

LIBRARY
OF THE
MASSACHUSETTS INSTITUTE
OF TECHNOLOGY

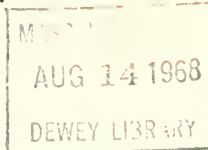
HD28
.M414
no. 339-
68

WORKING PAPER
ALFRED P. SLOAN SCHOOL OF MANAGEMENT

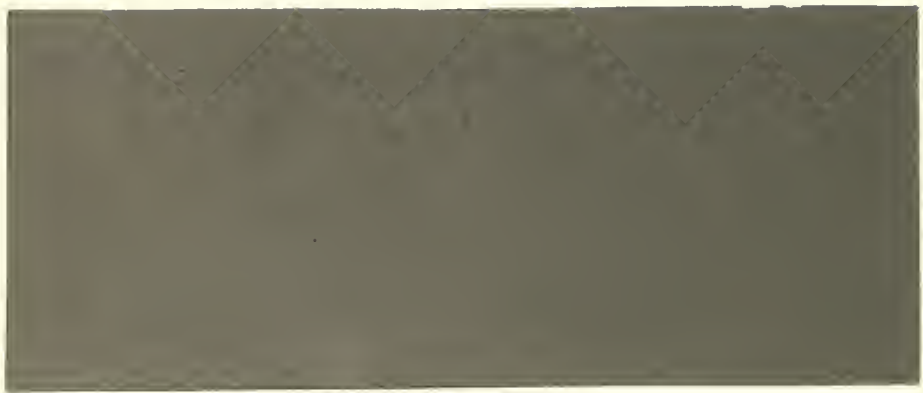
STOCHASTIC CONSUMER MODELS :
SOME EMPIRICAL RESULTS*

339-68

DAVID B. MONTGOMERY**



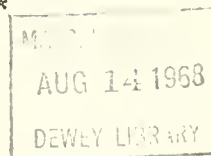
MASSACHUSETTS
INSTITUTE OF TECHNOLOGY
50 MEMORIAL DRIVE
CAMBRIDGE, MASSACHUSETTS 02139



STOCHASTIC CONSUMER MODELS:
SOME EMPIRICAL RESULTS*

339-68

DAVID B. MONTGOMERY**



*Paper presented at the Educators' Conference of the American Marketing Association, Denver, Colorado, August 29, 1968.

**The author is Assistant Professor of Management, Sloan School of Management, Massachusetts Institute of Technology.

FIVE
PAGE
1968

RECEIVED
SEP 4 1968
M. I. T. LIBRARIES

ABSTRACT

Four recent, generalized stochastic models of brand choice are empirically compared. The models are the Brand Loyal and Last Purchase Loyal Markov models, the heterogeneous form of the Linear Learning model, and the Probability Diffusion model. The models are first outlined and then compared with respect to their representation of consumers' brand choice probabilities. Each of the four models is then fitted to each of eight sets of brand choice data. The Learning and Diffusion models were found to be the best, with some advantage for the Diffusion model in the present empirical case.

STOCHASTIC CONSUMER MODELS: SOME EMPIRICAL RESULTS

David B. Montgomery
Sloan School of Management
Massachusetts Institute of Technology

I. Introduction

Since Kuehn's [Kuehn, 1958] pioneering application of the linear learning model to consumer brand choice, there has been considerable activity in the stochastic modeling of brand choice behavior. For a discussion of the many contributions to this area see [Montgomery and Urban, 1969, Chpt. 2].

Recently, several new, generalized stochastic models of brand choice have been developed. The purpose of the present paper is to report an initial empirical comparison of several such models. The models will be outlined in the second section of this paper. Since the models differ from one another in their representation of the structure and dynamics of consumers' brand choice probabilities, the third section of this paper will contrast the models with respect to this representation. Finally, the models will be compared as to their "goodness of fit" to several sets of consumer brand choice data.

