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ABSTRACT

Pulsed breakdown experiments have been performed in hydrogen and helium with a trace of mercury vapor. The breakdown field is theoretically calculated, assuming only free diffusion losses. One adjustable constant is introduced in the calculations. It is the ratio of the number of electrons in the cavity when ambipolar diffusion becomes appreciable to the number of electrons initially present. The theory is experimentally verified for hydrogen. In helium, the effect of the metastable atoms is such that the simple theory does not hold.

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I. Introduction

When a continuous-wave, high-frequency electric field is applied to a cavity the electrons oscillate, gaining energy from the field and losing energy through elastic and inelastic collisions. When an electron gains enough energy it will ionize, and in this way the number of electrons builds up. At the same time electrons are being lost through diffusion to the walls. The c-w breakdown field is that field for which the number of electrons produced per second equals the number lost per second through diffusion (1). Under these conditions, the electron density will build up to a finite value only after an infinite length of time. To make the breakdown occur within a finite time, it is necessary to have an over-voltage such that the number of electrons produced per second is greater than the number diffusing per second. Under these conditions the electron density will build up, slowly at first where free diffusion is the controlling factor, and then more rapidly as a positive ion space charge is built up which inhibits the diffusion of electrons. All indications are that the density build-up with space charge is many times more rapid than the density build-up without space charge. To a first approximation then, the controlling factor will be density build-up with free diffusion to a point where positive ion space charge becomes appreciable. Diffusion in the presence of positive ions will be neglected in the following theory for density build-up.

II. Theory

To a first approximation, the electron density as a function of time is given by

$$n = n_o \exp(\gamma_o t) \tag{1}$$

where γ_o is a characteristic value solution of the diffusion equation

$$\gamma_o = \upsilon_i - \frac{D}{\Lambda_o^2} \tag{2}$$

D is the diffusion coefficient, Λ_o is the diffusion length, and υ_i is the average ionization rate. The breakdown condition for the continuous-wave or steady-state case is $\gamma_o = 0$ or

$$\upsilon_{io} = \frac{D}{\Lambda_o^2} \quad . \tag{3}$$

The breakdown condition for the pulsed or transient case is

$$n_d = n_o \exp(\gamma_o \tau) = n_o \exp(\nu_1 - \frac{D}{\Lambda_o^2})\tau \tag{4}$$

where n_d is the density at which the positive ion space charge becomes predominant and τ is the pulse width required to reach this density.

The ionization rate ν_1 is an increasing function of the electric field E. γ_o may be rewritten as

$$\gamma_o = \nu_1 - \nu_{1o} = \frac{1}{\Lambda^2}(\frac{D}{\mu})\mu - \frac{1}{\Lambda_o^2}(\frac{D_o}{\mu})\mu \tag{5}$$

where ν_{1o} is the ionization rate for c-w breakdown, ν_1 is that for pulsed breakdown, and μ is the mobility. Λ is defined as in Eq. 3. In this form, γ_o may be calculated without regard to distribution function integrals. D/μ is very nearly a direct measure of the average energy of the electrons. For several distribution functions

$$\frac{D}{\mu} = \frac{2}{3}\bar{u} \tag{6}$$

where $\bar{u}$ is the average energy. ζ_o, the high frequency ionization coefficient, is defined through a characteristic value problem solution (1) as

$$\zeta_o = \frac{\nu_{1o}}{DE_o^2} \tag{7}$$

or for the case of parallel plates

$$\zeta_o = \frac{1}{\Lambda_o^2 E_o^2} \tag{8}$$

where E_o is the c-w breakdown field. It is now possible to introduce a $\zeta = 1/\Lambda^2 E^2$ since the characteristic value problem is still approximately the same. Hence

$$\gamma_o = \left[E^2\zeta \frac{D}{\mu} - E_o^2\zeta_o \frac{D_o}{\mu}\right]\mu \; . \tag{9}$$

The quantity γ_o may be calculated theoretically for certain gases (2,3). In general both ζ and D/μ are known as functions of E/p and the experimental values may be used.

