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The Significance of the Matrix Form in Accounting

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The matrix has received attention recently[1] as an accounting device. This attention has focussed upon the use of the matrix form operationally.[2] But it is clear that the operational usefulness is limited.[3]

The real significance of the matrix form in accounting is analytical insight not operational use. Pictorially and conceptually we have been given a new and a different view of traditional accounting. In order to illustrate this it is necessary to emphasize certain aspects of traditional double entry bookkeeping first.

In current bookkeeping practice we start with a mass of unsorted data, the journals, and sort this data via the ledgers, into the final form we desire. Any intermediate steps in the process are designed to reduce effort. It seems that little if any attention has been devoted to the possibility of increasing information by increasing the detail of the final information.

Yet increased detail is receiving much attention these days. The income statement is a detailed listing, by categories, of certain changes (normal operations) in the proprietorship or net worth account. The cash flow statement is also a listing by classes and categories of all the changes in the cash account, and the funds flow statement is a listing of the changes in an aggregation of accounts. A detailed listing of the causes of account changes is information which is often desirable but expensive to obtain.

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[1]While Arthur Cayley, The Principles of Bookkeeping by Double Entry (Cambridge, University Press, 1907) is the earliest article on this subject the best known is Richard Mattesich, "Towards an Axiomatization of Accountancy, with an Introduction to the Matrix Formulation of Accounting Systems," Accounting Research (October 1957), pp. 328-355.

[2]See for example A. Wayne Corcoran, "Matrix Bookkeeping," The Journal of Accountancy (March 1964), pp. 60-66.

[3]*Ibid.*, p. 62.

By considering the matrix forumlation, e.g., looking at the bookkeeping process from another point of view, an alternative approach to the bookkeeping process is seen. This is an intermediate step which generates additional information detail.

The matrix form is a square with as many rows and as many columns as there are accounts. Thus one row and one column is designated for each account. One of these contains all debits to that account and the other all credits. These rows and columns intersect to form cells. The total number of cells formed is the square of the number of accounts. If there are fifteen accounts, there will be 225 cells. In general if the number of accounts is n, the number of cells will be n^2.

Each cell represents a debit to one account and a credit to another. There is no duplication. For each account there are n cells containing debits to that account but each is also a credit to a different account. The same is true of the credits.[1] This is a unique aspect of this formulation. These cells represent all possible journal entries. For the accounts on which the matrix is based, there is no journal entry, a debit to one account and a credit to any other, which is not represented in the matrix. No matter how many journal entries there are for this set of accounts, during any period, they can be reduced to the n^2 various possibilities by summing all entries which are a debit and credit to the same accounts. In fact the matrix, since it includes all possible combinations, includes a combination which is not used. This is the diagonal of the matrix which contains a cell for each account which is both a debit and a credit to that account.[2]

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[1]There are only $n + n - 1 = 2n - 1$ cells related to each account. There is a cell which is counted twice because it is at the intersection of the row and column.

[2]See John G. Kemeny, Arthur Schleifer, Jr., J. Laurie Snell, and Gerald L. Thompson, Finite Mathematics with Business Applications (Englewood Cliffs: Prentice-Hall, Inc., 1962), p. 350 for a brief discussion of the number of cells available and useable.

If the accounting data of an organization is put in matrix form, a step that is essentially intermediate has been performed. This step is intermediate in the sense that it can be ignored and the traditional ledger data, account balances, can be calculated.

In the matrix form all of the data relating to each account is segregated by account. To calculate any account balance, it is only necessary to sum the row and column of that account and take their difference.[1] This difference and the beginning balance are combined to obtain the ending balance.

The matrix form emphasizes pictorially as well as analytically that there are n intermediate steps in double entry which have traditionally been ignored or overlooked. And this intermediate step makes available, as desired, additional detail information about every account. It is this and the additional possibilities this brings to mind that is the significance of the matrix approach. Once this approach and its possibilities are understood we can discard the physical matrix and its cells and utilize a more versatile mechanism to implement the concept.

