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SHOT NOISE IN BEAM TYPE TRAVELING-WAVE AMPLIFIERS

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Errata

The last expression in the numerator of Eq. 21, p. 7, should read $(\frac{\sqrt{3}}{2} - \frac{1}{2})$.

The notation in Fig. 3, p. 8, should read $x^2 - x\sqrt{3} - 1$.

Author's Correction

Substitute the following paragraph for the last paragraph of Part IV, p. 9:

The expression for γ^2 , Eq. (15), is frequency dependant, and can assume values greater than unity. The full consequence of this has not yet been examined. It would appear, however, that the increased shot noise at high frequencies and long transit angles is due to a species of amplification such as one has in a klystron. Measurements are now being made on the noise induced in a cavity by an electron beam, and it is hoped that the results will allow a direct check to be made on Eq. (15).



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ABSTRACT

The noise figure of a beam type traveling-wave amplifier is derived by computing the current and velocity fluctuations in the beam at the beginning of the helix and comparing the power induced in the growing wave to that due to thermal noise. The result of including both current and velocity fluctuations which are related to the initial velocity fluctuations at a space charge limited cathode is that the noise figure of the tube is inversely proportional to the gain parameter C . The d-c beam current and voltage have only secondary effects on the noise figure.

Notation

- e = electronic charge 1.59×10^{-19} coulomb
 m = electronic mass = 9.03×10^{-31} kg
 η = $e/m = 1.76 \times 10^{11}$ coulombs/kg
 ϵ = dielectric constant of space = $(36\pi)^{-1} \times 10^{-9}$ farad/m
 k = Boltzmann's constant = 1.37×10^{-23} joule/°K
 c = velocity of light in space = 3×10^8 m/sec
 I_0 = d-c beam current
 I = total circuit current (amps)
 J = total circuit current density (amps/m²)
 i = electronic convection current
 q = electronic convection current density
 u_0 = electron "d-c" velocity (m/sec) = $\sqrt{2\eta V_0}$
 v = fluctuating component of electron velocity
 V = voltage
 V_0 = d-c accelerating voltage
 ω = radian frequency
 t_D = transit time between cathode and anode
 t_c = transit time between anode and helix
 θ = transit angle = ωt
 σ = $(t_c/t_D + 1)$
 r = cathode radius (m)
 $E_{1,2,3}$ = axial field strength of various helix modes
 β_0 = helix propagation constant = ω/v_0
 v_0 = velocity of propagation on helix
 Γ_n = $j\beta_0 + \delta_n$ = propagation constant of nth mode
 C = gain parameter = $\left(\frac{EE^*}{\beta^2 P_0^*} \frac{I_0}{8V_0} \right)^{1/3}$
 X = $\frac{r\omega C}{\sigma} \sqrt{\frac{2\pi\epsilon u_0}{I_0}}$
 T_c = cathode temperature (°K)
 T = antenna temperature
 B = bandwidth (cps)
 F = noise figure
 γ^2 = space charge smoothing factor

SHOT NOISE IN BEAM TYPE TRAVELING-WAVE AMPLIFIERS

I. Introduction

Considerable effort is going into the development of traveling-wave amplifiers whose noise figures are low enough to allow their use in the input stages of microwave receivers. The criterion by which such tubes are measured is the noise figure of a silicon crystal used as a mixer, and the input circuit of the i-f amplifier. At 3000 Mc/sec it is possible to get a mixer-i-f noise figure of 9 to 10 db. Up to the present, the best noise figures that have been obtained in traveling-wave tubes are in the neighborhood of 12 db at 3000 Mc/sec, as reported by Kompfner (1) at Oxford and Field at Stanford.