2. The Models

The present paper will consider four recent stochastic models of brand choice: the Brand Loyal and Last Purchase Loyal models developed

by Morrison [Morrison, 1966], the heterogeneous linear learning model developed by Massy [Massy, 1967], and the Probability Diffusion model developed by Montgomery [Montgomery, 1966]. A key feature of each of these models is that they explicitly allow for heterogeneity among consumers in terms of the consumers' brand choice probabilities. That is, consumers may differ from one another with respect to their brand choice probabilities.

Each of these models considers a two brand market. Thus a multibrand market must be collapsed into a two brand market, say brands A and B, in order for these models to apply.¹ This is generally accomplished by letting brand A represent a particular brand or the consumer's favorite brand and then letting brand B represent all other brands. The probability that a particular consumer will purchase brand A rather than brand B on any purchase occasion t will be termed this consumer's brand choice probability and will be denoted by $P(A_t)$.

¹The heterogeneous linear learning model may be extended to multi-brand markets. This extension seems reasonably straight forward. The other models present more difficulty. The Brand Loyal and Last Purchase Loyal models cannot be extended without altering their basic structure. The Probability Diffusion model may be so extended at the cost of considerably greater mathematical complexity.

The models considered in this paper are what might be termed purchase incidence models. That is, they only consider $P(A_t)$ and not the timing of purchase occasions. For models which incorporate the phenomenon of interpurchase time, see [Massy, Montgomery, and Morrison, 1969, Chapters 8-11].

Brand Loyal Model

The Brand Loyal (BL) model is a first order Markov model which postulates brand choice probabilities (conditional upon the most recent brand purchase) of the form

$$(1) \quad P(A_{t+1} \mid A_t) = p$$

and

$$(2) \quad P(A_{t+1} \mid B_t) = kp,$$

where $0 < k < 1$.

Notice that the probability that the consumer will purchase brand A at purchase occasion $t+1$ depends upon what brand the consumer purchased at purchase occasion t . Thus, if the consumer purchased brand A at t , his probability of purchasing brand A at $t+1$ will be p . However, if he purchased brand B at t , his probability of purchasing brand A at $t+1$ will be kp , which is less than p . This impact of past brand purchases upon subsequent brand choice probability will be termed purchase event feedback and will be considered further in Section 3.

For further discussion it is convenient to cast the brand choice probabilities into a transition matrix. For the BL model the transition matrix for a given consumer is given by

		Brand Purchased at Occasion t+1	
		A	B
Brand Purchased at Occasion t	A	p	1-p
	B	kp	1-kp

Heterogeneity between consumers is introduced by assuming that p is distributed across the consumer population according to some probability law. In this model, the parameter p is an index of the consumer's brand loyalty. The model is termed the Brand Loyal model because, if p is high for a given consumer, he will be likely to purchase brand A at purchase occasion $t+1$ even if he purchased brand B at purchase occasion t . Thus his loyalty is directed toward a specific brand, brand A.

In the BL model, the parameter k is assumed to be constant for all consumers. Notice that if $k = 1$, the BL model reduces to a model which has no purchase event feedback. In this case the model is a heterogeneous Bernoulli model, the heterogeneity coming from the postulated distribution of p in the consumer population. Thus the heterogeneous Bernoulli model is just a special case of the Brand Loyal model. A similar result holds for the Linear Learning and Probability Diffusion models.

Last Purchase Loyal Model

The Last Purchase Loyal (LPL) model is also a heterogeneous, first order Markov model. In this model the transition matrix of a given consumer is given by

		Brand Purchased at Occasion t+1	
		A	B
Brand Purchased at Occasion t	A	p	1-p
	B	1-kp	kp

where $0 < k < 1$. Again, k is assumed to be constant for all consumers and p is postulated to be distributed across the population of consumers. In this model, if a consumer has a high value of p , he is likely to repeat the purchase of the brand he most recently bought. Thus in the LPL model, a consumer's loyalty is directed toward the brand last purchased.

