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# ROBUST REGRESSION AND SENSITIVITY ANALYSIS IN ESTIMATING MUTUAL FUNDS PERFORMANCE 1945 - 1964

by

Oded Berman and Eduardo Modiano*
M.I.T.

Working Paper 924-77 April, 1977

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Abstract

In estimating the performance of mutual funds ordinary least squares regression has been so far the most common statistical tool. In this paper an attempt is made to illustrate the application of a more sophisticated analysis, in particular, sensitivity anlaysis and robust regression. The paper includes results for a sample of 10 funds during the period 1945-1964.

## Acknowledgements

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## Introduction

The purpose of this paper is to challenge Jensen's results in his article "The Performance of Mutual Funds in the Period 1945-1964", using more sophisticated modern statistical techniques. These results concluded, from the analysis of 115 funds in the period 1945-1964, that portfolio managers had not superior forecasting capabilities. Out of the 115 funds investigated in the cited paper, 76 funds showed inferior forecasting capabilities.

Our work consists of analyzing a random sample of 10 such funds using sensitivity analysis and robust regression techniques implemented in the TROLL system.

This paper is composed of 7 sections:

Section 1: Derivation of Equilibrium Conditions in the Capital Asset Market.

Section 2: Jensen's Empirical Version of the CAPM

Section 3: The Database

Section 4: Sensitivity Analysis

Section 5: Residuals Analysis and Robust Regression

Section 6: Conclusions and possible Extensions

Section 7: Attachment A

1. Equilibrium conditions in the capital asset market

The theoretical results of the capital asset pricing models were derived independently by Sharpe [ 5], Lintner [ 3], Mossin [ 4], and Treynor [ 6]. A short derivation of these results based on Mossin [ 4] is presented below. The purpose of all the above models is to provide a theory of equilibrium of exchange in a market for risky assets and study the properties of this equilibrium.

Mossin assumes that the individual, as in a competitive market is a "price-taker" and has a preference ordering among possible portfolios. The solution of the problem at the individual level implicitly determines its demand for risky assets as a function of prices. The interaction of these individuals' demand schedules, under certain assumptions on the individual and market behavior, determines the prices of assets that equalize supply and demand for all assets.

All the cited models are based on the assumptions that:

a) all investors are risk averse and are single period expected utility of terminal wealth maximizers;

b) all investors have homogeneous expectations regarding investment opportunities. In Mossin this implies all individuals have identical probability distributions of the yields of the different assets;

c) all investors are able to rank portfolios solely on the basis of expected yields and variances. Borch [ I] proved that if no assumption is made about the probability distributions of yield, then individuals must have quadratic utility functions for yield. This implies rather unnatural assumptions about the market participants behavior toward risk;

d) there exists a riskless asset and all individuals are able to borrow or lend at the same riskless rate;

e) all transaction costs and taxes are zero;

f) all assets are infinitely divisible. Letting:

$n$ = number of assets

$p_j$ = price per unit of asset j, j=1,...,n-1

$p_n \equiv q$ = numeraire

$\mu_j$ = expected yield per unit of asset j, j=1,...,n-1 ($\mu_n = 1$ for the riskless asset)

$y_1^i$ = expected yield on individual i's portfolio

$\sigma_{kj}$ = covariance of yields of units of assets j and $k_j$, k=1,...,n-1 ($\sigma_{nk} = \sigma_{kn} = 0$ for all k)

$y_2^i$ = variance of yield on individual i's portfolio

$U^i(\tilde{y}^i)$ = individual i utility for yield

$f^i(y_1^i, y_2^i)$ $E[U^i(\tilde{y}^i)]$ derived utility for expected yield and variance for individual i with $f_1^i \equiv \frac{\partial f^1}{\partial y_1^i} > 0$ and $f_2^i \equiv \frac{\partial f^2}{\partial y_2^i} < 0$

$\bar{x}_j^i$ = before-exchange holdings of asset j in physical units by individual i, j=1,...,n-1, i=1,...,m

$\bar{x}_n^i$ = before-exchange holdings of the riskless asset by individual i

Decision variables:

$x_j^i$ = after-exchange holdings of individual i of risky asset j, in physical units j=1,...,n-1, i=1,...,m

$x_n^i$ = after-exchange holdings by individual i of riskless asset n in physical units, i=1,...,m

Under the above assumptions investor i solves:

$$\max f^i(y_1^i, y_2^i)$$

s.t.

$$(1) \qquad y_1^i = \sum_{j=1}^{n-1} \mu_j x_j^i + x_n^i$$

$$(2)\qquad y_2^i = \sum_{j=1}^{n-1}\sum_{\alpha=1}^{n-1} \sigma_{j\alpha} x_j^i x_\alpha^i$$

$$(3)\qquad \sum_{j=1}^{n-1} p_j x_j^i + x_n^i = \sum_{j=1}^{n-1} p_j \bar{x}_j^i + \bar{x}_n^i$$

Constraint (3) acts like a budget constraint equalizing before exchange and after exchange wealth. Under assumption (a) this is a concave problem over a convex set and the Kuhn-Tucker necessary and sufficient conditions for optimality become

$$(4)\qquad \frac{dy_2^i}{dy_1^i} = \frac{f_1^i}{f_2^i} = \frac{2\sum_{\alpha} \sigma_{j\alpha} x_\alpha^i}{\mu_j - p_j/q}$$

$$(5)\qquad \sum_{j=1}^{n-1} p_j(x_j^i - \bar{x}_j^i) + q(x_n^i - \bar{x}_n^i) = 0$$

Solving (4) and (5) for the $x_j^i$'s would determine individual i's demand for the n assets for this set of prices. In market equilibrium we must have equality between supply and demand for all assets.

$$(6)\qquad \sum_{i=1}^{m} x_j^i = \sum_{i=1}^{m} \bar{x}_j^i \equiv \bar{x}_j \qquad j=1,\ldots,n$$

However, the (mn+n) equations (4), (5), and (6) provide one redundant equation. Therefore we are left with (mn+n-1) equations and the unknowns are (mn)$x_j^i$'s and the (n-1) relative prices $p_j/q$. Counting equations and unknowns is the classical approach to determine where an equilibrium might exist.

Letting $\tilde{r}_j$ denote the rate of return on a unit of risky asset j we have

$$(7)\qquad E[\tilde{r}_j] = \left(\frac{\mu_j}{P_j} - 1\right) \qquad j=,\ldots,n-1$$

$$(8)\qquad r_n = \frac{1}{q} - 1$$

where $\beta_j$ is a measure of volatility (systematic risk) of the asset j defined as $\beta_j \equiv \mathrm{cov}(\tilde{r}_j, \tilde{r}_M)/\gamma_M^2$ where $\tilde{r}_M = \sum_{j=1}^{n-1} W_j \tilde{r}_j$ is the return on the market portfolio (a weighted average) and $\gamma_M^2$ is the variance of the market return.

By corresponding addition in (11) and the fact that

$$\beta_M = 1$$

we obtain the standard format of the asset market line, namely:

$$E[\tilde{r}_j]-r_n = \beta_j(E[\tilde{r}_M]-r_n) \tag{12}$$

This implies that all "fairly" priced assets should lie along this line.

## 2. Jensen's Empirical Version of CAPM

Equation (12) implies that expected return on any asset is equal to the risk-free rate plus a risk premium given by the product of its systematic risk and the risk premium on the market portfolio.

If a security analyst has "predictive capabilities" and therefore is able to predict future security prices he will be able to earn higher returns than those implied by (12) and the riskiness ($\beta_j$) of the portfolio.

However, Eq. (12) is stated in terms of expected returns which are unobservable quantities. Jensen [2 ] shows how (12) can be rewritten in terms of realized returns on any portfolio j and the market portfolio M. Also important for this study is Jensen's [2] conclusions after testing the single period model in a multiperiod world if $\beta_j$'s and generating functions are constant.

$$(12') \quad E[\tilde{r}_{jt}] = r_{Ft} + \beta_j[E(\tilde{r}_{Mt})-r_{ft}]$$

Jensen shows that

$$(13) \quad \tilde{r}_{jt} = E[\tilde{r}_{jt}] + b_j\tilde{\pi}_t + \tilde{e}_{jt} \qquad j=1,\ldots,n-1$$

where $b_j$ is a coefficient approximately equal to the measure of risk $\beta_j$ and $\tilde{\pi}_t$ is an unobservable "market factor" which affects the returns on all securities. The variables $\tilde{\pi}_t$ and $\tilde{\varepsilon}_{jt}$ are assumed to be independent normally distributed random variables with

$$E(\tilde{\pi}_t) = 0$$

$$E(\tilde{e}_{jt}) = 0 \qquad j=1,\ldots,n-1$$

$$\mathrm{cov}(\tilde{\pi}_t,\tilde{e}_{jt}) = 0 \qquad j=1,\ldots,n-1$$

$$\mathrm{cov}(\tilde{e}_{jt},\tilde{e}_{it}) = \begin{cases} 0 & i \neq j \\ \gamma^2(e_j) & i = j \end{cases}$$

Also to a close approximation we will have

$$(14)\quad \tilde{r}_{Mt} \cong E[\tilde{r}_{Mt}] + \tilde{\pi}_t$$

Substituting (14) and (13) in (12') we are able to express (12') in terms of ex-post returns. Hence (12') reduces to

$$(15)\quad \tilde{r}_{jt} - r_{Ft} = \beta_j[\tilde{r}_{Mt} - r_{Ft}] + \tilde{e}_{jt}$$

which implies that realized risk premiums on any security or portfolio can be expressed as a linear function of its systematic risk realized returns on the market portfolio and a random error $\tilde{e}_{jt}$ such that $E[\tilde{e}_{jt}] = 0$.

If the manager is a superior forecaster he will tend to systematically choose securities with $\tilde{e}_{jt} > 0$. In order to allow for the possibility that the portfolio selected earns more than the normal risk premium given its level of risk we simply regress (15) without constraining to pass through the origin. Hence we estimate

$$(16)\quad \tilde{r}_{jt} - r_{Ft} = \alpha_j + \beta_j[\tilde{r}_{Mt} - r_{Ft}] + \tilde{u}_{jt}$$

where $E[\tilde{u}_{jt}] = 0$.

Therefore we should expect $\alpha_j > 0$ if that portfolio manager has indeed superior forecasting capabilities and the capital asset pricing model holds. At the same time we should expect that random selection of buy and hold should yield a zero intercept ($\alpha_j = 0$). Conversely, if the manager is doing worse than random selection we should expect $\alpha_j < 0$.

Therefore Jensen [ 2] chose $\alpha_j$ to be a measure of performance of the funds. We should stick to his choice. By using least squares regression theory an estimate of the dispersion of the intercept $\alpha_j$ is obtained which permits evaluating the statistical significances of

the estimates $\tilde{\alpha}_j$ which is, under his assumptions t distributed with $n_j-2$ degress of freedom.

Definition of the variables

The variables used in estimating $\alpha_j$ and $\beta_j$ in (16) are defined more precisely below:

$\tilde{S}_t$ = Level of Standard and Poor Composite 500 price index at end of year t

$\tilde{D}_t$ = Estimate of dividends received on the market portfolio in year t measured by annual observations on the four quarter moving average of the dividends paid by the companies in the composite 500 index (stated on the same scale as the level of S & P 500 Index).

$\tilde{r}_{Mt} = \log_e\left(\frac{S_t+D_t}{S_{t-1}}\right)$ = The estimated annual continuously compounded rate of return on the market portfolio M for year t.

$NA_{jt}$ = Per share net asset value of the $j^{th}$ fund at end of year t.

$ID_{jt}$ = per share "income" dividends paid by the $j^{th}$ fund during year t.

$\tilde{CG}_{jt}$ = per share "capital gains" distributions paid by the $j^{th}$ fund during year t.

$\tilde{r}_{jt} = \log_e\left(\frac{\tilde{NA}_{jt}+\tilde{ID}_{jt}+\tilde{CG}_{jt}}{NA_{j,t-1}}\right)$ = The annual continuously compounded rate of return on the $j^{th}$ fund during year t, (adjusted for splits and stock dividends). This is net of all management fees and brokerage costs.

$r_t$ = yield to maturity of a one-year government bond at the beginning of year t (obtained from Treasury Bulletin yield curves)

$r_{Ft} = \log_e(1+r_t)$ = annual continuously compounded risk free rate of return for year t.

$n_j$ = number of observations of the $j^{th}$ fund ($10 \leq n_j \leq 20$).

3. The Database

The first stage in our analysis was to reproduce the database that Jensen used in his paper "The Performance of Mutual Funds in the Years 1945-1964" for the funds of our random sample (see Table 1). We used the same sources that Jensen used as recommended in that paper:

1) "Investment Companies, New York: Arthur Wiesenberg & Co." (for getting data for the funds)

2) "Standard and Poor Corporation, Trade and Securities Statistics: Security Price Index Record" (for obtaining data on the market)

3) "Treasury Bulletin" (for the risk free).

We ran least square regression for the ten funds and the results appear on Table 2. A comparison of these results with Jensen's results appears in Table 3. This comparison was not complete since Jensen does not provide in his paper the estimates for $\beta$. However, in another paper, "Risk, The Pricing of Capital Assets, and the Evaluation of Investment Portfolios", there are estimates for $\beta$, but only for the years 1955-1964 and those are the $\beta$ in Table 3.

