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THE RELATIVE STABILITY OF MONETARY VELOCITY AND THE INVESTMENT MULTIPLIER

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E R R A T A

1) Page 2, last par., line 3, should read: "between the stock of money and money income or consumption expenditure"

2) Page 6, line 1: Insert "[8]" after 197, and the word "below" after Table 2.

3) Page 8, line 4: Change the symbol I to K.

equation (5'): Change I to K.

last paragraph, second from last line: Change G+I to G+K.

equation (7a): Change I to K.

equation (7b): Change I to K.

4) Page 9, line 4: Change I to K.

line 14: Change I to K.

line 15: Change I to K.

line 21: Change I to K.

5) Page 10, line 2: Change I to K.

6) Page 10, line 3, should read: "On the basis of (i) to (iii) above. . ."

7) Page 11, line 10: the equation beginning with Y^d should be centered and on the left margin insert (8)

8) Page 11, line 12: shift the symbol C after the word "can".

9) Page 12, line 1, should read: "where ξ is a linear combination of ϵ, η_x, and η_z.

10) Page 13, 3rd line from the bottom should read: "error terms η_z and η_x. . . ."

11) Page 14, line 10, should read: ". . . to imports, to corporate retention, and to taxes."

12) Page 17, line 3: replace the word "results" with "procedure"

13) Page 24, line 10: the figure 437 should read 438

14) Page 26, line 8: replace the words "correlations and" with "correlations, or between"

15) Page 26, par. 2, line 2: replace the symbol M by M^f.

16) Page 27, line 1, should read: ". . . --except possibly for. . ."

17) Page 28, line 11, end: "in what might" should read "on what might"

18) Page 32, line 3, equation (a.4): the term a_2Z^a should read a_2X^a

19) Page 35, 2nd par., last line: "the first four rows" should read "the first three rows"

20) Page 37, heading of table: insert the word "of" after "Definitions"

in definition of M^f: replace the words "savings deposits" with "time deposits"

21) Page 38, definition of Y: replace the word "measured" with "measure of"

22) Page 39, equation (b.4): insert a minus sign in front of R.

23) Page 40, Table 2, footnote b, should read: "See [9, p. 190, Table II.1]. . . ."

24) Page 41, Table 3.b: the heading for the last column should be S_e^2 instead of S_y^2 .

25) Page 42, Table 4, heading: "and Consumption" should read "and Income or Consumption"

26) Page 43, Table 5, heading, first row: delete the words "(Using M*)

column headed M^f, first row: replace 2.5 with 2.0

third row, column headed Z^a: replace 1.3 with 1.2

third row, column headed M^f: replace 2.5 with 2.0

27) Page 43, footnote (a) to Table 5, add the following: "The larger value for the coefficient of M^* as compared with that of M^f is due to the fact that M^* is the potential supply of means of payments M, while M^f includes in addition to M also time deposits in commercial banks.

28) Page F-1, line 3: the sentence should begin "The contribution"

29) Page F-4, footnote 11, line 3, should read: "equations (b.5) and (b.7) for. . ."

30) Page F-6, Note 16, line 4: the symbol I should be replaced by Z^i; also, insert a comma after Z^a

31) Page F-8, Note 24, 3rd line from end: "of equation (4.3)" should read "of equations (4.2) or (4.4),"

32) Page F-9, Note 27, line four from end: add "are" at the end of the line

line two from end: replace "It" with "The residual variances in Table 4.b"

33) Page F-10, line 6: replace "the multiplier" with "a multiplier"

line 7: replace "the conclusion" with "a conclusion"

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Introduction

In their contribution to the staff studies for the Commission on Money and Credit [9], Friedman and Meiselman (hereafter referred to as F and M) report that, according to their tests, "Except for the early years of the Great Depression, money is more closely related to consumption than is autonomous expenditures", and that "The results are strikingly one-sided" [9 , pp. 165-166]. The purpose of this paper is (i) to call attention to a number of basic shortcomings in their procedure which make the results of their elaborate battery of tests essentially worthless; (ii) to show that the elimination of the shortcomings, insofar as feasible within the framework of their analysis, changes drastically their conclusions; and above all, (iii) to make it clear why the Friedman and Meiselman game of testing one-equation one-variable model in search of the highest correlation, fascinating as it might be, cannot be expected to throw any light on such basic issues as how our economic system works, or how it can be more effectively stabilized.

In order to establish these propositions, we first show in Section I that because of serious distortions in their formulation of the "income-expenditure" model and/or biases in their statistical procedure, the results reported by F and M contain very little, if any, information about the empirical usefulness of that model. No really adequate test of this model appears feasible within the constraint of a single equation with a single independent variable, arbitrarily imposed by F and M. It is shown, however, that insofar as the implications of the model concerning the relation between consumption and autonomous expenditure can be forced into such an artificial strait jacket, these implications receive very strong empirical support, even though the forcing is at the cost of a variety of misspecifications which tend to bias the results against the model. Even after an additional variable is introduced which eliminates, at least roughly, the possible influence of common trends, the impact effect of autonomous expenditures on current expenditure is shown to remain quite substantial.

In Section II, using the models proposed elsewhere by Friedman [7], [10], we suggest that the very high correlation between the stock of money and money consumption expenditure is to some extent spurious, although, again, after an additional variable is introduced to minimize the possible bias resulting from common trends, the net contribution of the money stock to the explanation of variations in current consumption remains significant.

On the basis of these results, we conclude that it is nearly impossible to answer the question posed by F and M as to which of the two models of income determination set up by them as contenders should win the prize for the highest correlation. In the concluding section, we go on to develop the reasons why an answer to this question, even if it could be obtained, would be of very little value for any conceivable purpose.

In order to conserve space, in what follows, various components of the national income account and other relevant variables will be frequently referred to by symbols whose definition is provided in Table 1.a.

I. The Stability of the Investment Multiplier

According to F and M the essence of the "income-expenditure theory" can be embodied in the equation

(1) $Y = \alpha + K'A$

where α and K' are constants, $K' > 1$ [9 , p. 170], and Y and A must satisfy the condition [9, p. 175]

(2) $Y = C + A.$

From (1) and (2) they derive

(3) $C = \alpha + K\ A, \quad K = K' - 1$

the equation on which all of their tests are based, with C defined as measured consumers' expenditure and A the sum of

measured net investment, net exports and government deficit (cf. Table 1.b, equation (b.1)).

Our contention that their results are irrelevant is based on three considerations.

(i) <u>Misspecification of the consumption function</u>

The income-expenditure model, as generally understood for at least two decades, does not imply equation (3). To establish this proposition, let us start with the most primitive and naive form of the consumption function given by

$$(4) \quad C = c_o + c_1 Y^d + \epsilon$$

where c_o and c_1 are constants, and ϵ is a random error term. Taking into account the identity

$$(2') \quad Y^d \equiv C + S,$$

substituting (2') into (4) and solving for C we obtain

$$(3') \quad C = \frac{c_o}{1-c_1} + \frac{c_1}{1-c_1} S + \frac{1}{1-c_1} \epsilon.$$

Equation (3') is of the form (3) and C, Y, and S satisfy the condition (2'), which is similar to (2). But, the independent variable in (3') is not A but S, which differs from A by corporate retained earnings adjusted for inventory valuation (R), the statistical descrepancy (H), excess of wage accruals over disbursement (W), and government foreign transfer payment (T_f)

(cf. equation (b.1)). Equation (3) above used by F and M in which the independent variable is A is valid only if Y^d is replaced by $Y^d + R + H + W + T_f$ in the consumption function (4). But this surely involves a grievous misspecification of the consumption function, for we are not aware of any author having suggested that current consumption is a linear function of current disposable income plus corporate savings plus statistical discrepancy plus excess of wage accrual over wage disbursement plus net government foreign transfer payments.[1/] [2/]

(ii) Treatment of war years

Even if one were to disregard the above misspecification, it is apparent from (3') that equation (3) could be expected to hold only if the parameters c_o and c_1 of the consumption function can be supposed stable. But these parameters could hardly be supposed stable over a period including the war years 1942-1946, especially when one recalls that C is defined to include purchases of consumers' durables. During this period consumers may have been pursuaded to consume abnormally small proportions of their income for patriotic reasons, and/or they may have changed their consumption habits in response to rationing and to unavailability of some goods.[3/] Hence any test including these years is worthless unless it has been shown that the results are largely invariant whether these years are included or omitted. In fact the omission of these years makes an overwhelming difference as is readily apparent from an inspection of F and M's own scatter diagrams on pp. 191

and 197, and from the results exhibited in Table 2. These results speak for themselves. Yet in their basic Table II-1, [8, p. 190], out of the six tests covering the period since 1929, three include the war years, and, not surprisingly, they are the most damaging to the income-expenditure model. Yet, in their contribution, F and M not even once mention the possibility that any of their results might be distorted by the inclusion of the war years!

