







STRUCTURE OF REPEAT BUYING FOR NEW PACKAGED GOODS\*

Manohar U. Kalwani and Alvin J. Silk\*\*

WP 1032-78 REV.

Revised July 1979

### THE MARKETING CENTER

Massachusetts Institute of Technology Alfred P. Sloan School of Management 50 Memorial Drive Cambridge, Massachusetts 02139 (617) 253-6615



### STRUCTURE OF REPEAT BUYING FOR NEW PACKAGED GOODS\*

Manohar U. Kalwani and Alvin J. Silk\*\*

WP 1032-78 LEV.

Revised July 1979

\*The authors are indebted to Jiri Nechleba and John McFadden for valuable assistance in data processing and analysis. Special thanks are due Gerald Eskin, Donald Morrison, and a JMR referee for helpful comments on an earlier draft of this paper. The assistance of the <u>Chicago Tribune</u> in supplying the consumer panel data analyzed here is also gratefully acknowledged.

\*\*Sloan School of Management, Massachusetts Institute of Technology, Cambridge, MA 02139

#### ABSTRACT

Results from empirical tests of three hypotheses concerning the structure of repeat buying for new packaged goods are reported in this paper. Maximum likelihood parameter estimates of the underlying depth of repeat buying model are presented for four new brands and found to support the hypotheses which offer a foundation for pre and early test market forecasting.

#### INTRODUCTION

Sustaining a satisfactory level of repeat buying is widely recognized as the key to the successful launching of a new packaged good. The volume of ongoing sales ultimately realized for a new brand is determined by the number of "adopters" or repeat buyers and their purchase frequency. However, repeat purchasing develops gradually with attrition continuing to occur among customers even after they have bought a new brand several times. Tauber (1977, p. 80) suggests that "this 'wear-out phenomenon' is probably due to factors such as boredom after continual use, eventual resistance to price, or the consumers' need to use a product a number of times to be able to tell if it really fits their needs."

Knowledge of such adoption phenomena constitutes the foundation for analytical efforts that are directed toward the assessment of new brands prior to test marketing (Silk and Urban 1978, Tauber 1977), and the subsequent forecasting of sales from early test market results. Building upon the ideas put forth in Eskin's depth of repeat model (Eskin 1973), PANPRO, Eskin and Malec (1976) have recently reported progress in understanding the process of how repeat buying develops.

They propose a model of the evolution of the proportion of repeat buyers wherein the attrition of previous buyers occurs systematically from the first repurchase onward and is described by a simple decay function. Eskin and Malec (1976) further postulate that the frequency of purchasing remains unchanged after the first repeat. The key implication of these hypotheses is that the on-going sales of a new packaged good can be obtained from a knowledge of the cumulative proportion of first repeaters and their purchase

-1-

.

frequency (or equivalently, their average interpurchase time).

This paper reports some further analysis and empirical findings concerning the structure of repeat buying for new packaged goods. The theoretical rationale for a set of hypotheses suggested by the work of Eskin (1973) and Eskin and Malec (1976) is examined. The depth of repeat model is used to analyze the structure of repeat buying--i.e., the conversion of triers into the first repeat class, the conversion of first repeaters into the second repeat class, and so on for higher repeat levels. The objective is to compare the time paths of the cumulative proportions across various repeat levels. Do these penetration curves have similar functional forms? Are the interpurchase times approximately the same across repeat levels? Does the cumulative proportion of consumers who convert from one repeat class to the next increase with depth of repeat? Model parameters estimated by a maximum likelihood method are used to test hypotheses related to these guestions.

The remainder of this paper is organized as follows. The first section describes the depth of repeat model when time is treated in discrete intervals. This section concludes with a brief discussion which establishes that the continuous analog of the discrete model is a familiar stochastic model based on a gamma mixture of exponential interpurchase times. A set of hypotheses suggested by Eskin (1973) and Eskin and Malec (1976) are presented next, and the likelihood expressions used to estimate model parameters are developed. Results from the empirical tests for four new products--two toothpastes and two coffees--are examined, and the implications of these results for consumer research on new packaged goods are discussed.