The matrix itself is not important. It is not needed as a mathematical form for any calculation. Once the importance and flexibility of the intermediate step is realized, the matrix concept along with indexing becomes one of the devices by which this intermediate step can be performed. The significance of the matrix form is the concepts embodied in it, not its physical form. The first and most obvious point is the intermediate step, the grouping of all journal entries into groups of like entries. Three useful concepts follow from this. They are indexing, subclassification, and building blocks.

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[1]For examples see A. Charnes, W. Cooper, and Y. Ijuri, "Breakeven Budgeting and Programming to Goals," Journal of Accounting Research (Spring 1963), p. 33; Corcoran, op. cit., p. 65, and Mattesich, op. cit., pp. 335 ff.

Indexing is a device to indicate a cell by its row and column in a matrix. It can be used also to indicate characteristics. Generally numbers are used, and their relation indicates the cell position. For example (1, 9) indicates, by convention, row one column nine. By stating a suitable convention and chart of accounts this could also mean debit cash, credit notes payable. A simple journal entry represents a type of indexing. The account debited and the account credited indicate jointly the cell in which the amount is to be placed.

Indexing is a convenient device which permits the use of matrix operations without the cumbersome physical matrix. Digital computers do not have, somewhere inside, large square or rectangular arrays, yet they can be programmed to perform linear algebra. While the computers do not have these arrays they do have their equivalent, cells, and these cells can be indexed. This index (name) identifies the cell. Consequently it can be called, manipulated, etc. A physical matrix which presents graphically the relation of cells is not necessary if the cells are indexed. This indexing allows identification of a desired cell no matter what its physical location to other cells. The changes (entries) to these cells must be indexed also. An understanding of indexing leads to a clear grasp of the relationship between the journal and the ledger.

The ledgers and the journals contain the same data, excluding journal explanations, but arranged differently. While this is known, it is obscured by the operational relation between these two. Generally, in current operations, the ledgers are made by copying the journals. It is possible, if the journal entries are physically separate, to construct ledgers by rearranging the journal entries.[1]

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[1]This cannot be done simultaneously, only sequentially. There are at least two methods; by individual accounts, or by debits (credits) for all accounts and then by credits (debits).

To unscramble journal entries back into the original journal another index, such as dates or sequential numbering, must exist.

I will use subclassification to mean a breakdown (partitioning) of the entries to an account on some other basis than the one implicitly contained in the original chart of accounts. Using the original chart of accounts a large but definite amount of detail is possible; for n accounts produce $n^2 - n$ separate types of entries. Detailed information for each account is possible because of the relation of the accounts to each other via the debit-credit (indexing) mechanism. Following this pattern, additional detail can be obtained only by adding more accounts. These additional accounts produce desired detail but possibly undesired (unnecessary or redundant) classes of information. This introduces unwarranted complexity. The accounts exist because they represent classes of desired information, not because they generate desirable details.[1]

It is a serious matter to create extra accounts to generate more detail only, not additional classes of desired information. The cell concept and the related accounts, in conjunction with indexing, can be utilized to develop subclassifications and generate additional detail. This additional detail can be developed for some accounts by the simple expedient of increasing the number of indices from two to three.

Assume that subsidiary account information about accounts receivable is desired. Each credit sale can be indexed by a debit to accounts receivable, a credit to sales, and a third index which shows the firm. Use of this third index will permit sorting credit-sales by purchaser.

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[1]A notable exception is the nominal accounts. They do provide information unavailable otherwise, but they also introduce some complexity.

The third index is not an additional dimension of the matrix, similar to debit and credit. It is merely a subclassification system relating to one account. This is one of the defects of this mechanism. Since every entry is related to two accounts, an additional index will provide a subclassification system appropriate to one of these accounts but not necessarily the other. If any entry type must be indexed for a subclassification system for each of the two accounts, a fourth index in addition to a third, would be necessary, except in the fortunate instance that both subclassifications are the same.

One possible application of the third index comes to mind immediately. This is the detail required for the income statement.[1] The income statement is, after all, a listing of the changes in the net worth account caused by operations. Furthermore this detail would not exist if only the real accounts were used. There would be a large number of debits and credits to cash or accounts payable and receivable, and corresponding credits or debits to net worth which would be indistinguishable without some further indication of their nature. This use of a third index is certainly feasible. Whether it is desirable depends upon convenience and efficiency.

