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Abstract

Why do shareholders vote for anti-takeover devices which apparently lower the value of their firm? We address this question by constructing an agenda-setting model in which rational, informed, and value-maximizing shareholders vote on requests for such devices made by a self-interested management with employment opportunities outside the firm. We find sufficient conditions for the value of the firm to decline as a result of a request, although it is approved by shareholders. In our model, the apparently paradoxical voting behavior occurs because the expected takeover premium is reduced more by rejection of the request than by approval.
1. Introduction

Why do shareholders vote for anti-takeover devices which apparently lower the value of their firm? We address this question by constructing a model in which rational, fully informed, and value-maximizing shareholders vote on requests for such devices made by a self-interested management with employment opportunities outside the firm. We describe conditions under which the value of the firm declines as a result of the request, although it is approved by shareholders. In our model, the apparently paradoxical voting behavior occurs because the expected takeover premium is reduced more by rejection of the request than by approval.

A large and increasing number of amendments to corporate charters are specifically designed to increase the cost of transferring control. DeAngelo and Rice (1983) report over 250 proposed amendments in 1974 through 1979. Linn and McConnell (1983) find generally increasing incidence of such proposals among NYSE firms over the period from 1960 to 1980. The Investor Responsibility Research Center lists over 200 in 1985 alone. The two major hypotheses describing the motives for instituting anti-takeover charter amendments are commonly described as "management entrenchment" and "shareholder interests." According to the management entrenchment view, incumbents are interested in job security and seek protection from the takeover market, to the detriment of shareholders. Two suggested explanations for shareholder approval of entrenching antitakeover devices are: (1) for a majority of shareholders, the costs of becoming informed about the effects of defensive charter amendments exceed any potential benefits, and uninformed shareholders consistently give their proxies to management; and (2) large shareholders wish to maintain friendly relations with management to ensure the benefits of future business, and large shareholders control sufficient shares to be pivotal in the vote. Shareholder irrationality is sometimes offered as a third alternative.

The stockholder interests hypothesis recognizes a free-rider problem in collective action by
shareholders (Grossman and Hart (1980), Jarrell and Bradley (1980)). Shareholders have difficulty colluding to extract larger premia from takeover bidders, so anti-takeover devices benefit shareholders by enforcing a level of collusion in takeover negotiations. Since defensive charter amendments benefit shareholders, there is no inconsistency in rational shareholders voting for them. However, there is little evidence that shareholders benefit from such amendments. DeAngelo and Rice (1983) find statistically insignificant negative abnormal returns around the public announcement of proposed antitakeover amendments. Linn and McConnell (1983) find positive returns around the board meeting date at which amendments are proposed, and insignificant negative returns around the proxy mailing date. Jarrell, Poulsen and Davidson (1985) find negative returns accompanying the announcement of "shark repellents", viz. supermajority, classified board and authorized preferred amendments.

In the context of our model, the management entrenchment and shareholder interests hypotheses do not necessarily lead to different predictions about shareholder value. Anticipatory takeover defenses raise the costs of acquiring control of the firm, thereby entrenching management, but lead to a higher premium for shareholders if a bid succeeds. We presume that different potential managers contribute different value to a given firm, by virtue of skill or experience. Only managers capable of producing higher value than the incumbent can mount successful takeover bids. Whether shareholders gain or lose as a result of the incumbent's defenses depends upon the quality of the incumbent management relative to potential bidders, and the incumbent's opportunities for employment outside the firm.

The paper is organized as follows. In section 2 we provide a brief overview of the model. The formal model is developed in section 3, and we summarize and discuss the results in section 4. Section 5 concludes. All proofs are confined to Appendix A. Appendix B consists of an example that illustrates our results.
2. Overview

The main implication of the results developed in sections 3 and 4 is that observations of a firm's value before and after the implementation of takeover defenses are not observations of the alternatives shareholders face when voting on these defenses. In our model, shareholders who approve defenses face an unobserved, inferior alternative if they choose rejection, and so rationally choose approval. Two central features of the model drive the results. First, the manager sets the agenda on takeover defenses monopolistically, and shareholders have only veto power over the managerial proposal. In particular, they are unable to amend the proposal or counter-propose before the vote. Second, the manager's request changes the subsequent decision-making environment.

