to collisions per unit volume in phase space, ∇ and ∇_v are gradients in configuration and velocity space, respectively, and F is the distribution function. If F and C are expanded in spherical harmonics and substituted into the transport equation, we obtain two equations by equating the zero and first order terms

$$C_0 = \frac{\partial F_0}{\partial t} + \frac{v}{3} \nabla \cdot \vec{F}_1 - \frac{e\vec{E}}{3mv^2} \cdot \frac{\partial(v^2 \vec{F}_1)}{\partial v}$$

$$\vec{C}_1 = \frac{\partial \vec{F}_1}{\partial t} + v\nabla F_0 - \frac{e}{m} \vec{E} \frac{\partial F_0}{\partial v} - \frac{e}{m} \vec{B} \times \vec{F}_1 . \qquad (19)$$

We consider separately the elastic and inelastic collisions. Let C_{el} represent the elastic collision term. Morse, Allis and Lamar (7) have computed the zero and first order components of this term:

$$C_{0,el} = \frac{m}{M} \frac{1}{v^2} \frac{\partial}{\partial v}\left(\frac{v^4 F_0}{\ell}\right)$$

$$\vec{C}_{1,el} = -\frac{v}{\ell} \vec{F}_1 \qquad (20)$$

where M is the mass of the gas molecule and ℓ the mean free path of the electron.

The inelastic term C_{in} shows no angular dependence and hence $C_{1,in} = 0$ and only $C_{0,in}$ is retained. Equation 19 then becomes

$$[0] \quad C_{0,in} + \frac{m}{M} \frac{1}{v^2} \frac{\partial}{\partial v}(v_c v^3 F_0) = \frac{\partial F_0}{\partial t} + \frac{v}{3} \nabla \cdot \vec{F}_1 - \frac{e}{3mv^2} \frac{\partial(v^2 \vec{E} \cdot \vec{F}_1)}{\partial v}$$

(21)

$$[1] \quad 0 = v_c \vec{F}_1 + \frac{\partial \vec{F}_1}{\partial t} + v\nabla F_0 - \frac{e}{m} \vec{E} \frac{\partial F_0}{\partial v} - \frac{e}{m} \vec{B} \times \vec{F}_1 \ .$$

If the electric field is given by $\vec{E} = \vec{E}_p \exp(j\omega t)$, the Fourier expansions become

$$\vec{F}_1 = \vec{F}_1^0 + \vec{F}_1^1 \exp(j\omega t)$$

and, with due caution in multiplying complex quantities

$$\vec{E} \cdot \vec{F}_1 = \frac{1}{2} \vec{E}_p \cdot (\vec{F}_1^1)_r + \vec{E}_p \cdot \vec{F}_1^0 \exp(j\omega t)$$

where $(\vec{F}_1^1)_r$ is the real part of $\vec{F}_1^1$. These are now substituted into Eq. 21 and the Fourier components equated term by term. Only the constant term of the first equation is needed but the d-c and a-c terms of the second must be kept. In the steady state $\partial F_0/\partial t = 0$.

$$[0,0] \quad C_{0,in} + \frac{m}{M} \frac{1}{v^2} \frac{\partial}{\partial v}(v_c v^3 F_0) - \frac{v}{3} \nabla \cdot \vec{F}_1^0 + \frac{e}{m} \frac{1}{3v^2} \frac{\partial}{\partial v}\left[\frac{v^2}{2} \vec{E}_p \cdot (\vec{F}_1^1)_r\right] = 0$$

$$[1,0] \quad v_c \vec{F}_1^0 + v\nabla F_0 - \frac{e}{m} \vec{B} \times \vec{F}_1^0 = 0$$

(22)

$$[1,1] \quad (v_c + j\omega) \vec{F}_1^1 - \frac{e}{m} \vec{E}_p \frac{\partial F_0}{\partial v} - \frac{e}{m} \vec{B} \times \vec{F}_1^1 = 0 \ .$$

These are the necessary equations for handling breakdown problems, which represent steady-state conditions for the electrons. The above equations are applicable for any orientation of $\vec{E}$ and $\vec{B}$. We shall only consider the cases when they are perpendicular or parallel to each other.