The building block concept is not new in accounting.[2] To follow this concept, the material at hand, in this case data, is reduced to the smallest unit size possible. The finished product, in this case information, is then built up from these building blocks. The initial size of the data will be accepted as given: it is the next step which is of importance. Traditionally, as noted previously, this has been a single step to the information desired. Contemplation of the

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[1] See Charnes, Cooper, and Ijuri, *op. cit.*, pp. 33, 35 for a partial simple example without additional indexing.

[2] See for example Billy E. Goetz, *Management Planning and Control* (New York: McGraw-Hill Book Company, Inc., 1949), p. 145.

matrix form brought the realization that a possible intermediate step existed. This intermediate step can be regarded as the construction of building blocks which are eventually aggregated to produce the desired information.[1]

Consideration of the information detail in the matrix form as building blocks suggests that since present information is constructed from these, it is possible to produce other information from different constructions. As an example of the production of other information from a new combination of these building blocks, consider the funds statement.[2] This is a statement of changes in a group of accounts defined as the fund. For this discussion a fund will be defined as current assets minus current liabilities.

The building blocks can be divided into sixteen groups as shown in Figure I.

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[1]The special journals perform this function to some degree. However they are restricted to a few types of journal entries and this severely limits the variety of special statements that can be made from them.

[2]See James J. Linn, "An Analysis of Double Entry," (Working Paper 87-64, Alfred P. Sloan School of Management, Massachusetts Institute of Technology, 1964), pp. 16-18, for the construction of a cash flow statement from these building blocks.

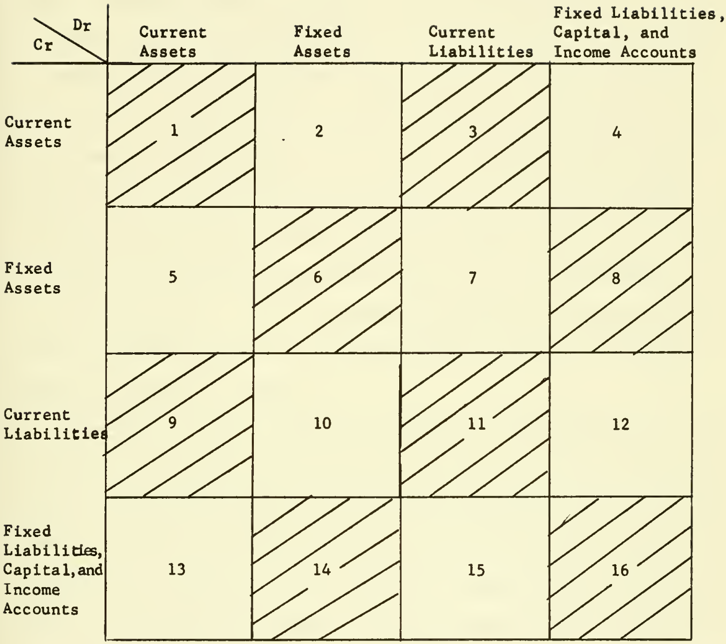
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Figure I

The matrix is used because it is an excellent summary picture. Each square represents a group of accounts: unneeded detail has been omitted. The shaded squares represent groups of building blocks which are not used to construct the funds statement. Block groups 1, 3, 9 and 11 represent internal fund changes which do not affect the net size of the fund and can be disregarded. Block groups 6, 8, 14, and 16 represent external-fund changes which produce no change in the fund and also can be disregarded. The remaining blocks represent changes which all affect the fund. Thus a fund statement can be internally generated from the data in block groups 2, 4, 5, 7, 10, 12, 13, and 15. Funds statements constructed by outsiders must be based on the other methods for constructing funds statements unless the detailed information is known.

For another example consider the funktionalen kontorechnung.[1] This is a scheme of transaction classification, more elaborate than the traditional system, by which the class of account (the five possible classes being the three real and the two nominal accounts) affected by the other class of transaction is indicated. Thus a funds flow statement would contain the headings under sources: increase in equities, decrease in assets and increase due to operations, while under uses would be: decrease in equities and increase in assets.

This of course is, in a real sense, another grouping of the building blocks formed by the use of index numbers. In this case the use of index numbers would eliminate the two major objections to the Thoms system; it is time consuming and costly, and assets and equities are not divided into significant sub-categories.