The two critical features, monopolistic agenda-setting and the real effect of the request itself, generate the voting conundrum discussed in the Introduction. The monopolistic agenda-setting power of the manager ensures that, for a properly-chosen request, the shareholders can do no better than to approve. In certain circumstances, the value of the firm following approval of a managerial request for takeover defenses will be lower than the pre-proposal value. Nevertheless, the value would be no greater, and may be lower, if the request is rejected. The wedge between pre-proposal value and the post-rejection value is induced by the real effect of the request, described below.

Previous research on the effects of the implementation of takeover defenses has compared the pre-proposal value of the firm with its post-proposal or post-vote value. In our model, the comparison for inferences about shareholder rationality in voting is, instead, the value under approval of the request versus the value under rejection.

In the next section, we offer a particular model to illustrate this central idea. Our model rests on a reduced form specification of an outside managerial labor market. This is our characterization of the mechanism that produces the real effect of the request for takeover defenses. Other mechanisms may exist with the same general property of inducing a change in the decision-making
environment. We find the outside labor market a natural way to introduce this real effect, since it
defines the manager's opportunities in the event of a successful takeover.

Two further aspects of the model are worth noting at the outset. First, in our model,
shareholders recognize the implications of their vote: they do not make mistakes or have regrets
with respect to their decision. Second, in some circumstances, the value of the firm will rise as a
result of a request for takeover defenses. However, the focus of this paper is to describe
circumstances in which the value of the firm will fall, to provide a rational choice answer to our
opening question.

3. Model

We are interested in managerial control of a firm with assets and financial structure fixed.
There is a continuum $M$ of different managerial types capable of running the firm, different types
contributing differently to the value of the firm. A manager's type is the common expectation of
owners and the outside market of the value added to the firm by that manager. Let $m \in M$ denote
the incumbent manager's type, and assume $M$ is the whole real line.\(^3\) The distribution $F(\cdot)$ of
types on $M$ is common knowledge and induces a distribution over possible values of the firm in
question. Because we take the value of the assets and the financial structure to be fixed, we can,
without loss of generality, let $F(\cdot)$ also describe the distribution over managers of possible firm
values. Assume that the support of $F$ is connected, and that $F$ is smooth. Let $f(\cdot)$ be the density
function. We additionally assume that \(\lim_{t \to \infty} [1 - F(t)]/f(t) < \infty\) : this is not a strong restriction.\(^4\)

The firm is owned by a set of shareholders holding fully diversified portfolios. Being fully
diversified, each shareholder is interested only in maximizing the expected value of the firm. This
expected value depends both on the incumbent manager's type and on the expected premium paid to
the owners, conditional on the firm being taken over by an alternative management.

Heuristically, the model of takeovers we have in mind is the following. The lifetime of the
firm is two periods. A manager of type \( m \) controls the firm in the first period. In the second period, Nature draws a potential alternative manager from the set \( M \), who can choose whether or not to make a takeover bid for the firm. If a bid is made and is successful, then the new manager controls the firm for the remainder of its life. If there is no bid or if there is an unsuccessful bid, the incumbent remains in control.\(^5\) Since no takeover bids are possible beyond the second period, the maximum value of the firm in this last period is determined by the value added by the manager then in control, i.e. the second period manager's type.

Given this model of takeovers in the second period, the first period expected value of the firm is completely determined by the sum of the incumbent manager's value added, \( m \), and the expected premium in the second period. Without loss of generality, let discount factors be equal to one for the manager and all shareholders. The expected premium depends on the probability of a successful bidder appearing in the second period, and the size of the premium which can be extracted from that bidder.

We suppose that there is no utility value to making a bid per se, so that, given the incumbent manager's type \( m \), only managers of types \( n > m \) find it worthwhile to make a takeover bid. If there is any cost to bidding, then no type \( n \leq m \) will make an offer. Hereafter, we assume there is such a cost, but, to avoid cluttering the notation, suppose the cost constant and take it to be implicit in \( m \). Hence, given the distribution \( F \), the probability that a manager minimally capable of making a successful takeover bid arrives is \([1-F(m)]\).\(^6\)