Diffusion Tensor

Integrating the [0,0] equations over velocity space in spherical coordinates, the second and fourth terms vanish at the limits. The first

term gives the total production rate of electrons, $\nu_1 n$, due to ionization. This is because excitations merely withdraw fast electrons to replace them by slow ones, whereas ionizations add an extra electron. The third term gives the divergence of a flow vector $\vec{\Gamma}$

$$\vec{\Gamma} = \int_0^\infty \frac{4\pi}{3} \vec{F}_1^0 v^3 dv \tag{23}$$

so that

$$n\nu_1 = \nabla \cdot \vec{\Gamma} . \tag{24}$$

Solving Eq. [1,0] for $\vec{F}_1^0$ we find

$$\vec{F}_1^0 = -\frac{v}{\nu_c} \frac{v_c^2 + \vec{\omega}_b \vec{\omega}_b \cdot + v_c \vec{\omega}_b \times}{v_c^2 + \omega_b^2} \nabla F_0 \tag{25}$$

and this must be substituted in Eq. 23 and, assuming that F_0 can be written as a product $n(x,y,z)f_0(v)$, we find that $\vec{\Gamma}$ is proportional to ∇n but not necessarily in the same direction. Accordingly it is possible to define a diffusion coefficient but it must be a tensor.

$$(D_{ij}) = \int_0^\infty \frac{\ell v}{3(1 + b^2)} \begin{pmatrix} (1 + b_x^2) & (b_x b_y - b_z) & (b_x b_z + b_y) \\ (b_x b_y + b_z) & (1 + b_y^2) & (b_y b_z - b_x) \\ (b_x b_z - b_y) & (b_y b_z + b_x) & (1 + b_z^2) \end{pmatrix} 4\pi v^2 f_0 dv \tag{26}$$

$$\Gamma_i = -\sum_j D_{ij} \frac{\partial n}{\partial j} ; \quad i,j = x,y,z$$

where $b_x = (\omega_b)_x / \nu_c$ and $b^2 = b_x^2 + b_y^2 + b_z^2$. This expression reduces to the ordinary coefficient $D = v^2/3\nu_c$ with no magnetic field when $b = 0$. If the magnetic field is taken along the z-axis, it reduces to

$$D_{ij} = \int_0^\infty \frac{\ell v}{3(1 + b^2)} \begin{pmatrix} 1 & -b & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 + b^2 \end{pmatrix} 4\pi v^2 f_0 dv \tag{27}$$

which is equivalent to Eq. 10 except that the present tensor has skew-symmetric terms which were not obtained by the random-walk definition.

Substituting in 24 we obtain the diffusion equation

$$D_{xx} \frac{\partial^2 n}{\partial x^2} + D_{yy} \frac{\partial^2 n}{\partial y^2} + D_{zz} \frac{\partial^2 n}{\partial z^2} + \upsilon_1 n = 0 \qquad (28)$$

which determines the spatial distribution of the electrons. One can use the normal diffusion coefficient D provided lengths are expanded at right angles to the magnetic field in the ratio $\sqrt{1 + b^2}$.

The solution of this equation depends on the boundary conditions. One must define an effective diffusion length Λ_e for the whole cavity which takes into account these expansions

$$\frac{1}{\Lambda_e^2} = \left(\frac{1}{\Lambda_x^2} + \frac{1}{\Lambda_y^2} \right) \frac{\upsilon_c^2}{\upsilon_c^2 + \omega_b^2} + \frac{1}{\Lambda_z^2} . \qquad (29)$$

The condition for the existence of a solution of Eq. 28 satisfying the boundary conditions is

$$D = \Lambda_e^2 \upsilon_1 \qquad (30)$$

where D is the ordinary diffusion coefficient. The effect of the magnetic field is equivalent to expanding the cavity in the ratio $\sqrt{\upsilon_c^2 + \omega_b^2}/\upsilon_c$ in all directions perpendicular to the magnetic field.

Differential Equation for F_0

The distribution function F_0 is obtained by eliminating $\vec{F}_1^0$ and $\vec{F}_1^1$ from Eqs. 22. One obtains in this way

$$C_{0,in} = \frac{v^2 F_0}{3\upsilon_c \Lambda_e^2} - \frac{1}{v^2} \frac{\partial}{\partial v} \left[\frac{m}{M} \upsilon_c v^3 F_0 + \frac{e^2 E_e^2}{3m^2 \upsilon_c} v^2 \frac{\partial F_0}{\partial v} \right] \qquad (31)$$

where the effective field E_e is defined by Eq. 14 when E is at right angles to B. Should E make any other angle with B, the component at right angles is reduced by Eq. 14, and the square of this added to the mean square along B. Introducing the energy variable $u = mv^2/2e$ and the inelastic ratio h $h = -C_{0,in}/\upsilon_c F_0$, Eq. 31 becomes

$$\frac{1}{\sqrt{u}\,\upsilon_c} \frac{\partial}{\partial u} \left[\frac{2m}{M} \upsilon_c u^{3/2} F_0 + \frac{2eE_e^2}{3m\upsilon_c} u^{3/2} \frac{\partial F_0}{\partial u} \right] = \left(h + \frac{2eu}{3m\upsilon_c^2 \Lambda_e^2} \right) F_0 . \qquad (32)$$

