

BASEMENT



LIBRARY  
OF THE  
MASSACHUSETTS INSTITUTE  
OF TECHNOLOGY









78  
WORKING PAPER  
ALFRED P. SLOAN SCHOOL OF MANAGEMENT

SAMPLE SIZE REQUIREMENTS FOR ZERO ORDER MODELS

Manohar U. Kalwani  
Massachusetts Institute of Technology

Donald G. Morrison  
Columbia University

WP1025-78

October 1978

MASSACHUSETTS  
INSTITUTE OF TECHNOLOGY  
50 MEMORIAL DRIVE  
CAMBRIDGE, MASSACHUSETTS 02139





SAMPLE SIZE REQUIREMENTS FOR ZERO ORDER MODELS

Manohar U. Kalwani  
Massachusetts Institute of Technology

Donald G. Morrison  
Columbia University

WP1025-78

October 1978



## ABSTRACT

This paper investigates the sensitivity of maximum likelihood estimates with a view to finding out how many individuals are needed and how many purchases are required for each individual to accurately estimate parameters for zero order models. Our results reveal that the estimation of the typically formulated original parameters requires about 2000 individuals with 5 purchases per consumer. In many zero order applications, however, knowledge of market share and loyalty index, which are both functions of the original parameters, should be adequate. Reduced sample sizes of about 400 with 5 purchase records per household are shown to be sufficient to estimate the transformed parameters, the market share and the loyalty index. Our numerical results use the beta distribution as the mixing distribution for the individual  $p$  values; however, the spirit of our results holds for arbitrary mixtures. Namely, much smaller sample sizes are required if we only need to know the location (market share) and shape (loyalty index) of the mixing distribution than if we need detailed knowledge of the original parameters.



## INTRODUCTION

Marketing researchers usually provide only point estimates of model parameters; that is, they do not take into account the effect of sampling variation on these parameter estimates. There are only a few exceptions (e.g., Shoemaker and Staelin [11], Van Mechelen [13]). In both these papers the authors reached the same conclusion, which is that the parameter estimates of the models studied are sensitive and that the coefficients of variation for commonly used sample sizes are large.

Shoemaker and Staelin [11] examined the effects of sampling variation on market share estimates of new consumer products in the Parfitt and Collins model [9]. Their results indicate that the coefficients of variation associated with the prediction of market shares for normally used sample sizes are in the range of 20% to 40%. Further, the sample size of over 2500 that is required for a coefficient of variation of .1 is much greater than that commonly used to estimate market share with the use of Parfitt and Collins model.

Van Mechelen [13] found the estimate of total buyers in a particular period in SPRINTER mod I [12] to be sensitive to small variations in input data. Specifically, he encountered coefficients of variation of over 20% given a small variance of 3% in the input data.

In this paper our objective is to investigate the sensitivity of maximum likelihood estimates in zero order models with a view to determine sample sizes required to estimate parameters within a given accuracy level (say,  $\pm 10\%$  of the true value). Zero order models have been frequently used

to describe brand switching behavior (Bass, Jeuland, and Wright [2], Kalwani and Morrison [6], and Massy, Montgomery, and Morrison [8]). Each consumer in these models is assumed to purchase a given brand (say, Brand 1) with probability,  $p$ , and Brand 0 representing the aggregate "all other" class with the complementary probability,  $1-p$ . These models allow for heterogeneity in the population by letting the  $p$  values differ across individuals. Since each person is a zero order process defined by a single parameter  $p$ , these zero order models are completely defined by  $f(p)$ , the distribution of  $p$  values across individuals in the population. A statistical question, which has received little attention arises: What are the data requirements for accurately estimating  $f(p)$ ? In this paper we address the issue of how many individuals need to be in the sample plus how many trials need to be observed for each individual to accurately estimate  $f(p)$ .

In some empirical settings the exact form of the purchase probability distribution,  $f(p)$ , is of interest and we need to estimate the original parameters of  $f(p)$  (e.g., in this paper this would amount to estimating the original parameters of the beta distribution). Our results reveal that the sample size requirements in such settings are excessive. However, in most applications of zero order models the properties of interest are not the parameters themselves but functions of these parameters like market share and a measure of the loyalty or switching rate (e.g., Hendry switching constant [6], Bass's correlation coefficient [1], and Sabavala and Morrison's loyalty index [10]). Our findings indicate that these transformed parameters - market share and loyalty rate - are "stable", that is, they vary much less than the original parameters.

The sample size requirements are determined for a coefficient of variation of .05. Assuming that the parameter estimates are normally

distributed, this implies that the true parameters are estimated within  $\pm 10$  percent of the true parameter values 95 percent of the time. The sample size requirements at other levels of accuracy can be easily calculated from a knowledge of the sample size required for the 10 percent accuracy level. For instance, sample size will have to be quadrupled to improve the accuracy to 5 percent. The variance of the estimator is inversely proportional to the sample size; hence, a quadrupling of the sample size is needed to reduce the standard deviation by a factor of 2.