The sources of noise in a traveling-wave tube are several: (a) shot noise arising from the random emission of electrons from the cathode; (b) partition noise caused by a portion of the electron beam being intercepted by the accelerating electrodes or the helix; (c) partition noise "of the second kind" caused by electrons at different cross sections of the beam having different coupling coefficients with the helix; (d) gas noise appearing as a modulation of the amplified signal caused by plasma oscillations of the ions trapped in the electron beam (2). The last three types of noise are more or less subject to control. Partition noise can be minimized by careful design of the electron gun and magnetic focusing system. Partition noise of the second kind can be reduced by confining the beam to a region close to the axis where the electric fields are nearly uniform. Gas noise can be minimized by careful outgassing and by the use of auxiliary electrodes, biased to reduce the ion concentration in the beam. Furthermore, gas noise does not set a limit on sensitivity since it appears only as a small modulation on an existing signal. Shot noise, on the other hand, presents the designer with a more or less fixed noise level beyond which he cannot go.

Both Pierce (3) and Kompfner (1) have computed the noise figure of a traveling-wave amplifier, considering only shot noise. Their results were identical and, in Pierce's notation (3),

$$F \simeq 80\gamma^2 V_0 C \quad (1)$$

where $\gamma^2 < 1$ is the space charge smoothing factor, V_0 is the d-c beam voltage, and C the gain parameter of the tube. This expression was derived on the assumption that the noise input to the helix is entirely due to the noise convection current in the beam, and that the mean square value of the current is given by

$$\overline{i_n^2} = \gamma^2 2eI_0B \quad (2)$$

Equation (1) has been widely used as a design equation for developing low noise tubes. It indicates that the greatest improvements are to be had by a reduction in beam voltage V_0 . In principle, one can make a helix to operate at any voltage, so that if we assume the helix impedance $E_z E_z^* / \beta_0^2 P_0$ to be a constant, then $C \propto (I_0/V_0)^{1/3}$, and Eq.(1) can be written as

$$F \propto V_0^{2/3} I_0^{1/3} \quad (3)$$

It is the purpose of this paper to inquire into some of the basic assumptions used in deriving Eq.(1). We will first derive expressions for the actual noise current i_n and the noise velocity fluctuations v_n of the beam at the beginning of the helix. Then the amplitude E_1 of the growing wave will be computed, and the noise figure derived.

II. Noise Current and Velocity in the Beam

The treatment followed here will be along the line laid down by Llewellyn (4), Llewellyn and Peterson (5), and Peterson (6). These authors have discussed parallel-plane electron flow extensively, and cylindrical flow slightly. The discussion that follows will be based on the assumption of flow between infinite parallel planes. It is not entirely clear that such a derivation can be applied to a thin, cylindrical beam. One justification for this procedure is that no other analysis is available at the moment. A second justification arises from the nature of the "Pierce gun" (7) commonly used to produce the beams in traveling-wave tubes. In the region between cathode and anode, the electrostatic fields are arranged so that their values at the edges of the beam are those that would exist if the beam were part of the space charge-limited flow between an outer spherical cathode and inner spherical anode. This is a kind of "infinite" flow, and one presumes that the differences in geometry between parallel planes and concentric spheres will change the magnitudes slightly but not the basic phenomena.

The heart of Llewellyn's method is to reduce a complicated electronic system to a series of diodes in which the input convection current and velocity modulation of the electrons of one diode are the corresponding output quantities of the preceding diode. This is illustrated in Fig. 1.

According to Llewellyn and Peterson (5) we may write the following system of linear equations if the a-c quantities are small:

$$\left. \begin{aligned} V_{ab} &= A^* I + B^* q_a + C^* v_a \\ q_b &= D^* I + E^* q_a + F^* v_a \\ v_b &= G^* I + H^* q_a + I^* v_a \end{aligned} \right\} \quad (4)$$