(iii) Inclusion of induced components in the "independent" variable and the resulting least squares bias.

Whether the "independent" variable of equation (3) is S, as called for by the standard consumption function, or the variable A, arbitrarily picked by F and M, one cannot get a reliable estimate of the parameters of this equation by the method of least squares applied by F and M--except in the limiting and empirically irrelevant case in which the consumption function (4) holds with no error. This is because both S and A cannot be regarded as autonomous in the sense that they are uncorrelated with the residual error of the consumption function (4)[4/].

This point and its implications can readily be established with the help of the accounting identities set forth in Table 1.b. It is seen from equation (b.1) that, aside from the minor reconciliation items $H + W + T_f$, personal saving S can be expressed as

$$S \simeq \text{Private Domestic Investment} + \text{Government Expenditure} + \text{Exports} - [\text{Net Taxes} + \text{Imports} + \text{Corporate Saving}]$$

The three components in the square bracket could not possibly be regarded as autonomous in the sense defined above. The movements of each of these three components are closely related to that of consumption (either directly as in the case of imports or through income as in the case of taxes) which in turn is clearly correlated with the error term ϵ of the consumption function. Since S thus includes terms correlated with ϵ, it will in general be itself correlated with ϵ. It is well known that under these conditions direct regression of C on S will yield biased estimates of the coefficients as well as of the variance of the error term ϵ. It can also be verified from the definition (b.1) in Table 1.b that this same conclusion applies if S were replaced by A as done by F and M. The bias in the coefficient will be upward if the correlation of S (or A) and ϵ is positive, and downward in the opposite case. In our case there can be little doubt that this correlation will be negative and that the bias is therefore definitely downward. This is because the expression in brackets, which is positively correlated with ϵ, appears in S with a negative sign. In other words fluctuations in ϵ will tend to be accompanied by fluctuations in the same direction in imports, net taxes and corporate retained earnings, and hence by fluctuations in S in the opposite direction.[5/] Indeed, one could very well imagine circumstances in which the downward bias resulting from this negative association between S and ϵ might be strong enough so that the regression coefficient of S (or A) on C and hence also the correlation coefficient might be zero or even negative [See also 3, Appendix B].

This conclusion may usefully be illustrated by a simple algebraic example. Consider an economy in which there are no corporations and no foreign trade, while net investments, I, and government expenditure, G, are entirely autonomous and net tax collection, T, is closely related to before-tax income. Under these conditions we have

(5') $S = A = I + (G - T)$

and

(6') $T = t_o' + t_1' (Y^d + T) + \eta'$

where the t's are constants and η' is a random error term, implying

(6) $T = t_o + t_1 Y^d + \eta, \qquad 0 < t_1 < 1$

(For the sake of simplicity let us also suppose that ϵ and η are uncorrelated.)

The four equations (5'), (6), (2'), and (4) form a system in the four endogenous variables C, Y^d, S and T and one exogenous variable G + I. If we solve this system for C and Y^d we find

(7a) $$C = \frac{c_1}{1-c_1+t_1}(G+I) + \frac{1+t_1}{1-c_1+t_1}\epsilon - \frac{c_1}{1-c_1+t_1}\eta + \text{constant}$$

(7b) $$Y^d = \frac{G+I+\epsilon-\eta}{1-c_1+t_1} + \text{constant}$$

From these equations it is apparent that C and Y^d will be positively correlated with autonomous expenditure, as one would expect under the income-expenditure model. The size of the correlation would depend on the variance of G + I relative to that of the two error terms (and not just ϵ). But what does the model imply about the correlation between consumption (or income) and the government deficit D=G-T? Close consideration will indicate that this correlation is likely to be very much smaller, and might even be zero or negative. This is because the relation between a change in D and the simultaneous change in C depends entirely on the source of the change in D. If D changes because of a change in G then the association will be positive; but the association will be negative if the change in D occurs due to changes in either I or ϵ with G (and η) constant. To illustrate, a fall in I will reduce income causing a <u>decrease</u> in consumption and also in taxes and hence an <u>increase</u> in D; similarly a fall in the error term ϵ will <u>decrease</u> consumption and hence income and taxes leading again to an <u>increase</u> in the deficit. Analogous considerations apply to the relation between S and C. If the change in C arises from a change in I + G then S and C will tend to vary in the same direction. But a change in C may also arise from a change in ϵ; this will change Y^d in the same direction but to a smaller extent because of the tax bite, and hence change S in the <u>opposite</u> direction.[6/] Thus, while the model definitely implies a positive correlation between Y or C and <u>autonomous</u> expenditure it has no definite implication

about the correlation between C and S. It might even be negative depending on the variance of I + G relative to that of ϵ (and η). [7/] [8/]

On the basis of (i) and (iii) above we submit that the laborious battery of tests presented by F and M is basically irrelevant for the purpose of assessing the empirical usefulness of the income-expenditure framework as generally understood.

It does not follow from the above considerations that the model has no testable empirical implications. On the contrary, as is apparent from the above example, the model does imply a strong positive association between income or even consumption and autonomous components of the income account, but this relation differs from (3) in a number of important respects. For one thing, because only a part of S can be taken as autonomous expenditure, it follows that the sum of C and the autonomous expendiutre will not add up to income, at least as long as income is defined as the argument of the consumption function (a condition which is in turn necessary to derive (3) from the consumption function).

Having removed this irrelevant constraint, let us inquire what are the conditions under which the rudimentary consumption function (4) will imply that consumption can be expressed as a linear function of the autonomous expenditure (and no other variable). For this purpose, it is unfortunately necessary to dwell on the detailed structure of the national income accounting, and recognize that S is the sum of two parts, the

difference between NNP and C, denoted by Z, and the difference between personal disposable income and NNP, denoted by X:

$$S \equiv Y^d - C \equiv (N - C) + (Y^d - N) \equiv Z + X$$

As can be seen from equation (b.3) and (b.4) in Table 1.b, Z and X can be expressed as the sums of a number of entries in the national income accounts, some of which can be supposed as autonomous and others clearly induced. Let Z^a and X^a denote the autonomous parts and Z^i and X^i the induced portions of Z and X respectively. We can thus rewrite equation (2') as $Y^d \equiv C + Z^a + X^a + Z^i + X^i$.

The question we wish to answer then becomes: under what conditions C can be expressed as a linear function of autonomous expenditures Z^a and X^a? It can be verified that the condition in question is that the induced components Z^i and X^i can be expressed as the following linear functions (one or more coefficients, of course, could be zero):[9/]

$$\text{(9a)} \quad Z^i = z_y Y^d + z_c C + z_x X^a + z_z Z^a + z_i X^i + z_o + \eta_z$$

$$\text{(9b)} \quad X^i = x_y Y^d + x_c C + x_x X^a + x_z Z^a + x_i Z^i + x_o + \eta_x$$

Under these conditions (and only under these conditions) equations (8), (9), and (4) can be solved to yield an expression for C linear in Z^a and X^a,

$$\text{(10)} \quad C = \alpha_o + \alpha_z Z^a + \alpha_x X^a + \xi$$

where ξ is a linear combination of ϵ. η_x and η_z.
If we make the further assumptions that $z_x = z_z$ and $x_x = x_z$, then the coefficients α_z and α_x will be equal and (10) reduces to

$$\text{(10a)} \quad C = \alpha_o' + \alpha_a' (Z^a + X^a) + \xi'$$

We have thus finally arrived at an equation which, like F and M's equation (3), relates consumption to a single independent variable. It differs from equations (3) and (3') in two important respects aside from the difference in the argument: (i) its coefficients, unlike those of (3) and (3'), depend not only on those of the consumption function (4) but also on those of equations (9a) and (9b); (ii) similarly the error term depends not only on that of (4) but also on η_z and η_x.