As we can see in Table 3 we did not get exactly the same point estimates although they are very close. The magnitude of the standard deviation of $\hat{\alpha}$ only strengthens our decision that the differences are not significant.

| Jensen's ID Number | Name | Number of Observations |
|---|---|---|
| 168 | Composite Fund, Inc. | 15 |
| 198 | Incorporated Investors | 20 |
| 206 | Istel Fund, Inc. | 11 |
| 218 | Massachusetts Life Fund | 16 |
| 220 | Mutual Investment Fund, Inc. | 18 |
| 224 | National Securities - Dividend Series | 14 |
| 236 | The George Putnam Fund of Boston | 20 |
| 249 | Television Electronics Fund, Inc. | 16 |
| 253 | United Science Fund | 14 |
| 1191 | Group Securities - Aerospace Science Fund | 14 |

Randomly Selected Open End Mutual Funds

Table 1

2: R168-RFT = A+B*(RMT-RFT)

NOB = 15 NOVAR = 2
RANGE = 1950 TO 1964
RSQ = 0.91237 CRSQ = 0.90563 F(1/13) = 135.345
SER = 0.0303 SSR = 1.190E-02 DW(0) = 2.02

| COEF | VALUE | ST ER | T-STAT |
|---|---|---|---|
| A | -0.00396 | 0.00982 | -0.40347 |
| B | 0.59524 | 0.05117 | 11.63380 |

2: R198-RFT = A+B*(RMT-RFT)

NOB = 20 NOVAR = 2
RANGE = 1945 TO 1964
RSQ = 0.94178 CRSQ = 0.93855 F(1/18) = 291.1[illegible]
SER = 0.0495 SSR = 4.405E-02 DW(0) = 2.14

| COEF | VALUE | ST ER | T-STAT |
|---|---|---|---|
| A | -0.06101 | 0.01361 | -4.48214 |
| B | 1.27541 | 0.07474 | 17.06390 |

2: R206-RFT = A+B*(RMT-RFT)

NOB = 11 NOVAR = 2
RANGE = 1954 TO 1964
RSQ = 0.90638 CRSQ = 0.89598 F(1/9) = 87.132
SER = 0.0423 SSR = 1.610E-02 DW(0) = 2.42

| COEF | VALUE | ST ER | T-STAT |
|---|---|---|---|
| A | 0.01545 | 0.01532 | 1.00820 |
| B | 0.71708 | 0.07682 | 9.33447 |

2: R220-RFT = A+B*(RMT-RFT)

NOB = 18 NOVAR = 2
RANGE = 1947 TO 1964
RSQ = 0.92091 CRSQ = 0.91597 F(1/16) = 186.304
SER = 0.0336 SSR = 1.811E-02 DW(0) = 2.72

| COEF | VALUE | ST ER | T-STAT |
|---|---|---|---|
| A | -0.03370 | 0.00996 | -3.38447 |
| B | 0.76129 | 0.05578 | 13.64930 |

Table 2

```
2:  R224-RFT = A+B*(RMT-RFT)

NOB = 14       NOVAR = 2
RANGE =  1951 TO 1964
RSQ =    0.75912      CRSQ =     0.73905      F(1/12) =     37.818
SER =    0.1201       SSR =      0.173        DW(0) =  2.44

COEF              VALUE           ST ER          T-STAT

A               -0.05281         0.03911        -1.35029
B                1.28227         0.20851         6.14965


2:  R236-RFT = A+B*(RMT-RFT)

NOB = 20       NOVAR = 2
RANGE =  1945 TO 1964
RSQ =    0.88847      CRSQ =     0.88228      F(1/18) =    143.395
SER =    0.0388       SSR = 2.714E-02         DW(0) =  1.75

COEF              VALUE           ST ER          T-STAT

A               -0.00721         0.01068        -0.67472
B                0.70248         0.05866        11.97480


2:  R249-RFT = A+B*(RMT-RFT)

NOB = 16       NOVAR = 2
RANGE =  1949 TO 1964
RSQ =    0.51341      CRSQ =     0.47866      F(1/14) =     14.772
SER =    0.1552       SSR =      0.337        DW(0) =  2.60

COEF              VALUE           ST ER          T-STAT

A               -0.00824         0.04959        -0.16609
B                1.00809         0.26229         3.84340


2:  R253-RFT = A+B*(RMT-RFT)

NOB = 14       NOVAR = 2
RANGE =  1951 TO 1964
RSQ =    0.89848      CRSQ =     0.89002      F(1/12) =    106.205
SER =    0.0597       SSR = 4.276E-02         DW(0) =  1.88

COEF              VALUE           ST ER          T-STAT

A               -0.02516         0.01943        -1.29442
B                1.06789         0.10362        10.30560
```

Table 2 (Cont'd.)

2: R1191-RFT = A+B*(RMT-RFT)

NOB = 14 NOVAR = 2
RANGE = 1951 TO 1964
RSQ = 0.71749 CRSQ = 0.69395 F(1/12) = 30.476
SER = 0.1456 SSR = 0.254 DW(0) = 1.85

| COEF | VALUE | ST ER | T-STAT |
|---|---|---|---|
| A | -0.08209 | 0.04739 | -1.73213 |
| B | 1.39501 | 0.25270 | 5.52052 |

Table 2 (Cont'd.)

```
1:   R218-RFT = A+B*(RMT-RFT)

NOB = 16        NOVAR = 2
RANGE =   1949 TO 1964
RSQ =    0.86535       CRSQ =    0.85573       F(1/14) =     89.974
SER =    0.0322        SSR = 1.451E-02         DW(0) =   2.04
```

| COEF | VALUE | ST ER | T-STAT |
|---|---|---|---|
| A | -0.00398 | 0.01028 | -0.38734 |
| B | 0.51682 | 0.05449 | 9.48546 |

Table 2 (Cont'd.)

| Fund | Jensen's Estimates | | | Our Estimates | | | |
|---|---|---|---|---|---|---|---|
| | $\hat{\alpha}$ | $\sigma(\hat{\alpha})$ | $\hat{\beta}$ | $\hat{\alpha}$ | $\sigma(\hat{\alpha})$ | $\hat{\beta}$ | $\sigma(\hat{\beta})$ |
| 168 | -.0022 | .01 | .594 | -.00396 | .0098 | .595 | .0511 |
| 198 | -.0615 | .0127 | 1.262 | -.0610 | .0136 | 1.2754 | .074 |
| 206 | .0165 | .015 | .716 | .0154 | .0153 | .7170 | .0768 |
| 220 | -.0332 | .0121 | .821 | -.03370 | .0099 | .7612 | .055 |
| 224 | -.0411 | .0238 | 1.085 | -.0528 | .0391 | 1.282 | .2085 |
| 236 | -.0050 | .0108 | .704 | -.0072 | .0106 | .7024 | .0586 |
| 249 | -.0155 | .0209 | 1.060 | -.00824 | .04959 | 1.00809 | .2622 |
| 253 | -.0249 | .0199 | 1.065 | -.02516 | .0194 | 1.0678 | .10362 |
| 1191 | -.0805 | .050 | 1.409 | -.08209 | .0473 | 1.395 | .2527 |
| 218 | -.0014 | .01 | .512 | -.00378 | .0102 | .5168 | .0549 |

Comparison Between Jensen's Least Square Regression and Our Paper's Results

Table 3

## 4. Sensitivity Analysis

Any least squares regression model contains many assumptions like independence, homoscedasticity, error distribution, etc. There are some classic methods to check these assumptions, and the model builder should use them. In addition the output of the model is a consequence of the input (the data) that is used. Therefore it seems essential that the model builder should investigate also what changes of the output of the model would be the result of perturbing the data, or in other words, performing sensitivity analysis.

The ideal situation should be that a slight input perturbation would lead to small output changes.

In this paper we decided to analyze changes in the parameter estimates $(\hat{\alpha},\hat{\beta})$ when observations for one year are deleted from the time series. We shall describe first in more detail the sensitivity analysis approach, and then analyze the results that we obtained with our model.

### The Sensitivity Analysis

The model that we investigate is

$$r_{jt}-r_{Ft} = \alpha_j + \beta_j(r_{Mt}-r_{Ft})$$

where

$j \in$ {set of the funds in the model}

$T'_j$ = max {1945, the year when the fund was created}

$T''_j$ = min {1964, the year when and if the fund was cancelled}

Let $\underline{\delta}_j^{TOTAL} = (\alpha_j^{TOTAL},\beta_j^{TOTAL})$, where total means that the regression model includes all the year $T'_j,\ldots,T''_j$.

Let $\underline{\delta}_j^i = (\alpha_j^i,\beta_j^i)$ be the estimates of $\alpha_j$ and $\beta_j$ when year i is deleted $i \in \{T'_j,\ldots,T''_j\}$.

We are interested in the quantity $(\hat{\underline{\delta}}_j^{TOTAL} - \hat{\underline{\delta}}_j^{i})$, $\forall j$ and $\forall i$.

Let

$$X = \begin{bmatrix} 1 & r_{MT'_j} & - & r_{FT'_j} \\ & \vdots & & \\ 1 & r_{MT''_j} & - & r_{FT''_j} \end{bmatrix}$$

and let $H = X(X^T X)^{-1} X^T$.

Then it is not difficult to prove that:

$$\hat{\underline{\delta}}_j^{TOTAL} - \hat{\underline{\delta}}_j^{i} = \frac{(X^T X)^{-1} \underline{X}_i r_i}{1 - h_{ii}}$$

where $\underline{X}_i$ denote the $i^{th}$ row of X, $r_i$ the residual $r_{ji} - r_{Fi} - \underline{X}_i \hat{\delta}_j^{TOTAL}$, and $h_{ii}$ is the $i^{th}$ diagonal element of H.

We want also to indicate that it is possible to investigate other perturbation of the data. One can perturb any element of the matrix X or perturb the assumption of homoscedacticity of the various observations (by considering a variance of $\sigma^2/P_i$ and investigate small changes in $P_i$).

Because of computer time limitations we concentrated on investigating the quantity $(\hat{\underline{\delta}}_j^{TOTAL} - \hat{\underline{\delta}}_j^{i})$ which seemed to us the most important. We used a macro program that was implemented in the troll system in order to do the actual computations.

The results of the sensitivity analysis appear in Table 4. For each of the funds in our sample of 10 funds the rows (1 and 2) represents the two estimators $(\hat{\alpha}, \hat{\beta})$ and the column represents the year deleted. For example, the number -.001589 for fund 168 that appears in Row 1, Column 2 is the difference between the estimate for least squares and the estimate where 1951 is deleted. The most interesting results are the changes of sign of the estimate for $\alpha$, that occured in

some cases. For fund 168 - Composite Fund, Inc., we see that if we delete the year 1957 our estimate for $\alpha$ would become positive (from -.00369 to .0004136). Also for fund 249 - Television-Electronic Fund, Inc., the estimate for $\alpha$ becomes positive (from -.00824 to .00542) after deleting the year 1951, and (to .00198) after deleting the year 1962. For the rest of the funds there were changes in the magnitudes but a change of sign did not happen.

According to the results summarized in Table 5 we may notice that the removal of 1962 observation reveals a major trend towards more positive intercept term ($\hat{\alpha}^{1962}$). More precisely for 8 out of the 10 funds studied performance improved ($\hat{\alpha}^{1962} - \hat{\alpha}^{LS} > 0$). In case that the 1962 data points are outliers, this would indicate that 80% of the portfolio managers in the sample did better relative to the estimates where 1962 is not regarded as an extraordinary year. It is conceivable that the reduction in performance imposed by 1962 is due to erratic movements in the capital markets in this year.

It is peculiar to 1962 that elimination of its observations reveals opposite movements in the magnitudes of the performance estimate ($\alpha$) and the volatility coefficient ($\beta$) for all the 10 funds. This suggests that 80% of the funds studied became less risky once 1962 is deleted. There is a possibility that portfolio managers made unsuccessful moves into riskier stocks in 1962. In order to test this hypothesis we should proceed into analyzing the dynamic composition of the sample portfolios over a period of time including 1962.

There may be some empirical support for considering classifying 1962 as an outlier. In any case, the peculiarity of this year cannot be neglected in future studies. Table 5 illustrates these remarks.

Even though looking at movements in the performance coefficient

($\alpha$) of individual funds is insightful to detect particular trends, in order to be consistent we should look also at averages. The average $\bar{\alpha}_{LS}$ is -.0266. Upon deletion of 1962 the average performance measure $\bar{\alpha}^{1962}$ increased to -.02503.

In order to determine the significance of the change a robust estimate of the standard deviation of $\alpha$ was used as suggested by Breiman [8].

$$\sigma^*_{\hat{\alpha}} = \frac{\sqrt{\pi}}{2} * \text{MAD}$$

$$\text{MAD} = \sum_{i=1}^{10} \frac{|\hat{\alpha}_{LS_i} - \bar{\hat{\alpha}}_{LS}|}{10}$$

Since $\sigma^*_{\alpha} = .0313$ we are tempted to conclude that the estimate of the average performance of portfolio managers does not improve significantly if 1962 is deleted ($|\bar{\hat{\alpha}}_{LS} - \bar{\hat{\alpha}}^{1962}|/\sigma^*_{\hat{\alpha}_{LS}}$ is small). The average is still negative confirming Jensen's assertion about the inability of portfolio managers in the average to pick "winners". Deleting years 1951, 1956, and 1957 was also relevant for some funds.