Given the aforementioned criterion of  $\pm 10$  percent accuracy level at 95 percent confidence level, the resulting sample sizes required for the estimation of the original, untransformed model parameters exceed 2000 when only 5 purchase records are available for every household. These sample size requirements are much larger than most researchers' intuitions would indicate and have not been met in most published studies. When the transformed, more stable parameters (market share and loyalty rate) are estimated, however, sample size requirements given 5 purchases per consumer are less than 400 for the U shaped purchase probability distribution typically encountered in empirical research.

The remainder of this paper is organized as follows: first we discuss the use of market share and loyalty rate as measures of market response in applications of zero order models. This is followed by a presentation of the overall methodology including the likelihood expressions for estimation of the original as well as the transformed parameters. Next, the findings from the simulated data are reported. Finally, the results of the paper are summarized in the concluding section.

MEASURING MARKET RESPONSE

As mentioned earlier, knowledge of the purchase probability distribution completely defines a zero order model. That is, various response measures like market share, repeat-purchase and switching probabilities--conditional as well as unconditional--can be easily obtained given the exact form of the purchase probability distribution. In most applications of zero order models, however, the marketing researcher may only be interested in obtaining the brand share and a measure of brand loyalty (e.g., Hendry switching constant [6], Bass's correlation coefficient [1], and Sabavala and Morrison's loyalty index [10]). It turns out that, assuming beta heterogeneity on p, other response measures like conditional and unconditional switching as well as repeat-purchase probabilities can be easily obtained from a knowledge of the market shares and a measure of brand loyalty.

The beta distribution due to its flexibility to take different shapes and mathematical tractability is often used to represent the functional form of the purchase probability distribution. The beta distribution takes bell, U, J, or reverse-J shape according to the values of its parameters,  $\alpha$  and  $\beta$ . The functional form of the beta distribution is given by

$$f(p) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\alpha-1}(1-p)^{\beta-1}, \quad \text{for } 0 < p < 1, \alpha, \beta > 0, \quad (1)$$

where  $\Gamma$  denotes the gamma function. The mean and variance of the beta distribution are



$$E [p] = \frac{\alpha}{\alpha+\beta} ,$$

$$\text{and, } \text{VAR}[p] = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$$

The unconditional probability of repeat buying Brand 1 is given by

$$P(1,1) = \int_0^1 p^2 f(p) dp = \frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)} . \quad (2)$$

The unconditional probability of switching from Brand 0 to Brand 1 is given by

$$P(0,1) = \int_0^1 (1-p) p f(p) dp = \frac{\alpha\beta}{(\alpha+\beta)(\alpha+\beta+1)} . \quad (3)$$

The conditional probabilities  $P(1|1)$  and  $P(0|1)$  can be obtained by dividing the two unconditional probabilities in equations (2) and (3) by market share of Brand 1 which for a beta heterogeneity is  $\alpha/\alpha+\beta$  . In essence, then, switching and repeat purchase probabilities can be easily obtained from a knowledge of the parameters of the mixing beta distribution,  $\alpha$  and  $\beta$ . In the remainder of this section, we show that the conditional as well as the unconditional switching and repeat purchase probabilities can be easily obtained from a knowledge of the market share and loyalty index.

Kalwani and Morrison [6] show that an essential property of the Hendry System is that switching between two brands  $i$  and  $j$  will be proportional to their shares  $S_i$  and  $S_j$ , that is

$$P(i,j) = K_w S_i S_j, \quad j \neq i, \quad (4)$$

where  $K_w$  is independent of  $i$  and  $j$ . From equation (4), it is easy to show that the repeat purchase probability is given by

$$P(i,i) = S_i - \sum_{j \neq i} P(i,j) = S_i - K_W S_i (1 - S_i) \quad (5)$$

The conditional probabilities of purchasing brand  $i$  on the second purchase occasion are given by

$$P(i|j) = \begin{cases} K_W S_i & j \neq i, \\ 1 - K_W (1 - S_i) & j = i. \end{cases} \quad (6)$$

It is not difficult to show that if the mixing distribution is Dirichlet (multivariate extension of beta) then equation (4) will hold (see Bass, Jeuland and Wright [2]). For the case of beta heterogeneity, Kalwani and Morrison [6] find that

$$K_W = \frac{\alpha + \beta}{\alpha + \beta + 1}, \quad (7)$$

and further, relate the Hendry switching constant to Bass's correlation coefficient [1]