Equation (10a) is directly testable as soon as one specifies which components of Z and X can be regarded as "reasonably" autonomous. We have actually carried out such a test for the period 1929-1958 excluding war years with Z^a and X^a defined by equations (b.5) and (b.7) in Table 1.b. Our choice of the autonomous components is no doubt debatable. Its main justification is that we have endeavoured to stick as close as possible to the definition of A adopted by F and M, eliminating only those components whose induced nature seemed beyond reasonable doubt.[10/] [11/]

Before we comment on our results we must call attention to the large and serious number of misspecifications that are involved in deriving the one variable equation (10a) from the income-expenditure model.

(a) The consumption function (4) is so crude that for at least a decade hardly any one, least of all Mr. Friedman, would regard it as an acceptable approximation--except possibly for elementary classroom exercises.

(b) Equations (9a) and (9b) must be regarded as equally crude approximations. As is documented by countless empirical studies, many of the components of Z^i and X^i depend on other variables besides those included: e.g., inventory investment depends not only on current income, but also on initial inventories and lagged income, and possibly also on the availability of credit and terms thereof; imports depend on relative prices; corporate profits on some measure of the rate of utilization of capacity and its rate of change, and so on. It is precisely the endeavour to take into account these more complex interrelations that has lead to the formulation of more or less extensive models such as those of Klein [14] of Duesenberry, Eckstein and Fromm [6], of Suits [18], and of Liu [15], to mention but a few.12/ Dropping variables known to be relevant, as we have done in order to arrive at equation (10a), can only be at the cost of increasing the variance of the error term η_z and η_x, and hence finally of increasing the variance of ξ and decreasing the correlation between C and $(Z^a + X^a)$.

(c) The coefficients of equations (9), especially (9b), and hence those of (10) will depend on forms and rates of taxation. Treating these coefficients as constant over a period of substantial tax change again produces an upward bias in the variance of the error term ξ.[13/]

(d) The assumption leading from (10) to (10a) is also unwarranted. It can be shown that α_z may be expected to be smaller than α_x. This is because the impact of Z^a on personal income tends to be lower than that of X^a as a result of leakages related to imports, and to corporate retention, and to taxes. Thus, the restrictions that these two coefficients be equal again has the effect of biasing down the extent to which the variance of C appears to be explained by the model.

In spite of all these biases, it is apparent from Table 3a, row (3.1a) that hypothesis (10a) fits the data reasonably well in terms of the correlation coefficient as well as the size of the regression coefficient. Comparison with (2.3) of Table 2 shows that replacing F and M's variable A with the more relevant variable $X^a + Z^a$, and thereby at least reducing the downward bias implicit in their procedure, reduces the unexplained variance from roughly 600 to 69, a reduction of nearly 90 per cent. Note also, by comparing (3.1a) with (2.4), that the variance left unexplained by (10a) is less than half as large as that left unexplained by the F and M's money equation for the same period.

Some of the misspecifications listed under (a) to (d) above can be removed or made less serious, though at the cost

of violating F and M's arbitrary restriction to a single independent variable. In particular we can remove the misspecification pointed out under (d) by regressing C on Z^a and X^a rather than on their sum. The outcome, reported in Table 3a, row (3.2a), shows that the coefficients of both variables are highly significant, that the coefficient of X^a is larger than that of Z^a as expected (though the coefficient of X^a may be on the high side), and that removal of this misspecification reduces the unexplained variance by another 40 per cent.

In the last two rows of the table we present the results of a set of tests which consists in adding the variable C_{t-1}. These two tests can be justified and supported by two rather different sets of considerations. First, they can be regarded as the outcome of replacing the naive consumption function (4) with the less naive but still quite simple formulation

$$(4') \quad C_t = c_o + c_1 Y_t^d + c_2 C_{t-1}.$$

In this version C_{t-1} performs a role similar to that of such variables as the highest previous income, permanent income or net worth, which have been advocated and successfully tested by students of the consumption function over the last fifteen years. It can be readily verified that replacing (4) by (4') leads to equations of the form (10) or (10a), but including the additional variable C_{t-1}. Second, the introduction of C_{t-1} reduces the danger that the high correlation reported in Tables 2 and 3 may be just the spurious results of common trends

in all the variables. The partial correlation of the autonomous expenditures, given C_{t-1}, provides a less questionable, and probably overconservative,[14/] estimate of the net contributions of these variables to variations in consumption.

It is seen from an inspection of (3.3a) and (3.4a) that the introduction of C_{t-1} reduces further the unexplained variance by some 1/3 to 1/2, but that otherwise the results confirm those of equations (3.1a) and (3.2a). In particular the partial correlations of the autonomous expenditures remain uniformly quite substantial.[15/]

In Table 3.b we report the results of one final battery of tests which were inspired by a criticism kindly offered by Professor Friedman to an earlier draft of this paper. He has suggested that the tests we have reported so far do not quite come to grips with the basic issue with which F and M are concerned. This issue is which of the two alternative models does a better job of accounting for the behavior of a broad measure of income such as NNP, taking as a datum certain autonomous variables, to wit, autonomous expenditure in the income-expenditure model, and the money supply in the "rival" model. The only reason for the choice of C rather than NNP as the dependent variable in the F and M tests was to avoid a bias in the correlation coefficient coming from the inclusion of A in NNP. But, in our tests, we have no reason to use consumption as the dependent variable since the sum of C and Z^a or $Z^a + X^a$ does not add up to NNP. Instead, the relation between

NNP and C is given by

$$NNP = C + Z = C + Z^i + Z^a.$$

Now our results tell us only how well one can account for C given the value Z^a (and X^a), but not how well one can account for NNP, because it leaves one part of NNP, namely Z^i, still unaccounted for. Hence Mr. Friedman has understandably suggested that, to be faithful to the spirit of the F and M tests, the dependent variable in our tests should not be C, but rather the entire induced component, i.e., $C + Z^i$.

Fortunately this suggestion can be carried out with very little difficulty. The four equation system (4), (8), (9a), and (9b) can be solved simultaneously to give solutions for the four endogenous variables, C, Y^d, Z^i and X^i, as linear functions of Z^a and X^a. From these solutions, the entire induced component, $C + Z^i$, can itself be expressed as a linear function of Z^a and X^a. Hence to carry out this last test all we need to do is to replace C with a new dependent variable $C^f \equiv C + Z^i$ and correlate C^f with Z^a and X^a (or just with their sum $Z^a + X^a$ if we impose the misspecification $z_x = z_z$ and $x_x = x_z$). Note that since we have defined X^i as inventory investment minus imports (cf. Table 1.b, equation (b.8)) our new dependent variables is roughly equal to the value of domestically produced consumption (C-I) plus the change in inventories, a very sensible approximation to the induced part of NNP.[16]

It is apparent from Table 3.b that the results of this latest test do not change in any way our previous conclusions, especially if one uses as a criterion of performance the variance of the residual error. Note that, because $C^f + Z^a \equiv NNP$, and Z^a is assumed given, this variance can now also be interpreted as the variance of the error in predicting NNP given the autonomous components Z^a and X^a. When the dependent variable is the sum $Z^a + X^a$, the residual variance of (3.1b) is even smaller than in (3.1a). When Z^a and X^a are introduced separately, as they should be, the coefficient of each remains highly significant and of a very reasonable order of magnitude while the residual variance is halved, implying that, given the autonomous components, the error of prediction of NNP has a standard deviation of only about $6 billion. Finally, the residual variances in (3.1b) and (3.2b) are respectively one-third and one-fifth of the residual variance of the money equation (2.4) (though a comparison of these variances is no longer the relevant one as we shall point out in the next section). With the addition of C_{t-1} in (3.3b) and (3.4b), the coefficient of Z^a and X^a remains highly significant and sensible, and the residual variance is reduced further, though it remains somewhat higher than in (3.3a) and (3.4a).[17/]

We have thus demonstrated that the relatively low correlation coefficients between C and A obtained by F and M are due to several very serious misrepresentations of the Keynesian theory and its observable implications: inclusion of the second World War years; the use of induced components in their

independent variable; neglect of the necessity of separating various components of the exogenous variable; and the oversimplification of the consumption function. We have shown that autonomous expenditure, provided they are reasonably defined, can account for the variations in NNP up to a rather small error when some of the oversimplification and misspecifications are removed, although it is impossible to remove many of them due to the strait jacket imposed by F and M rules of the game. We will now turn to a review of their test of the money velocity.