Linear Learning Model

The Linear Learning (LL) model also assumes that there is purchase event feedback. That is, it assumes that the purchase made at purchase occasion t influences the brand choice probability at purchase occasion $t+1$. In particular, the brand choice probability at $t+1$ is taken to be a linear function of the brand choice probability at t . If brand A was purchased at t , then the probability of the consumer purchasing brand A at $t+1$ is given by

$$(3) \quad P(A_{t+1}) = \alpha + \beta + \lambda P(A_t).$$

However, if brand B was purchased at purchase occasion t , then the brand choice probability at $t+1$ is given by

$$(4) \quad P(A_{t+1}) = \alpha + \lambda P(A_t).$$

The parameters α , β , and λ in equations (3) and (4) must all lie in the zero to one interval. Thus we see that a purchase of brand A at t increases the probability of purchasing brand A at $t+1$ by an amount β over what it would have been if brand B had been purchased at occasion t . It is this increment in the probability of purchasing brand A due to a recent purchase of brand A which is referred to as learning.

In the linear learning model it is assumed that all consumers have the same parameters α , β , and λ . In initial applications of the model it was also assumed that all consumers have the same initial brand choice probability $P(A_1)$ when we first begin to observe them. Even in this case consumers will differ from one another in terms of their brand choice probabilities after one or more purchases, since their actual brand purchases will alter their brand choice probabilities over a sequence of purchase occasions according to Equations (3) and (4). Massy [Massy, 1967] has provided an important generalization of the linear learning model which allows consumers to differ from one another in terms of their initial brand choice probabilities $P(A_1)$. This heterogeneity in $P(A_1)$ is incorporated by postulating a distribution of $P(A_1)$ in the consumer population.

Probability Diffusion Model

The Probability Diffusion (PD) model developed by Montgomery, [Montgomery, 1966] is perhaps best presented by a graphical example. In Figure 1, we have depicted the PD representation of the brand choice probability of two hypothetical consumers over a sequence of five purchase occasions. We see from the figure that consumers are again explicitly considered to differ from one another in terms of their brand choice probabilities. Furthermore, each consumer's brand choice probability may change from purchase occasion to purchase occasion. However, in the PD model, the change in a consumer's brand choice probability does not result from purchase event feedback. That is, the brand a consumer chooses at occasion t does not alter the brand choice probability at $t+1$. The PD model postulates that the change in a consumer's brand choice probability results from other factors such as market conditions or fluctuations in the consumer's brand preferences. This is in direct contrast to the three previous models which, while allowing for changes in a consumer's choice probability between purchase occasions, assume that these changes are totally the result of purchase event feedback i.e. brand experience.

[Insert Figure 1 about here]

3. Contrast of Brand Choice Representation

In this section we consider how each model represents the brand choice probability of individual consumers. Before discussing the models in this context it will be useful to discuss the characteristics we shall use to make this contrast. These characteristics will be

separated into intra-consumer and inter-consumer characteristics.

Intra-Consumer Representation of Brand Choice Probability

The following characteristics relate to the representation of brand choice probability for any given individual consumer.

Non-Stationarity. Non-stationarity refers to whether a consumer's brand choice probability may change between purchase occasions.

Purchase Event Feedback. Purchase event feedback represents the influence which actual brand purchases have upon the brand choice probability. In a zero-order model there will be no purchase event feedback. In a first-order model, the most recent brand purchased influences the brand choice probability for the next purchase, etc.

Values a Consumer's Brand Choice Probability May Attain. This characteristic relates to how many distinct values between zero and one a given consumer's brand choice probability may attain as represented in a model. This characteristic may vary from a single value to an infinite number of alternative values, depending upon the model.

Directions in Which an Individual's Response Probability May Change. This characteristic refers to whether an individual's brand choice probability may increase or decrease between purchase occasions when it begins at any given value. If a model does not allow a consumer's brand choice probability to change between purchase occasions (i.e. it assumes stationarity), then it clearly has a zero value on this characteristic. Models which allow the brand choice probabilities to change (i.e. assume non-stationarity) may have one or at most two feasible directions of change starting at any given value.