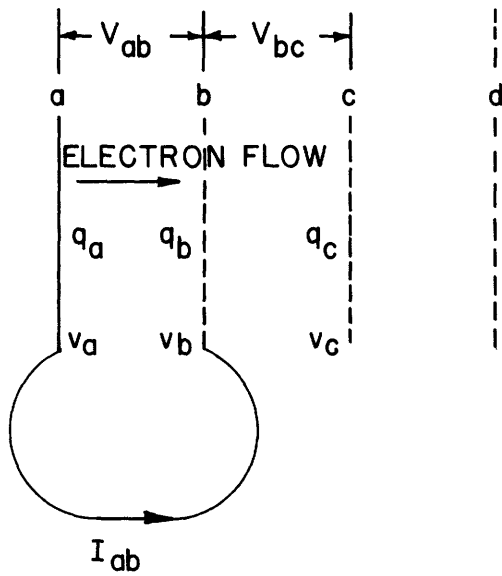


Fig. 1 Electron flow through a series of parallel plane electrodes.

where the coefficients A^* through I^* are defined in Table II of Ref. (5). The structure we shall consider here is the gun or cathode-anode space, and the drift tube between the anode and helix, Fig. 2. We shall assume that the radio-frequency impedance of the sections a-b and b-c are zero, so that the voltages V_{ab} , $V_{bc} \approx 0$. Then Eq.(4) can be written

$$\left. \begin{aligned} q_b &= \left(E^* - \frac{B^* D^*}{A^*} \right) q_a + \left(F^* - \frac{C^* D^*}{A^*} \right) v_a \\ v_b &= \left(H^* - \frac{B^* G^*}{A^*} \right) q_a + \left(I^* - \frac{C^* G^*}{A^*} \right) v_a \end{aligned} \right\} . \quad (5)$$

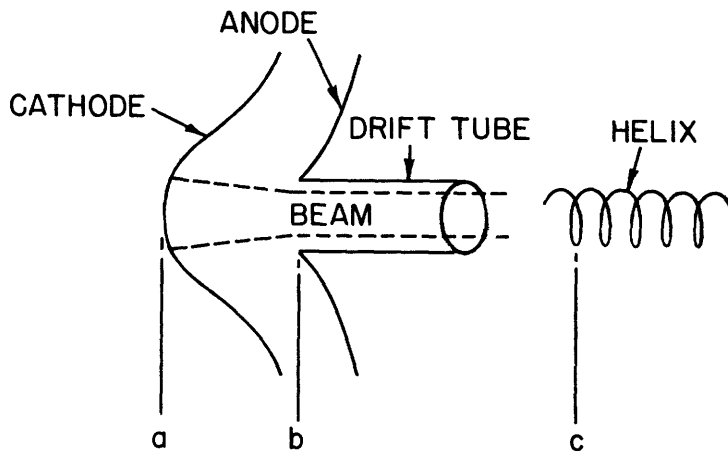


Fig. 2 Input end of traveling-wave amplifier.

Thus, if the noise currents and velocity are defined at a, we may finally compute their magnitude at c by applying Eq.(5) twice, using the proper values for the coefficients in each case.

The noise problem can be handled in this elementary manner if we assume an electron flow in which all electrons in a given cross section of the beam have the same velocity. It will be assumed that the flow in the anode-cathode region is completely space charge limited with the virtual cathode lying in the plane of the cathode. Evaluating the coefficients from Table II of Ref. (5), we find

$$\left. \begin{aligned} q_b &\simeq \frac{2j\epsilon\theta_b}{\eta t_b^2} e^{-j\theta_b} v_a \\ v_b &\simeq -e^{-j\theta_b} v_a \end{aligned} \right\} \quad (6)$$

where $\theta_b = \omega t_b \gg 1$.

We now repeat the process for the drift tube. Here, we also assume a large transit angle θ_c , but since the electrons are traveling at full beam voltage V_o , we assume a small space charge density ($\zeta \ll 1$, Ref. (4)). Then, using the proper coefficients, we get from Eqs.(5) and (6)

$$\left. \begin{aligned} q_c &\simeq \frac{2j\epsilon\omega v_a}{\eta t_b} e^{-j(\theta_b+\theta_c)} \\ v_c &\simeq -v_a \left(\frac{2t_c}{t_b} + 1 \right) e^{-j(\theta_b+\theta_c)} \end{aligned} \right\} \quad (7)$$

Thus, the convection current q_c and the velocity modulation v_c are functions of the initial velocity fluctuation v_a at the cathode, but not of the initial current q_a .