The four terms of this equation are readily interpreted: the first, with $2m/M$, represents energy lost to recoil of the atoms; the second, with E_e^2, is the energy gain due to the field; h gives the loss of electrons through inelastic collisions; and the last term, in $1/\Lambda_e^2$, the loss through diffusion to the walls. It is noted that the magnetic field and the frequency are entirely contained in the effective quantities E_e and Λ_e. The pressure enters only in ν_c and indirectly through ν_c into E_e and Λ_e. The pressure cancels out of the first and third terms, and the only experimental parameters are thus E_e/p in the second term and $p\Lambda_e$ in the fourth. The other quantities are universal constants such as e/m and m/M, and atomic constants such as the excitation and ionization cross sections included in h. It follows that all the breakdown data should plot on a single curve when $E_e\Lambda_e$ is plotted against E_e/p. This exact law is, however, of very limited applicability. The effective quantities E_e and Λ_e depend on ν_c, which is a function of the electron's energy. It is therefore in general impossible to make the effective values the same at all energies as the law requires. For helium and hydrogen the collision frequency ν_c is very nearly constant so the effective values are significant as shown in Fig. 6.
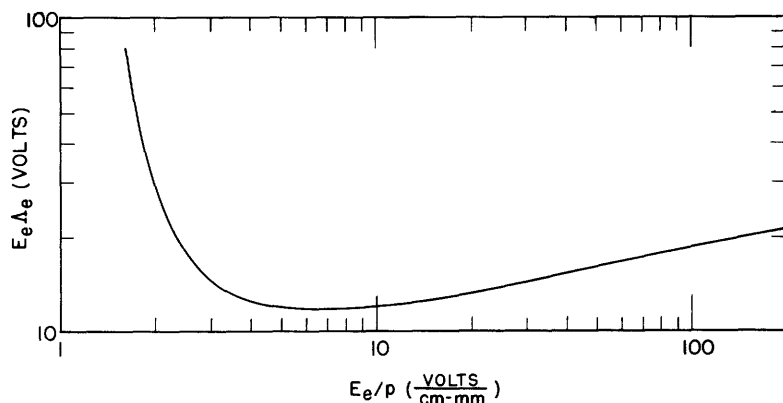

Fig. 6.
Theoretical curve for the effective breakdown in helium.

Because inelastic impacts set in discontinuously at the excitation potential u_x, it is necessary to solve Eq. 32 in two parts: below u_x, h = 0 and above u_x recoil and diffusion may generally be neglected. The solution of this equation in the completely analogous nonmagnetic case has been carried out by MacDonald and Brown (8).

The Boltzmann theory can be compared with the simple average-electron theory in the case of helium containing traces of mercury so that all excitations result in ionizations. We must then neglect the recoil and excitation terms. The solution of Eq. 32 is then expressible in terms of Bessel functions of order $\pm 1/4$. Setting the coefficients so that the function vanishes at u_1

$$F_0 = w^{-1/4} \left[J_{1/4}(w_1) J_{-1/4}(w) - J_{-1/4}(w_1) J_{1/4}(w) \right]$$