-2-

#### DEPTH OF REPEAT MODEL

The present analysis of repeat buying follows the work of Fourt and Woodlock (1960), Massy (1969), and Eskin (1973). Depth of repeat classes are defined as the penetration or cumulative proportion of consumers who repeat a  $J^{th}$  time (J=1,2,3...) given that they had previously made J-1 repeat purchases. Note that the repeat class (or level) J=1 refers to first repeaters, the repeat class J=2 refers to second repeaters, and so on for higher repeat levels.

Two postulates underlie the penetration model proposed by Fourt and Woodlock (1960). First, there is a ceiling on the proportion of consumers who convert from one repeat class to the next. Second, the number of consumers who enter the next repeat level in each time period is a constant fraction of the remaining consumers who will eventually convert into the next repeat class. Fourt and Woodlock (1960, p. 32) cite empirical evidence to support both of these postulates:

> Observation of numerous annual cumulative penetration curves shows that (1) successive increments to these curves decline, and that (2) the cumulative curves seem to approach a limiting penetration of less than 100 per cent of households -- frequently far less.

Consider repeat level J where  $J=1,2,3,\ldots$ . Given the above two postulates it follows that the proportion of consumers, P(L), who convert into the modeled repeat level J during the L<sup>th</sup> time period is given by:<sup>1</sup>

$$P(L) = F(L) - F(L-1) = (1-\rho) (\alpha - F(L-1)), \quad L=1,2,3,\ldots, \quad (1)$$



where F(L) is the cumulative proportion of consumers who convert into a particular repeat level, J, by the L<sup>th</sup> time period since their previous purchase; $\rho$  is a constant; and  $\alpha$  denotes the cumulative proportion who will eventually convert into the repeat class J. Thus,  $\alpha$  represents the ceiling on penetration. Solving the above equation recursively (with F(0) = (0)), one obtains:

$$P(L) = (1-\rho) (\alpha - F(L-1)) = \alpha \rho^{L-1}(1-\rho), \qquad L=1,2,3,...$$
(2)

$$F(L) = \sum_{L=1}^{L} \alpha \rho^{L-1} (1-\rho) = \alpha (1-\rho^{L}), \qquad L=1,2,3,... \quad (3)$$

Figure 1 provides a graphic display of the penetration function, F(L), over time. An examination of the above expressions for P(L) and F(L) reveals that the purchasing behavior of consumers who enter repeat class J is given by a geometric distribution. The average interpurchase time,  $\tau$ ,

INSERT FIGURE 1 HERE



of these consumers who will eventually convert into some repeat level J is given by :<sup>2</sup>

$$E[L] = \tau = \frac{L=1}{\sum_{k=1}^{\infty} \alpha \rho^{k-1}(1-\rho)} = (\frac{1}{1-\rho}), \qquad (4)$$

which can be rearranged as:<sup>3</sup>

$$p = \left(\frac{\tau - 1}{\tau}\right). \tag{5}$$

Note that the expression for interpurchase time given in equation (4) assumes that data on conversions into the  $J^{th}$  repeat level are available over an infinite period of time. In practice, however, the period over which conversion rates are observed is bound to be limited. Assuming that the conversion into the  $J^{th}$  repeat class is observed for T periods, the expression for average interpurchase time,  $\tau_{T}$ , becomes:

$$\tau_{T} = \frac{\sum_{L=1}^{T} (1-\rho)}{\sum_{L=1}^{T} \alpha \rho^{L-1} (1-\rho)} = (\frac{1}{1-\rho}) - (\frac{T\rho^{T}}{1-\rho^{T}}) .$$
(6)

Hence, the average interpurchase time,  $\tau_T$ , for truncated data is smaller than the theoretical average interpurchase time,  $\tau$ .