The transit times t_b and t_c may be written in terms of V_o , I_o , and the drift tube length l . In a space charge limited, parallel plane diode, the transit time is (8)

$$t = \frac{3d}{\sqrt{2\eta V_o}} \quad (8)$$

where d is the spacing. The d-c current density is

$$J_o = \frac{4\epsilon\sqrt{2\eta} V_o^{3/2}}{9d^2} \quad (9)$$

In the drift tube, the transit time is simply

$$t = \frac{l}{\sqrt{2\eta V_o}} \quad (10)$$

Using Eqs.(8),(9) and (10), Eqs.(7) may be rewritten

$$\left. \begin{aligned} q_c &= j \frac{v_a \omega}{\eta} \sqrt{\frac{\epsilon J_o u_o}{V_o}} e^{-j(\theta_b + \theta_c)} \\ v_c &\simeq -\sigma v_a e^{-j(\theta_b + \theta_c)} \end{aligned} \right\} \quad (11)$$

where $\sigma = (\tau_c/t_p + 1)$.

Thus, according to Eqs.(11), the current and velocity fluctuations of the beam are proportional to the velocity fluctuations of the electrons leaving the virtual cathode, and are independent of the input current fluctuations, if the flow is space charge limited. The problem of determining the initial velocity fluctuation has been treated by Rack (9) and Pierce (10). They give the mean squared value as

$$\overline{v_a^2} = \frac{\eta k T_c B}{I_o} (4 - \pi) \quad . \quad (12)$$

From Eqs.(12) and (11) we can write the expression for the mean squared noise current

$$\overline{i_n^2} = (\pi r^2)^2 \overline{q^2} = \pi r^2 (4 - \pi) \frac{2kT_c B \omega^2 \epsilon}{u_o} \quad . \quad (13)$$

We can now compute γ^2 by dividing Eq.(13) by the shot noise mean squared current

$$\overline{i_s^2} = 2eI_o B \quad . \quad (14)$$

Then

$$\begin{aligned} \gamma^2 &= \frac{\omega^2 \epsilon k T_c (4 - \pi)}{e u_o J_o} \\ &= \frac{2.04 \times 10^{-15}}{\pi} \frac{\omega^2 T_c}{u_o J_o} \quad . \end{aligned} \quad (15)$$

Up to this point we have considered only a pencil of current within a diode of infinite extent. The cross section of such a pencil is constant, and we do not have to specify at what point r is measured. If we are concerned with practical devices, the beam of electrons will generally have a larger diameter at the cathode than it will have within the helix. The effect of this convergent action upon the noise has not yet been solved. As a first approximation, however, we can use the following reasoning: the noise in the beam is primarily due to the fluctuations at the cathode, and the changing cross section of the beam probably has only secondary effects.

Therefore, in the following, r will be the cathode radius. If we choose $T_c = 1000^\circ\text{K}$, $\omega = 6 \times 10^{10}$, $r = 0.06 \text{ inch} = 1.5 \times 10^{-3} \text{ m}$, $u_o = 2 \times 10^7 \text{ m/sec}$ ($V_o \simeq 1000 \text{ volts}$), then

$$\gamma^2 = \frac{0.82}{I_{ma}}$$

As a final step, before investigating the way in which the beam induces noise in the helix, let us substitute Eq.(15) into Pierce's noise figure expression, Eq.(1):

$$\begin{aligned} F &= 80\gamma^2 V_o C = \frac{1.6 \times 10^{-10} \omega^2 V_o^{1/2} C r^2}{\sqrt{2\eta} I_{oma}} \\ &= 2.7 \times 10^{-16} \omega^2 V_o^{1/2} C r^2 I_{oma}^{-1} \end{aligned} \quad (16)$$

This makes

$$F \propto V_o^{1/6} I_o^{-2/3} \left(\frac{E_z E_z^*}{\beta_o^2 P_o} \right)^{1/3} \quad (17)$$

which is quite different from the relation indicated by Eq.(3). In both Eq.(3) and Eq.(17) the noise figure is proportional to the helix impedance; a tube with weaker coupling to the beam will have a better noise figure. On the other hand, Eq.(17) indicates that the voltage dependence is weak, and the noise figure varies as $I_o^{-2/3}$, while Eq.(3) indicates an $I_o^{1/3}$ variation.