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[1] H. Peter Holzer and Hans-Martin Schonfeld, 'The "Funktionale Kontorechnung" of Walter Thoms,' Accounting Review, XXXIX, No. 2 (April 1964), pp. 405-413.

In its traditional form double entry is uniquely suited to the physical matrix form. This is because of the debit-credit mechanism. To illustrate this uniqueness I will introduce another mechanism in place of the debit-credit mechanism and show the problems of using this with a physical matrix. This substitute mechanism will be increase and decrease. These are the only possible effects a transaction can have on any account. However, the two accounts affected by any transaction can be affected identically or oppositely.

Journal entries can be divided into two broad groups on the basis of how they affect the accounting identity, $A \equiv L + P$ or $A \equiv E$, (whichever is preferred). A transaction, as reflected in the journals, either causes a change in both sides of this identity, an increase or a decrease, or it causes no change, a transfer between two accounts, both on one side or the other of the identity. Now this must be incorporated into a matrix.

Consider a square matrix; the rows and columns labeled to correspond to accounts. Instead of signifying debit and credit the columns and rows will signify increase and decrease. Thus each cell represents an increase to one account and a decrease to another. Only one of the two broad groups of transactions, the second, can be represented in this matrix. The other broad group cannot because there is no way an increase to two accounts can be placed in this matrix. To include this second group two additional matrices must be introduced. One would be increase in both dimensions and the other decrease in both dimensions. These could be combined physically as shown in Figure II and Figure III. In Figure II only the three surfaces would be used,(i.e., one of the three coordinates of any point would be zero). Let A, B, and C represent increase in one account and decrease in another, increase in both accounts, and decrease in both accounts