JBETA 168

1950-1964

Table 4

| ROW | COLUMN 1 | COLUMN 2 | COLUMN 3 | COLUMN 4 |
|---|---|---|---|---|
| 1 | -0.000408 | -0.001589 | -0.000393 | -0.000724 |
| 2 | -0.006267 | -0.007356 | -0.000369 | 0.00272 |

| ROW | COLUMN 5 | COLUMN 6 | COLUMN 7 | COLUMN 8 |
|---|---|---|---|---|
| 1 | -0.000484 | -0.001153 | 0.002011 | -0.004136 |
| 2 | 0.016032 | -0.017147 | -0.005033 | [illegible] |

| ROW | COLUMN 9 | COLUMN 10 | COLUMN 11 | COLUMN 12 |
|---|---|---|---|---|
| 1 | 6.261286E-07 | 0.002593 | 0.004401 | 0.00071 |
| 2 | -0.000216 | -0.003829 | -0.016678 | 0.004182 |

| ROW | COLUMN 13 | COLUMN 14 | COLUMN 15 |
|---|---|---|---|
| 1 | -0.001243 | -0.00011 | 0.001783 |
| 2 | 0.005921 | -0.000301 | -0.000435 |

JBETA 198

1945-1964

| ROW | COLUMN 1 | COLUMN 2 | COLUMN 3 | COLUMN 4 |
|---|---|---|---|---|
| 1 | 0.000301 | 0.000717 | -0.001757 | -0.001175 |
| 2 | 0.015815 | -0.003398 | 0.004392 | 0.003022 |

| ROW | COLUMN 5 | COLUMN 6 | COLUMN 7 | COLUMN 8 |
|---|---|---|---|---|
| 1 | 0.001204 | 0.000893 | -0.000644 | -0.00048 |
| 2 | 0.002196 | 0.01724 | -0.003862 | -0.000766 |

| ROW | COLUMN 9 | COLUMN 10 | COLUMN 11 | COLUMN 12 |
|---|---|---|---|---|
| 1 | 0.002261 | -1.838141E-05 | -0.000704 | 0.006763 |
| 2 | -0.008808 | 0.000653 | -0.013136 | -0.016494 |

| ROW | COLUMN 13 | COLUMN 14 | COLUMN 15 | COLUMN 16 |
|---|---|---|---|---|
| 1 | -0.003461 | 2.129217E-05 | 0.002257 | -0.001026 |
| 2 | 0.017855 | -0.004658 | -0.002807 | 0.004032 |

| ROW | COLUMN 17 | COLUMN 18 | COLUMN 19 | COLUMN 20 |
|---|---|---|---|---|
| 1 | -0.003249 | -0.002087 | -0.000588 | 0.001561 |
| 2 | -0.024438 | 0.010542 | -0.002203 | 0.000309 |

1954-1964

Table 4 (Cont'd).

JBETA 206

| ROW | COLUMN 1 | COLUMN 2 | COLUMN 3 | COLUMN 4 |
|---|---|---|---|---|
| c 1 | -0.000122 | -0.001486 | -0.001364 | -0.003871 |
| 2 | 0.011225 | -0.0153 | 0.002897 | 0.017671 |

| ROW | COLUMN 5 | COLUMN 6 | COLUMN 7 | COLUMN 8 |
|---|---|---|---|---|
| 1 | 0.000556 | -0.001042 | 0.009086 | 1.377515E-05 |
| 2 | 0.02797 | 0.001184 | -0.03115 | 6.699041E-05 |

| ROW | COLUMN 9 | COLUMN 10 | COLUMN 11 |
|---|---|---|---|
| 1 | 0.002743 | -0.00404 | -0.000755 |
| 2 | -0.012248 | -0.010161 | 5.716674E-06 |

JBETA 220

1947-1964

| ROW | COLUMN 1 | COLUMN 2 | COLUMN 3 | COLUMN 4 |
|---|---|---|---|---|
| 1 | -0.001038 | -0.000309 | 0.002604 | -0.000476 |
| 2 | 0.002788 | 0.000852 | 0.004897 | -0.011992 |

| ROW | COLUMN 5 | COLUMN 6 | COLUMN 7 | COLUMN 8 |
|---|---|---|---|---|
| 1 | 0.000164 | -0.001433 | 0.001795 | 0.000277 |
| 2 | 0.001094 | -0.002338 | -0.007323 | [illegible] |

| ROW | COLUMN 9 | COLUMN 10 | COLUMN 11 | COLUMN 12 |
|---|---|---|---|---|
| 1 | -0.000879 | 0.00167 | 0.001013 | -0.000139 |
| 2 | -0.021239 | -0.00438 | -0.005377 | 0.010277 |

| ROW | COLUMN 13 | COLUMN 14 | COLUMN 15 | COLUMN 16 |
|---|---|---|---|---|
| 1 | -0.000839 | 0.0011 | 0.001108 | -0.00571 |
| 2 | 0.001172 | -0.004525 | 0.009456 | 0.029731 |

| ROW | COLUMN 17 | COLUMN 18 |
|---|---|---|
| 1 | 0.000712 | 0.001085 |
| 2 | 0.002873 | 0.000129 |

1949-1964

218

ETA - DATE REVISED: 9/10/76
PERIODICITY(16) DATA FROM 1 1 TO 2 16

JBETA = MATMULT(DVBETA,HMIDIAG)

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 0.000881 | -0.001482 | -0.001033 | 2.805846E-05 |
| 1 | 5 | 0.001085 | -0.000459 | -0.000317 | 2.343883E-05 |
| 1 | 9 | 0.001383 | -0.000217 | -0.002322 | 0.002249 |
| 1 | 13 | 0.001249 | -0.001354 | -0.000323 | 0.000546 |
| 2 | 1 | 0.001076 | -0.026773 | -0.004994 | 2.764065E-05 |
| 2 | 5 | -0.004248 | 0.012431 | -0.005467 | -6.280633E-05 |
| 2 | 9 | -0.006897 | 0.022301 | 0.003806 | -0.008733 |
| 2 | 13 | 0.008091 | 0.006603 | -0.000915 | -0.000169 |

Table 4 (Cont'd.)

1951-1964

| ROW | COLUMN 1 | COLUMN 2 | COLUMN 3 | COLUMN 4 |
|---|---|---|---|---|
| 1 | -0.004705 | -0.002679 | 0.003688 | 0.001282 |
| 2 | -0.024531 | -0.003762 | -0.013712 | -0.057174 |

| ROW | COLUMN 5 | COLUMN 6 | COLUMN 7 | COLUMN 8 |
|---|---|---|---|---|
| 1 | -0.001576 | 0.001499 | -0.017328 | 0.000461 |
| 2 | -0.023535 | -0.003473 | 0.085916 | 0.159612 |

| ROW | COLUMN 9 | COLUMN 10 | COLUMN 11 | COLUMN 12 |
|---|---|---|---|---|
| 1 | -0.00059 | -0.003211 | -0.002962 | 0.020531 |
| 2 | 0.000703 | 0.012047 | -0.01918 | -0.099658 |

| ROW | COLUMN 13 | COLUMN 14 |
|---|---|---|
| 1 | 0.001392 | 0.003638 |
| 2 | 0.004584 | 0.000508 |

JBETA 2 3 6

1945-1964

| ROW | COLUMN 1 | COLUMN 2 | COLUMN 3 | COLUMN 4 |
|---|---|---|---|---|
| 1 | -0.000247 | 0.002765 | -0.002924 | -0.001259 |
| 2 | -0.012939 | -0.013106 | 0.00731 | 0.003237 |

| ROW | COLUMN 5 | COLUMN 6 | COLUMN 7 | COLUMN 8 |
|---|---|---|---|---|
| 1 | 0.001354 | -0.000264 | -0.001001 | -0.000598 |
| 2 | 0.00247 | -0.005093 | -0.006004 | -0.000955 |

| ROW | COLUMN 9 | COLUMN 10 | COLUMN 11 | COLUMN 12 |
|---|---|---|---|---|
| 1 | 0.001157 | -0.000463 | -0.00072 | 0.000293 |
| 2 | -0.004506 | 0.016458 | -0.01344 | -0.000714 |

| ROW | COLUMN 13 | COLUMN 14 | COLUMN 15 | COLUMN 16 |
|---|---|---|---|---|
| 1 | 0.000624 | -7.449285E-05 | 0.001489 | 0.005982 |
| 2 | -0.003221 | 0.016297 | -0.001852 | -0.023513 |

| ROW | COLUMN 17 | COLUMN 18 | COLUMN 19 | COLUMN 20 |
|---|---|---|---|---|
| 1 | 0.001219 | -0.00576 | -0.001017 | -0.000237 |
| 2 | 0.009167 | 0.029102 | -0.003809 | -4.694585E-05 |

1949-1964

JBETA 249

Table 4 (Cont'd.)

| ROW | COLUMN 1 | COLUMN 2 | COLUMN 3 | COLUMN 4 |
|---|---|---|---|---|
| 1 | -0.002329 | -0.000771 | -0.013662 | 0.019742 |
| 2 | -0.002637 | -0.014165 | -0.068003 | 0.018316 |

| ROW | COLUMN 5 | COLUMN 6 | COLUMN 7 | COLUMN 8 |
|---|---|---|---|---|
| 1 | 0.003408 | -0.002164 | -0.001935 | 0.003449 |
| 2 | -0.013259 | 0.059291 | -0.034224 | -0.009095 |

| ROW | COLUMN 9 | COLUMN 10 | COLUMN 11 | COLUMN 12 |
|---|---|---|---|---|
| 1 | 0.00177 | -0.000356 | 0.007245 | 0.001849 |
| 2 | -0.008783 | 0.033802 | -0.011564 | -0.00725 |

| ROW | COLUMN 13 | COLUMN 14 | COLUMN 15 | COLUMN 16 |
|---|---|---|---|---|
| 1 | -0.001171 | -0.009438 | -0.00209 | -0.001611 |
| 2 | -0.007518 | 0.045995 | -0.006022 | 0.000425 |

1951-1964

JBETA 253

| ROW | COLUMN 1 | COLUMN 2 | COLUMN 3 | COLUMN 4 |
|---|---|---|---|---|
| 1 | 0.000651 | -0.004738 | 0.003061 | -0.00048 |
| 2 | 0.003393 | -0.006653 | -0.011382 | 0.021384 |

| ROW | COLUMN 5 | COLUMN 6 | COLUMN 7 | COLUMN 8 |
|---|---|---|---|---|
| 1 | -0.001112 | 0.006482 | 0.001766 | 2.789286E-05 |
| 2 | -0.016611 | -0.015018 | -0.008756 | 0.008452 |

| ROW | COLUMN 9 | COLUMN 10 | COLUMN 11 | COLUMN 12 |
|---|---|---|---|---|
| 1 | 0.009234 | 0.000361 | -0.001965 | -0.008243 |
| 2 | -0.011 | -0.001356 | -0.012725 | 0.040003 |

| ROW | COLUMN 13 | COLUMN 14 |
|---|---|---|
| 1 | -0.000717 | -0.003006 |
| 2 | -0.002361 | -0.00042 |

Table 4 (Cont'd.)

JBETA 1191

1951-1964

| ROW | COLUMN 1 | COLUMN 2 | COLUMN 3 | COLUMN 4 |
|---|---|---|---|---|
| 1 | -0.002753 | -0.004585 | 0.017403 | -0.004148 |
| 2 | -0.01435 | -0.006438 | -0.072138 | 0.13497 |

| ROW | COLUMN 5 | COLUMN 6 | COLUMN 7 | COLUMN 8 |
|---|---|---|---|---|
| 1 | -0.001992 | 0.011384 | -0.007923 | -0.000259 |
| 2 | -0.029749 | -0.026376 | 0.039283 | -0.0724 |

| ROW | COLUMN 9 | COLUMN 10 | COLUMN 11 | COLUMN 12 |
|---|---|---|---|---|
| 1 | 0.007359 | 0.00722 | -0.005247 | -0.006915 |
| 2 | -0.008766 | -0.027088 | -0.033974 | 0.03556 |

| ROW | COLUMN 13 | COLUMN 14 |
|---|---|---|
| 1 | -0.007002 | -0.00131 |
| 2 | -0.02306 | -0.000183 |

| Fund no. | $\hat{\alpha}_{LS}$ | $\hat{\alpha}^{1962}$ | $\hat{\alpha}^{1962}-\hat{\alpha}_{LS}$ | $\hat{\beta}_{LS}$ | $\hat{\beta}^{1962}$ | $\hat{\beta}^{1962}-\hat{\beta}_{LS}$ | Years of Major Change |
|---|---|---|---|---|---|---|---|
| 168 | -.00369 | -.00245 | +.001243 | .595 | .58909 | -.005921 | 1957,1951,1962 |
| 198 | -.0610 | -.05992 | +.002087 | 1.2754 | 1.26486 | -.010542 | 1956,1957,1961 |
| 206 | .01545 | .012707 | -.002743 | .7170 | .72925 | +.012248 | 1957,1960,1962 |
| 218 | -.00398 | -.002626 | +.001354 | .5168 | .5102 | -.006603 | 1957,1960,1962 |
| 220 | -.03370 | -.02799 | +.00571 | .7612 | .73147 | -.029734 | 1949,1956,1962 |
| 224 | -.05281 | -.07341 | -.020531 | 1.282 | 1.38164 | +.099638 | 1951,1957,1962 |
| 236 | -.00721 | -.00576 | +.00576 | .7024 | .6733 | -.029102 | 1947,1960,1962 |
| 249 | -.00824 | .001198 | +.009438 | 1.00809 | .96204 | -.045995 | 1951,1952,1962 |
| 253 | -.02516 | -.016917 | +.008243 | 1.0678 | 1.0278 | -.040003 | 1956,1957,1962 |
| 1191 | -.08209 | -.075175 | +.006915 | 1.395 | 1.3614 | -.03356 | 1953,1956,1957 |
| AVER. | -.0266 | -.02503 | +.0016 | .932 | .923 | -.009 | |

Summary of the Most Important Changes in Sensitivity Analysis

Table 5

5a. Residual analysis

Concerning returns of individual securities there seemed to be considerable evidence that their distribution belongs to a Stable class of distributions which have finite means but infinite variances Fama [10]). However, it is currently accepted that returns are normally distributed but that variances are non-stationary over time.

Least-squares regression will provide maximum-likelihood estimates of the coefficients if the postulated market model for (13) holds. However, an analysis of the residuals plotted in Figure 1 shows some slight evidence that stationarity through time may not hold for all funds. Except for funds 168 and 200, the plots do not indicate a horizontal "band" of residuals. According to Draper & Smith [9] either long-term or short-term time effects might be influencing the data. The fact that the variance might not be constant over time implies that a weighted least squares analysis should have been used. This seems to provide enough support for using robust regression techniques, described below, that iterate to the optimal estimate through weighted least squares steps.