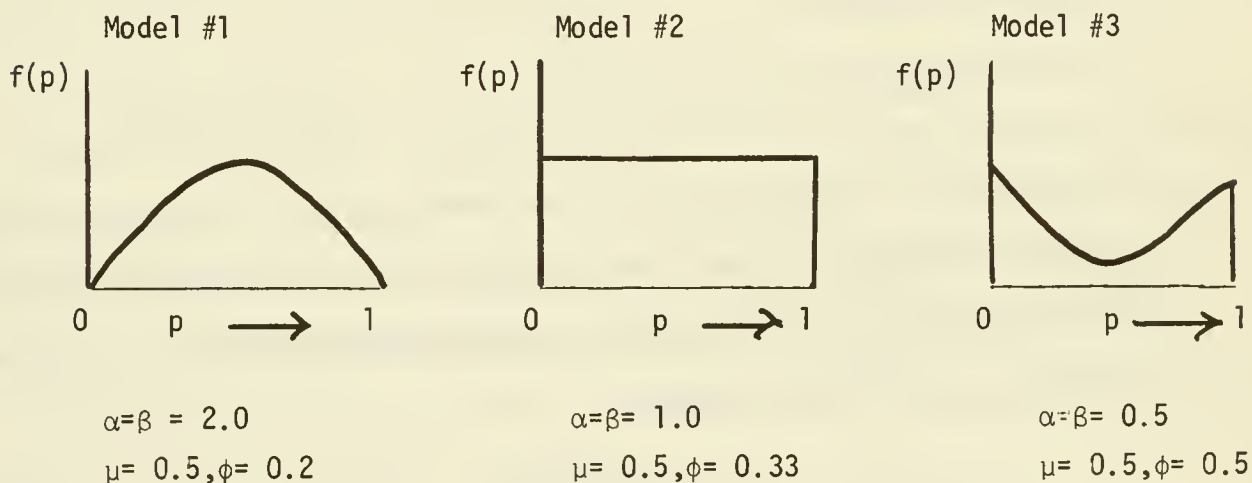
$$(1 - K_W) = \rho = \left( \frac{1}{\alpha + \beta + 1} \right), \quad (8)$$

where  $\rho$  is the correlation of successive purchases of a brand. Sabavala and Morrison [10] suggest the use of  $\phi = 1/(\alpha + \beta + 1)$  as a measure of loyalty and term it the Loyalty (or Polarization) Index. They state that  $\phi$  is a measure of the strength of preference for and against a brand. A purchase probability distribution concentrated at the extremes,  $p = 0$  and  $p = 1$ , will have a high value of  $\phi$ .

In summary, then, a marketing researcher may wish to estimate  $\phi$  (or  $\rho$ , or  $K_W$ ) as a measure of loyalty (or switching) rate. Along with market share, denoted

by  $\mu$ ),  $\phi$  provides estimates of switching and repeat purchase probabilities as shown in equations (4) through (6).

In this paper, we determine the sensitivity of maximum likelihood estimates of zero order models using  $\alpha, \beta$  and  $\mu, \phi$  parameterizations. We perform the sensitivity analysis for three zero order models which cover different shapes of the purchase probability distribution, namely, bell, uniform, and U. These three models are displayed in Figure 1 along with true values of the original parameters  $\alpha, \beta$  and the transformed parameters  $\mu, \phi$ .



METHODOLOGY

In this section, we first present the beta-binomial distribution to develop an expression for the unconditional probability of  $j$  purchases of Brand 1 on  $k$  trials. Note that the actual purchase records of consumers are assumed to be organized in a frequency distribution form giving the number of households,  $N_j$ , who make  $j$  purchases of Brand 1 on  $k$  purchase occasions. Given this data and the expression for unconditional probability of  $j$  successes on  $k$  trials, we develop the likelihood expressions with  $\alpha, \beta$  and  $\mu, \phi$  parameterizations. We conclude this section by presenting our data simulation procedure.

### Beta-Binomial Distribution

Following the representation of zero order models in the previous section, for a consumer who has a purchase probability  $p$  of buying Brand 1, the number of purchases,  $j$ , of Brand 1 on  $k$  trials is distributed binomial with parameters  $p$  and  $k$ . That is,

$$P(j;p;k) = \binom{k}{j} p^j (1-p)^{k-j}, \quad 0 \leq j \leq k.$$

As indicated earlier the probability of purchasing Brand 1 is allowed to vary over the population of consumers according to the beta distribution with parameters  $\alpha$  and  $\beta$ .

$$f(p;\alpha,\beta) = p^{\alpha-1} (1-p)^{\beta-1} / B(\alpha,\beta), \quad 0 < p < 1, \text{ and } \alpha, \beta > 0,$$

where  $B(\alpha,\beta) = \Gamma(\alpha)\Gamma(\beta)/\Gamma(\alpha+\beta)$  is the beta function. The marginal distribution of  $j$  purchases of Brand 1 on  $k$  trials is obtained by compounding the binomial and beta distributions. The unconditional probability of  $j$  purchases of Brand 1 on  $k$  trials is given by

$$P(j;k,\alpha,\beta) = \binom{k}{j} \frac{B(\alpha+j,\beta+k-j)}{B(\alpha,\beta)}, \quad j = 0, 1, \dots, k. \quad (9)$$

This is the beta-binomial distribution whose mean and variance are given by

$$E[j] = k \frac{\alpha}{\alpha+\beta},$$

$$\text{and } \text{VAR}[j] = \frac{k\alpha\beta(\alpha+\beta+k)}{(\alpha+\beta)^2(\alpha+\beta+1)}.$$

The expression for the unconditional probability of  $j$  successes on  $k$  trials in equation (9) can also be written in terms of the transformed parameters

$\mu$  and  $\phi$  by making the substitutions for  $\alpha$  and  $\beta$  as follows:

$$\alpha = \frac{\mu(1-\phi)}{\phi}, \text{ and } \beta = \frac{(1-\mu)(1-\phi)}{\phi} \quad (10)$$