Thus far it has been assumed that the consumers entering a particular repeat level are homogeneous. Empirical evidence, however, indicates that the early entrants into a repeat class tend to be heavier buyers of the product than later entrants (Fourt and Woodlock 1960, Parfitt and Collins 1968, Massy 1969, and Eskin 1973). Fourt and Woodlock (1960, pp. 33-34) note that the estimated penetration levels based on equation (3) fit the data well except that the predictions for distant time periods tend to be too low. The poor fit is attributed to heavier buyers converting into a repeat class earlier than light buyers. The addition of a trend factor,  $\delta$ , provides for this "stretch out" of penetration, which leads to the following adjustments in equations (2) and (3):

$$P(L) = \alpha \rho^{L-1}(1-\rho) + \delta, \qquad L=1,2,3,...$$
(7)

$$F(L) = \alpha(1-\rho^{L}) + \delta L, \qquad L=1,2,3,....$$
(8)



This modification allows the ceiling to be a linear function of time instead of a fixed quantity. The expression for average interpurchase time given a finite observation time period, <u>T</u>, now becomes:

$$r_{T}^{\delta} = \frac{L=1}{\sum_{\substack{\nu=1\\ \nu \in \nu}}^{T} \pi_{\nu} \frac{L-1}{(1-\rho)} + \delta} = \frac{\alpha \{\frac{1-\rho}{1-\rho} - T\rho^{T}\} + \frac{\delta T (T+1)}{2}}{\alpha (1-\rho^{T}) + \delta T} .$$
(9)

Empirical evidence presented by Fourt and Woodlock (1960) and Eskin (1973) suggests that the numerical value of the "stretch out" factor,  $\delta$ , is quite small compared to the conversion proportion term,  $\alpha$ . Therefore, the effect of the  $\delta$  term in equation (9) above is likely to be minor, especially when the observation period over which the purchase data are available is of limited duration (say, 12 or 24 weeks).

The continuous analog of the discrete depth of repeat model discussed above is the familiar NBD model (Ehrenberg 1959) which assumes exponentially distributed interpurchase times (or equivalently, a Poisson distribution of purchase events across successive time periods of equal length) for an individual household, and gamma heterogeneity over the population.

Following the NBD model, the density function of time to conversion into the J<sup>th</sup> repeat class is distributed as a negative exponential; hence:

$$f(t) = \lambda e^{-\lambda t}, \qquad \lambda > 0. \qquad (10)$$

Assuming that the mixing distribution of  $\lambda$  is gamma,<sup>4</sup> with  $\mu$  as the scale parameter and  $\nu$  as the shape parameter, the expressions for f(t) and F(t) are

$$f(t) = \frac{\nu}{\mu} (\frac{\mu}{\mu + t})^{\nu + 1},$$
 (11)

and,

$$F(t) = 1 - \left(\frac{\mu}{\mu + t}\right)^{\vee}.$$
 (12)

A refinement of this model allows for a "zero group" of consumers who are not in the market for the new product and are to be excluded from the "relevant population". Kalwani and Silk (1978) used a likelihood ratio test to determine the desirability of this refinement. The unconstrained model which allows for the "zero group" provided a fit superior to that of the constrained model in each of the four cases that were tested. The expressions for f(t) and F(t) with this modification are given by:

$$f(t) = A_{\mu}^{\nu} \left(\frac{\mu}{\mu + t}\right)^{\nu+1} , \qquad (13)$$

and,

$$F(t) = A\{1 - (\frac{\mu}{\mu + t})^{\nu}\}.$$
 (14)

Penetration estimates from equation (14) are compared below with those from the discrete case (equation (8)) to see if the latter model (which is simpler and easier to interpret) performs as well as the continuous model.

### HYPOTHESES ON THE STRUCTURE OF REPEAT BUYING

For the depth of repeat model set forth above, the cumulative proportion of consumers who convert into the J<sup>th</sup> repeat level within L periods of previous purchase is given by:

$$F_{J}(L) = \alpha_{J}(1-\rho_{J}^{L}) + \delta_{J}L, L=1,2,...,$$
 (15)

where the suffix J denotes the repeat class, and the time variable L is measured in time periods since the last purchase. As indicated earlier, one purpose here is to compare the penetration curves represented by equation (15) across various repeat levels. How do the parameters  $\alpha$ ,  $\rho$ , and  $\delta$  vary with J? Eskin (1973) proposed three hypotheses regarding the patterns in

 $\alpha$ ,  $\rho$ , and  $\delta$  across repeat levels and tested them with purchase data for six <u>established</u> products.