II. Stability of the Relation between Money Supply and Income or Consumption

The correlation between C and the stock of money reported by F and M is uniformly so high that one cannot fail to be impressed, especially if one holds, as we do, that money is an important factor contributing to the determination of the level of income. Yet, there are also ample grounds for holding that the causal links from the money supply,--or more precisely from the proximate determinants of the money supply under the control of the monetary authority--to money income are quite complex and some times tortuous. One is therefore inclined to suspect that their high correlation may be partly spurious in the sense of overstating the strength and tightness of the causal mechanism from the money supply to the level of income or consumption. We will consider here two possible sources of bias in their test procedure, one suggested by Friedman's

own theory of income determination, and the other related to the existence of feedbacks from income to the stock of money.

The core of Friedman's monetary model of income determination is the quantity theory of money. His well known and elegant formulation of this theory [8 , pp. 4-17] is so general that few would quarrel with it except in matters of detail. But for purposes of empirical applications some further specifications are needed, which Friedman has supplied in later articles [7, esp. pp.335-338 and 350], [10, esp. pp.59-63] In these contributions his formulation of the demand for money has been narrowed down to

$$(11) \quad \frac{M^D}{P\pi_p} = \gamma \left(\frac{N_p^*}{P\pi_p}\right)^{\delta} + \eta^*$$

where δ and γ are constants, and η^* is a random error term.

In order to use Friedman's theory in interpreting the results of F and M, we must establish the relation between (11) and the demand for money implicit in the F and M tests, namely

$$(12) \quad M^D = g_1 N + g_o + \epsilon$$ 18/

where g_o and g_1 are constants and ϵ is a random error term.

If δ is unity, it is obvious that (11) reduces to

$$(13) \quad M^D = \gamma_1 N_p^* + \gamma_o + \eta$$

where $\gamma_1 = \gamma, \gamma_o = 0, \eta = \eta^* P\pi_p$.

Friedman, relying on a set of time series data covering a span of about a century, has contented that δ is distinctly larger than unity and close to 1.8. He has rationalized this finding on the ground that money is a superior good. Both his finding and his rationalization are, however, open to considerable question. At the factual level, it may be pointed out that, using a definition of money either less inclusive (excluding savings deposits in commercial banks) or more inclusive (including all forms of savings deposits and not merely in commercial banks) and following his procedure, from the turn of the century to date, one gets a value of δ much closer to unity than to 1.8.[19/] At the theoretical level, the inventory model of the transactions demand for money suggests that, for a given pattern of payment habits and a given rate of interest, δ would tend to be somewhat less than unity. It is quite conceivable that the mechanism to which Friedman appeals and that stressed by the inventory theoretical approach may be simultaneously at work, making the value of δ not much different from unity (once proper adjustment is made for the effects of variations in interest rates). However, it must be admitted that there is no reason for δ to be exactly unity, though it may be close to it. To the extent that δ is not unity (13) must be regarded as an approximation obtained by first expanding the right-hand side of (11) by the Taylor's series around the mean of $\frac{N^*_p}{P\pi_p}$, and then disregarding all but the constant and the first term of the expansion.[20/]

N_p^* is not an observable magnitude. Following Friedman, however, we may approximate N_p^* by an exponentially weighted moving average of past values of N, or

$$(14) \quad N_{pt}^* \propto N_{pt} \equiv \beta(1-\rho) \sum_{\tau=0}^{\infty} \rho^{\tau} N_{t-\tau}$$

where β is an adjustment factor for the time trend in N, a number slightly greater than unity.[21/] Friedman thinks that ρ is about .7. When (14) is substituted into (13) we have

$$(15) \quad M_t^D = \gamma_1 \beta(1-\rho) \sum_{\tau=0}^{\infty} \rho^{\tau} N_{t-\tau} + \gamma_o + \eta$$

If we further assume that

$$(16) \quad M_t^D = M_t$$

where M is the supply of money, and that M is completely autonomous, then (16) can be substituted into (15), which can then be solved for N_t in terms of M and $N_{t-\tau}$, $\tau = 1,2...$ We thus obtain

$$(17) \quad N_t = \frac{1}{\gamma_1\beta(1-\rho)} M_t - \frac{\rho}{\beta(1-\rho)}\beta(1-\rho) \sum_{\tau=1}^{\infty} \rho^{\tau-1} N_{t-(\tau-1)} - \frac{\gamma_o}{\gamma_1\beta(1-\rho)} - \frac{\eta}{\gamma_1\beta(1-\rho)}$$

$$= \frac{1}{\gamma_1\beta(1-\rho)} M_t - \frac{\rho}{\beta(1-\rho)} N_{p,t-1} - \frac{\gamma_o}{\gamma_1\beta(1-\rho)} - \frac{\eta}{\gamma_1\beta(1-\rho)}$$

where M_t and $N_{p,t-1}$ are both predetermined variables. This

implication is the one suggested by Friedman himself in his earlier article [7 , p. 350].

It appears from (17) that in order to make their test at least roughly consistent with Friedman's own model of how money affects income through the demand for money, F and M should have added to their test equation the variable $N_{p,t-1}$ and judged the importance of money from the over-all fit of this equation and the partial correlation of M. This same conclusion can be reached by a rather different route by others who, like ourselves, agree as to the importance of money but have serious reservations about the specific and rather simple minded mechanism proposed by Friedman. One may suspect that, since both N and M are dominated by marked time trends, there exists a very real possibility that the high correlation between them may be partly the spurious result of the common trend. The introduction of another trend-dominated variable like $N_{p,t-1}$ would tend to reduce this danger. Clearly that part of the movement of N_t which can be accounted for by a lagged variable like $N_{p,t-1}$ cannot be properly attributed to current variations in the money supply, and therefore a more reliable, though conceivably overconservative, measure of the impact effect of M will be provided by its partial correlation with N, given $N_{p,t-1}$.[22/] In short the addition of $N_{p,t-1}$ would seem an effective testing device from more than one point of view. In terms of the Friedman model the coefficient of this variable should be negative, and for this reason as well as because it eliminates a misspecification,

the addition of this variable should tend to increase the multiple correlation as well as the partial correlation of the variable M. Those who suspect some spuriousness would also expect the multiple correlation to increase, but would expect the coefficient of $N_{p,\ t-1}$ to be positive, and the partial correlation of M to decrease and to provide a more realistic measure of the initial impact of M on income.

The first two rows of Table 4 present the result of this test.[23/] Row (4.1) gives the residual variance of NNP given money supply alone as 437, and comparison of this row with the first two rows of Table 3.b reveals that this residual variance is seven to ten times larger than the residual variance of NNP given autonomous expenditure. Introduction of the additional variable $N_{p,t-1}$ does reduce the residual variance by nearly two-thirds, as can be seen from row (4.2), though it remains substantial, and is three to four times as large as for (3.1b) and (3.2b). Furthermore, the coefficient of $N_{p,t-1}$ is <u>positive</u>, contrary to the implication of Friedman's theory, and the partial correlation between N_t and M^f, given $N_{p,t-1}$, is dramatically smaller than the simple correlation, though it is by no means negligible. These results are consistent with the view that F and M's high correlations are somewhat misleading and that money affects income through a mechanism which is quite different from the simple one envisaged by Friedman and which leaves room for "slippage".[24/]

As mentioned earlier a second likely source of bias in F and M's tests, or even in that of equation (17), is that,

under the institutional arrangements prevailing during the period covered by the tests, M was at least partly induced, and in consequence positively correlated with the error term of (13) or (17). 25/ We should like to suggest a test which should at least reduce the danger of this bias. It consists in replacing M with M*, defined as the estimated maximum amount of money (in the conventional definition) that could be created by the banking system on the basis of the reserves supplied by the money authority, account being taken of reserve requirements and currency holding habits [5] [19]. 26/ There should be little question but that the authority is, by and large, in a position to control M* autonomously. It cannot however directly set M and there is a good deal of evidence that the movements of M, given M*, have tended to be partly caused by movements of income through the mechanism of movements in interest rates (modified by the rediscount rate) inducing changes in bank borrowings and excess reserves [5] [11] [19]. To be sure even replacing M with M* may not completely dispose of the bias problem. The money authority could--and to some extent probably did--follow policies under which M* would be partly induced by changes in aggregate demand, e.g., a policy of stabilizing interest rates or maintaining free reserves at some stated level. Nonetheless it seems highly likely that M* is more nearly autonomous than M and that the substitution is therefore a step in the right direction.