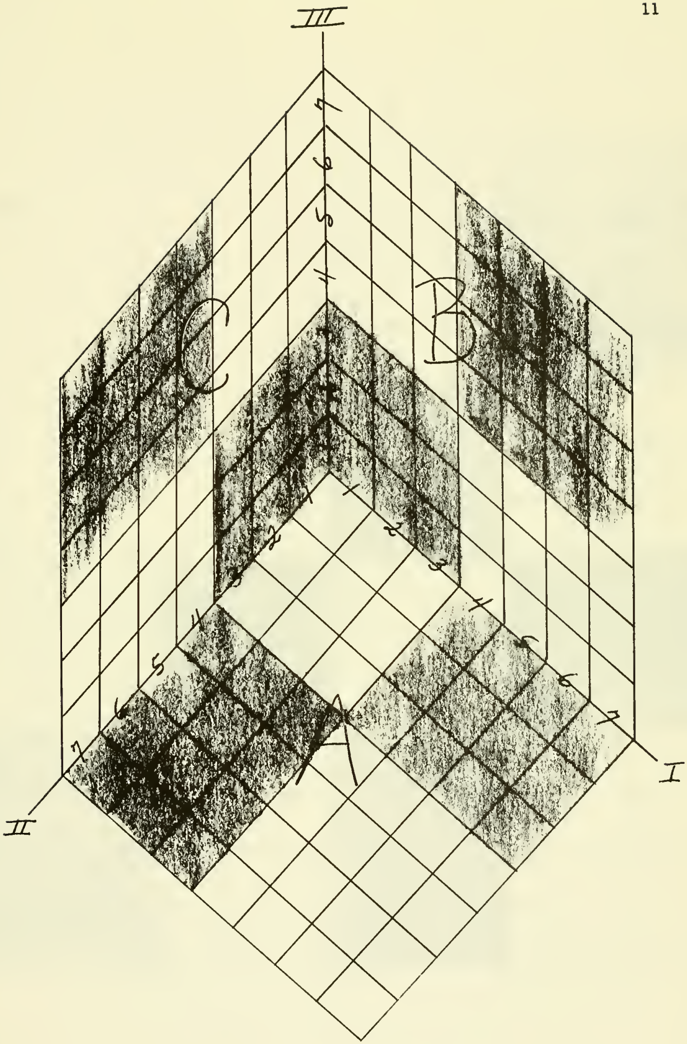

Figure II

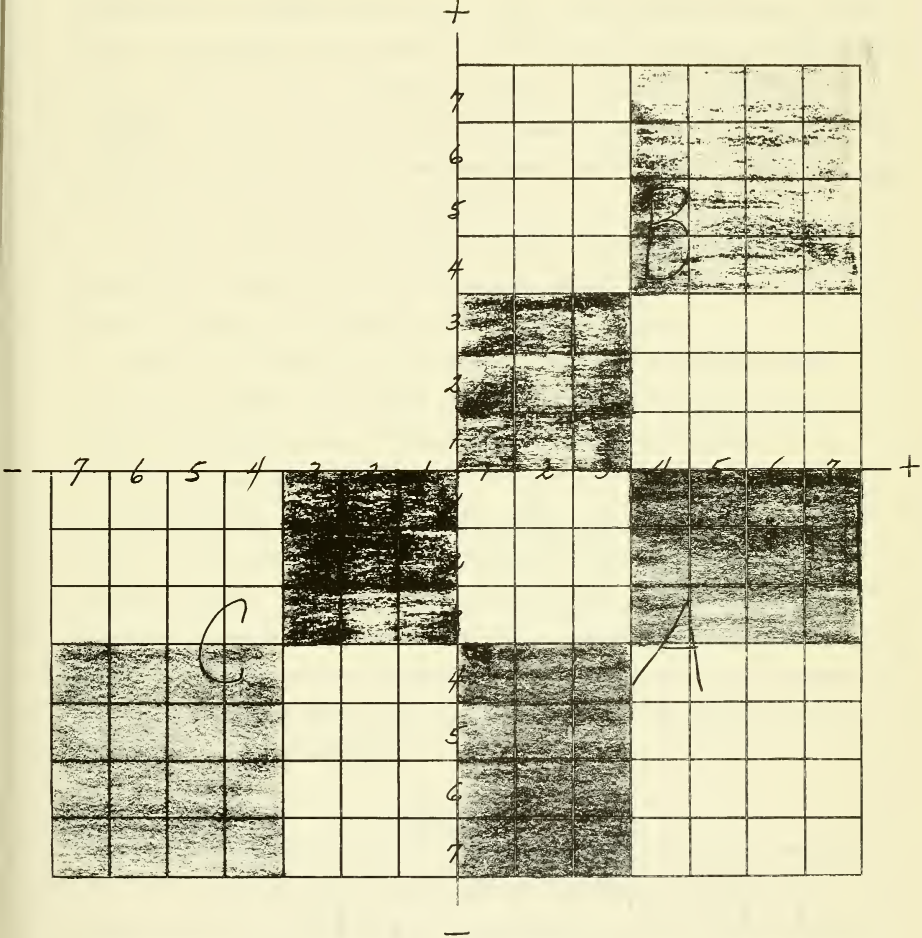

Figure III

respectively. The shaded areas are those cells in which transactions cannot occur: there is the assumption that accounts 1, 2 and 3 are asset accounts and accounts 4, 5, 6, and 7 are equity accounts. In surface A for example transactions involving an increase in one account and a decrease in another can take place only if both accounts are either asset accounts or equity accounts. Otherwise, the conditions of the identity are violated.

A similar condition occurs in surface B and C. The accounts affected by transactions here must be an asset account and an equity account; one of each. Again, if otherwise, the accounting identity will be violated.

Figure III is Figure II in one dimension. It is also a coordinate system with one sector unused. The comments on Figure II are valid here. This also requires a coordinate system of three dimensions. The third index sign + or - is implicit in the sector indicated.

Since it is possible to develop a physical form for this system of double entry, it is also possible to develop an analytical form as was done for the traditional system. In an analytic form the same number of cells are required for the new system as for the traditional form. A and either B or C together contain the total number of cells under the traditional system. In the analytic form of the new system either B or C can be dispensed by combining them with the introduction of signed quantities.

Conclusions

The significance of the matrix form of double entry is the concepts embodied in it not its physical form. These concepts are indexing for manipulation and sorting of data, subclassifications for desired detail in the information produced, and building blocks as the materials for constructing information. These concepts and the physical matrix form are not unique to the traditional form of double entry. They are applicable to one other form of double entry, at least.

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