Eskin's first hypothesis was that for a new product the parameter  $\rho_{\rm J}$  has approximately the same value across all the repeat levels. This implies that the average interpurchase times are approximately equal across all repeat classes. The average interpurchase time for a particular new product is bound to be larger than the average interpurchase time for the product class which, of course, depends on the size of the offering. Also, the average interpurchase time depends on the proportion of consumers who are completely loyal to the test brand. The larger the proportion of buyers that are committed to the new product and buy it every time, the smaller will be the average interpurchase time. In testing this hypothesis with data for six established products, Eskin (1973, p. 127) found that  $\rho_{\rm J}$  "does not fluctuate excessively nor does it exhibit a strong trend".

A second hypothesis put forth by Eskin (1973) states that  $\delta_J$  takes on the same value across all repeat levels. In this case, Eskin found that  $\delta_J$ 's "vary in a relatively small range but do tend to exhibit a negative trend over J." As indicated earlier, both Eskin (1973) and Fourt and Woodlock (1960) found that the magnitude of the  $\delta$  term is small across all repeat levels. In other words, in the expression for penetration at the J<sup>th</sup> repeat class given by equation (15), the first term --  $\alpha_J(1-\rho_J^L)$  -- forms the major contribution, especially at higher repeat levels.

A third hypothesis suggested by Eskin (1973) deals with the conversion proportion terms,  $\alpha_J$ 's. Eskin (1973) postulated that  $\alpha_J$ 's could be obtained from a geometric distribution of the form:

$$\alpha_{\rm J} = \alpha_{\rm \infty} (1 - \gamma^{\rm J}), \qquad J = 2, 3, \dots, (16)$$

-8-

where the limit,  $\alpha_{\infty}$ , is usually unity or slightly less than unity. He found that the relationship given by equation (16) provided a good fit to the data, with the estimates of  $\alpha_{\infty}$  being less than unity for each of the six established products that were studied. When equation (16) was fitted to a data base that consisted of observations for an unspecified cross-section of new products, Eskin and Malec (1976) obtained an estimate of 0.636 for the parameter  $\gamma$ . With  $\alpha_{\infty} = 1$ , this implies:  $\alpha_{2} = 60\%$ ,  $\alpha_{3} = 74\%$ ,  $\alpha_{4} = 84\%$ ,  $\alpha_{5} = 90\%$ , etc.

The systematic pattern in conversion proportion terms across a variety of new packaged goods can be interpreted as follows. Consumers will try a new product on the basis of its expected performance. The first or trial purchase of a new product is ordinarily followed by numerous usage experiences in which a consumer evaluates the product's qualities. The new product is repurchased (generally to the exclusion--partial or whole--of other products that the consumer was previously buying) only if the consumer is satisfied with the test product, as compared with previously-used products. Commitment to the new product increases as it is repurchased again and again, and ultimately the consumer "adopts" the product.

What are the practical implications of these three hypotheses? The first hypothesis implies that, for a given new product, the mean interpurchase times (or equivalently, the parameter  $\rho_J$ ) of second, third, fourth, etc., repeaters is the same as those of first repeaters. The second hypothesis states that for a new product the parameter  $\delta_J$  is constant across repeat levels. The final hypothesis on the systematic pattern in the values of  $\alpha_2$ ,  $\alpha_3$ ... is applicable across new packaged goods.

Assuming that the magnitude of  $\delta_J$  is small across repeat levels, the above hypotheses taken together imply that for a new product the penetration

-9-



for various classes of repeat (see equation (1)) can be obtained from knowledge of the average interpurchase times of first repeaters (or equivalently,  $\rho_1$ ) and the cumulative proportion of first repeaters,  $\alpha_1$ . Further, to the extent that the interpurchase time of a new brand resembles those of existing brands in the product class, the cumulative repeat proportions for various repeat classes can be estimated solely from the cumulative proportion of first repeaters.