III. Calculations

A. Hydrogen. D/μ is shown as a function of E_e/p in Fig. 1, where E_e is the effective field (4). By effective field is meant that part of the electric field in phase with the electrons which actually transfers energy to the electrons (5)

$$E_e^2 = E^2 \frac{\nu_c^2}{\nu_c^2 + \omega^2} . \tag{10}$$

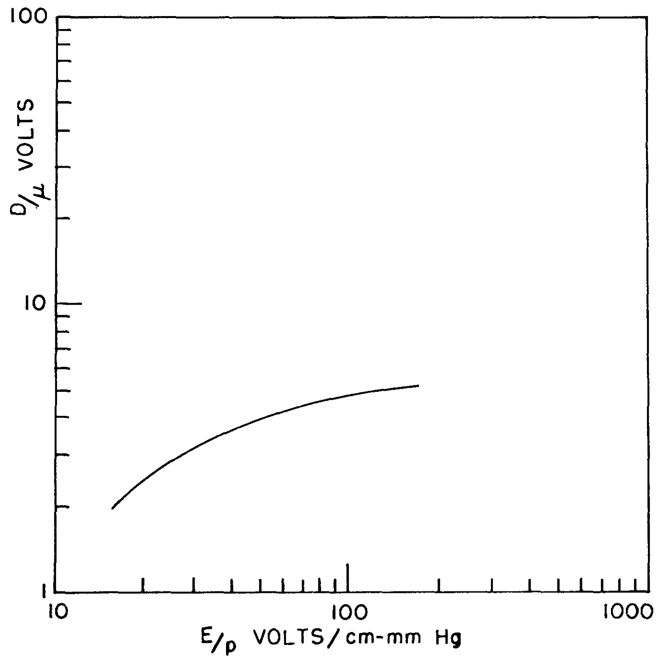

Fig. 1. Experimental D/μ as a function of E_e/p for hydrogen.

For hydrogen

$$E_e^2 = \frac{E^2}{1 + \left(\frac{31.8}{p\lambda}\right)^2} . \tag{11}$$

The experimentally determined ζ curve is shown in Fig. 2. The c-w breakdown curve is shown in Fig. 3. The mobility for hydrogen is calculated to be

$$\mu = 2.97 \times 10^5/p \tag{12}$$

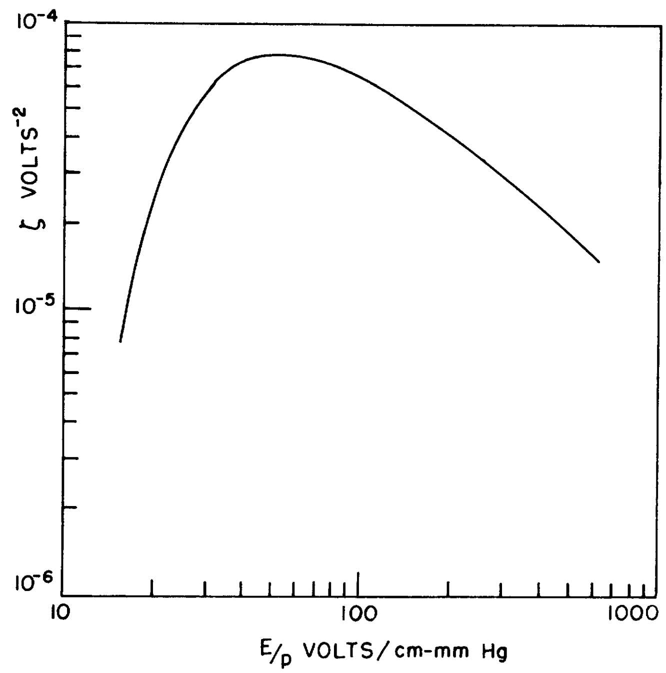

Fig. 2. The experimental high frequency ionization coefficient, ζ, in hydrogen as a function of E/p for a cavity Λ of 0.1 cm.