### Parameter Estimation

Given the purchase frequency data (i.e.,  $N_j$ 's) the likelihood function can be written as

$$\ell(N_0, N_1, \dots, N_k; \alpha, \beta) = \frac{N!}{N_0! N_1! \dots N_k!} (P_0)^{N_0} (P_1)^{N_1} \dots (P_k)^{N_k}, \quad (11)$$

where the term  $P_j$  represents the probability of  $j$  purchases of Brand 1 on  $k$  trials and its value is given in equation (9). The maximum likelihood estimates of  $\alpha$  and  $\beta$  can be obtained by maximizing the above likelihood expression with respect to  $\alpha$  and  $\beta$ . Since maximizing a monotonic transform of  $\ell(\cdot)$  does not change the values of the maximum likelihood estimates, the constant term in  $\ell(\cdot)$  is replaced by unity and for computational convenience the logarithm of the altered likelihood function is maximized. The transformed log-likelihood function  $L(\cdot)$ , is given by

$$L(N_0, N_1, \dots, N_k; \alpha, \beta, k) = \log \left\{ (P_0)^{N_0} (P_1)^{N_1} \dots (P_k)^{N_k} \right\} = \sum_{j=0}^k N_j \log P_j \quad (12)$$

Substituting for  $P_j$  from equation (9) and expressing the beta functions in terms of the gamma functions, the above expression simplifies to the following log-likelihood function

$$L(N_0, N_1, \dots, N_k; \alpha, \beta, k) = \sum_{j=0}^k N_j \left[ \sum_{r=0}^{j-1} \log(\alpha+r) + \sum_{r=0}^{k-j-1} \log(\beta+r) - \sum_{r=0}^{k-1} \log(\alpha+\beta+r) \right]. \quad (13)$$

Substituting for  $\alpha$  and  $\beta$  in terms of  $\mu$  and  $\phi$ , the above log-likelihood function can be rewritten as

$$L(N_0, N_1, \dots, N_k; \mu, \phi, k) = \sum_{j=0}^k N_j \left[ \sum_{r=0}^{j-1} \log(\mu(1-\phi)+r\phi) + \sum_{r=0}^{k-j-1} \log((1-\mu)(1-\phi)+r\phi) - \sum_{r=0}^{k-1} \log(1-\phi+r\phi) \right]. \quad (14)$$

The maximum likelihood estimates of  $\mu$  and  $\phi$  can now be found by maximizing the log-likelihood function in equation (14) with respect to  $\mu$  and  $\phi$ .

The likelihood functions  $L(\cdot)$  given in equations (13) and (14) are quite complex and closed form analytical solutions are not available. Therefore numerical optimization is used to determine the maximum likelihood estimates of the parameters. The computer program used for this purpose, namely, "modified pattern search," is based on the pattern search procedure developed by Hookes and Jeeves [3]. The program provides a general optimization procedure for any function with a vector of  $n$  parameters. It was developed for Kalwani's doctoral dissertation and is explained in detail in Kalwani [4].

### Simulated Data Generation

Simulated data are used to determine sample size and purchase sequence length requirements for estimating the model parameters. The first step in the procedure is to generate 50 samples of size  $N$ --equal to 100, 300, or 500--

and a purchase sequence length  $k$ --equal to 5, 10, or 20. In other words, for each of the three zero order models displayed in Figure 1, 50 samples are generated for nine (3 values of  $N$  times 3 values of  $k$ ) different sample specifications starting with purchase sequence length of 5 with sample size of 100 and ending with purchase sequence length of 20 with sample size of 500.

The output of each sample specification (say, sample size = 300 and purchase sequence length = 10) for each of the 50 simulations is a purchase frequency distribution which gives the number of consumers,  $N_j$ , who make  $j$  (where  $j = 0, 1, \dots, k$ ) purchases of Brand 1 on  $k$  choice occasions.

The second step in the procedure involves the estimation of the model parameters--  $\alpha, \beta$  or  $\mu, \phi$  --for each of the 50 simulated purchase frequency distributions. Next the means, standard deviations, and coefficients of variation of the maximum likelihood estimates from the 50 simulations are computed. This second step is implemented for each of the nine  $N, k$  sample specifications.

### FINDINGS

This section contains the findings from the Monte Carlo simulation runs for each of the three zero order models displayed in Figure 1. The results reported here cover three purchase sequence lengths--5, 10, and 20. Note that for many frequently bought grocery and household items, 5 purchases, for many households, would cover a quarter, 10 would extend over half a year, and 20 would span a year. The final sample size requirements for each of these three purchase sequence lengths are based on combining our findings on sample size requirements using sample sizes of 100, 300, and 500.

Since the U shaped purchase probability distribution has been found to fit empirical data well (see Kalwani and Morrison [7]), we use Model #3 (see Figure 1) to illustrate our findings. Table 1 displays coefficients of variation associated with estimation of the parameters of Model #3. In the  $\alpha, \beta$  parameterization the two parameters have equal stability and the coefficients of variation displayed in Table 1 are averages of those for  $\alpha$  and  $\beta$ . In case of the  $\mu, \phi$  parameterization the parameter  $\mu$  is more stable than the parameter  $\phi$ . The coefficients of variation of the parameter  $\phi$  displayed under the  $\mu, \phi$  parameterization are about four-thirds of the coefficients of variation of the  $\mu$  parameter. Therefore, it is the  $\phi$  parameter which determines the sample size requirements and Table 1 contains coefficients of variation of  $\phi$ .