where
$$w = ju/\Lambda_e E_e \ .$$

The breakdown condition then reduces to

$$(1/4)! \ J_{1/4}(w_1) = 2w_1^{1/4}$$

or

$$u_1 = 2.273 \ \Lambda_e E_e \ .$$

This corresponds to using for the mean energy in Eq. 16

$$\bar{u} = u_1/3.44 \ .$$

III. Nonuniform Fields

To study the effect of the magnetic field on diffusion alone the electric and magnetic fields are oriented in the same direction, and in order to reduce diffusion along the magnetic field it is necessary to perform the experiment in a cavity whose height is greater than the radius. In such a cavity the electric field may no longer be considered uniform and a correction to the computations must be made in a manner which has been shown by
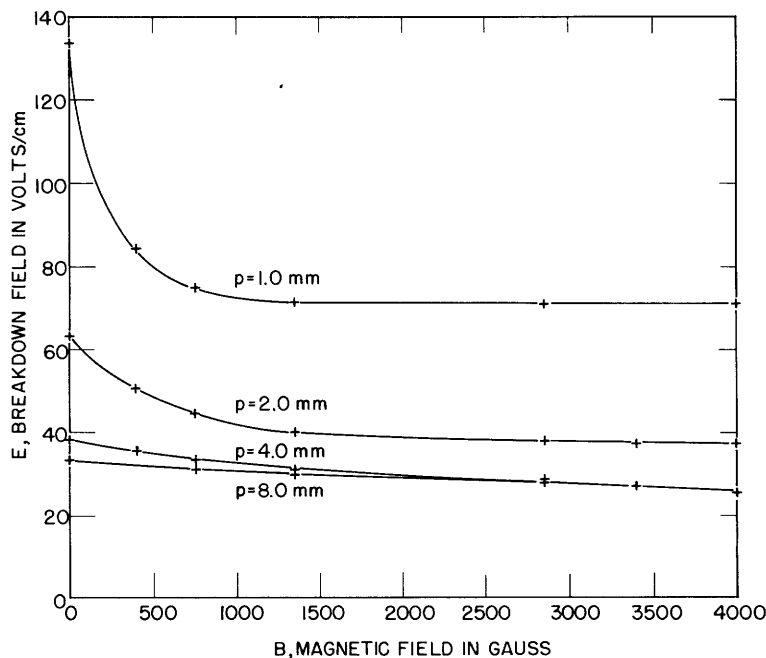

Fig. 7.
Breakdown of helium in parallel electric and magnetic fields in a cylindrical cavity; diameter = 7.32 cm, height = 4.60 cm. The solid curves are theoretical and the points are experimental.

Herlin and Brown (9). In the presence of the longitudinal magnetic field the equivalent diffusion length of the cylinder, from Eq. 29 is

$$\frac{1}{\Lambda_e^2} = \frac{v_c^2}{v_c^2 + \omega_b^2}\,(\frac{2.405}{R})^2 + (\frac{\pi}{L})^2\ .$$

Using this and the nonuniform field correction to the Boltzmann theory one obtains the agreement with experiment shown in Fig. 7. This result confirms the predicted effect of the magnetic field upon diffusion.

The breakdown measurements shown in Fig. 4 were made in a flat cavity with the magnetic field transverse to the axis. The effect of the latter is to require the solution of the diffusion equation in an elliptical cylinder whose diffusion length is then given by

$$\frac{1}{\Lambda_e^2} = \frac{v_c^2}{v_c^2 + \omega_b^2}\left[(\frac{\pi}{L})^2 + \frac{1}{2}(\frac{2.405}{R})^2\right] + \frac{1}{2}(\frac{2.405}{R})^2\ .$$

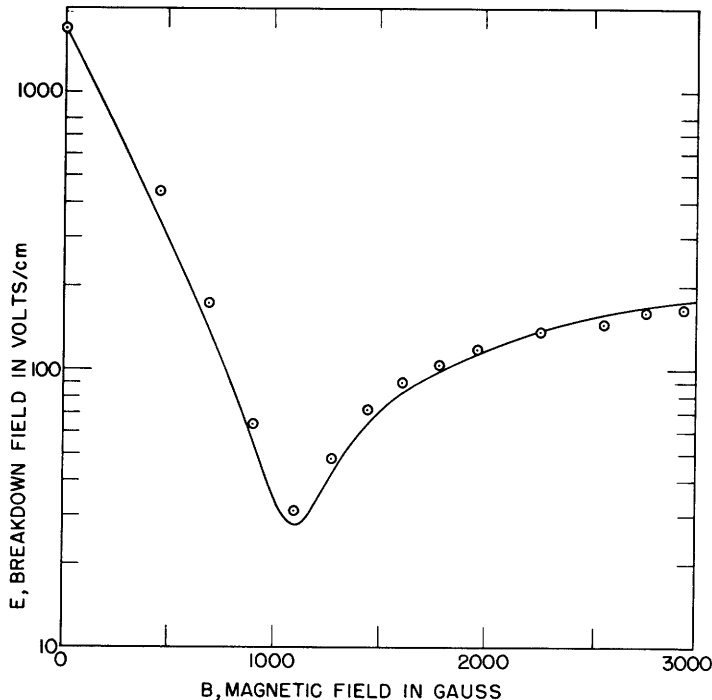Using this and the nonuniform field correction gives the theoretical curves shown in Fig. 8.

Fig. 8.
Breakdown of helium at 1-mm pressure in a cylindrical cavity; diameter = 7.60 cm, height = 0.318 cm. Solid line is obtained from the Boltzmann theory and points are experimental.