The results of this substitution of M^* for M^f are shown in Rows (4.3) and (4.4) of Table 4. When $N_{p,t-1}$ is not present the error variance is quite large, many times larger than it is when the independent variable is M^f, tending to confirm the suspicion that some of the high correlation between N and M^f is due to the induced nature of M^f. When $N_{p,t-1}$ is present, however, the difference between the two partial correlations and the residual variances is not large enough to warrant any definite conclusion.

It is of some interest to note that whether one uses M^* or the more dubious variable M, the variance of the error of prediction of NNP, though modest, is still three to four times larger than that resulting from using Z^a and X^a, and six times larger than that resulting from using Z^a, X^a, and C_{t-1} (cf. row (3.4b)).[27/]

III. Some Implications and Conclusions

We have thus shown that the "strikingly one-sided" results of F and M are largely accounted for by their strikingly one-sided procedure. Once we rely on a less partisan approach, the income-expenditure model can readily meet the challenge on F and M's own chosen ground, namely, the size of the correlation coefficient. Indeed, from our tests, this model comes out somewhat ahead of the "rival", though we do not doubt that with some ingenuity F and M would be able to better the score for their favorite champion.

We should like to make it clear however that we regard the game of who can produce the highest correlation a very

sterile one--except possible for its entertainment value. Even in terms of the issue posed by F and M--which of the two "rival" models is more successful in accounting for the movement of NNP, given the relevant autonomous variables--the relevant measure of "success" is the variance of the residual error and not the correlation coefficient, which depends on the ratio of this variance to another variance, and which can be radically changed by a mere transformation of variables in many cases.[28/] We have shown that in terms of the residual variance the income-expenditure model appears distinctly more successful than its rival. But regardless of which model is more successful even in terms of the residual variance, of what possible value would the contest be? Conceivably it might be relevant if the issue before us were which of the two <u>mutually exclusive</u>, <u>small</u> sets of variables does a better job of prediction. But this is most definitely not <u>the</u> issue, as must be obvious from the fact that both M and Z^a and X^a are contemporaneous with the variable to be predicted, and not known in advance. None of the experiments performed in F and M's or in our paper throws any light on how well, or even how, the relevant "independent" variables could be predicted in advance.

But can it not be maintained that these tests do shed a good deal of light on the issue as to how income or consumption can be most effectively controlled? Even on this score the answer must be largely negative. In the first place, it is readily apparent that none of the hypotheses tested should

seriously be regarded as a behavioral or structural relation. If the "independent variables" used in each equation were truly exogenous, then these equations could be regarded as grossly misspecified "reduced forms". But even this interpretation is open to question since our independent variables are at best "autonomous" in the sense of not being correlated with the error term of the consumption function[29/], and this is quite different from being exogenous to the economic system. Hence, the covariation observed over a period in which our "independent variables" were not used for control purposes can throw but little direct light in what might happen if we endeavored to manipulate them to stabilize the economy. Clearly such an endeavor would change significatnly the underlying structure. This fundamental problem exists in addition to the already formidable issue of how the "independent" variables used in our tests could be effectively and reliably controlled.

Second, and more important, there is absolutely no justification for treating autonomous expenditures and money supply as mutually exclusive stabilization devices. Indeed there is no justification for F and M's juxtaposition of the income-expenditure framework and the quantity theory model as mutually exclusive hypothesis. It is well known that, if broadly understood as a theory of the demand for money, the quantity theory is, far from being inconsistent, actually an important part of the mechanism of income determination in the income-expenditure framework as generally understood.[30/]

As is shown in the Appendix, there may be an inconsistency in the implications of the two models only if the quantity theory is stated in an extreme form, to wit, that the relation between money supply and income is so stable that the multiplier mechanism has nothing independent to contribute to the determination of income. But the test designed and carried out in the Appendix lead us to the conclusion that this "extreme quantity model does not have one shred of support".

We conclude therefore that if we are concerned with advancing knowledge of how to reduce economic instability, whether by rules, built-in-stabilizers, and/or by ad hoc measures, there is little point in pursuing the game of testing one-equation-one-independent-variable models in seach for the highest correlation, fascinating as the game might be. We need instead to buckle down to the unended and unending labor of learning more about the structure of our economy. This applies in particular to the task of charting the complex and still ill understood channels through which money and the tools of monetary policy affect economic activity. We trust that Mssrs. Friedman and Meiselman will share this point of view and will continue to apply their talents and intimate knowledge of matters monetary in the pursuit of this challenging task.

APPENDIX:

A Crucial Experiment on the "Extreme Quantity Theory"

In the main body of our paper we repeatedly criticized F and M for juxtaposing the income-expenditure and the quantity of money models as mutually inconsistent theories. We went on to point out that both in Keynes' original formulation and in its many widely accepted elaborations, the income-expenditure framework assigns an important role to money in the determination of real and money income. Some of the best known contributions to the literature of the last twenty five years beginning with Hicks' classical essay [13] have been concerned with clarifying the relation between the multiplier mechanisms and the quantity of money mechanisms in the process of income determination, and the outcome of these endeavours has found its way into the standard economic textbooks[31/].

There is, however, a definite contrast between the income-expenditure framework and the extreme quantity theory position espoused by Friedman in at least some of his writings and pronouncements. According to this extreme view, the relation between income and the money supply is so stable that the other side of the blade, the condition of equilibrium in the commodity market--embodied, e.g., in Hicks' I-S curve--has nothing to contribute to the process of income determination regardless of whether or not there exists a stable consumption function and a stable multiplier effect of autonomous expenditure. We propose to show that these assertions have observable implications which are so different from those of the income-expenditure theory that it is possible to perform a sharp test,

at least of the most extreme version of the Friedman's theory.

To this end we first note that extreme quantity theory asserts the existence of a stable relation between NNP and the exogenously given money supply which implies

(A.1) $N = m_o + mM^* + u; \quad m > 0, \quad E(M^*u) = 0$, and $Var(u)$ small.

Furthermore the proposition that M^* affects NNP independently of the multiplier mechanism means that u is uncorrelated with autonomous expenditures, or

(A.2) $E(uZ^a) = E(uX^a) = 0$

It must be stressed that it is precisely the specification (A.2) which gives empirical content to the extreme quantity theory in contrast to the income-expenditure framework. For this framework certainly does not deny that m is positive; though it might imply that u would be non-negligible, this is not an easily quantifiable specification. But the income-expenditure model definitely implies a strong positive correlation between u and autonomous expenditure, a point to which we shall presently return.

Let us now substitute in (A.1) the identity $N \equiv C^f + Z^a$, and solve for C^f. We obtain

(A.3) $C^f = m_o + mM^* - Z^a + u.$

It now follows from the specification (A.2) that the application

of the direct least squares method to the following regression equation

$$(A.4) \quad C^f = a_o + a_1Z^a + a_2Z^a + a_3M^*$$

will yield unbiased estimates of the coefficients of (A.3), or

$$E(a_1) = -1; \quad E(a_2) = 0; \quad E(a_3) = m.$$

From equation (4.3) of Table 4 we can also infer that m should be in the order of 2.5. If we were to use M^f instead of M^*, then, according to equation (4.1), the value of m should be around 2. It should be noted however that this substitution weakens the power of the test, biasing it in favor of the quantity theory, since M is likely to be partly induced and hence correlated with u, and this would tend to bias the coefficient m upward and the coefficient of Z^a downward.