-----  
INSERT TABLE 1 HERE  
-----

The coefficients of variation displayed in Table 1 as well as those for the other two models were found to be consistent with the "inverse square root of n relationship" which provides a check on the reliability of the modified pattern search program for obtaining the estimates of the model parameters.<sup>1</sup> That is, the coefficient of variation obtained for a sample size n times larger than another sample is  $(1/\sqrt{n})$  times the coefficient of variation for the latter sample.

The "inverse square root of n relationship" can be used to obtain the sample sizes required for any desired level of accuracy in parameter estimation.<sup>2</sup> Table 2 displays the sample sizes required for various models when the purchase sequence lengths of the simulated samples are 5, 10 and 20.



These sample sizes have been obtained for a coefficient of variation of .05. The implication of selecting this ratio as 5%, is that 95% of the parameter estimates can be expected to fall within 10% of the true parameter value (i.e., in the range  $\pm 0.1\%$ ).

-----  
INSERT TABLE 2 HERE  
-----

An examination of the findings displayed in Table 2 reveals that it is easier to estimate parameters for a U shape purchase probability distribution (Model #3) than for a uniform (Model #2) or bell shape (Model #1). This result holds good across both the  $\alpha, \beta$  and  $\mu, \phi$  parameterizations.

More importantly, however, the results displayed in Table 2 reveal that the sample sizes required for estimating the original parameters  $\alpha$  and  $\beta$  within  $\pm 10\%$  of their true values are excessive. Even in case of the "easiest to estimate" U shape purchase probability distribution a panel size of about 2000 is required given 5 purchases per each household. On the other hand, the sample size requirements are considerably smaller in case of the  $\mu, \phi$  parameterization. This indicates that the transformed parameters  $\mu$  and  $\phi$  are more stable than the original parameters  $\alpha$  and  $\beta$ . Therefore, in applications where knowledge of the market share,  $\mu$  and the loyalty index (or Hendry switching constant, Bass's correlation coefficient) would suffice, it is much more efficient to estimate them directly rather than computing them indirectly from estimates of  $\alpha$  and  $\beta$ .

Many researchers in the past have used sample data with purchase sequence lengths of 5 or less when estimating the original parameters of the purchase probability distribution. To the extent that inadequate sample sizes have been used to estimate model parameters, the results ob-

tained in such models are suspect. Note, however, that in case of Model #3 a sample size of 400 with 5 purchase records per household is sufficient to estimate the transformed parameters  $\mu$  and  $\phi$  within  $\pm 10\%$ .

Two cautionary comments are in order. Since maximum likelihood estimates are invariant, we can obtain maximum likelihood estimates of the original parameters  $\alpha$  and  $\beta$  from maximum likelihood estimates of the transformed parameters  $\mu$  and  $\phi$  (see equation (10)). Note, however, that the accuracy levels of the estimates of  $\alpha$  and  $\beta$  will in general be different from those of the transformed parameters  $\mu$  and  $\phi$ . Specifically, while reduced sample sizes are needed to estimate  $\mu$  and  $\phi$  (say, within  $\pm 10\%$ ) by searching in the  $\mu, \phi$  space, the estimates of  $\alpha, \beta$  obtained therefrom will be less accurate than estimates of  $\mu$  and  $\phi$ . Therefore, in zero order applications where estimates of  $\alpha$  and  $\beta$  are needed within  $\pm 10\%$ , the prior search in the  $\mu, \phi$  space does not help in reducing the data requirements: the results displayed in Table 2 under the  $\alpha, \beta$  parameterization still hold good and indicate the sample size requirements.

Our second caveat is in connection with the effect of the market share of the brand under consideration namely, Brand 1, on the sample size requirements. We have obtained data requirements for three different zero order models keeping the brand share constant at 50%. Obviously, the sample size requirements will be larger for smaller shares of Brand 1. Our findings elsewhere confirm this intuitively appealing proposition (Kalwani and Morrison [5]). This increase in sample size requirements for the three purchase sequence lengths is small - around 10% to 20% - for the initial reduction in share of Brand 1 from 50% to 25%. However, as the share of Brand 1 reduces to around 12%, the increase in sample size requirement is around 50% to 100%. The reader is referred to Kalwani and Morrison [5] for further details.

## CONCLUSIONS

In this paper, the sensitivity of maximum likelihood estimates in zero order models was investigated with a view to determine sample sizes required to estimate parameters within a given accuracy level (within  $\pm 10\%$  of the true value). We used simulated data from bell, uniform, and U shaped purchase probability distributions to determine sample size requirements for three purchase sequence lengths, namely 5, 10, and 20. Our primary conclusion is that the maximum likelihood estimates of the original parameters of the purchase probability distribution are sensitive and that the coefficients of variation for commonly used sample sizes are large. Specifically, our findings reveal that for the U shaped purchase probability distribution which is the model that fits empirical data well, the sample size required with a purchase sequence length of 5 is around 2000.