5b. Robust Regression- Summary

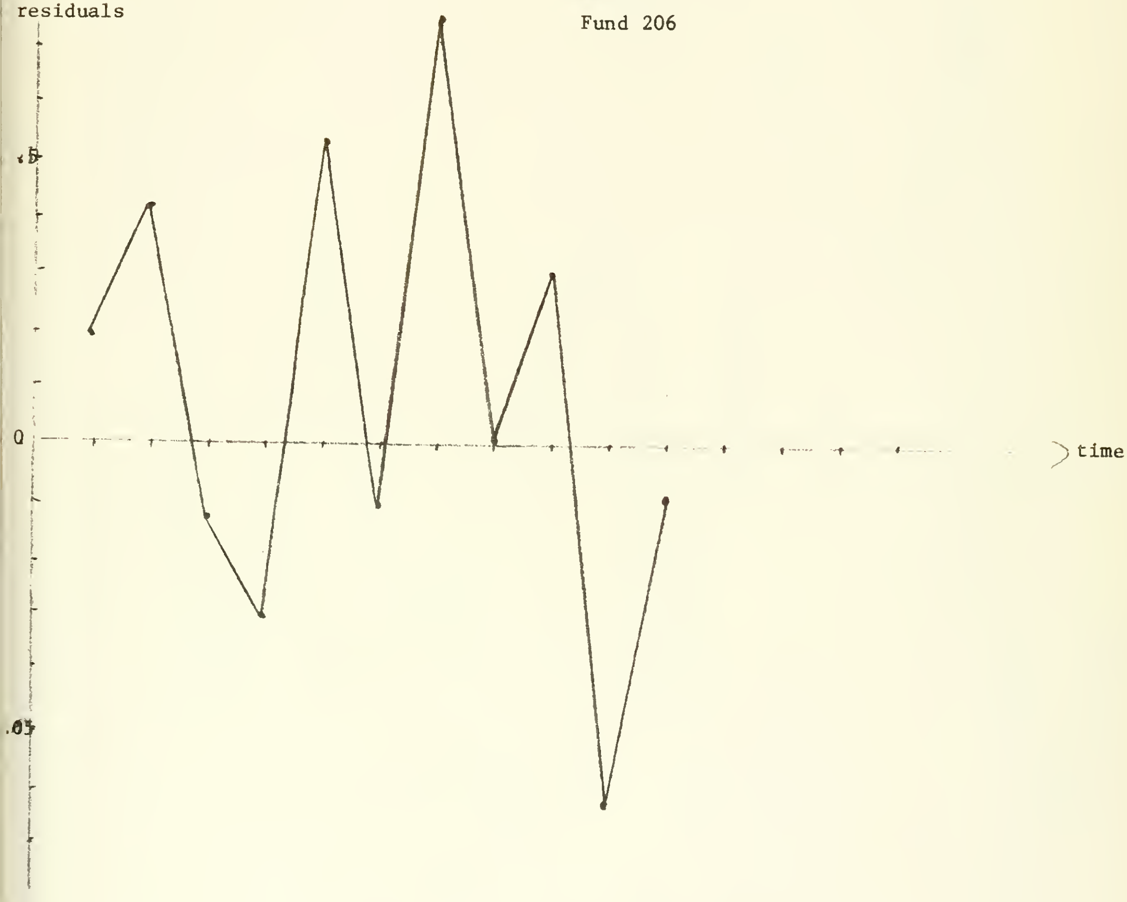
Robust Regression is a new technique designed to outperform ordinary least squares when the errors in a regression model have non-Gaussian distributions with longer tails.

Robust regression is intended to meet two conditions of robust or "resistant" techniques

1) the coefficients should not be unduly influenced by any small portion of the data.