Inter-Consumer Representation of Brand Choice Probability

Heterogeneity. As was noted earlier, heterogeneity of brand choice probability in a consumer population means that each consumer is represented in the model as having his own individual brand choice probability which need not be identical to that of any other consumer. This is an extremely important characteristic for a model to possess since if the model ignores heterogeneity between consumers, there is a danger that the model will indicate greater purchase event feedback than may actually exist. For a numerical illustration of this danger see [Montgomery, 1967]. It is the incorporation of consumer heterogeneity that represents one of the most significant contributions of the models discussed in this paper.

Contrast of the Models

Since each of the models discussed in Section 2 allows for heterogeneity of brand choice probability in the consumer population, we will confine our present contrast to the representation of brand choice probability at the level of the individual consumer. The Brand Loyal and Last Purchase Loyal models, both being Markov models, have similar characteristics in terms of their representation of an individual consumer's brand choice probability. Since these characteristics are true in general for Markov models, the present discussion will relate to Markov models in general. Furthermore, since the Bernoulli model is a special case of the Brand Loyal, Linear Learning, and Probability Diffusion models and since it has been of interest (Frank, 1962], it will be included in the present contrasting of models.

In order to make our discussion more concrete, we shall examine how each of the models (Bernoulli, Markov, Linear Learning, and Probability Diffusion) represents the brand choice probability of a hypothetical consumer who has the brand purchase sequence ABB on three successive purchase occasions. The representation of his brand choice probability is given in Figure 2.

[Figure 2 about here]

Bernoulli Model. In the Bernoulli model, an individual consumer retains his initial brand choice probability throughout all his purchases. See Figure 2(a). Thus the Bernoulli model postulates a consumer who has a stationary brand choice probability which experiences no purchase event feedback.

Markov Model. The Markov model allows a consumer's brand choice probability to change between purchase occasions. In this model the changes which occur are due entirely to purchase event feedback, since if we know the brand he purchased at t , we then know his brand choice probability at $t+1$. As indicated in Figure 2(b), the consumer may only have one of two distinct values of brand choice probability. If his most recent purchase was brand A, then this brand choice probability will be p_A . If it was brand B, then his brand choice probability will be p_B . In addition, from any given value the consumer's brand choice probability may only change in one direction. Consider the consumer in Figure 2(6). At his first purchase his brand choice probability was p_A . Since he purchased brand A at this initial purchase occasion, his brand choice probability remained at p_A . However, on the second purchase occasion he purchased brand B. This then reduces his brand choice probability to p_B for the third purchase occasion. Thus when the consumer's brand choice

probability is p_A , it can only remain at p_A or decrease to p_B , depending upon his next brand purchase. Similarly for p_B .

Linear Learning Model. Within the upper and lower limits established for brand choice probability in the Linear Learning model, a consumer's brand choice probability will be increased whenever he purchases brand A and decreased whenever he purchases brand B. This is illustrated in Figure 2(c). Thus the linear learning model also allows for non-stationarity in a consumer's brand choice probability, the non-stationarity being due entirely to purchase event feedback. In fact, the linear learning model assumes that a purchase will always change the brand choice probability. (Exceptions, of course, occur at the lower and upper extreme values established for $P(A_t)$.) We see from the figure that the consumer's brand choice probability changes in discrete increments. Thus the consumer's brand choice probability may only assume one of a finite number of discrete values. In contrast to the Markov model, however, the learning model allows for changes to occur in either direction starting from a particular value $P(A_t)$. Which direction it will actually change depends upon which brand is purchased.

Probability Diffusion Model. The Probability Diffusion model might be termed a zero-order model in that it does not incorporate purchase event feedback. As we see in Figure 2(d), it does, nevertheless, allow for non-stationarity in the consumer's brand choice probability. In contrast to the previous models, the PD model allows a consumer's brand choice probability to assume any value between zero and one. This provides a more appealing representation of brand choice probability than the discrete values in the previous models. As in the Linear Learning model,

a consumer's brand choice probability may change in either direction between successive purchase occasions.

The above discussion is summarized in Table 1.

[Insert Table 1 about here]

4. An Empirical Comparison

In this section we compare the four models in terms of their "goodness of fit" to consumer brand choice data.