Consider now the implications of the income-expenditure framework. We have already shown that from this model, at the cost of a number of misspecifications, we can derive the equation

$$(A.5) \quad C^f = n_o + n_1Z^a + n_2X^a + v$$

where v is a linear combination of ϵ, η_z, and η_x. In specifying the properties of the error term v, it is necessary to distinguish two forms of the theory. The more extreme, though perhaps popular, version states that the money supply does not directly affect the C^f component of NNP and hence can

affect income only through its effects on various components of autonomous expenditure, via interest rates and availability effects. This formulation, which we shall call the "special" income-expenditure theory, implies therefore the specification

(A.6) $E(vM^*) = 0.$

If M^* is replaced by M, then the correlation with v would tend to be positive though presumably still reasonably small.

One can however hold to a more general formulation of the theory under which M, besides working through autonomous expenditures, could have a direct effect on consumption given income. This means that M^* could be correlated with the error term ϵ and hence with v, implying finally $E(vM^*) \gtrsim 0$.

Our own view is that the money supply is unlikely to affect consumption directly. But C^f is consumer expenditure (which includes consumer durables) plus changes in inventory minus imports, and we would not be surprised to find some direct effects of M^* on C^f, and hence a slight positive correlation between v and M^*.

It follows from the above consideration that, according to the income-expenditure theory, provided Z^a and X^a are truly autonomous and hence uncorrelated with v, the regression coefficients a_1 and a_2 of the test equation (A.4) will be unbiased estimates of the coefficients n_1 and n_2 of (A.5), i.e.,

$$E(a_1) = n_1; \quad E(a_2) = n_2.$$

From equation (3.2b) of Table 3.b we can infer that n_1, and hence a_1, should be in the order of 1.2 and a_2 in the order of 3.8. As for the coefficient a_3 we must conclude that:

under the special theory: $E(a_3) = 0$

under the general theory: $0 < E(a_3) << m (\simeq 2.5)$

Some words of caution are needed before we proceed to present the results of the test. Although in the main body of the paper we have treated Z^a and X^a as truly autonomous, this assumption involves some misspecification in the light of a fuller statement of the income-expenditure theory. Even in the crudest version of this model, to equation (A.5) (or to the five equations, (4), (8), (9a), (9b) and the definitional equation $C^f \equiv C + Z^i$, leading up to (A.5)), one should add at least three more equations, namely: (i) an equation relating Z^a to the interest rate r (and possibly to the money supply); (ii) a demand for money equation relating M to NNP (i.e., $C^f + Z^a$); and (iii) a money supply equation relating M to M* and r. Since these three equations involve two new endogenous variables, M and r, and we are now treating Z^a as endogenous, we have altogether a system of eight equations in eight endogenous variables (Y^d, C, C^f, X^i, Z^i, Z^a, r, M).

With the standard assumption that an increase in r reduces both investment and the demand for money, the above system implies a tendency for negative correlation between Z^a and the error term v of equation (A.5). Under these conditions a_1 will tend to underestimate n_1. This, together with positive correlation between Z^a and M^*, leads to an upward bias in the

least squares estimate of the true coefficient of M^* [4, Appendix I]. Hence, a moderately positive value of a_3 of the regression equation (A.4) is consistent even with the special theory. Finally, one might note that under the extreme quantity theory, if there is error of measurement in Z^a, its coefficient might be attenuated and hence be algebraically larger than minus one. However the attenuation can never reverse the sign, and therefore a_3 should be significantly negative in any event, even if not quite exactly minus unity.

In summary, under the extreme quantity theory, a_1 must be negative and close to minus one, a_2 approximately zero and a_3 around 2.5. Under the income expenditure theory, a_1 should be around 1.2 instead of -1 and a_2 around 3.8 instead of zero; as for a_3 it should be very close to zero if the special theory is valid, and moderately positive but well below 2.5 under the general theory. These implications are summarized in the first four rows of Table 5.

Rows 5 and 6 give the acutal leasts square estimates. It is apparent that when the money supply is measured by M* the extreme quantity model does not have one shred of support, and the results are instead strikingly consistent with the income expenditure model, and more particularly with the special version of this model. In other words, the evidence supports the hypothesis that the money supply does affect income only by way of its effects on autonomous expenditure. Even when M^* is replaced by M, the extreme quantity model does not fare any better. While the results are now less strikingly in favor of the special version of the income-expenditure model,

it must be remembered that the use of M tends to bias the results against this model.

In the last two rows of the table we present the results of a final test which aims at giving the extreme quantity theory model one more break. As pointed out in footnote 24 the results of equation (4.2) and (4.4) suggest that the adjustment of NNP to the money supply may be gradual, so that (A.1) should include the additional variable $N_{p,t-1}$. We have therefore added this variable to the test equation (A.4). With this addition, under the extreme quantity theory the coefficients a_1 and a_2 should still be respectively -1 and zero, while a_3 should be around .5 if M* is used and around .9 if the money supply is measured by M^f. On the other hand under the income-expenditure model the coefficients a_1 and a_2 should be of the order of magnitude of the coefficient of Z^a and X^a in equation (3.4b), or .8 and 2.8 respectively. The latter inference is only approximate since in (3.4b) the lagged variable is C_{t-1} rather than $N_{p,t-1}$, and for this reason the test is in fact appreciably biased against the income-expenditure model. But it is apparent that even in terms of this test the extreme quantity theory is completely unsupported while the income expenditure-theory comes out unscathed in its general version and, by and large, even in its special version.

Table 1

Definition Symbols and Accounting Relations

a. Definitions of Symbols (All variables measured in billions of current dollars unless otherwise specified)

A : Autonomous expenditure. Also autonomous expenditure as defined by F and M (see equation b.1 below)

C : Consumption expenditure.

$C^f \equiv C + Z^i$: Induced expenditure.

E : Exports

G : Total government purchases of goods and services.

H : Statistical discrepancy.

I : Imports.

$K \equiv K_1 + K_2$: Net private domestic investment

K_1: Net investment in plant and equipment, and in residential houses.

K_2: Net changes in inventory.

M^*: Maximum currency plus demand deposit that can be created given the supply of reserves by the Federal Reserve System excluding borrowed reserves.

M^D: Demand for money.

M^f: Currency outside banks plus demand deposit adjusted plus savings deposit in commercial banks.

M^S: Supply of money.

N : Net national product.

N_p^*: Permanent net national product.

P : Population

π_p: Permanent price level.

Q : Subsidies less current surplus of government enterprises.

R : Corporate retained earnings after taxes plus inventory valuation adjustment.

S : Personal saving $\equiv Y^d - C$.

$T \equiv T_b + T_c + T_p + T_s - T_g - T_i - Q$

T_b: Indirect business tax $\equiv T_b^1 + T_b^2$.

$T_b^1 \equiv T_b - T_b^2$.

T_b^2: Property tax portion of indirect business taxes.

T_c: Corporate profit tax accruals.

T_f: Foreign transfer payment by government.

T_g: Government transfer payment $\equiv T_g^1 + T_g^2$.

T_g^1: Unemployment insurance benefits

$T_g^2 \equiv T_g - T_g^1$.

T_i: Net interest paid by government

T_p: Personal tax and nontax payment.

T_s: Contribution to social insurance.

W : Excess of wage accruals over disbursement: may be ignored because it is negligibly small.

$X \equiv Y^d - N \equiv X^a + X^i$

X^a: autonomous portion of X.

X^i: induced portion of X.

Y : Some measured income.

Y^d: Disposable personal income.