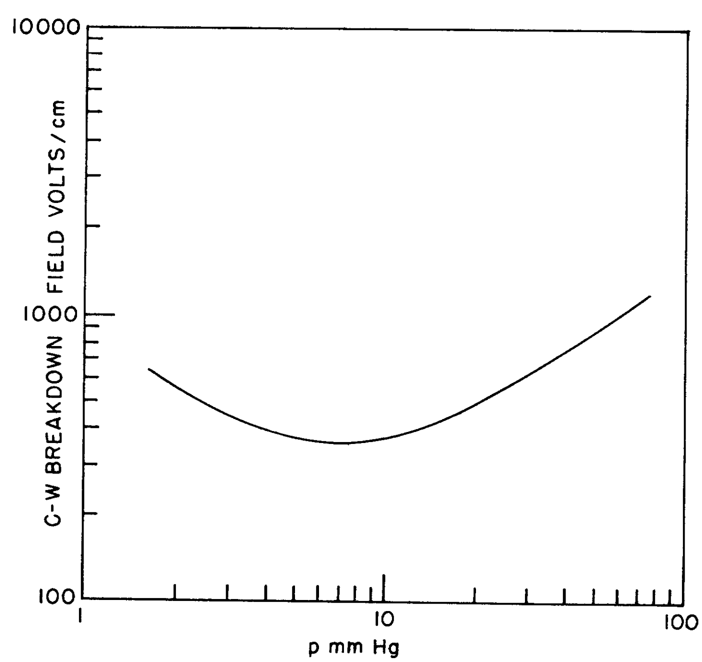

Fig. 3. The experimental c-w breakdown field in hydrogen as a function of pressure for a cavity Λ of 0.1 cm.

using a value (6) of $\nu_c = 5.93 \times 10^9 p$. The quantity $\ln n_d/n_o$ is an adjustable constant and should be a function to some extent of the sensitivity of the detecting apparatus. The value used for computing was $\ln n_d/n_o = 13.4$ or $n_d/n_o = 10^6$. This is the correct order of magnitude since ambipolar diffusion starts at about 10^9 electrons/cc and a radioactive source gave an initial ionization of about 10^3 electrons/cc.