The Chi Square Goodness of Fit Measure

Each of the models are fitted to consumer brand choice data using minimum chi square procedures. The BL and LPL models are fitted for the case of an arbitrary distribution of p in the consumer population using a minimum chi square procedure developed by Blanchard and Montgomery [Blanchard and Montgomery, 1968]. Fitting these models under an arbitrary distribution as opposed a specific distribution of p enhances their comparison to the LL and the PD models in terms of their relative goodness of fit.

The chi square goodness of fit procedures yield a X^2 statistic from which we may assess the fit of a model to the data. In general terms, the chi square statistic arises from a function of the form

$$(5) \quad X^2 = \sum_{i=1}^I \frac{[O_i - E_i(\Delta)]^2}{E_i(\Delta)}$$

where $i = 1, \dots, I$ indexes a set of mutually exclusive and collectively exhaustive categories of brand choice responses.

O_i = observed number of consumers purchasing brand A in brand choice response category i

$E_i(\underline{\Delta})$ = the (model generated) expected number of consumers purchasing brand A in brand choice category i. Note that this expected number depends upon the parameter vector $\underline{\Delta}$.

$\underline{\Delta}$ = vector of parameters in the model. The vector will have d elements

The X^2 statistic is distributed as a chi square random variable having I-d-1 degrees of freedom under the hypothesis that the model generated the O_i . It should be noted that the brand choice categories i, the parameter vector $\underline{\Delta}$, and the expected number $E_i(\underline{\Delta})$ are all specific to a particular model. Thus we see that X^2 is the weighted squared deviations of the model's predictions from the actual data, the weights being the reciprocals of the model's predictions. The methods used to develop X^2 for the LL model is given in [Massy, 1967] and the method used for the PD model is given in [Montgomery, 1963].

Since the X^2 statistic has a different number of degrees of freedom for each model, we shall base our comparison upon the p-level of the statistic. The p-level is given by

(6)

$$\text{p-level} = \int_{\hat{X}^2}^{\infty} f[X^2(\text{df for the model})] dX^2$$

where \hat{X}^2 is the observed value of the statistic for a particular model. Thus the p-level tells us the probability of observing deviations of the model from the data at least as large as the ones we've observed when, in fact, the model generated the data. Thus, the closer the p-level is to 1.0, the better the fit of the model. The closer the p-level is to 0.0,

the worse the fit of the model. Since the p-level adjusts for different degrees of freedom for each of the models, it may be used directly in the comparison.

Strictly speaking, the p-levels reported below for the PD model represent upper bounds on the p-level. See [Montgomery, 1968]. While a lower bound may also be established, the large sample size and the degree of consumer heterogeneity in the empirical case reported below make the upper bound representative of the fit of the PD model in this case.

The Empirical Case

The data base for this initial comparison of models is composed of the dentifrice purchase records for January, 1958 to April, 1963 from the M.R.C.A. National Consumer Panel.² These data span the August 1, 1960 endorsement of Crest by the American Dental Association, an endorsement which had considerable market impact.

The data used to compare the models consists of the five brand purchases prior to the endorsement and the five brand purchases subsequent to the endorsement. Thus by using the relatively stable pre-endorsement period and the very unstable post endorsement period, we shall have an opportunity to assess the models in two markedly different market situations.

The population of consumer households was segmented into four groups by average interpurchase time in the before ADA period. Thus we have eight sets of brand choice data (four before and four after) on which to

²The author is indebted to Dr. I.J. Abrams of M.R.C.A. for making these data available at nominal cost and to the Stanford Graduate School of Business for supplying the funds to obtain these data.

compare the results.

The Empirical Results

The results from fitting each model to each of the eight sets of data are reported in Table 2.³ The BL model provides a reasonably good fit in four cases, a moderately good fit in one case, and a very poor fit in three cases. The LPL model, with one exception, provides a very poor fit. Thus the BL model would appear to be the better of the two Markov models in this case. It should be noted that this is similar to Morrison's [Morrison, 1966] conclusions using coffee purchasing data. The LL model provides a good fit in all but two of the cases, while the PD model provides a good fit in all but one case. Examination of the p-levels in Table 2 suggest that the LL and PD models perform the best with respect to our goodness of fit criterion.