#### PARAMETER ESTIMATION

Maximum likelihood methods were used to estimate model parameters for each repeat level. It is well-known (see Rao (1965)) that under quite general regularity conditions, maximum likelihood estimates are best asymptotically normal (BAN). That is, they are consistent, asymptotically normal, and asymptotically efficient. In addition, maximum likelihood estimates are invariant. Eskin (1973) employed a least squares approach to estimate model parameters. The fact that the dependent variable in the regression equation is defined as a cumulative proportion, and hence is nondecreasing, makes the presumption of uncorrelated error terms tenuous. Detecting autocorrelation, Eskin (1973) applied a generalized least square procedure to obtain consistent parameter estimates. In some preliminary analyses of the products studied here, the same generalized least squares approach followed by Eskin was investigated but the results obtained were unsatisfactory. Estimates of  $\alpha$ that exceeded unity were found for some repeat levels, and some estimates of  $\delta$  were negative. Measurement errors in the penetration data might account for these difficulties. The estimates were also unstable due to small sample sizes for two of the new products considered, especially at higher repeat levels.

-10-

The difficulty with employing the maximum likelihood method is that it is not possible to obtain closed-form analytical solutions. Therefore, numerical optimization is required to obtain the maximum likelihood estimates. For this purpose, a general optimization procedure developed by Kalwani (1975) was used. The computer program (written in FORTRAN IV) executes an accelerated pattern search technique. It permits the search to be restricted to the feasible range of solutions; it was not necessary to impose such restrictions for any of the four new products investigated here.

The likelihood expression that is maximized is determined as follows. Consider any one of the repeat levels, J. Let  $n_L$  denote the number of consumers who enter repeat level J within L time periods since their previous purchase. Furthermore, let  $\bar{n}_L$  denote the number of consumers who had L time periods available to convert into the J<sup>th</sup> repeat level but did not do so. Then  $\sum_{L=1}^{52} n_L = n$  represents the total number of buyers who convert into repeat level J within 52 weeks of their entry into the (J-1)<sup>th</sup> repeat class. Similarly,  $\sum_{L=1}^{52} \bar{n}_L = (m-n)$  denotes the total number of consumers who have not entered the repeat class J given that altogether, m consumers made (J-1) purchases of the new product.

The likelihood expression given purchase data  $n_L$  and  $\bar{n}_L$  for, say L = 1, ...52, is given by

$$\ell$$
  $(n_{L}'s, \bar{n}_{L}'s; \alpha, \rho, \delta) = \prod_{L=1}^{52} (P[L])^{n_{L}} \prod_{L=1}^{52} (1-F[L])^{\bar{n}_{L}},$  (17)

where,

$$P(L) = \alpha \rho^{L}(1-\rho) + \delta,$$
  
 $F(L) = \alpha(1-\rho^{L}) + \delta L.$ 

The left multiplicand in equation (17) represents the joint probability of

-11-

finding  $n_L$  (L=1,2,...,52) consumers who converted into the J<sup>th</sup> repeat class within L time periods of their previous purchase. Similarly, the right multiplicand forms for the joint probability of observing that  $\bar{n}_L$  (L=1,2,...,52) consumers who failed to enter repeat class J within L time periods of their previous purchase.

#### FINDINGS

#### Data Base

The results reported below are based on an analysis of purchase data for four new products, two brands of toothpaste--UltraBrite and Plus White-and two brands of freeze-dry coffees--Maxim and Taster's Choice. The source of purchase information is the panel data collected by <u>Chicago Tribune</u>'s Family Survey Bureau, which contains purchase records for 530 households. The largest amount of purchase information was available for Ultra Brite, where the observations covered a three-year period following its introduction. For Plus White and the two brands of coffees, the purchase data extended over a period of about 2 years following the introduction of each of these products. Findings are reported only for those repeat levels where the sample sizes are at least 30.

#### Results

Maximum likelihood estimates of parameters are displayed in Tables 1 - 4, and permit the three aforementioned hypotheses to be evaluated.