Figure 1

Fund 206



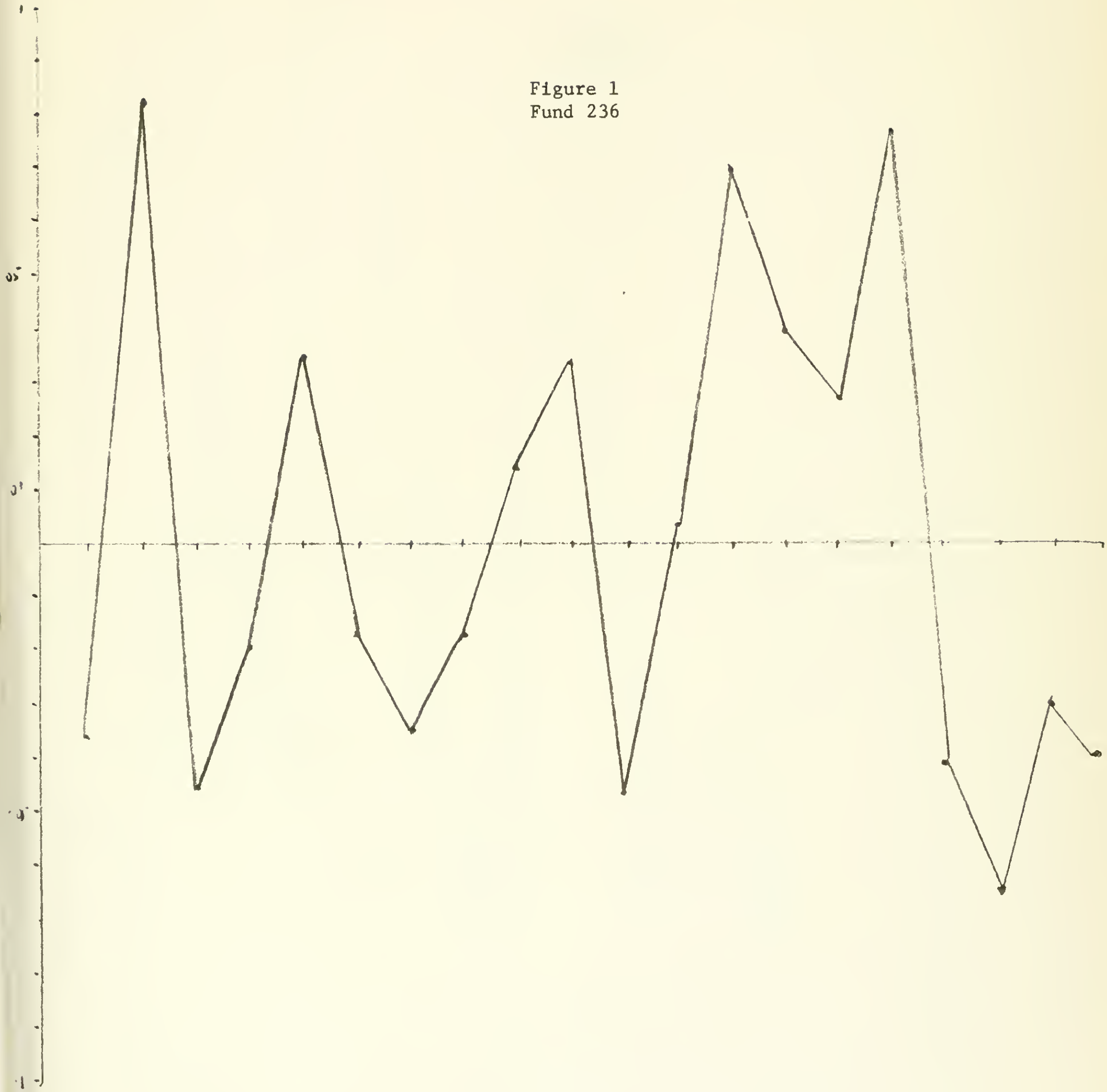


Figure 1
Fund 236

Figure 1
Fund 1198

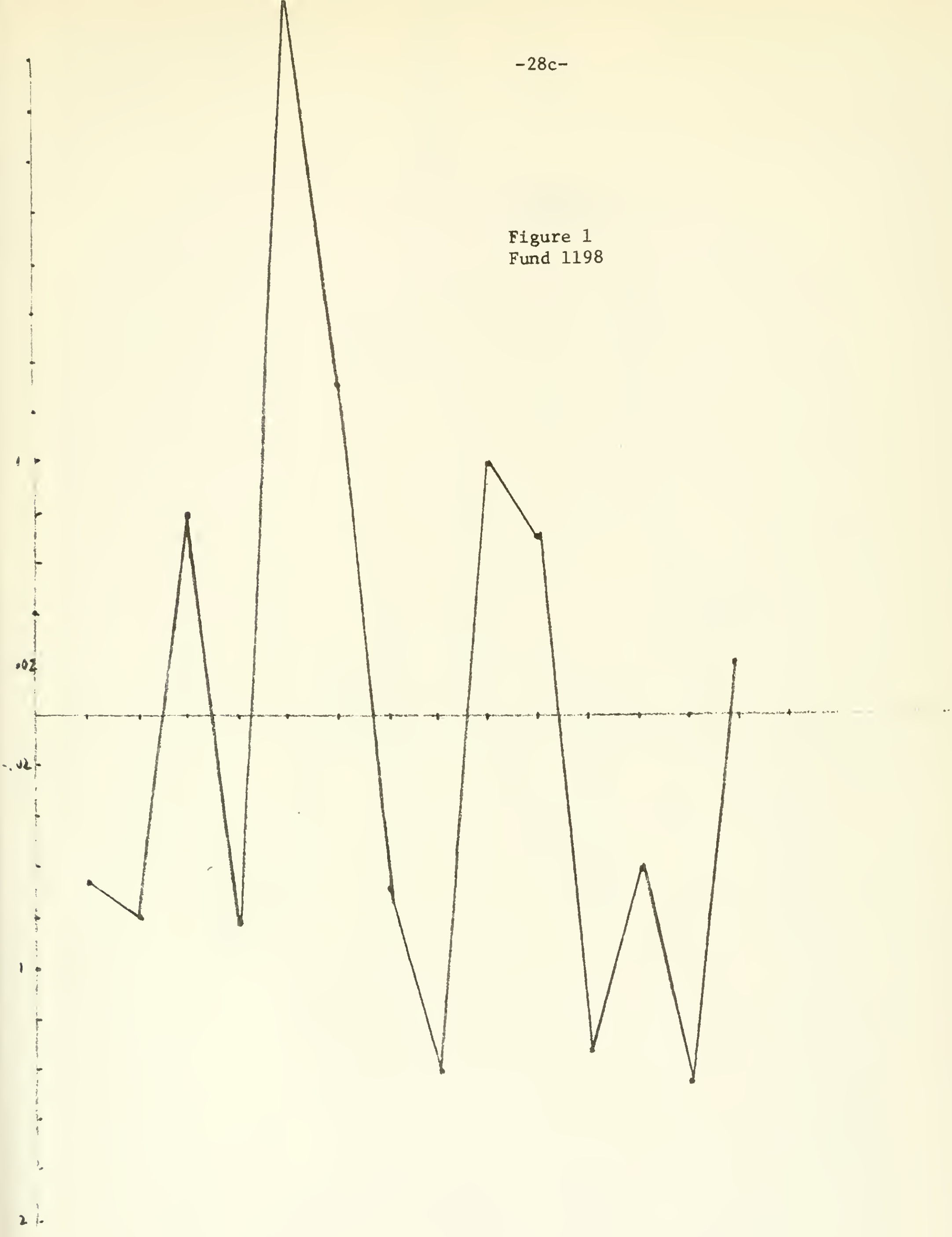



Figure 1
Fund 224

Figure 1
Fund 249

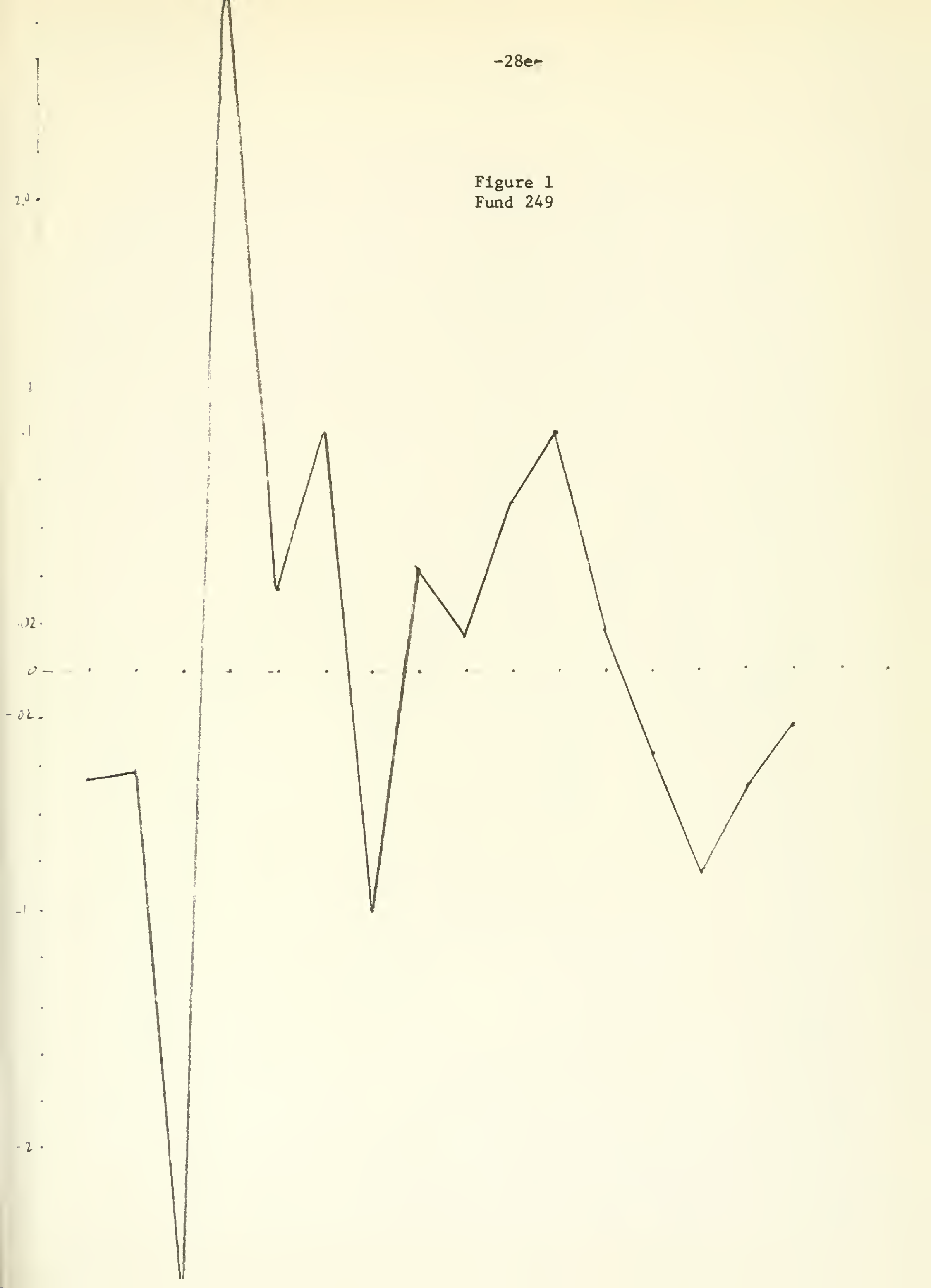


Figure 1
Fund 218

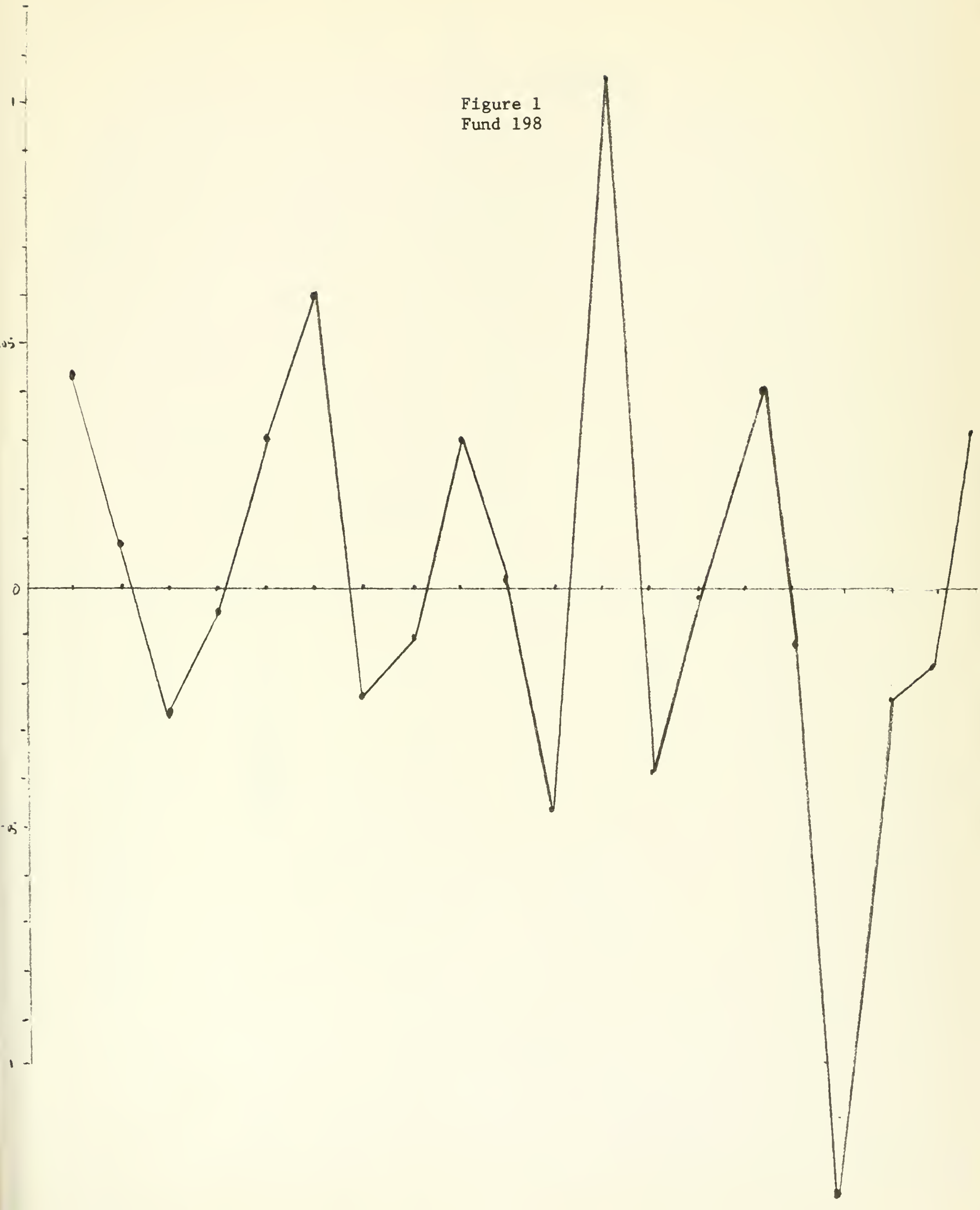



Figure 1
Fund 198

Figure 1
Fund 253

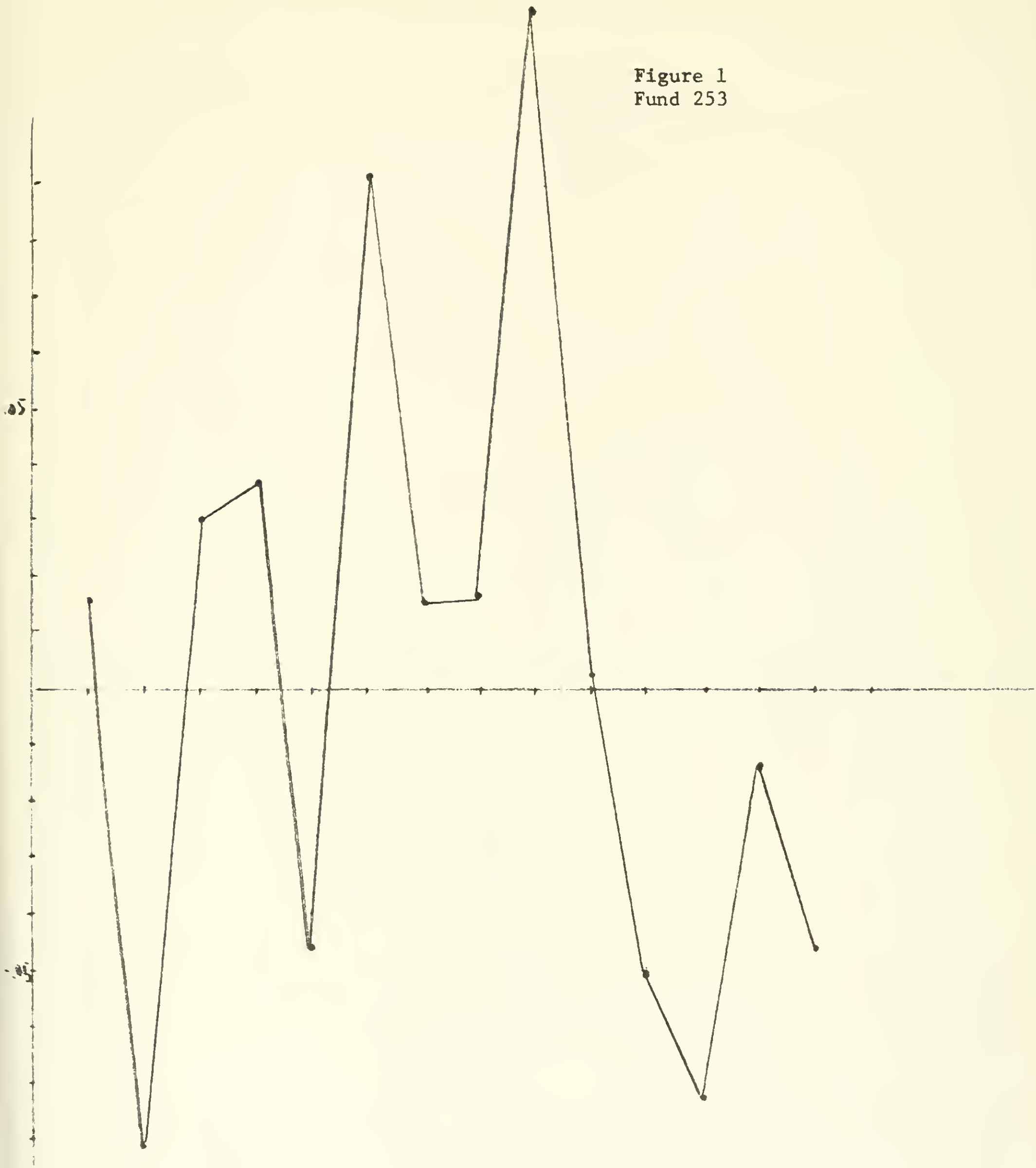


Figure 1
Fund 168

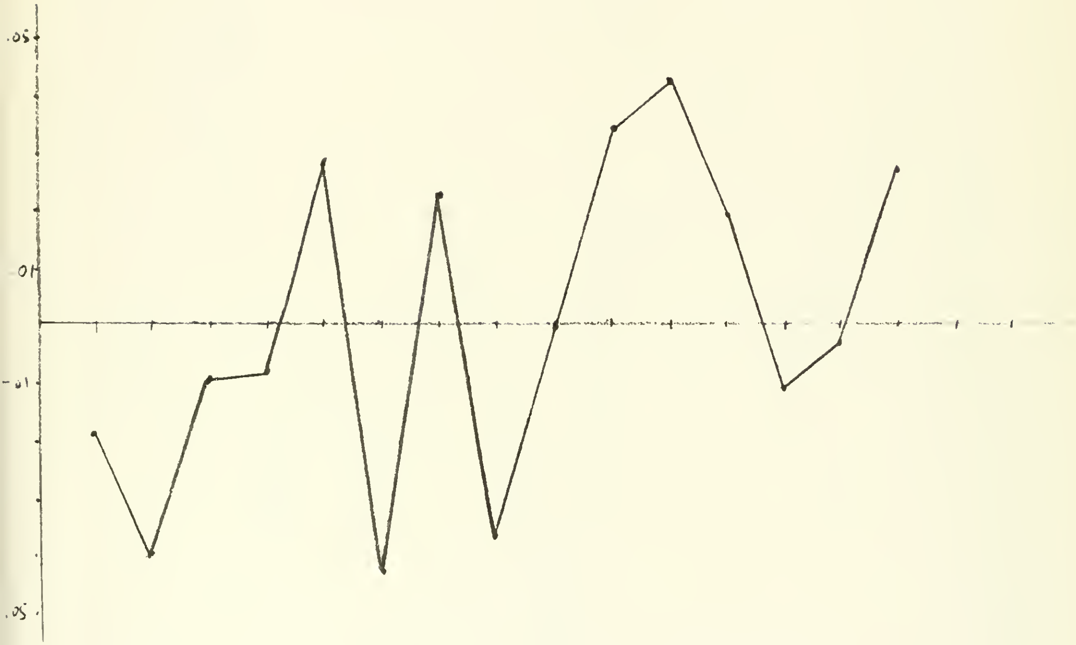


Figure 1
Fund 220

2) minor inaccuracies in the model should cause only minor errors in the final results.

This technique is performed iteratively by means of reweighted least squares.

The method starts with an initial set of coefficients. It then computes the residuals and reweights the data observations, using a technique similar to a minimization of a criterion function. The weights thus obtained are used in computing new coefficients which in turn produce new residuals and a new set of weights.

The user specifies a weighting function in the εRobust macro implemented in the TROLL system used in this work. If we let the data vector Y stand for the dependent variable (in the current study $Y = (\tilde{r}_j - r_{Ft}))$ and the data matrix X for the dependent variables (in this application, column 1 is filled with 1's and column 2 is $(\tilde{r}_{mt} - r_{ft}))$ the fitting of the linear model is accomplished by searching for $\hat{\alpha}$ and $\hat{\beta}$ which minimize

$$\sum_{i=1}^{n} \rho_c\left(\frac{r_{j_i} - r_{FT_i} - \hat{\alpha} - \hat{\beta}(rMT_i - r_{FT_i})}{s}\right)$$

or equivalently

$$\sum_{i=1}^{n} \rho_c\left(\frac{Y_i - X_i\hat{\beta}}{s}\right)$$

where s is a scale estimate and $\rho_c(\cdot)$ is the loss function.

The "robust" macro implemented in TROLL permits specifying three different families of loss functions. Of these only the Huber criterion was used and will be briefly illustrated here. For details see [7 ]. The Huber loss function was chosen because it is the only convex criterion available and therefore convergence to a global minimum is guaranteed.

$\rho'(\cdot) = \psi_c(x) = \max(-c, \min(c, x))$

The sensitivity and stability of the fit, coefficient estimates and residuals may be examined by varying c and plotting the results.

In order to start the process it is necessary to specify an initial guess for the coefficient estimates. The system default choice is to use the coefficient estimates derived from the well known and widely used least absolute residuals (LAR) criterion function ($\rho(t) = |t|$, or $\rho' = \psi_o(x)$).

The LAR method exists also as macro that may be invoked by the user in the TROLL system. It performs linear regression using a linear programming algorithm for minimizing the sum of absolute residuals.

The LAR criterion was thoroughly used in this study as a means of challenging the robustness of the Jensen's capital asset pricing model results and at the same time saving computer time when compared to complete robust regression

A complete robust regression run using Huber loss function was obtained for fund 218. This fund was chosen because its least square $\alpha$ value is nearer the borderline. The results are shown in Table 6 for various values of the constant c. We may notice that $\alpha$ is positive at $c = 0$ (LAR) but becomes negative at $c = 1$ and thereafter. A Huber trace for $\beta$ is shown in Table 7, which indicates stability of the coefficient $\beta$ at the various values of c. The Huber Trace feature available in the TROLL system, unfortunately does not plot the intercept term.