In many zero order applications, however, knowledge of the market share and loyalty index (or Hendry switching constant, or Bass's correlation coefficient) may suffice. We demonstrated that, assuming zero order process and beta heterogeneity, conditional as well as unconditional switching and repeat purchase probability can be easily obtained from a knowledge of the market share and loyalty index. From a statistical viewpoint we found that it is much more efficient to estimate these two transformed parameters rather than the original parameters. Specifically, for the U shaped purchase probability distribution sample size of 400 given 5 purchases per household was found to be adequate to estimate these two transformed parameters within 10% of their true value.

FOOTNOTES

1. The reliability of the numerical optimization program is crucial to the accuracy of sample size requirements developed in this paper. It should be pointed out that the modified pattern search program has been checked very thoroughly at both Columbia University and M.I.T. Several tests were carried out to measure the magnitude of the numerical error in our computer program. The number of consumers who make  $j$  purchases of Brand 1 and  $(k-j)$  purchases of Brand 0 were obtained theoretically using equation (9). This was done for a variety of parameter values and sample sizes. The modified pattern search program was then used to estimate the known parameter values which were used to generate the theoretical data in the first place. The true parameter values were reproduced within an accuracy of 1 in 10,000.
2. To illustrate the computation of sample size requirements in Table 2, consider the entry for purchase sequence length of 5 in case of Model #3 with  $\alpha, \beta$  parameterization. The sample size requirement of 2005 represents an average based on three figures from Table 1 as shown below:

$$\frac{1}{3} \left( \frac{.214}{.05} \right)^2 (100) + \frac{1}{3} \left( \frac{.134}{.05} \right)^2 (300) + \frac{1}{3} \left( \frac{.101}{.05} \right)^2 (500) = 2005.$$

Table 1

COEFFICIENTS OF VARIATION\* FOR DIFFERENT SAMPLE SPECIFICATIONS OF MODEL #3

SAMPLE SIZE	Parameterization					
	$\alpha, \beta$			$\mu, \phi$		
	Purchase Sequence Length			Purchase Sequence Length		
	5	10	20	5	10	20
100	.214	.180	.194	.095	.076	.077
300	.134	.107	.086	.059	.047	.037
500	.101	.081	.075	.045	.037	.032

\*In case of the  $\alpha, \beta$  parameterization the numbers in the table represent an average of the coefficient of variations of the estimates of  $\alpha$  and  $\beta$ . In case of the  $\mu, \phi$  parameterization, since the parameter  $\phi$  is less stable than the parameter  $\mu$ , the numbers in the table represent coefficient of variation of the estimate of  $\phi$ .

Table 2

SAMPLE SIZES REQUIRED FOR MEETING  
THE ACCURACY CRITERION OF  $\pm 10\%$

Model *	Parameterization					
	$\alpha, \beta$			$\mu, \phi$		
	Purchase Sequence Length			Purchase Sequence Length		
	5	10	20	5	10	20
Model #1	3563	2865	1496	1822	1332	708
Model #2	2626	1795	1158	903	627	371
Model #3	2005	1328	1176	394	254	201

\* For Model #1:  $\alpha=\beta=2$ ; for Model #2:  $\alpha=\beta=1.0$ ; for Model #3:  $\alpha=\beta=0.5$ .