\_\_\_\_\_

### INSERT TABLES 1 - 4 HERE

\_\_\_\_\_

-12-



The first hypothesis was that the parameter,  $\rho$ , would be approximately the same across repeat levels for each of the four products. For Ultra Brite, where the sample size is largest, Table 1 reveals very little variation across the four repeat levels with the estimated  $\rho$ 's fluctuating around .94. The results for Plus White (Table 2) and the two coffees (Tables 3 and 4) appear less consistent; but the variability is still slight. The ranges of the estimated  $\rho$ 's for these three products are as follows: Plus White, .89 - .93; Maxim, .84 - .92; and Taster's Choice, .88 - .92. Note, however, that some variation in  $\rho$  is expected for the two coffees, since  $\rho$  is related to the interpurchase time, which is affected by variations in the consumption of coffee.

The second hypothesis was that the parameter  $\delta$  would be approximately the same across repeat levels for each of the four products. Results show that the estimated values of  $\delta$  are small in magnitude, generally less than 0.2 percent, which is consistent with the empirical experience of Fourt and Woodlock (1960).

The final hypothesis concerns the conversion proportion terms,  $\alpha_{J}$ 's. As noted previously, on the basis of the empirical results reported by Eskin and Malec for a cross-section of new products, it was postulated that these terms would systematically increase to  $\alpha_{\infty}$ , which is expected to be slightly less than unity. More specifically, given Eskin and Malec's estimate of  $\gamma = .636$ for equation (16), it was expected that the  $\alpha_{J}$ 's would take the following values:  $\alpha_{2} = 60\%$ ,  $\alpha_{3} = 74\%$ ,  $\alpha_{4} = 84\%$ , etc.

The maximum likelihood estimates of the  $\alpha_J$  terms for the four new products are displayed in Tables 1 through 4. While the specific estimates of  $\alpha_2$ ,  $\alpha_3$ , and  $\alpha_4$  for Ultra Brite in Table 1 of the  $\alpha_J$ 's are not exactly equal



to the hypothesized values, they do exhibit a nondecreasing pattern. Table 5 displays the differences between estimated and hypothesized values of the  $\alpha_J$ 's. Note that the deviations are all within approximately 10 percent of the hypothesized values.


#### INSERT TABLE 5 HERE

\_\_\_\_\_

Finally, presented below is a comparison of penetration estimates from the unconstrained version of the continuous model with those from the discrete model. Ultra Brite has been chosen to illustrate the results since this is the case for which the largest amount of purchase information was available for estimation purposes.

### INSERT TABLE 6 HERE

Table 6 displays the penetration for time intervals (since last purchase) of 12, 24, 36, and 52 weeks. The estimated penetrations based on the discrete and unconstrained version of the continuous models are very close to the actual penetration levels, particularly for the 52 week interval. (Note that the results shown in Table 6 relate to the goodness of fit rather than the predictive accuracy of the discrete and continuous models.) These results indicate that the discrete model (which has parameters that are easier to interpret) provides a fit as good as the unconstrained continuous model. The two key parameters of the discrete model are  $\rho$  and  $\alpha$ . As indicated earlier, the parameter  $\rho$  is directly related to average interpurchase time, and the



parameter  $\alpha$  simply represents the cumulative proportion of consumers who convert from a given repeat level to the next.

#### DISCUSSION AND CONCLUSIONS

In this paper, results were reported from some empirical tests of three hypotheses set forth by Eskin (1973) and Eskin and Malec (1976) on the structure of repeat buying. Maximum likelihood estimates of model parameters were developed for four new packaged goods--two toothpastes and two coffees--and compared for consistency with the hypothesized patterns. The first hypothesis predicted that for a given new product the parameter  $\rho_J$ , a measure of average interpurchase time, would be constant across repeat levels. Overall, the parameter estimates obtained were found to be consistent with this hypothesis -- especially in the case of Ultra Brite and to a lesser but still supportive degree for the other three new brands. As hypothesized, the parameter  $\delta_J$  displayed little variation across repeat levels and further, its magnitude was generally found to be very small.

The empirical tests of the final hypothesis on systematic patterns in cumulative conversion proportion terms  $\alpha_2, \alpha_3, \ldots$  revealed that while the estimates of these parameters deviated somewhat from their hypothesized values, they did exhibit the postulated non-decreasing pattern (see Table 5 for differences between estimates and hypothesized values of  $\alpha_J$ 's).