Robust runs are lengthy and expensive and the results for fund 218 suggested that we should proceed with the other 9 funds using only the LAR code which is also the first step in the robust run.

```
C = 0      SSR = .015154      SAR = .357784
R = 1     WSSR = .000036     WSAR = .02055

NOB = 16     NOVAR = 2  STEP = 4  SCALE = 1
RSQ = .859324  WRSQ = .996233   SER = .032901  WSER = .001599
SR = -.021548   LHSMEAN = .056672   SUMW = 2.12504   RIDGE = 0
F(1/14) = 85.5195  WF(1/14) = 3702.92

COEFS

                      BETA            COEF           STD ERR          T STAT

A                   0.007047        0.002313        0.001097         2.10905
B                   0.854306        0.47467         0.007811        60.7691

                      MEAN            PRIOR

A                   1.              0.
B                   0.117358        0.


C = 1      SSR = .01475       SAR = .358359
R = .5    WSSR = .000948     WSAR = .283399

NOB = 16     NOVAR = 2  STEP = 4  SCALE = 1
RSQ = .863079  WRSQ = .925134   SER = .032459  WSER = .022278
SR = -.057305   LHSMEAN = .056672   SUMW = 13.7266   RIDGE = 0
F(1/14) = 88.2486  WF(1/14) = 173.126
MAXEIG = 14.1551  MINEIG = .860061  COND# = 16.4583

COEFS

                      BETA            COEF           STD ERR          T STAT

                   -0.004698       -0.001542        0.005921         0.260395
                    0.943327        0.520872        0.040663        12.957

                      MEAN            PRIOR

                    1.              0.
                    0.117358        0.
```

Table 6 - Robust Regression Results for Fund 218

%
C = 39 SSR = .014505 SAR = .359879
R = .025 WSSR = .014505 WSAR = .359879

COEFS

| | BETA | COEF | PRIOR |
|---|---|---|---|
| A | -0.01213 | -0.003981 | 0. |
| B | 0.930242 | 0.516825 | 0. |

%
C = 9 SSR = .014505 SAR = .359879
R = .1 WSSR = .014505 WSAR = .359879

%
C = 4 SSR = .014505 SAR = .359879
R = .2 WSSR = .014505 WSAR = .359879

%
C = 2.33333 SSR = .014602 SAR = .358368
R = .3 WSSR = .012024 WSAR = .341831

%
C = 1.5 SSR = .014714 SAR = .358406
R = .4 WSSR = .00938 WSAR = .31751

Table 6 (Cont'd.)

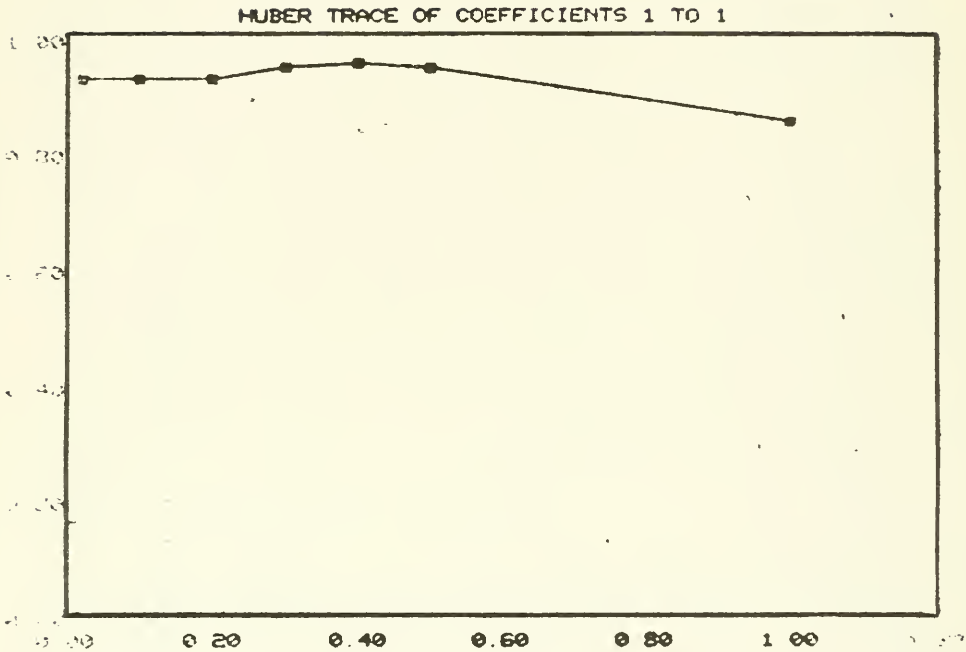



Table 7 - Huber Trace for Fund 218

6. Least Absolute Residuals Regression Results

The results of the LAR regression are shown on Table 8. Results are also summarized in Table 9, where they are compared to the least square regression results. The most important results are the change of sign of the $\hat{\alpha}$ for the two funds 206 and 218. For the fund 206 - "Istel Fund Inc", $\hat{\alpha}$ changed from .01545(the LS) to -.00198 (LAR). For the fund 218 - "Massachusetts Life Fund" we have a corresponding change of $\hat{\alpha}$ from -.00398 to .007047. For the fund 249 - "Television-Electronics Funds, Inc.", we have significant change both in $\hat{\alpha}$ and in $\hat{\beta}$. They changed correspondingly from -.00824 and 1.00809 to -.013519 and .860861. For the rest of the funds we have also changes in the magnitude of $\hat{\alpha}$ and $\hat{\beta}$ although the sign of $\hat{\alpha}$ has not changed. Standard deviation for estimates of $\alpha$ are included in the table for the purpose of evaluating the significance of the distance between the two estimates. In all cases the LAR estimates are within one standard deviation of the LS estimates. We avoided using t statistics because under LAR we don't have the necessary normality assumptions.

Table 9 shows that 5 out of the 10 funds studied moved slightly into poorer performance measure while the other 5 showed slight improved performance. In the average performance is poorer under this criterion. However the reduction in the average performance coefficient is again not significant.

Also the least absolute residuals regression reveals a lower average systematic risk suggesting that the funds seem to be more conservative in their investment policies, offering investors portfolios with smaller systematic risk. The average systematic risk (.905) is in this situation significantly smaller than under least squares (.932) already lower than the market portfolio systematic risk (1.).

Table 8

```
LAR REGRESSION RESULTS FOR R168-RFT ON NEWX
NOB = 15  NOVAR = 2
RSQ =        0.909657  MLRSQ =        0.891042  ROBRSQ =        0.702687
SER =        0.030716  MEDAR =        0.025795  SSR =        0.012265  SAR =        0.340007

      COEF              ST ER

     -0.010121          0.009969
      0.618414          0.05195

                   COVARIANCE MATRIX
   9.938861E-05
     -0.000314          0.002699

LAR REGRESSION RESULTS FOR R198-RFT ON NEWX

NOB = 20  NOVAR = 2
RSQ =        0.936541  MLRSQ =        0.950317  ROBRSQ =        0.974095
SER =        0.05165   MEDAR =        0.022609  SSR =        0.048019  SAR =        0.683438

      COEF              ST ER

     -0.078259          0.014211
      1.32339           0.078034

                   COVARIANCE MATRIX
      0.000202
     -0.000646          0.006089

LAR REGRESSION RESULTS FOR R206-RFT ON NEWX

NOB = 11  NOVAR = 2
RSQ =        0.888617  MLRSQ =        0.916569  ROBRSQ =        0.9996
cSER =       0.046137  MEDAR =        0.001566  SSR =        0.019158  SAR =        0.298469

      COEF              ST ER

     -0.001989          0.016714
      0.806601          0.083792

                   COVARIANCE MATRIX
      0.000279
     -0.000776          0.007021
```

Table 8 (Cont'd.)

```
LAR REGRESSION RESULTS FOR R220-RFT ON NEWX

NOB = 18  NOVAR = 2
RSQ =        0.917204  MLRSQ =       0.926489  ROBRSQ =       0.94661
SER =        0.034419  MEDAR =       0.017164  SSR =        0.018954  SAR =        0.45275

      COEF          ST ER

      -0.025617      0.010188
       0.747295      0.057067

               COVARIANCE MATRIX
       0.000104
      -0.000352      0.003257

LAR REGRESSION RESULTS FOR R224-RFT ON NEWX

NOB = 14  NOVAR = 2
RSQ =        0.715     MLRSQ =       0.777596  ROBRSQ =       0.838219
SER =        0.130662  MEDAR =       0.045695  SSR =        0.204872  SAR =        1.04724

      COEF          ST ER

      -0.039042      0.042538
       0.993832      0.226806

               COVARIANCE MATRIX
       0.001809
      -0.005509      0.051441
LAR REGRESSION RESULTS FOR R236-RFT ON NEWX

NOB = 20  NOVAR = 2
RSQ =        0.882418  MLRSQ =       0.868259  ROBRSQ =       0.878495
SER =        0.039869  MEDAR =       0.026782  SSR =        0.028611  SAR =        0.612753

      COEF          ST ER

      -0.007049      0.010969
       0.654843      0.060234

               COVARIANCE MATRIX
       0.00012
```

Table 8 (Cont'd.)

```
LAR REGRESSION RESULTS FOR R249-RFT ON NEWX

NOB = 16  NOVAR = 2
RSQ =        0.490649  MLRSQ =      0.660068  ROBRSQ =      0.832663
SER =        0.158785  MEDAR =      0.042667  SSR =      0.352978  SAR =      1.45994

     COEF          ST ER

  -0.013519       0.050735
   0.860861       0.268357

               COVARIANCE MATRIX
   0.002574
  -0.008479       0.072016
LAR REGRESSION RESULTS FOR R253-RFT ON NEWX

NOB = 14  NOVAR = 2
RSQ =        0.894869  MLRSQ =      0.896855  ROBRSQ =      0.733593
SER =        0.06075   MEDAR =      0.04061   SSR =      0.044286  SAR =      0.616769

     COEF          ST ER

  -0.020154       0.019777
   1.10375        0.10545

               COVARIANCE MATRIX
   0.000391
  -0.001191       0.01112
LAR REGRESSION RESULTS FOR R1191-RFT ON NEWX

NOB = 14  NOVAR = 2
RSQ =        0.656597  MLRSQ =      0.576796  ROBRSQ =     -0.552044
SER =        0.160501  MEDAR =      0.076523  SSR =      0.309128  SAR =      1.41404

     COEF          ST ER

  -0.092227       0.052252
   1.09521        0.278601

               COVARIANCE MATRIX
   0.00273
  -0.009313       [illegible]
```

| FUND | $\hat{\alpha}_{LAR}$ | $\hat{\alpha}_{LS}$ | $\hat{\alpha}_{LAR}-\hat{\alpha}_{LS}$ | $\sigma(\hat{\alpha}_{LS})$ | $\sigma(\hat{\alpha}_{LAR})$ | $\hat{\beta}_{LAR}$ | $\hat{\beta}_{LS}$ | $\hat{\beta}_{LAR}-\hat{\beta}_{LS}$ |
|---|---|---|---|---|---|---|---|---|
| 168 | -.01021 | -.00396 | -.00625 | .00982 | .00996 | .61841 | .59524 | .0231 |
| 198 | -.07825 | -.06101 | -.01724 | .01361 | .01421 | 1.3233 | 1.2754 | .0479 |
| 206 | -.00198 | .01545 | -.01743 | .01532 | .01671 | .80660 | .71908 | .0875 |
| 218 | .00704 | -.00398 | .0110 | .01028 | .00231 | .85436 | .51682 | .3375 |
| 220 | -.02561 | -.03370 | .008 | .00996 | .01018 | .74729 | .76129 | -.01399 |
| 224 | -.03904 | -.05281 | .0137 | .03911 | .04253 | .99383 | 1.2822 | -.2884 |
| 236 | -.00704 | -.00721 | .0016 | .01068 | .01096 | .65484 | .70248 | -.0476 |
| 249 | -.01351 | -.00824 | -.0052 | .04959 | .05073 | .86086 | 1.0080 | -.1394 |
| 253 | -.02015 | -.02516 | .0050 | .01943 | .01977 | 1.1037 | 1.0678 | .0358 |
| 1191 | -.09222 | -.08209 | -.0101 | .04739 | .05225 | 1.0952 | 1.3950 | -.2998 |
| AVERAGE | -.0280 | -.0266 | -.0014 | | | .905 | .932 | .027 |

Comparison of LAR and LS Results

Table 9

## 7. Conclusions

In estimating the performance of portfolio managers ($\alpha$) and the systematic risk ($\beta$), a sample of ten randomly open-ended mutual funds was chosen. The sensitivity analysis indicates that two funds (168 and 249) were sensitive enough to show superior forecasting capabilities of its managers under deletion of yearly observations as compared to inferior performance when the whole set of data is used. However, the years showing considerable effect to change the sign of the estimate $\alpha$ were not the same for these funds.

In a particular analysis of each individual fund it was found that some years provoke major deviations from the estimates based on considering the complete database. One such year that seems to bias the results is 1962. Deletion of these observations implies major magnitude changes for all 10 funds. Also 80% of the funds indicate better performance of its managers forecasting capabilities. For all 10 funds, whenever the performance estimate improved, the systematic risk decreased and vice-versa. This suggests further research should be undertaken in the composition of these funds during this period. In the average portfolio, managers show better performance but not significantly better, under deletion of 1962.

An analysis of the residuals for the complete set of observations in 8 out of 10 funds suggested some time effects in the variances. Robust regression techniques seem to be appropriate to deal with these circumstances.