REFERENCES

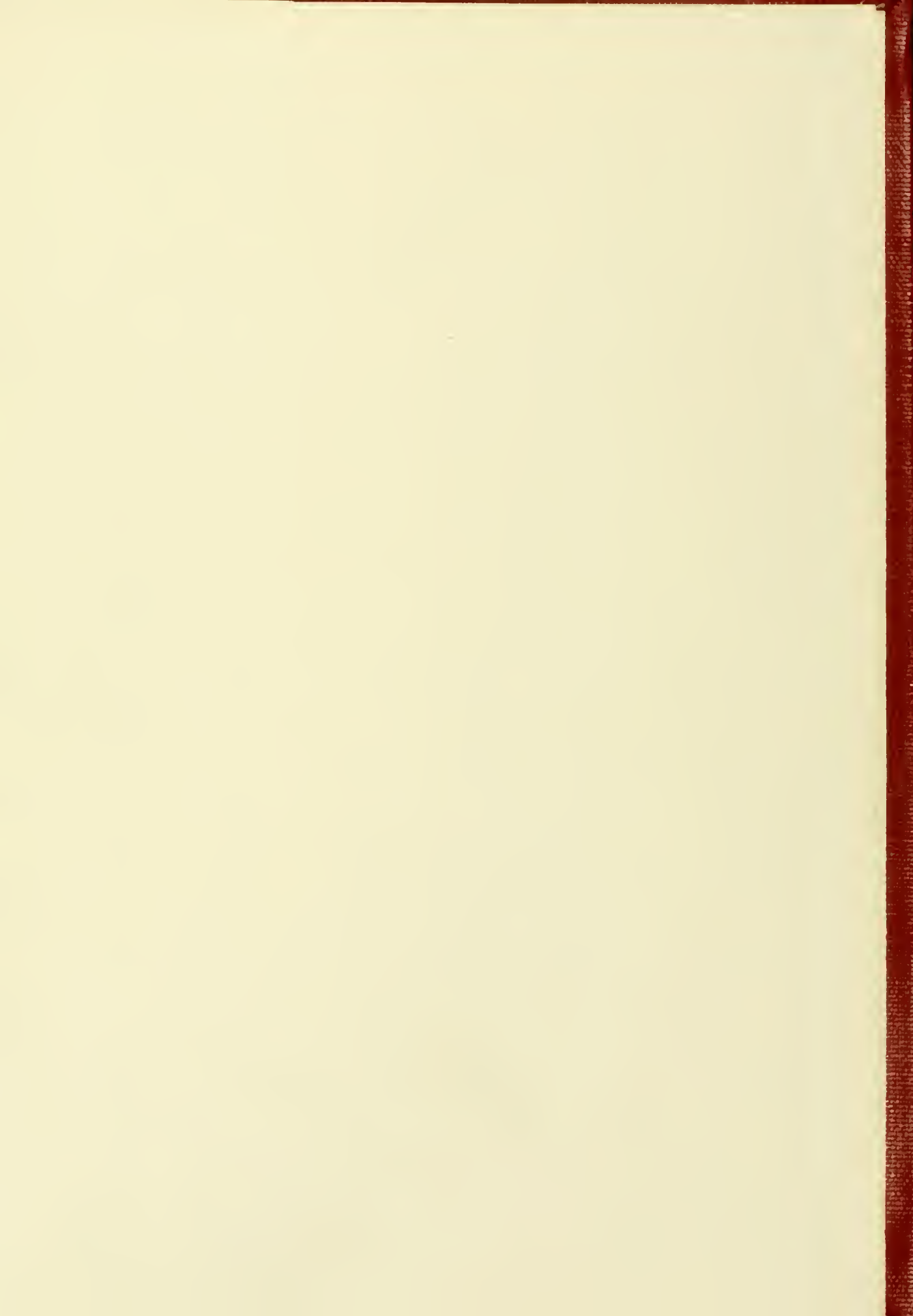
1. Bass, Frank M., "The Theory of Stochastic Preference and Brand Switching," Journal of Marketing Research, 11 (February 1974), pp. 1-20.
2. Bass, Frank M., Abel Jeuland, and Gordon P. Wright, "Equilibrium Stochastic Choice and Market Penetration Theories: Derivations and Comparisons," Management Science, 22 (June 1976), pp. 1051-63.
3. Hookes, Robert and T.A. Jeeves, "Direct Search Solution of Numerical and Statistical Problems," Journal of the Association of Computing Machinery, 8 (
4. Kalwani, M.U., "Zero Order Models: Specification Data Requirements and Estimation," Unpublished Ph.D. dissertation, Graduate School of Business, Columbia University, 1975.
5. Kalwani, M.U. and Donald G. Morrison, "Sample Size and Number of Trials Required for Parameter Estimation in Zero Order Binary Models," M.I.T., Sloan School of Management Working Paper No. 885.76, December 1976.
6. Kalwani, M.U. and Donald G. Morrison, "A Parsimonious Description of the Hendry System," Management Science, 23 (January 1977), pp. 499-509.
7. Kalwani, M.U. and Donald G. Morrison, "Estimating the Proportion of 'Always Buy' and 'Never Buy' Consumers: A Likelihood Ratio Test with Sample Size Implications," Journal of Marketing Research, 14 (November 1977), pp. 601-6.
8. Massy, W.F., D.B. Montgomery, and D.G. Morrison, Stochastic Models of Buyer Behavior, M.I.T. Press: Cambridge, MA 1970.
9. Parfitt, J.H. and B.J.K. Collins, "The Use of Consumer Panels for Brand Share Prediction," Journal of Marketing Research, 6, (November 1968), pp. 405-12.
10. Sabavala, Darius and Donald G. Morrison, "A Model of TV Show Loyalty," Journal of Advertising Research, 17 (December 1977), pp. 35-43.
11. Shoemaker, Robert and Richard Staelin, "The Effects of Sampling Variation on Sales Forecasts for New Consumer Products," Journal of Marketing Research, 13 (May 1976), pp. 138-43.