[Insert Table 2 about here]

One method of summary comparison of the models is to examine the number of times each model provided the best fit to the data. This result is presented in Table 3 for the Before period, the After period, and overall. From the table we see that in terms of providing the best fit among the four models, the LL model is best in the relatively stable Before period, while the PD model is best in the unstable After period.

[Insert Table 3 about here]

³ The author would like to thank Prof. William F. Massy of Stanford and Prof. Morgan Jones of UCLA who computed the chi square for the Linear Learning model and Mr. Jacques Blanchard who assisted in the computations for the other models.

Another method of overall comparison of the models is to develop summary measures of their performance in both the before and after periods. To illustrate the development of such a measure, consider the groups in the Before period. The X^2 statistic developed for each of these groups will be independent of all the others. Thus, we may form an overall X^2 statistic for the before period by summing the chi square statistics for each of the Before groups. The degrees of freedom for this overall X^2 statistic will just be the sum of the degrees of freedom from each group. A similar procedure was used to develop an overall measure of fit for the After groups. These results are reported in Table 2 as Sum of Before and Sum of After. We see from the table that on this criterion, the PD model provides the best overall fit in both the before and after periods. Note the remarkably good overall fit of the PD model in the unstable after period and the fact that by the present criterion it provides the best overall fit in the before period. The difference between this result and the result in Table 3 for the before period is due to the fact that the overall measures take account of the magnitudes of the deviations of the model from the data, whereas the Table 3 method only uses an ordinal property of the data.

While we must be careful not to over generalize our results from this one comparative study, the following would seem to be justified conclusions in the present case:

- (1) Both the LL and the PD models provided reasonably good fits to the data
- (2) The LPL model did not provide an adequate fit to the data.

The alternative Markov model (BL) was found to be much better.

- (3) The BL model was generally dominated by the LL and PD models. The more flexible representation of brand choice probability in the LL and PD models probably accounts for this.
- (4) The PD model clearly dominated the other models in the transient after period.

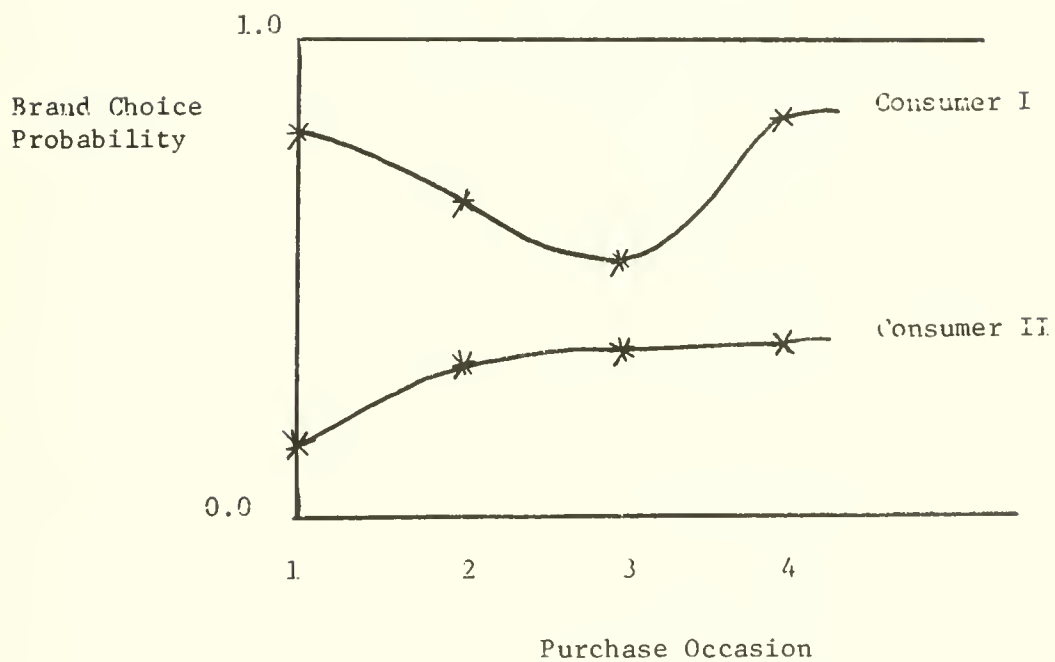
Much more needs to be done by way of comparing models before we will have confidence in our ability to generalize.