The robust technique used was Least Absolute Residuals. Two funds (206 and 218) revealed different signs for the performance of portfolio managers and compared to least square estimates. In terms of the average performance, it becomes poorer but again not significantly. The system-

atic risk decreased significantly in the average indicating more conservative portfolios than under the least squares assumptions.

Figure 2 summarizes the effects of both techniques. Riskier stocks show lower performance coefficients under all techniques. Higher $\beta$ funds seem to be more sensitive to the introduction of least absolute residuals regression than lower $\beta$ funds.

Our results about average poor performance of portfolio managers do not suggest major departures from Jensen's conclusions. Individually, however, 4 out of the 10 funds studied showed opposite performance to what was estimated by Jensen. (Three funds show better performance, one shows poorer performance leaving with a net better performance in two funds.) At least at the individual fund level, this result suggests that one should be careful when evaluating the strength of results derived uniquely on the basis of least squares regression. In [2] page 415, Jensen concludes "the evidence on mutual fund performance discussed above indicates not only that these 115 mutual funds were on average not able to predict security prices, well enough to outperform a buy-the-market-and hold policy, but also that there is very little evidence that any individual fund was able to do so significantly better than that which we expected from mere random choice."

Our results do not seem to improve the significance of the $\alpha$'s. However, we have to realize that our sample is relatively small; the number of observations were for each fund in the range of 15 to 20 yearly data, and only one special loss function was used (LAR). For comparative purposes it would also be interesting to have a code for performing sensitivity analysis for robust regression runs.

Even though the concentration of this work was in illustrating the application of new statistical techniques in joint tests of the capital asset pricing model, and portfolio managerial performance. These powerful techniques can be very useful also in obtaining more stable estimates of the systematic risk of the portfolios (β). Both Jensen's results [2] and our preliminary findings do suggest that the β's are far more important determinants of portfolio returns. A much more complete work with an enlarged sample would provide us with stronger conclusions about the significance of performance coefficients α's (individuals and average). While the estimation of the β's were not the primary motivation of this initial work we suspect that robust techniques and sensitivity analysis would be also rewarding in refining the estimates of the systematic risk. The latter seems to be the current trend in research in portfolio performance.

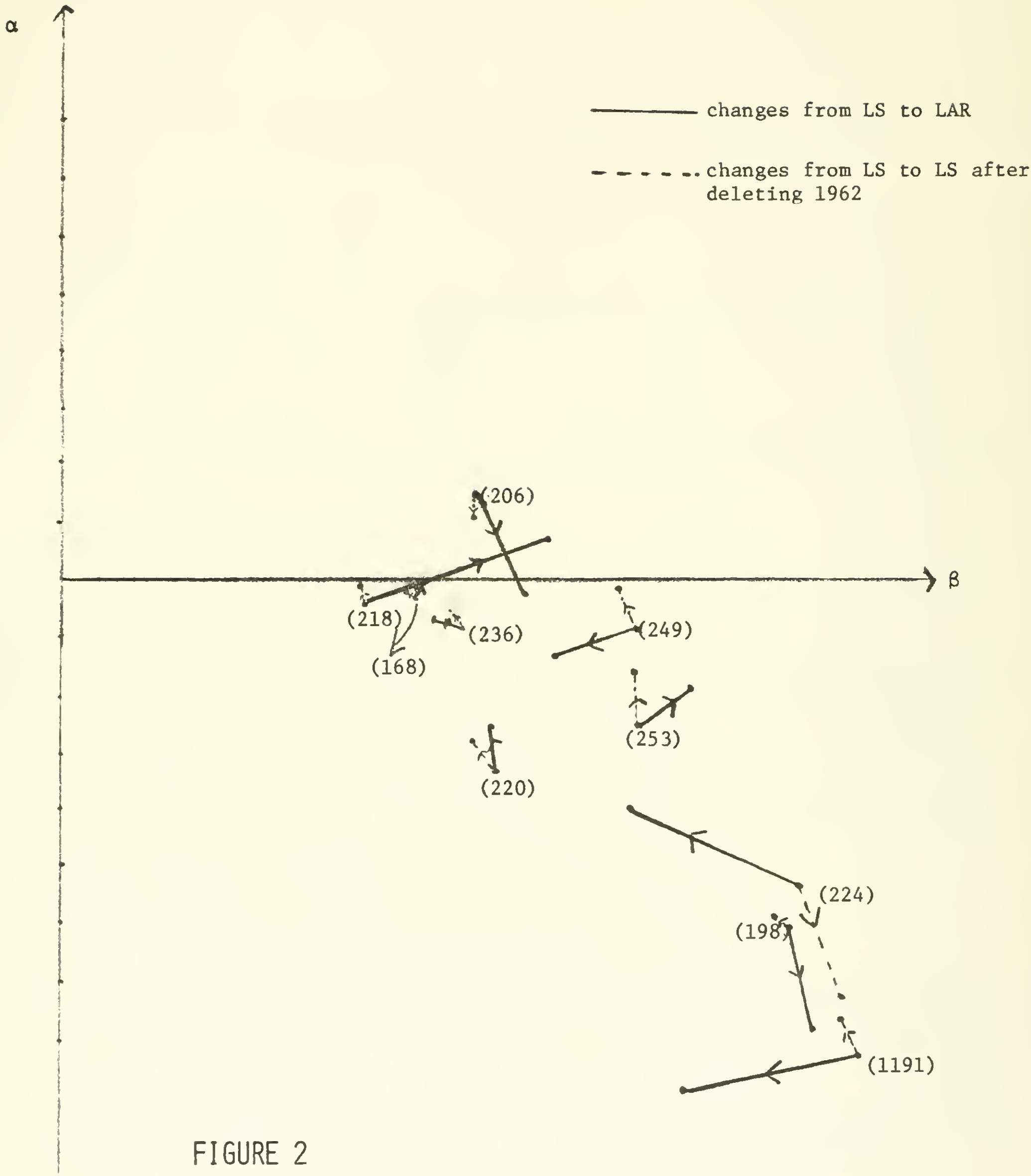

α
β
changes from LS to LAR
changes from LS to LS after deleting 1962
(206)
(218)
(168)
(236)
(249)
(253)
(220)
(224)
(198)
(1191)

FIGURE 2

Attachment A

NAV168 - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1949 TO 1964

NAV168 = NAV168/2

| | | | | |
|---|---|---|---|---|
| 1949 | 5.605 | 6.16 | 6.33 | 6.6 |
| 1953 | 6.18 | 7.79 | 8.23 | 8.16 |
| 1957 | 6.84 | 7.94 | 8.32 | 8.25 |
| 1961 | 9.16 | 8.07 | 8.65 | 9.19 |

DIV168 - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1950 TO 1964

| | | | | |
|---|---|---|---|---|
| 1950 | 0.155 | 0.235 | 0.2 | 0.2 |
| 1954 | 0.185 | 0.195 | 0.18 | 0.195 |
| 1958 | 0.2 | 0.2 | 0.21 | 0.21 |
| 1962 | 0.21 | 0.21 | 0.22 | |

CG168 - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1950 TO 1964

CG168 = CG

| | | | | |
|---|---|---|---|---|
| 1950 | 0.16 | 0.18 | 0.175 | 0.175 |
| 1954 | 0.215 | 0.30655 | 0.47 | 0.4 |
| 1958 | 0.375 | 0.375 | 0.367 | 0.4 |
| 1962 | 0.4 | 0.33 | 0.44 | |

R168 - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1950 TO 1964

R168 = LOG((NAV168+DIV168+CG168)/NAV168(-1))

| | | | | |
|---|---|---|---|---|
| 1950 | 0.144289 | 0.090724 | 0.097032 | -0.006842 |
| 1954 | 0.281595 | 0.114101 | 0.068101 | -0.093046 |
| 1958 | 0.219041 | 0.113575 | 0.056884 | 0.17153 |
| 1962 | -0.053825 | 0.129962 | 0.129912 | |

NAV198 - DATE REVISED: 9/16/76
c2 ANNUAL DATA FROM 1944 TO 1964

NAV198 = NAV198/6

| | | | | |
|---|---|---|---|---|
| 1944 | 3.95167 | 5.07333 | 3.965 | 3.62833 |
| 1948 | 3.33667 | 3.74 | 4.83 | 5.36 |
| 1952 | 5.69 | 5.18 | 7.87 | 9.53 |
| 1956 | 9.9 | 7.01 | 9.69 | 10.11 |
| 1960 | 8.41 | 8.84 | 6.75 | 7.39 |
| 1964 | 7.79 | | | |

DIV198 - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1945 TO 1964

DIV198 = DIV198/6

| | | | | |
|---|---|---|---|---|
| 1945 | 0.128333 | 0.141667 | 0.191667 | 0.225 |
| 1949 | 0.22 | 0.238333 | 0.24 | 0.22 |
| 1953 | 0.21 | 0.22 | 0.24 | 0.26 |
| 1957 | 0.25 | 0.19 | 0.18 | 0.18 |
| 1961 | 0.16 | 0.16 | 0.16 | 0.16 |

CG198 - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1945 TO 1964

CG198 = CG198/6

| | | | | |
|---|---|---|---|---|
| 1945 | 0.5 | 0.208333 | 0.041667 | 0. |
| 1949 | 0. | 0.15 | 0.17 | 0.18 |
| 1953 | 0. | 0.12 | 0.13 | 0.56 |
| 1957 | 0.45 | 0.27 | 0.57 | 0.7 |
| 1961 | 0.33 | 0.23 | 0.45 | 0.67 |

R198 - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1945 TO 1964

R198 = LOG((NAV198+DIV198+CG198)/NAV198(-1))

| | | | | |
|---|---|---|---|---|
| 1945 | 0.36662 | -0.161901 | -0.026407 | -0.018545 |
| 1949 | 0.171271 | 0.333091 | 0.177825 | 0.127683 |
| 1953 | -0.054165 | 0.460547 | 0.229476 | 0.117666 |
| 1957 | -0.250017 | 0.370135 | 0.113992 | -0.084587 |
| 1961 | 0.103813 | -0.213574 | 0.169898 | 0.153956 |

NAV206 - DATE REVISED: 9/15/76
ANNUAL DATA FROM 1953 TO 1964

NAV206 = NAV203*2

| | | | | |
|---|---|---|---|---|
| 1953 | 21.09 | 28.03 | 31.1 | 30.71 |
| 1957 | 25.76 | 33.33 | 31.92 | 34.98 |
| 1961 | 38.9 | 33.72 | 36.08 | 37.56 |

DIV206 - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1954 TO 1964

DIV206 = NEWPER(DIV203,1,1954)

| | | | | |
|---|---|---|---|---|
| 1954 | 0.85 | 1.1 | 1.3 | 1.3 |
| 1958 | 1.13 | 1.13 | 0.4 | 1.23 |
| 1962 | 1.13 | 1.18 | 1.22 | |

CG206 - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1954 TO 1964

CG206 = CG203

| | | | | |
|---|---|---|---|---|
| 1954 | 0.7 | 1.2 | 0.86 | 1.15 |
| 1958 | 1.4 | 3.65 | 0. | 2.41 |
| 1962 | 3.35 | 0.33 | 2.15 | |

R206 - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1954 TO 1964

R206 = LOG((NAV206+DIV206+CG206)/NAV206(-1))

| | | | | |
|---|---|---|---|---|
| 1954 | 0.338298 | 0.17528 | 0.055351 | -0.084913 |
| 1958 | 0.330799 | 0.096317 | 0.102913 | 0.195668 |
| 1962 | -0.018159 | 0.108646 | 0.126123 | |

Attachment A (Cont'd.)

NAV - DATE REVISED: 9/03/76
ANNUAL DATA FROM 1948 TO 1964

| | | | | |
|---|---|---|---|---|
| 1948 | 6.25 | 6.69 | 6.77 | 7.005 |
| 1952 | 7.385 | 7.155 | 8.75 | 9.565 |
| 1956 | 9.5 | 8.785 | 10.715 | 10.65 |
| 1960 | 10.53 | 11.98 | 10.92 | 11.65 |
| 1964 | 12.31 | | | |

DT = DT(2)

| | | | | |
|---|---|---|---|---|
| 1948 | 0. | 0.247 | 0.256 | [illegible] |
| 1952 | 0.261 | 0.265 | 0.282 | 0.3 |
| 1956 | 0.31 | 0.333 | 0.34 | 0.345 |
| 1960 | 0.355 | 0.35 | 0.35 | 0.35 |
| 1964 | 0.36 | | | |

CGT - DATE REVISED: 9/03/76
ANNUAL DATA FROM 1949 TO 1964

| | | | | |
|---|---|---|---|---|
| 1949 | 0.0312 | 0.09125 | 0.12575 | 0.0464 |
| 1953 | 0.075 | 0.1 | 0.1175 | 0.1175 |
| 1957 | 0.08 | 0.135 | 0.175 | 0.18 |
| 1961 | 0.21 | 0.18 | 0.22 | 0.24 |

R218 = LOG((NAV+DT+CGT)/NAV(-1))

| | | | | |
|---|---|---|---|---|
| 1949 | 0.108775 | 0.061907 | 0.08881 | 0.093606 |
| 1953 | 0.014785 | 0.243973 | 0.131779 | [illegible] |
| 1957 | -0.032306 | 0.241974 | 0.041586 | 0.038226 |
| 1961 | 0.174695 | -0.045249 | 0.112478 | 0.102696 |

Attachment A (Cont'd)

NAV22000 — DATE REVISED: 9/16/76
ANNUAL DATA FROM 1946 TO 1964

| | | | | |
|---|---|---|---|---|
| 1946 | 7.19 | 6.9 | 6.47 | 7.19 |
| 1950 | 7.61 | 8.08 | 8.07 | 7.53 |
| 1954 | 9.35 | 9.83 | 9.56 | 8.09 |
| 1958 | 9.98 | 9.91 | 9.3 | 10.72 |
| 1962 | 8.8 | 9.71 | 10.26 | |

DIV220 — DATE REVISED: 9/16/76
ANNUAL DATA FROM 1947 TO 1964

| | | | | |
|---|---|---|---|---|
| 1947 | 0.275 | 0.2575 | 0.2855 | 0.2284 |
| 1951 | 0.222 | 0.2065 | 0.2355 | 0.26 |
| 1955 | 0.27 | 0.333 | 0.293 | 0.323 |
| 1959 | 0.3 | 0.3 | 0.3 | 0.27 |
| 1963 | 0.25 | 0.25 | | |

CG220 — DATE REVISED: 9/16/76
ANNUAL DATA FROM 1947 TO 1964

| | | | | |
|---|---|---|---|---|
| 1947 | 0. | 0.2285 | 0.0895 | 0.4216 |
| 1951 | 0.428 | 0.3435 | 0.1795 | 0.29 |
| 1955 | 0.38 | 0.377 | 0.267 | 0.237 |
| 1959 | 0.26 | 0.21 | 0.3 | 0.14 |
| 1963 | 0.26 | 0.35 | | |

R220 — DATE REVISED: 9/16/76
ANNUAL DATA FROM 1947 TO 1964

R220 = LOG((NAV22000+DIV220+CG220)/NAV22000(-1))

| | | | | |
|---|---|---|---|---|
| 1947 | -0.002088 | 0.008083 | 0.156356 | 0.1387[illegible] |
| 1951 | 0.137301 | 0.064692 | -0.015611 | 0.273659 |
| 1955 | 0.114092 | 0.043788 | -0.100029 | 0.264548 |
| 1959 | 0.047931 | -0.010142 | 0.196556 | -0.151821 |
| 1963 | 0.149594 | 0.111929 | | |

Attachment A (Cont'd.)