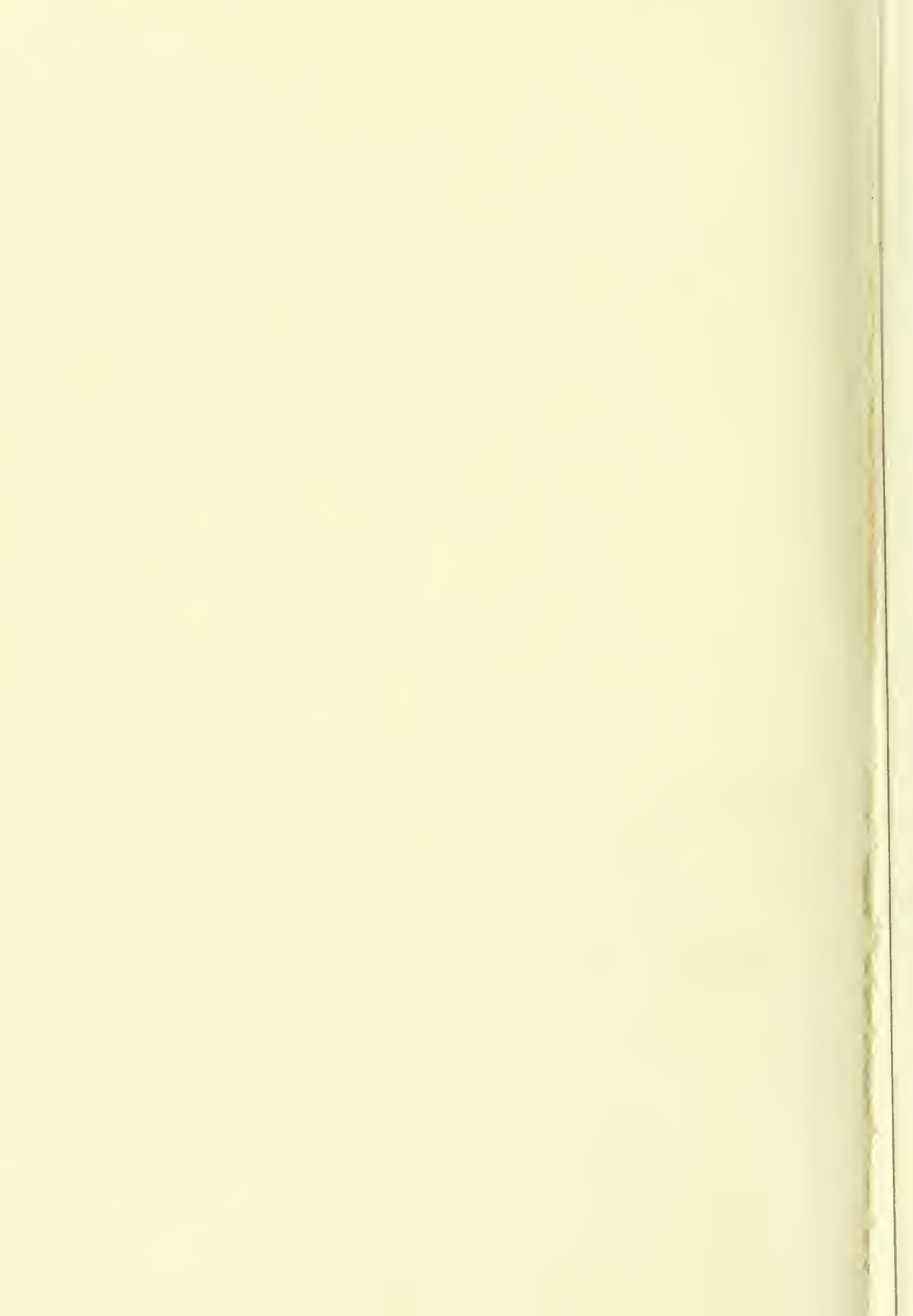
12. Urban, Glen L., "SPRINTER Mod I: A Basic New Product Analysis Model," Proceedings, National Conference of the American Marketing Association (1969), pp. 139-50.
13. Van Mechelen, Claude J., "Measuring the Sensitivity of SPRINTER Mod I," Working Paper 75-15, Center for Managerial Economics and Econometrics University Sait Ignace, University of Antwerp (December 1975).











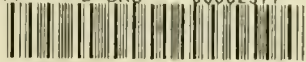


Date Due BASEMENT

<del>DEC 10 '79</del>		
APR 1 1980		
APR 08 1980		
DEC 22 '80	JUL 13 '88	
FEB 1 1981	JAN 16 1989	
MAR 22 '81		
NOV 27 '81		
MAY 03 '83		
MAY 11 '88		
JUN 15 '88		

Lib-26-67

HD28.M414 no.1020- 78  
Mass, Nathanie/Monetary and real cause  
735644 D\*BKS 00062377



3 9080 001 159 885

HD28.M414 no.1021- 78  
Serghiou, Serg/A theoretical model for  
735643 D\*BKS 00062378



3 9080 001 159 901

HD28.M414 no.1022- 78  
Kobrin, Stephe/Firm and industry facto  
735640 D\*BKS 00070034



3 9080 001 248 530

HD28.M414 no.1023- 78  
Beckhard, Rich/Explorations on the tea  
822509 D\*BKS 00060868



3 9080 001 141 461

HD28.M414 no.1024- 78  
Beckhard, Rich/Core content of the tea  
822510 D\*BKS 00060869



3 9080 001 141 487

HD28.M414 no.1025- 78  
Kalwani, Manoh/Sample size requirement  
735639 D\*BKS 00062374



3 9080 001 159 836

HD28.M414 no.1026- 78  
Kalwani, Manoh/Estimating zero order m  
736029 D\*BKS 00069326



3 9080 001 240 891

HD28.M414 no.1027- 78  
Schmalensee, R/On the use of economic  
822500 D\*BKS 00060870



3 9080 001 141 495

HD28.M414 no.1028- 78  
Zannetos, Zeno/Intelligent information  
735636 D\*BKS 00062379



3 9080 001 159 927

HD28.M414 no.1029- 78  
Katz, Ralph. /Time and work : towards  
735635 D\*BKS 00062376



3 9080 001 159 869