In conclusion, we note that even though the LL and PD models differ significantly in their representation of the mechanism whereby a consumer's brand choice probability may change between purchase occasions, both models provide rather good fits to the data. This suggests that both the learning and the diffusion phenomena are important aspects of the brand choice process. What seems needed is a model which incorporates both phenomena.⁴

⁴In a personal communication Prof. Morgan Jones of UCLA reports progress on such a model.

FIGURE 1

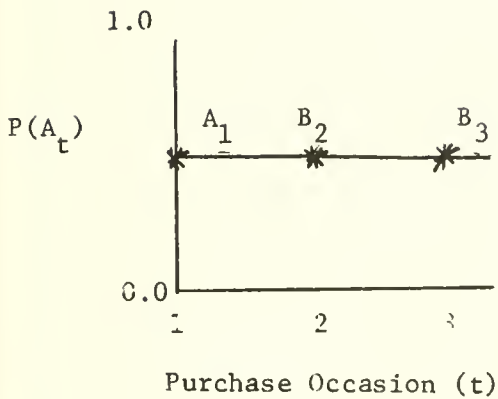
Brand Choice Probability
in the Probability Diffusion Model
Two Hypothetical Consumers



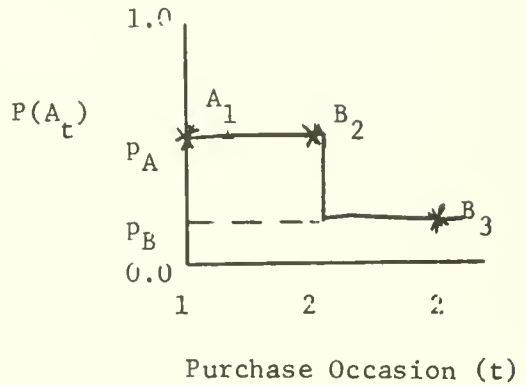
Note: *'s denote purchases of one of the two brands.

Intra-Consumer Representation of
Brand Choice Probability in the Models

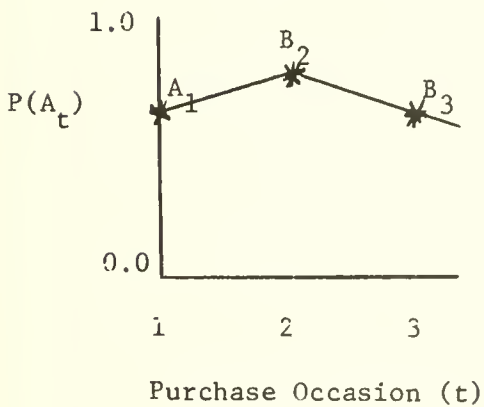
(a) Bernoulli Model



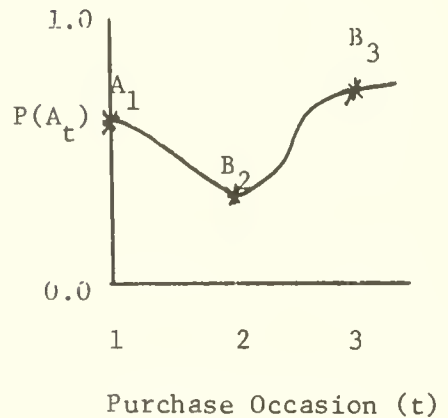
(b) Markov Model



(c) Linear Learning Model



(d) Probability Diffusion Model



Note: *'s denote a purchase occasion. The subscripted A's and B's denote the brand actually purchased at each purchase occasion.