While these results are generally encouraging, attention needs to be drawn to the limited number of repeat levels that were used to test these hypotheses. In spite of the fact that the purchase records for each

of the four products extended over a period of at least two years, the small size of the Chicago Tribune Panel yielded only a few repeat levels with sample sizes of 30 or more.

The aforementioned hypotheses carry important implications for the tasks of making either pre-test market or early test market forecasts of the time path and equilibrium level of penetration for new brands. Given that these hypotheses about repeat buying hold, it follows that penetration levels for various repeat classes depend primarily on two factors: the average interpurchase time of first repeaters ( or equivalently the parameter  $\rho_1$ ) and the cumulative proportion of first repeaters. A reasonable initial estimate of  $\rho_1(\text{or }\tau_1)$  could be obtained for a new brand by examining the interpurchase times of existing brands in the product class (taking into account, of course, any differences in package sizes). Then, the repeat sales for the new brand could be forecast from a knowledge of the cumulative proportion of triers who repeat buy it at least once. Research is underway to link these hypotheses about repeat buying to measurement methodologies that are used to assess new brands prior to test marketing (Silk and Urban, 1978).

-16-

#### FOOTNOTES

- Note that in equation (1) the number of consumers purchasing the new packaged good (or the number of buyers entering a particular repeat class) is not influenced by the number who have already purchased the new product (i.e., F(L-1)).
- 2. The expression for variance can be obtained in a manner similar to the derivation of expected value of L.

$$VAR[L] = \frac{\sum_{l=1}^{\infty} L^{2} \alpha \rho^{L-1} (1-\rho)}{\sum_{l=1}^{\infty} \alpha \rho^{L-1} (1-\rho)} - (E[L])^{2} = \frac{\rho}{(1-\rho)^{2}}.$$

- 3. Gerald Eskin (personal correspondence) has drawn to our attention that (5) above corrects a typographical error that appears in equation (4) of Eskin and Malec (1976, p. 231).
- 4. This is the key difference between the discrete and continuous models. In the discrete case, the "stretch out" factor is used to allow for the heterogeneity between early and late entrants into a repeat class. The use of the gamma distribution in the continuous case allows for more flexibility in describing consumer heterogeneity.

### ULTRA BRITE TOOTHPASTE

REPEAT	SAMPLE	PARAMETER ESTIMATES				
LEVEL	SIZE	α	ρ	δ		
1	134	.5410	. 9394	.0001		
2	86	.6786	.9474	.0000		
3	66	.7116	.9446	.0018		
4	49	.7560	.9363	.0000		

### PLUS WHITE TOOTHPASTE

REPEAT	SAMPLE	PARAMETER ESTIMATES					
LEVEL	SIZE	α	ρ	δ			
1	64	. 3929	.9264	.0000			
2	39	.5378	.8928	.0014			
3	31	.7991	,9042	.0000			

# MAXIM COFFEE

REPEAT	SAMPLE	PARAMETER ESTIMATES				
LEVEL	SIZE	a.	ρ	δ		
1	118	. 3027	.8394	.0025		
2	75	.6191	.8837	.0014		
3	54	.7605	.9190	.0000		

# TASTER'S CHOICE COFFEE

REPEAT	SAMPLE	PARAMETER ESTIMATES				
LEVEL	\$IZE	α	ρ	δ		
1	150	.6425	.9235	.0000		
2	99	.6694	.8852	.0014		
3	72	.8092	.8755	.0003		