IV. Experimental Apparatus and Procedure

The block diagram of the experimental equipment is shown in Fig. 9. The source of microwave energy is a 10-cm tunable c-w magnetron whose frequency is monitored by a calibrated wavemeter. The power fed into the cavity is varied by a balanced power divider and measured through a

directional coupler by a bolometer, consisting of a thermistor and a bal-
anced power bridge. The slotted section and detector are used to determine
the electrical constants of the cavity (10). The cavity, which is vacuum-
tight, is loop-coupled to the transmission line on either side. The out-
put loop feeds into a variable calibrated attenuator, a crystal detector
and a sensitive microammeter, which are adjusted to measure the power fed
into the cavity at resonance. The magnetic field across the cavity is
applied by an adjustable electromagnet. The magnetic field for a given
gap size is calibrated as a function of the current through the coils by
the use of a snatch coil and a ballistic galvanometer or by a compensated
torque type fluxmeter. The pressure in the cavity during the experiment is
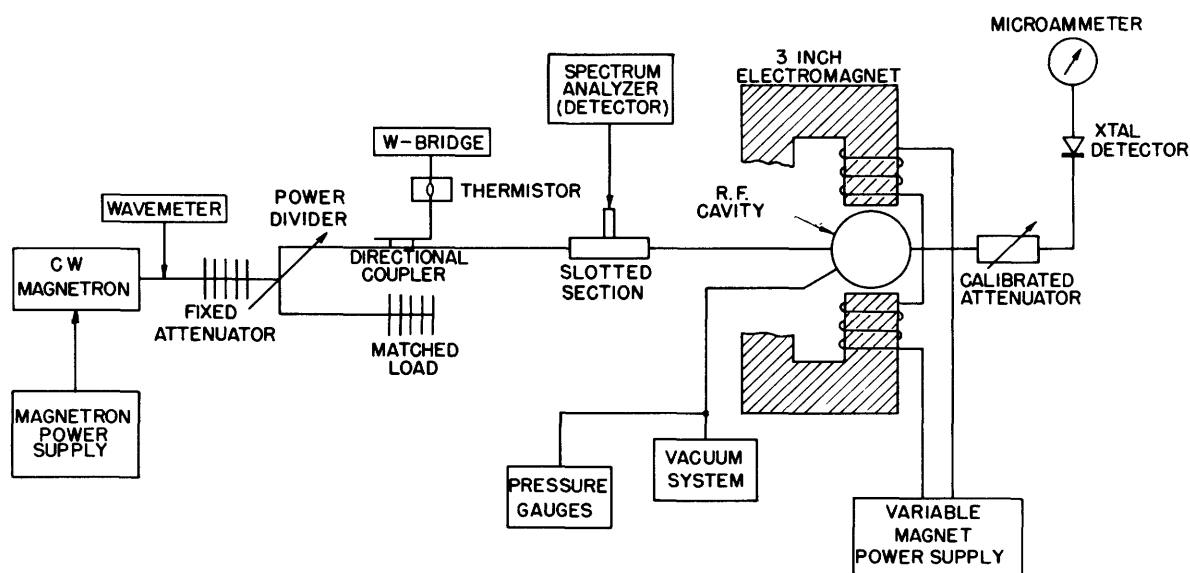measured by a McLeod gauge.

Fig. 9.
Block diagram of the apparatus.

The breakdown experiment was performed at fixed values of pressure
while the current through the coils and hence the magnetic field was set
at different values. The power fed into the cavity was increased from
nearly zero until the needle on the microammeter rose to a maximum and
then suddenly dropped. This occurred at breakdown since the sudden increase
in electron density detuned the cavity and produced a mismatch in the line.
The setting of the calibrated attenuator and the maximum reading of the
microammeter measured the power fed into the cavity. The breakdown field
was then calculated from the measured constants and geometrical dimensions
of the cavity.

V. Conclusions

The two principal effects of a magnetic field on high-frequency break-down of gases, the energy resonance with transverse fields and the reduction of diffusion, have been demonstrated experimentally and explained theoretically. The diffusion effect was shown to exist by itself when the electric and magnetic fields were parallel. The resonance phenomenon could not be separated because of the presence of diffusion at all times. Nevertheless this effect was brought into major prominence in breakdown by reducing the diffusion loss in a large cavity.

The development of the diffusion tensor in the presence of a magnetic field resulted in a more general diffusion equation. This led to the concept of the effective diffusion length, which together with the effective field, extended here to include the magnetic field, served to generalize the theory. The correspondence of the Boltzmann theory and the average electron theory was shown.

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