NAV224 - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1950 TO 1964

| | | | | |
|---|---|---|---|---|
| 1950 | 3.81 | 3.89 | 3.83 | 3.26 |
| 1954 | 4.42 | 5.08 | 4.69 | 2.71 |
| 1958 | 4.06 | 4.1 | 3.43 | 3.79 |
| 1962 | 3.56 | 4.28 | 4.85 | |

DIV224 - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1951 TO 1964

| | | | | |
|---|---|---|---|---|
| 1951 | 0.308 | 0.298 | 0.271 | 0.27 |
| 1955 | 0.297 | 0.299 | 0.269 | 0.21 |
| 1959 | 0.21 | 0.2 | 0.2 | 0.19 |
| 1963 | 0.18 | 0.2 | | |

CG224 - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1951 TO 1964

| | | | | |
|---|---|---|---|---|
| 1951 | 0. | 0.18 | 0.15 | 0.04 |
| 1955 | 0.13 | 0.28 | 0.3 | 1. |
| 1959 | 0.06 | 0.1 | 0.04 | 0.04 |
| 1963 | 0. | 0.1 | | |

R224 - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1951 TO 1964

R224 = LOG((NAV224+DIV224+CG224)/NAV224(-1))

| | | | | |
|---|---|---|---|---|
| 1951 | 0.096978 | 0.102064 | -0.03968 | 0.372197 |
| 1955 | 0.21988 | 0.036528 | -0.357895 | 0.665081 |
| 1959 | 0.073579 | -0.094579 | 0.161206 | 0. |
| 1963 | 0.225388 | 0.185044 | | |

Attachment A (Cont'd.)

NAV236 - DATE REVISED: 9/15/76
ANNUAL DATA FROM 1944 TO 1965

NAV236 = NAV236/2

| | | | | |
|---|---|---|---|---|
| 1944 | 7.385 | 8.42 | 7.81 | 7.64 |
| 1948 | 7.07 | 7.805 | 8.79 | 9.21 |
| 1952 | 9.56 | 9.01 | 11.79 | 12.75 |
| 1956 | 12.44 | 10.85 | 13.64 | 14.[illegible] |
| 1960 | 14.57 | 17.05 | 14.44 | 15.51 |
| 1964 | 16.03 | 7.5 | | |

DIV236 - DATE REVISED: 9/15/76
ANNUAL DATA FROM 1945 TO 1964

DIV236 = DIV236/2

| | | | | |
|---|---|---|---|---|
| 1945 | 0.22 | 0.265 | 0.31 | 0.32 |
| 1949 | 0.345 | 0.395 | 0.375 | 0.375 |
| 1953 | 0.375 | 0.375 | 0.4 | 0.43 |
| 1957 | 0.43 | 0.42 | 0.42 | 0.43 |
| 1961 | 0.43 | 0.43 | 0.435 | 0.45 |

CG236 - DATE REVISED: 9/15/76
ANNUAL DATA FROM 1945 TO 1964

CG236 = CG236/2

| | | | | |
|---|---|---|---|---|
| 1945 | 0.18 | 0.135 | 0.09 | 0.08 |
| 1949 | 0.055 | 0.055 | 0.25 | 0.25 |
| 1953 | 0.25 | 0.3 | 0.45 | 0.53 |
| 1957 | 0.34 | 0.44 | 0.52 | 0.55 |
| 1961 | 0.62 | 0.17 | 0.48 | 0.54 |

R236 - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1945 TO 1964

R236 = LOG(R236)

| | | | | |
|---|---|---|---|---|
| 1945 | 0.17757 | -0.025257 | -0.009003 | 0.017556 |
| 1949 | 0.148883 | 0.168777 | 0.112333 | 0.100820 |
| 1953 | 0.097814 | 0.324589 | 0.142818 | 0.240723 |
| 1957 | -0.068139 | 0.289935 | 0.119910 | 0.0838 |
| 1961 | 0.216946 | -0.125437 | 0.116551 | 0.105802 |

NAV249 - DATE REVISED: 9/15/76
ANNUAL DATA FROM 1948 TO 1964

NAV249 = NAV249/4

| | | | | |
|---|---|---|---|---|
| 1948 | 2.4 | 2.585 | 2.96 | 2.23 |
| 1952 | 3.6 | 3.4 | 5.31 | 5.78 |
| 1956 | 5.9 | 4.86 | 7. | 8.13 |
| 1960 | 7.73 | 8.87 | 6.96 | 7.62 |
| 1964 | 8.11 | | | |

DIV249 - DATE REVISED: 9/15/76
ANNUAL DATA FROM 1949 TO 1964

DIV249 = DIV249/4

| | | | | |
|---|---|---|---|---|
| 1949 | 0.07155 | 0.1378 | 0.1478 | 0.1524 |
| 1953 | 0.1451 | 0.16 | 0.1732 | 0.183 |
| 1957 | 0.18 | 0.1638 | 0.1625 | 0.17 |
| 1961 | 0.15 | 0.14 | 0.15 | 0.17 |

CG249 - DATE REVISED: 9/15/76
ANNUAL DATA FROM 1949 TO 1964

CG249 = CG249/4

| | | | | |
|---|---|---|---|---|
| 1949 | 0.02595 | 0.1122 | 0.1147 | 0.0564 |
| 1953 | 0.1049 | 0.215 | 0.2561 | 0.279 |
| 1957 | 0.276 | 0.225 | 0.3125 | 0.32 |
| 1961 | 0.36 | 0.26 | 0.3 | 0.33 |

R249 - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1949 TO 1964

R249 = LOG(R249)

| | | | | |
|---|---|---|---|---|
| 1949 | 0.111279 | 0.216545 | -0.171904 | 0.530047 |
| 1953 | 0.013793 | 0.514055 | 0.156536 | 0.095939 |
| 1957 | -0.104231 | 0.418926 | 0.206433 | 0.011008 |
| 1961 | 0.19347 | -0.186615 | 0.147973 | 0.122148 |

NAV253 - DATE REVISED: 9/15/76
ANNUAL DATA FROM 1950 TO 1964

NAV253 = NAV253/2

| | | | | |
|---|---|---|---|---|
| 1950 | 2.61 | 3.065 | 3.105 | 2.91 |
| 1954 | 4.405 | 5.2 | 5.5 | 4.5 |
| 1958 | 6.195 | 7.38 | 6.9 | 7.89 |
| 1962 | 6.2 | 7.12 | 7.47 | |

DIV253 - DATE REVISED: 9/15/76
ANNUAL DATA FROM 1951 TO 1964

DIV253 = DIV253/2

| | | | | |
|---|---|---|---|---|
| 1951 | 0.1 | 0.105 | 0.09 | 0.1 |
| 1955 | 0.15 | 0.14 | 0.12 | 0.11 |
| 1959 | 0.1 | 0.13 | 0.11 | 0.08 |
| 1963 | 0.1 | 0.1 | | |

CG253 - DATE REVISED: 9/15/76
ANNUAL DATA FROM 1951 TO 1964

CG253 = CG253/2

| | | | | |
|---|---|---|---|---|
| 1951 | 0.065 | 0.06 | 0.075 | 0.065 |
| 1955 | 0.11 | 0.19 | 0.21 | 0.19 |
| 1959 | 0.2 | 0.18 | 0.2 | 0.18 |
| 1963 | 0.16 | 0.2 | | |

R253 - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1951 TO 1964

R253 = LOG(R253)

| | | | | |
|---|---|---|---|---|
| 1951 | 0.213131 | 0.064741 | -0.009709 | 0.45136 |
| 1955 | 0.214708 | 0.114357 | -0.129902 | 0.360955 |
| 1959 | 0.214876 | -0.023305 | 0.172612 | -0.199967 |
| 1963 | 0.174224 | 0.087362 | | |

Attachment A (Cont'd.)

NAV1191 - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1950 TO 1964

| | | | | |
|---|---|---|---|---|
| 1950 | 3.72 | 4.07 | 4.09 | 4.21 |
| 1954 | 9. | 10.28 | 10.85 | 7.54 |
| 1958 | 9.43 | 9.31 | 8.8 | 9. |
| 1962 | 6.66 | 6.678 | 6.96 | |

DIV1191 - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1951 TO 1964

| | | | | |
|---|---|---|---|---|
| 1951 | 0.19 | 0.18 | 0.22 | 0.27 |
| 1955 | 0.33 | 0.33 | 0.28 | 0.28 |
| 1959 | 0.23 | 0.14 | 0.07 | 0.085 |
| 1963 | 0.065 | 0.085 | | |

CG1191 - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1951 TO 1964

| | | | | |
|---|---|---|---|---|
| 1951 | 0. | 0.03 | 0. | 0.11 |
| 1955 | 0.39 | 0.47 | 0.06 | 0.04 |
| 1959 | 1.5 | 0.12 | 0.6 | 0. |
| 1963 | 0.2 | 0.28 | | |

R1191 = LOG((NAV1191+DIV1191+CG1191)/NAV1191(-1))

| | | | | |
|---|---|---|---|---|
| 1951 | 0.135545 | 0.054972 | 0.079854 | 0.80111[illegible] |
| 1955 | 0.20067 | 0.125106 | -0.319837 | 0.257045 |
| 1959 | 0.157629 | -0.02722 | 0.094276 | -0.288423 |
| 1963 | 0.041614 | 0.092474 | | |

Attachment A (Cont'd.)

S - DATE REVISED: 9/03/76
ANNUAL DATA FROM 1944 TO 1964

| | | | | |
|---|---|---|---|---|
| 1944 | 13.28 | 17.36 | 15.3 | 15.6 |
| 1948 | 15.2 | 16.76 | 20.41 | 23.77 |
| 1952 | 26.57 | 24.81 | 35.28 | 45.18 |
| 1956 | 46.67 | 39.99 | 55.21 | 59.99 |
| 1960 | 58.11 | 71.55 | 63.1 | 75.02 |
| 1964 | 84.75 | | | |

D - DATE REVISED: 9/03/76
ANNUAL DATA FROM 1945 TO 1964

| | | | | |
|---|---|---|---|---|
| 1945 | 0.66 | 0.71 | 0.84 | 0.93 |
| 1949 | 1.14 | 1.47 | 1.41 | 1.41 |
| 1953 | 1.45 | 1.54 | 1.64 | 1.74 |
| 1957 | 1.79 | 1.75 | 1.83 | 1.95 |
| 1961 | 2.02 | 2.13 | 2.28 | 2.5 |

RMT - DATE REVISED: 9/03/76
ANNUAL DATA FROM 1945 TO 1964

RMT = LOG((S+D)/S(-1))

| | | | | |
|---|---|---|---|---|
| 1945 | 0.305222 | -0.080956 | 0.053447 | 0.05[illegible] |
| 1949 | 0.163504 | 0.266577 | 0.210025 | 0.16[illegible] |
| 1953 | -0.011736 | 0.413627 | 0.269734 | 0.062[illegible] |
| 1957 | -0.110684 | 0.353719 | 0.113082 | 0.001166 |
| 1961 | 0.235899 | -0.092477 | 0.202973 | 0.151023 |

Attachment A (Cont'd.)

R - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1945 TO 1964

| | | | | |
|---|---|---|---|---|
| 1945 | 2.44 | 2.21 | 2.21 | 2.45 |
| 1949 | 2.42 | 2.2 | 2.39 | 2.74 |
| 1953 | 2.8 | 2.69 | 2.68 | 2.88 |
| 1957 | 3.34 | 3.24 | 3.91 | 4.37 |
| 1961 | 3.89 | 4.08 | 3.89 | 4.15 |

RFT - DATE REVISED: 9/16/76
ANNUAL DATA FROM 1945 TO 1964

RFT = LOG(1+R/100)

| | | | | |
|---|---|---|---|---|
| 1945 | 0.024107 | 0.021859 | 0.021859 | 0.024205 |
| 1949 | 0.023911 | 0.021761 | 0.023618 | 0.02703 |
| 1953 | 0.027615 | 0.026544 | 0.026446 | 0.028392 |
| 1957 | 0.032854 | 0.031885 | 0.038355 | 0.042771 |
| 1961 | 0.038162 | 0.039989 | 0.038162 | 0.040661 |

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