III. Excitation of the Helix by a Modulated Beam of Electrons

Equation (1) was derived by Pierce on the assumption that the beam entering the helix had a fluctuating current but not a fluctuating velocity. According to Eqs.(11), however, the beam has both a fluctuating current and a fluctuating velocity. We shall therefore repeat Pierce's derivation, and include the effect of the velocity fluctuations.

The expressions relating the axial field strengths of the three forward waves to the input conditions in a traveling-wave tube are (3)

$$\left. \begin{aligned} E_1 + E_2 + E_3 &= E \\ \frac{E_1}{\delta_1} + \frac{E_2}{\delta_2} + \frac{E_3}{\delta_3} &= -\frac{u_o}{\eta} v \\ \frac{E_1}{\delta_1^2} + \frac{E_2}{\delta_2^2} + \frac{E_3}{\delta_3^2} &= \frac{2V_o}{j\beta_o I_o} i \end{aligned} \right\} \quad (18)$$

where $u_0 = \sqrt{2\eta V_0}$, E is the field at the beginning of the helix, $j\beta_0$ is the phase constant of the helix, and

$$\left. \begin{aligned} \delta_1 &= \beta_0 C \left(\frac{\sqrt{3}}{2} - \frac{j}{2} \right) \\ \delta_2 &= \beta_0 C \left(\frac{-\sqrt{3}}{2} - \frac{j}{2} \right) \\ \delta_3 &= j\beta_0 C \end{aligned} \right\} \quad (19)$$

If we let $E = 0$, and solve for E_1 , the amplitude of the growing wave, we get

$$\begin{aligned} E_1 &= \frac{\beta_0 C \left[\frac{2V_0 C I_0}{j I_0} \left(\frac{1}{2} - j \frac{\sqrt{3}}{2} \right) - \frac{2V_0}{u_0} v \left(\frac{\sqrt{3}}{2} - \frac{j}{2} \right) \right]}{3 \left(\frac{1}{2} + j \frac{\sqrt{3}}{2} \right)} \\ &= \frac{2\beta_0 C V_0 \frac{v}{u_0} \left[\frac{1 C u_0}{j I_0 v} \left(\frac{1}{2} - j \frac{\sqrt{3}}{2} \right) - \left(\frac{\sqrt{3}}{2} - \frac{j}{2} \right) \right]}{3 \left(\frac{1}{2} + j \frac{\sqrt{3}}{2} \right)} \end{aligned} \quad (20)$$

Into this equation, we may substitute the relation between the fluctuation current density q_c and the velocity fluctuation v_c of Eqs.(11), letting $i = \pi r^2 q_c$, where r is the beam radius:

$$E_1 = \frac{2V_0 \beta_0 C v_a \sigma \left[\frac{r a C}{\sigma} \sqrt{\frac{2\pi \epsilon u_0}{I_0 \eta}} \left(\frac{1}{2} - j \frac{\sqrt{3}}{2} \right) - \left(\frac{\sqrt{3}}{2} + \frac{j}{2} \right) \right]}{3 u_0 \left(\frac{1}{2} + j \frac{\sqrt{3}}{2} \right)} \quad (21)$$

The quantity $\sigma \geq 1$ and < 3 for ordinary geometries, i.e. the drift tube is less than six times as long as the cathode-anode distance. Let us assume some typical values and compute the magnitude of the first term in the brackets. Let $\omega = 6 \times 10^{10}$, $r = 0.06$ inch $= 1.5 \times 10^{-3}$ m, $C = 0.01$, $u_0 = c/25$, $I_0 = 10^{-4}$ amp, then

$$\omega r C \sqrt{\frac{2\pi \epsilon u_0}{\eta I_0}} = 5.5 \quad .$$

Thus the contributions of the current and velocity fluctuations are of the same order of magnitude, and neither can be neglected.