# DIFFERENCES BETWEEN ESTIMATES AND HYPOTHESIZED VALUE OF $\alpha_{J}^{}\mbox{'s}$

HYPOTHESIZED	OBSERVED-HYPOTHESIZED VALUE						
VALUE	ULTRA- BRITE	PLUS WHITE	MAXIM	TASTER'S CHOICE			
.60	.0786	0622	.0191	.0694			
.74	0284	0409	.0205	.0692			
.84	0840	n.a.	n.a.	n.a.			
	HYPOTHESIZED VALUE .60 .74 .84	HYPOTHESIZED  OB    VALUE  ULTRA- BRITE    .60  .0786    .74 0284    .84 0840	HYPOTHESIZED  OBSERVED-HYPOT    VALUE  ULTRA- BRITE  PLUS WHITE    .60  .0786 0622    .74 0284 0409    .84 0840  n.a.	HYPOTHESIZED  OBSERVED-HYPOTHESIZED  MAXIM    VALUE  ULTRA- BRITE  PLUS WHITE  MAXIM    .60  .0786 0622  .0191    .74 0284 0409  .0205    .84 0840  n.a.  n.a.			

n.a. denotes "not available".

### OBSERVED AND ESTIMATED PENETRATIONS (%) FOR ULTRA BRITE

REPEAT LEVEL	T = 12 WEEKS		T = 24 WEEKS		T = 36 WEEKS			T = 52 WEEKS				
	OB-	ESTIMATED		OB- ESTIM		ATED OB-		ESTIMATED		OB- E	ESTIM	ESTIMATED
	VED	DIS.	CONT.	SER- VED	DIS.	CONT.	SER- VED	DIS.	CONT.	SER-	DIS.	CONT.
l	30.3	28.7*	28.8	43.3	42.2	42.2	48.8	48.8	48.8	52.8	52.5	52.8
2	32.8	32.4	32.6	50.7	49.3	49.2	58.2	58.2	58.1	64.2	63.8	64.2
3	36.0	37.4	37.4	59.3	57.4	57.2	67.4	68.5	68.4	76.7	76.8	76.7
4	43.3	41.3	41.5	62.7	60.0	59.8	68.7	68.5	68.2	73.1	73.1	73.1

\* A total of 28.7% of first purchasers entered the first repeat class within 12 weeks of trying the new product.



# PENETRATION FOR REPEAT LEVEL J



.

•

-24-

.

#### REFERENCES

- 1. Ehrenberg, A.S.C. (1959), "The Pattern of Consumer Purchases," <u>Applied</u> <u>Statistics</u>, 8, 26-41.
- Eskin, Gerald J. (1973), "Dynamic Forecasts of New Product Demand Using a Depth of Repeat Model," <u>Journal of Marketing Research</u>, 10 (May), 115-129.
- Eskin, Gerald J., and John Malec (1976), "A Model for Estimating Sales Potential Prior to the Test Market," <u>Proceedings</u>, Educators' Conference, Series No. 39, American Marketing Association, 220-233.
- Fourt, Louis A., and Joseph W. Woodlock (1960), "Early Prediction of Market Success for New Grocery Products," <u>Journal of Marketing</u>, 24 (October), 31-38.
- Kalwani, M.U. (1975), "Zero-Order Models: Specification, Data Requirements, and Estimation," Unpublished Ph.D. dissertation, Graduate School of Business, Columbia University.
- Kalwani, Manohar U., and Alvin J. Silk (1978), "Structure of Repeat Buying for New Packaged Goods," Working Paper No. 1032-78, Sloan School of Management, M.I.T., (October).
- 7. Massy, William F. (1969), "Forecasting the Demand for New Convenience Products," Journal of Marketing Research, 6 (November), 405-412.
- 8. Parfitt, J.H., and B.J.K. Collins (1968), "Use of Consumer Panels for Brand-Share Prediction," Journal of Marketing Research, 5 (May), 131-145.
- 9. Rao, C.R., Linear Statistical Inference and its Applications. New York: John Wiley & Sons.
- Silk, Alvin J., and Glen L. Urban (1978), "Pre-Test Market Evaluation of New Packaged Goods: A Model and Measurement Methodology," <u>Journal of Marketing Research</u>, 15 (May), 171-191.
- 11. Tauber, Edward M. (1977), "Forecasting Sales Prior to Test Market," <u>Journal</u> of Marketing, 41 (January), 80-84.





> ACME BOOKBINDING CO., INC. SEP 6 1983

100 CAMBRIDGE STREET CHARLESTOWN, MASS.



1031-78