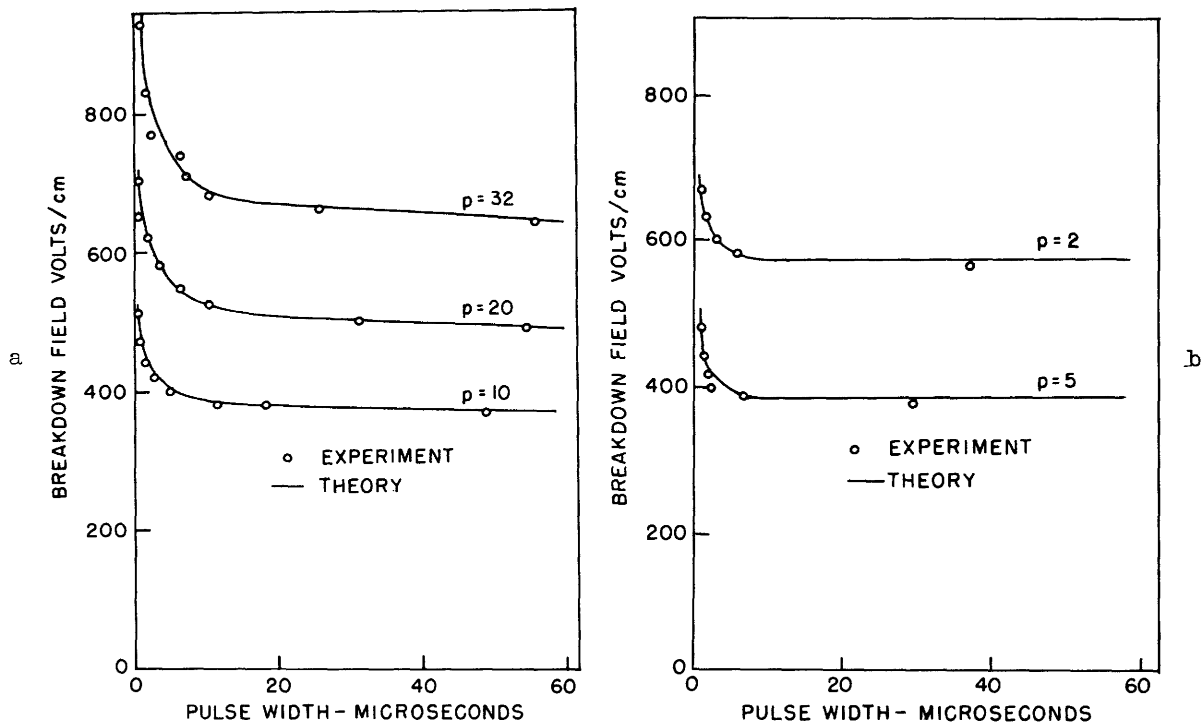

Fig. 4. Experimental and theoretical breakdown fields for various pressures in hydrogen as a function of the pulse width in microseconds for a cavity Λ of 0.1 cm.

The experimental and theoretical curves are shown in Fig. 4. Figure 4a refers to the region to the right of the minimum in the breakdown curve, Fig. 3. In this region there are many collisions per oscillation and the breakdown field is lowered with decreasing pressure, since more energy is gained by the electrons between collisions. Figure 4b refers to the region to the left of the minimum in the breakdown curve. In this region there are many oscillations per collision and the energy transfer efficiency, as represented by the effective field, Eq. 10 is lower. Hence a stronger field must be applied to cause breakdown for lower pressures. For long pulse widths, the values of the breakdown field approach steady state values.

IV. Calculation of Limits

For the calculations of Section I to hold, it is necessary that both the number of electrons and their energy decay between repetition cycles.

If these conditions are not satisfied, preceding cycles will contribute to breakdown and eventually a steady state will be reached.

A. Density Pulsation. The time required for the electrons to diffuse from the cavity while the pulse is off will be governed by the ambipolar diffusion constant (7), D_a, between the maximum density and the transition density of 10^7 electrons/cc. From the transition density to the source density, 10^3 electrons/cc, the process will be free diffusion. Of the two processes, ambipolar diffusion will be the longer, and for a first approximation, free diffusion may be neglected. If a maximum density of 10^{10} electrons/cc is assumed, the condition that the electrons diffuse within a time t will be

$$t > 7\frac{\Lambda^2}{D_a} .$$

At a pressure of 50 cm, D_a is of the order of magnitude of 7. Hence

$$t > 0.01 .$$

This corresponds to a repetition rate of about 100 cps and will be slightly too high since free diffusion was neglected. It was observed that at repetition rates below about 80 cycles there was no effect from other cycles.

B. Energy Pulsation. It is necessary that there be no carry-over of energy from one cycle to the next, i.e. the energy of the electrons must drop to thermal energy during the off time. Energy pulsation occurs when (8)

$$t > 2M/3m\, \upsilon_r \qquad (16)$$

where υ_r is the frequency of recoil. Assuming $\upsilon_r \sim \upsilon_c = 5.93 \times 10^9 p$ for hydrogen, the condition for energy pulsation becomes for p = 10 mm, $t > 2 \times 10^{-8}$ sec. This limit may be neglected as compared to the density pulsation calculated in IV A.

V. Experimental Procedure

A block diagram of the microwave apparatus which was used is shown in Fig. 5. The audio oscillator output was fed into a Hewlett-Packard square wave generator which sharpened the sine wave trigger pulse for the pulse-forming circuit. The output from this generator was a square wave variable from about one microsecond to several hundred microseconds and any repetition rate could be used in conjunction with it. The square pulse turned the modulator on and off. Because of capacity and inductance in the modulator, the output had a rise time of about 1/4 μsec and hence pulse widths below one μsec could only be estimated. The r-f power from the c-w magnetron was fed into the cavity through a power divider and attenuator. A crystal and A/R scope were used to observe the output from the cavity. The pulse widths were measured on the A/R scope which was equipped with a crystal-controlled calibration circuit and pulse widths could be measured to an