TABLE 1

Brand Choice Probability Representation

Characteristic of Brand Choice Probability	<u>Model</u>			
	Bernoulli	Markov	Learning	Diffusion
Inter Consumer Heterogeneity	YES	YES	YES	YES
Intra Consumer				
Non-Stationary	NO	YES	YES	YES
Purchase Event Feedback	NO	YES	YES	NO
Values it May Attain	ONE	TWO	SEVERAL	INFINITE NUMBER
Directions it May Change	NO CHANGE	ONE	TWO	TWO

TABLE 2

Comparison of p-levels
M.R.C.A. Dentifrice Data

Group*	Sample Size	p-level of model		LL	PD
		BL	LPL		
BEFORE 0-30 DAYS	751	.37	.00	.69	.54
31-45 DAYS	618	.13	.01	.32	.28
46-60 DAYS	503	.30	.00	.03	.04
OVER 60 DAYS	1163	.00	.00	.33	.61
SUM OF BEFORE		.01	.00	.13	.17
AFTER 0-30 DAYS	751	.01	.25	.92	.76
31-45 DAYS	618	.04	.00	.00	.66
46-60 DAYS	503	.33	.00	.60	.94
OVER 60 DAYS	1163	.32	.01	.41	.63
SUM OF AFTER		.00	.00	.15	.95

*The Before and After designates brand purchasing data before and after the A.D.A. endorsement of Crest. The range of days given represents the range of average interpurchase times for households included in each of these groups.

TABLE 3

Best Fitting Model
M.R.C.A. Dentifrice Data

Period	<u>Model</u>			
	BL	LPL	LL	PD
Before	1	0	2	1
After	0	0	1	3
Overall	1	0	3	4

References

1. J. Blanchard and D. B. Montgomery [1968], "On Testing A Class of Markov Response Models", forthcoming working paper, Sloan School of Management, M.I.T.
2. R.E. Frank [1962], "Brand Choice as a Probability Process", Journal of Business, XXXV, pp. 43-56.
3. A. A. Kuehn [1958], "An Analysis of the Dynamics of Consumer Behavior and its Implications for Marketing Management", unpublished Ph.D. dissertation, Graduate School of Industrial Administration, Carnegie - Mellon University.
4. W. F. Massy [1967], "Linear Learning Models of Brand Choice", Working Paper 133, Graduate School of Business, Stanford University.
5. W. F. Massy, D. B. Montgomery, and D.G. Morrison [1969], Stochastic Models of Consumer Behavior (Cambridge, Mass.: The M.I.T. Press), forthcoming.
6. D. B. Montgomery [1966], "A Probability Diffusion Model of Dynamic Market Behavior", Working Paper 205-66, Sloan School of Management, M.I.T.
7. _____ [1967], "Stochastic Modeling of the Consumer", Industrial Management Review, VIII, pp. 31-42.
8. _____ [1968], "A Stochastic Response Model with Application to Brand Choice", Working Paper 238-67, Sloan School of Management, M.I.T.
9. D. B. Montgomery and G. L. Urban [1969], Management Science Models in Marketing, (Englewood Cliffs, New Jersey: Prentice-Hall), forthcoming.
10. D. G. Morrison [1966], "Testing Brand Switching Models", Journal of Marketing Research, III, pp. 401-409.

BAS-10-11

Date Due

~~DEC 31 77~~

DEC 31 77

~~MAY 29 78~~
~~JUN 02 78~~

NOV 7 1955

MIT LIBRARIES



332-68

3 9080 003 904 924

MIT LIBRARIES



334-68

3 9080 003 904 981

MIT LIBRARIES



335-68

3 9080 003 873 962

HD

MIT LIBRARIES



336-68

3 9080 003 873 772

etc

MIT LIBRARIES



337-68

3 9080 003 904 973

MIT LIBRARIES



338-68

3 9080 003 873 921

MIT LIBRARIES



339-68

3 9080 003 905 277

MIT LIBRARIES



340-68

3 9080 003 905 269

MIT LIBRARIES



341-68

3 9080 003 874 309