The noise figure of the tube will be defined as (3)

$$F = \frac{|E_{1T}|^2 + |E_{1n}|^2}{|E_{1T}|^2} \approx \frac{|E_{1n}|^2}{|E_{1T}|^2} \quad (22)$$

The thermal noise due to the antenna is

$$|E_{1T}|^2 = \frac{8V_o \beta^2 C^3 kTB}{9I_o} \quad (23)$$

The noise due to the beam is

$$|E_{1n}|^2 = \frac{4\beta_o^2 C^2 V_o^2 \sigma^2 \left(\frac{v_a}{u_o}\right)^2}{9} (X^2 + 1 - X\sqrt{3}) \quad (24)$$

where

$$X = \frac{r\omega C}{\sigma} \sqrt{\frac{2\pi\epsilon u_o}{\eta I_o}} \quad .$$

Then

$$F = \frac{\sigma^2 I_o \overline{v_a^2}}{4\eta C kTB} (X^2 + 1 - X\sqrt{3}) \quad (25)$$

Substituting Rack's value for $\overline{v_a^2}$, Eq.(12), we find

$$F = \frac{0.107\sigma^2 T_c}{CT} (X^2 + 1 - X\sqrt{3}) \quad (26)$$

If $T_c = 1000^\circ K$, $T = 300^\circ K$

$$F = \frac{0.357}{C} (X^2 + 1 - X\sqrt{3})\sigma^2 \quad (27)$$

The quantity in the brackets cannot be made equal to zero for any real value of X. Fig. 3 is a plot of the function vs. X. There is a rather broad minimum for $1/2 < X < 1 1/2$. In this region we find, then, that $F \propto 1/C$. That is, the absolute voltage or current has little effect on the noise figure; and, the coupling C between beam and helix is the critical parameter.

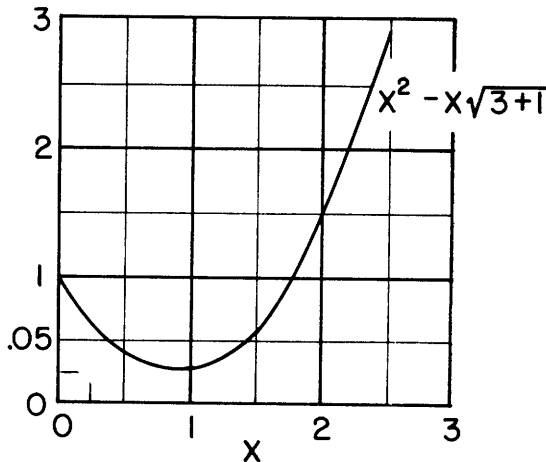


Fig. 3

IV. Conclusion

Since the noise figure of a traveling-wave amplifier turns out to be mainly dependent on the gain parameter C , there is little or nothing to be gained by building tubes to operate at very low voltages. The choice of operating current and voltage becomes a "practical" one; that is, ease of construction, power consumption, magnetic focusing field, and other similar factors are the ones to be considered.

The fact that $F \propto 1/C$ puts a premium upon helix impedance. The impedance can be increased only by reducing the helix diameter. Thus it appears that the best tubes will have small helix and small beam diameters.

The expression for γ^2 , Eq.(15), can obviously assume values greater than unity, which is physically impossible. This is due to the assumptions used by Rack in computing \bar{v}_a^2 , Eq.(12). These assumptions are that the effect of the electrons turned back by the virtual cathode may be neglected (which is nearly true if the current density is large) and that the final (anode) velocity is large compared to the thermal velocities. At the moment, no attempt has been made to extend the analysis to the case of low current densities.

Acknowledgment

The approach to the problem used in this paper was first indicated by J. R. Pierce at the I.R.E. Electron Tube Conference, Princeton, N. J., June 1949.

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