$Z \equiv N - C \equiv Z^a + Z^i$

Z^a: autonomous portion of Z

Z^i: induced portion of Z

b. Definitional and Accounting Identities

(b.1) $S \equiv K + (G - T) + (E - I) - R - (H + W + T_f)$

$\equiv A - R - (H + W + T_f)$

(b.2) $S \equiv (N - C) + (Y^d - N) \equiv Z + X \equiv Z^a + Z^i + X^a + X^i$

(b.3) $Z \equiv K_1 + K_2 + G + E - I$

(b.4) $X \equiv R - T_b - T_c - T_p - T_s + T_i + T_g + Q - H - W$

For the purpose of the empirical tests reported in this paper, the following definitions were used:

(b.5) $X^a \equiv - T_b^2 + T_i + T_g^2 + Q - H - W$

(b.6) $X^i \equiv - R - T_b^1 - T_c - T_p - T_s + T_g^1$

(b.7) $Z^a \equiv K_1 + G + E$

(b.8) $Z^i \equiv K_2 - I$

(b.9) $C^f \equiv C + Z^i$

Table 2

Effects of Including the Years 1942-46
On Regression and Correlation Coefficients

Using C as the Dependent Variable [a]

Row No.	Period	Coefficient of Constant	A	M^f	R^2	S_e^2
(2.1)	1929-58 inclusive	71 (17)	3.94 (.79)		.49 [b]	3284
(2.2)	1929-58 inclusive	-1.2 (7.0)		1.35 (.06)	.94 [b]	386
(2.3)	1929-58 excluding 1942,43,44,45,and 46	50.6 (7.2)	6.70 (.39)		.92	601
(2.4)	1929-58 excluding 1942,43,44,45,and 46	2.0 (5.0)		1.37 (.04)	.98	174

[a] Variance of C, 1929-58 inclusive = 6440
Variance of C, 1929-58 excluding 42-46 = 7515.

[b] See [p,190, Table II.1]. Slight differences are due to revisions of national income statistics by the Department of Commerce.

Table 3

Effects of the Failure to Distinguish Exogenous and Endogenous Components of A. Regression, Multiple and Partial Correlation Coefficients, 1929-58. Excluding 1942-46 [a/]

a. Using C as the Dependent Variable [b/]

Row No.	Coefficient of					R^2	S_e^2
	constant	Z^a+X^a	Z^a	X^a	C_{t-1}		
(3.1a)	42.1 (2.5)	1.60 (.03)				.992	69
(3.2a)	49.3 (2.6)		1.34 (.07) [.947]	3.87 (.56) [.686]		.995	41
(3.3a)	14.8 (4.3)	.64 (.14) [.486]			.64 (.09) [.677]	.997	23
(3.4a)	24.1 (4.6)		.68 (.12) [.615]	2.10 (.46) [.498]	.52 (.09) [.626]	.998	16

b. Using C^f as the Dependent Variable [c/]

Row No.	Coefficient of					R^2	S_y^2
	Constant	Z^a+X^a	Z^a	X^a	C_{t-1}		
(3.1b)	40.4 (2.5)	1.49 (.03)				.991	66
(3.2b)	47.8 (2.5)		1.23 (.06) [.944]	3.83 (.54) [.706]		.995	37
(3.3b)	20.3 (5.8)	.79 (.19) [.436]			.47 (.12) [.385]	.994	43
(3.4b)	33.1 (6.2)		.84 (.16) [.570]	2.79 (.62) [.489]	.30 (.12) [.239]	.996	29

a/ Figures in parenthesis are standard errors of respective coefficients, and figures in brackets are partial correlation coefficient squared.

b/ Variance of C = 7515

Table 4

Spuriousness in Correlations between Money Stock and Consumption, Regression Coefficients, Multiple and Partial Correlation Coefficients,[a] 1929-58. Excluding Years 1942-46.

a. Using N_t as the Dependent Variable [b]

Row No.	Coefficient of Constant	M^f	M^*	$N_{p,t-1}$	R^2	S_e^2
(4.1)	-15.8 (8.0)	2.05 (.07)			.974	438
(4.2)	-15.4 (4.8)	.91 (.18) [.527]		.69 (.11) [.651]	.991	160
(4.3)	-28.9 (24.3)		2.45 (.24)		.808	3226
(4.4)	-23.0 (5.6)		.51 (.11) [.487]	1.01 (.05) [.950]	.990	173

b. Using C_t as the Dependent Variable [c]

Row No.	Coefficient of Constant	M^f	M^*	C_{t-1}	R^2	S_e^2
(4.5)	- 2.4 (2.0)	.37 (.09) [.445]		.79 (.07) [.841]	.997	25
(4.6)	- 5.5 (16.8)		1.63 (.16)		.796	1536
(4.7)	- 6.5 (2.1)		.19 (.04) [.472]	.97 (.03) [.987]	.997	23

[a] Figures in parenthesis are standard errors of respective coefficients, and figures in brackets are partial correlation coefficients squared.

[b] Variance of N_t = 16765.

[c] Variance of C_t = 7515.

Table 5

Test of the "Extreme Quantity Theory"

According to		Coefficient of Z^a (a_1)	X^a (a_2)	M^* (a_3^*)	M^f (a_3)	$N_{p,t-1}$
Extreme Quantity Model (Using M^*)		$0 >> a_1 \geq -1$	0	2.5[a]	2.5[a]	-
Income-Expenditure Model Special Version		1.2	3.8	0[a] Except for Bias	small[a]	-
Income-Expenditure Model General Version		1.3	3.8	$0 \leq a_3^* << 2.5$[a]	$0 \leq a_3 < 2.5$[a]	-
Least Squares Estimates	1	1.24 (.07)	3.90 (.57)	-.03 (.06)	-	-
	2	1.07 (.10)	3.18 (.58)	-	.25 (.12)	-
	3	.96 (.21)	3.15 (.76)	.04 (.08)	-	.14 (.10)
	4	.72 (.17)	2.30 (.63)	-	.32 (.11)	.17 (.07)

[a] Theoretical coefficients for M^* and M^f are mutually exclusive.

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FOOTNOTES

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**As this paper was about to be completed, we became aware of the paper by Professor Donald Hester of Yale University which deals with similar problems with somewhat different emphasis. We have had the benefit of discussing with him our mutual problems. See Donald Hester [12].

1/ As far as we can see, this last item--net government foreign transfer payment--was overlooked by F and M.

Footnotes continued

2/ While in the traditional formulation, consumption is supposed to depend on disposable income, it has been argued more recently that the argument of the consumption function should be (long term, expected, or "permanent") total accrued income, including corporate earnings, whether distributed or not. But the extreme short run variability of undistributed corporate income makes it unlikely that consumption would significantly respond to it from year to year. cf. [3, Section III and IV].

3/ Other reasons for omitting war years are given in footnotes 8 and 13 below.

4/ Although F and M make extensive use of the terms "autonomous" and "induced" in their paper, they have failed to communicate clearly, at least to us, just what is the relevant criterion for distinction. As will soon become apparent, for the purpose of their test the relevant criterion of classification is whether or not a variable is supposed to be correlated with the error term ϵ of the consumption function. Accordingly, in what follows, we shall call autonomous those variables that are expected to be uncorrelated with ϵ and any other variable "induced". Autonomous variables in this sense are not necessarily "exogenous" in the usual sense of being determined entirely outside the economic system and therefore uncorrelated with the error term of any structural equation. Thus exogenous variables are autonomous, but not all autonomous variables are necessarily exogenous.

Footnotes continued

5/ There is a strong temptation to argue that the downward bias in the regression of C on S arises directly from the positive association between consumption and the sum of imports, net taxes and corporate saving which enter into S with a negative sign. However, this correlation can create a bias only through the mechanism of the resulting negative correlation between S and ϵ pointed out in the text. This can be readily seen by considering the limiting case in which the residual error in the consumption function is zero. In this case the correlation between C and S must be unity, no matter what might be the correlation between C and some components of S.

6/ From (7a) and (7b) one finds

$$\frac{\partial S}{\partial \epsilon} = \frac{\partial Y^d}{\partial \epsilon} - \frac{\partial C}{\partial \epsilon} = \frac{1}{1-c_1+t_1} - \frac{(1+t_1)}{1-c_1+t_1} = \frac{-t_1}{1-c_1+t_1} < 0$$

7/ The condition for the correlation between C and S to be negative is

$$\frac{c_1(1 - t_1)}{(1+t_1)t_1 \dfrac{\mathrm{var}(\eta)}{\mathrm{var}(\epsilon)} + c_1(1-c_1)} < \frac{\mathrm{var}(\epsilon)}{\mathrm{var}(I+G)}$$

If we remember that t_1 is the marginal variation of total taxes, with respect to the variation in disposable personal income, and therefore a fairly large number, say .3 or .4, the possibility that the above condition is satisfied cannot be ruled out.