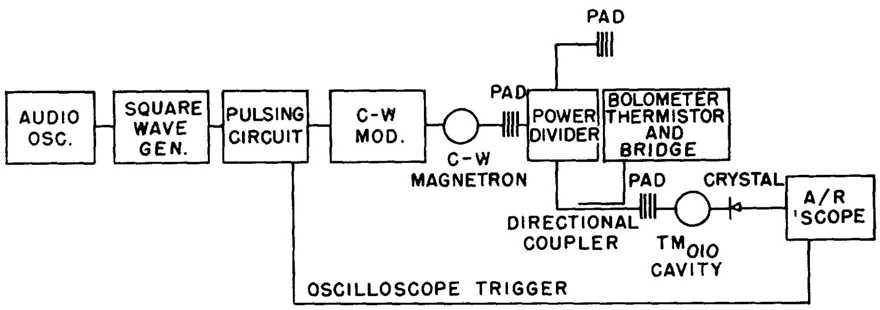

Fig. 5. Block diagram of the microwave experimental equipment.

accuracy of ± 0.5 μsec. The power incident on the cavity was measured by a directional coupler and a thermistor bridge. Because of the low average power required for breakdown, it was necessary to put an attenuator between the directional coupler and the cavity. The power measurements were accurate to about 10 percent. Measurements of the attenuation and the Q of the cavity were made by standard microwave techniques and the electric field was calculated from the incident power by the usual methods (9). The cavity which resonated at 10.7 cm and had a height of 1/8 inch was made of oxygen-free high conductivity copper, silver-soldered together. An all-glass vacuum system, including a two-stage oil diffusion pump and liquid nitrogen trap, was used to evacuate the cavity before allowing the gas to enter. A mercury manometer measured the pressure to an accuracy of ± 1 mm. A radioactive source supplied initial ionization to the cavity.

The breakdown experiment consisted of filling the cavity with gas at a measured pressure and increasing the magnetron power until breakdown was observed on the A/R oscilloscope for the particular pulse width chosen. Breakdown was observed on the oscilloscope as a decrease in the power transmitted through the cavity. The decrease in power does not occur until the density of positive ions has increased to a point where ambipolar diffusion is the controlling factor. The electrons diffuse much more slowly and the density builds up much more rapidly than when free diffusion is the controlling factor. This rapid build-up causes the resonant frequency to shift and the transmitted power to drop. Pictures of this rapid build-up have been taken (10) and all indications are that it takes place within 10^{-8} sec. Hence, the assumption that the majority of the time is spent in build-up during free diffusion seems to be a good approximation. The indications of breakdown are distributed at random depending on when an electron enters the cavity. However, for any given power level there is a minimum time at which breakdown is observed. This time is taken as the build-up time. For very short pulses where the power level is too low to read with accuracy, the pulse width is held constant and the position of the minimum time noted for various power levels.

VI. Discussion

Breakdown under pulsed conditions has been measured and computed for the case of hydrogen assuming ionization and free diffusion to be the balancing factors. Similar experiments in air indicate that it behaves qualitatively as hydrogen. Breakdown in air cannot be computed easily, since υ_c is not a simple function of energy.

Considerable work (3) has been done in the gas discharge field with Heg gas, i.e. helium with a trace of mercury vapor. This is a convenient gas to work with since mercury is ionized by collisions of the second kind with metastable helium atoms. This means that all helium atoms which are excited to 19.8 volts and collide with mercury atoms will produce ionization. All inelastic collisions lead to ionization. However, it is necessary in the pulsed case under study that every helium metastable find a mercury atom within the pulse width. It was found experimentally that when the pulse width was so short that few metastables encountered mercury atoms, the breakdown field approached that for pure helium, while at long pulses, the field dropped and approached that for Heg gas. The theory as developed for hydrogen is no longer applicable in this case.

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