Footnotes continued

8/ It should be noted that the correlation between S (or A) and ϵ will be particularly strong and negative when the economy is operating near or at the full employment. This is because, given income, an accidental increase in ϵ must be offset by the corresponding decrease in S, and the condition of given income is more likely to be binding in the full employment situation, even in terms of current prices.

9/ If more detailed components of Z^i and X^i are expressed separately as linear functions of more dtailed components of Z, X, C, and Y^d, similar considerations will also apply.

10/ With respect to those components of X which are related to government expenditure and receipts our choice was largely dictated and supported by Ando, A., Brown, E. C., and Adams, E., [2].

11/ While we confine ourselves to reporting results using the specific definition of autonomous expenditure given by equations (b.4) and (b.6) for the entire period 1929-58, some sporadic experiments we have made suggest that alternative "reasonable" choices of the autonomous components or of sub-periods would not change the results appreciably. In particular, if we substitute GNP for NNP in our formulation, but consider capital consumption allowances to be exogenous, then

Footnotes continued

capital consumption allowances will be included in Z^a with a positive sign, and in X^a with the negative sign, leaving $Z^a + X^a$ unchanged. Even in equation (10), this change in the definition does not alter our empirical results in any significant way. For further evidence on these points see also Hester [12].

12/ Even though each of these models may be criticized on a number of grounds, they do endeavour to deal with the problem of correctly specifying the implications of the income-expenditure approach, as presently understood.

13/ This is incidentally another reason why war years had better be omitted or analyzed separately.

14/ See footnote 15 below.

15/ The regression coefficients of Z^a and X^a in (3.4a) or of $(Z^a + X^a)$ in (3.3a) provide estimates only of the immediate or impact "multiplier" effect of a change in these variables on consumption. The long-run effect of autonomous expenditures on consumption (and hence finally on income) may be considered as being appreciably larger, at least if one accepts the interpretation of the consumption function (4') as reflecting a gradual adjustment of consumption to changes in income (whatever might be the "true" mechanism behind this gradual adjustment). Under this "distributed lag" model, the long-run multiplier on consumption of a permanent change in $(Z^a + X^a)$

Footnotes continued

implied by (3.3a) is given by the ratio of the coefficient of $(Z^a + X^a)$ to one minus the coefficient of C_{t-1}. In the present instance this long-run multiplier is thus given by $1.60/(1 - .36) = 1.78$, a figure very similar to the direct estimate obtained from (3.1a). Similarly from (3.4a) we find the long-run effect of Z^a and X^a to be respectively $.68/(1 - .52) = 1.42$ and $2.10/(1 - .52) = 4.37$. These figures are only moderately larger than the direct estimates obtained in (3.2a).

16/ This interpretation of C^f is only approximate since I does include imports going into fixed investment, government expenditure and reexports. Ideally one would want to subtract this part of imports from I and add it back to Z^a though in practice this suggestion would be extremely hard to carry out.

17/ It should be noted that, since $C^f + Z^a = NNP$, from Table 3.b, we can derive the estimate of the net national product multipliers for autonomous expenditures. In the case of Z_a the multiplier is one plus its coefficient in row (3.2b), or approximately 2.2. For X^a, it is directly its coefficient in row (3.2b), or approximately 3.8. In row (3.4b), because the remaining variable is C_{t-1} rather than C^f_{t-1}, we can obtain only rough approximations for the long run multipliers, 2.2 for Z^a, 4.0 for X^a.

18/ The further problem created by the substitution of C for N may be postponed momentarily.

Footnotes continued

19/The following table gives the average of ratios of the more commonly used definitions of money to GNP for three periods. Data are taken from the U.S. Bureau of the Census, Historical Statistics of the United States, 1960, and various issues of the Federal Reserve Bulletin.

	Currency Plus Demand Deposit Adjusted	Currency plus Demand Deposit Adjusted plus Savings Deposit in Commercial Banks	Currency plus Demand Deposit Adjusted Plus Savings Deposit in Commercial Banks Plus Savings Deposits in Mutual Saving Banks
1895-1905	.315	.366	.466
1950-1955	.348	.469	.535
1956-1963	.280	.428	.499

20/Note that, if δ is not unity, (13) should contain a term involving the price level, as would be true for (4) if the consumption function is properly stated in real terms. We ignore this consideration as is done by F and M in most of their tests.

21/Strictly speaking, this calculation should be performed separately for NNP in real terms and for prices, and N_{pt} should be obtained as the product. However, since Friedman applies the same weights to both series, equation (14) should provide a close approximation.

22/See footnote 24 below.

Footnotes continued

23/ The series for money used in the calculation is that given in [9 , p. 262, Appendix II-B2], except for M*, defined below.

24/ The regression coefficients of equation (4.2) might also be interpreted as reflecting a rather complex process of gradual adjustment of N_t to the money supply. To see this, suppose that $N_{p,t}$ adjusts but gradually to the money supply so that we may write

$$\text{(i)} \quad N_{p,t} - N_{p,t-1} = g(hM_t - N_{p,t-1})$$

where g is the speed of adjustment and h ($\simeq \frac{1}{\gamma_1}$) is the equilibrium value of velocity of money in terms of permanent income. Using definition (14), we can rewrite (i) as

$$\beta(1-\rho)N_t + \beta(1-\rho)\rho\, N_{p,t-1} = g(hM_t - N_{p,t-1}) + N_{p,t-1}$$

$$\text{(ii)} \quad N_t = \frac{gh}{\beta(1-\rho)} M_t + \left[\frac{1-g}{\beta(1-\rho)} - \rho\right] N_{p,t-1}$$

The difference between (ii) and (17) is that, according to (ii), for reasonable values of g and ρ, the coefficient of $N_{p,t-1}$ is likely to be positive while (17) predicts it to be negative. Also, the value of the coefficient of M_t implied by (ii) is g times that predicted by (17). In other words, the impact effect implied by (ii) is g times that predicted by (17). It should be recognized however that while this is a possible and reasonable interpretation of equation (4.3) it implies a theory of the relation between money and income quite different from the simple-minded demand equation of Friedman embodied in (11).

Footnotes continued

25/ This possibility and its implications are set forth in general terms by Donald Hester [12].

26/ To the extent that M* is scaled to the conventional definition of M rather than M^f, the substitution of M* for M^f introduces a misspecification to Friedman's strictest formulation.

27/ In most of F and M tests in which the independent variable is M, the dependent variable is C, not N. F and M justifies this choice on the ground that, to avoid upward bias in the correlation coefficient, the appropriate dependent variable in the "income-expenditure" model is C, and that they wish to compare the two models "running over the same mile". As we have seen, the only meaningful measure for performance of these models is the residual variance of NNP given the autonomous variables. This variance can be obtained by using indifferently N or C_f as the dependent variable in the income-expenditure model (since the resulting residual variance will be identical) and by using N as the dependent variable in the money model. Hence the relevant test consists in comparing residual variance in Table 4.a and Table 3.b. Nonetheless, for the sake of completeness, we give in Table 4.b the results of money regression using C as the dependent variable, although these regressions do not yield the residual variance of NNP given M^f or M^* and therefore hard to interpret.

It could at best be interpreted as a measure of the accuracy of F and M's prediction for C plus autonomous expenditure

Footnotes continued

from M^* or M^f on the assumption that autonomous expenditure is already known without error. But note that, under this interpretation, F and M must admit that C plus autonomous expenditures, which is a broad measure of the level of employment, depends not only on M but also on autonomous expenditures. This would imply the multiplier of at least one for autonomous expenditures--the conclusion which F and M would presumably find distasteful. See also appendix at the end of this paper.

28/ For instance in our Tables 3 and 4, for each of the equations containing the dependent variable lagged, we can drastically reduce the correlation coefficient by changing the dependent variable to a first difference, without thereby changing the error variance or the estimates of the regression coefficients. In addition, the recent systematic experiments of Ames and Reiter confirm what econometricians have known for some time, namely that obtaining high correlation is not much of a trick when dealing with American time series data [1].

29/ More generally with the error terms of the structural equations accounting for the component of C^f.

30/ See, for instance, Modigliani [16], particularly Section 6.

31/ See e.g., the chapter dealing with the "Synthesis of Monetary and Income Analysis" in Samuelson's Economics [17], beginning with the fourth edition, and the less formally articulated treatment in earlier editions.

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