The qualification "minimally" above refers to the possibility that the incumbent will contest a bid by erecting takeover defenses, as described below. We denominate takeover defenses in terms of the increment in price, above \( m \), necessary to acquire the firm. Thus, if takeover defenses are used, the potential bidder's type \( n \) must exceed \( m \) by at least the amount of takeover defenses. Being a manager of type \( n > m \) is necessary but not sufficient for making a successful bid for the firm.
The manager is endowed with two control variables, $x$ and $y$, both takeover defenses. Since we view defenses as an increment in price, as described above, both $x$ and $y$ are nonnegative real variables: $(x, y) \in \mathbb{R}_+^2$, and are measured in the same units as $m$. In the absence of any defense, managerial types $n > m$ will successfully bid for the firm: if there is any defense, a successful bidder must be capable of overcoming the defenses in addition to improving on $m$. Therefore, given defenses $x$ and $y$, a takeover bid must be at least equal to $[m+x+y]$ in order to win. As yet there is nothing to distinguish the two types of defense and, indeed, from any potential bidder's perspective the two sorts are indistinguishable. They are, however, quite different from the incumbent manager's perspective.

Defense $x$ is anticipatory: it can be put in place only in the first period, prior to any potential bidder appearing. For convenience, direct costs of implementing $x$ are normalized to zero. However, the manager is obliged to ask the shareholders explicitly for a level of $x$. Majority voting among the shareholders determines whether or not the manager's request is granted. Amendments are not permitted, so the shareholders can only accept or reject the manager's proposal. There is an indirect cost borne by the manager for requesting anticipatory defense, regardless of whether this request is granted. This cost is in terms of his outside value -- the utility payoff he can expect to receive in the second period, conditional on being ousted from the firm. Our presumption is that efforts on the part of incumbent managers to erect takeover defenses in the absence of an explicit offer is an indication that the manager is, for instance, more interested in personal security than the welfare of the shareholders; and this lowers his outside market value. Let $x?$ denote a request for a level of anticipatory defense. If $x?$ is approved by shareholder vote, then the implemented level of this defense is $x = x?$; and if it is not approved then $x = 0$. Evidently, $x? = 0$ implies $x = 0$. Note however that the above discussion implies that the states of the world $[x = 0 \mid x? = 0]$ and $[x = 0 \mid x? > 0]$ are distinct: this is crucial to our model.

The second type of takeover defense available to the manager, $y$, is responsive. Responsive
defense is implemented only to fight an explicit takeover bid in the second period, conditional on a bid materializing. Unlike anticipatory defenses, the manager implements responsive defense, without obtaining shareholder approval.\(^7\) There are, however, direct costs to engaging in a takeover fight. Let \(c(y)\) be the direct costs, measured in units of utility, borne by the incumbent in implementing a responsive defense of \(y\). Assume \(c(0) = 0, c'(y) > 0\) and \(c''(y) > 0\) for all \(y > 0\). For technical reasons, it is convenient to assume \(\lim_{y \to 0} c'(y) = 0\): none of our principal results depend on this.\(^8\)

In the heuristic discussion of the takeover process above, given defenses \(x\) and \(y\), we argued that a successful bidder must pay at least \(m+x+y\) to acquire the firm. Will any more be paid? Because shareholders are interested only in maximizing wealth, it is sufficient to pay \(\varepsilon > 0\) more than \(m+x+y\) to persuade them to remove the incumbent. No acquirer wishes to pay any more than necessary to take over the firm. Therefore, in the limit, epsilon will be zero. Hence, the premium that the incumbent secures for the shareholders -- conditional on losing a takeover battle despite defenses \(x\) and \(y\) -- is precisely \([x+y]\). Moreover, again taking \(x\) and \(y\) as fixed, only a manager of type \(n > [m+x+y]\) will make an offer, and any such offer will succeed. Recalling that \(F\) is the distribution over managers, the probability of takeover in the second period is therefore given by \([1 - F(m+x+y)]\).

To derive the expected premium, consider first the incumbent's objective function. Since he is surely in control of the firm in the first period and the investment and financial structure of the enterprise is taken as given, it is necessary only to examine his second period payoff. It is necessary to describe this second period payoff under two employment alternatives: within the firm, or, if a bid succeeds, outside the firm. If there is no takeover bid in the second period, the incumbent stays in control and receives utility \(V(m)\). Conditional on losing control of the firm, and given \(x\) and \(y\), the manager's net second period utility is:

\[
(1) \quad \omega(x, y \mid x?) = W(x+y, x? \mid m) - c(y),
\]
where \( W(\cdot) \) is the manager's gross outside value given \( x, y \) and \( x? \). Assume \( W_1 > 0, W_2 < 0 \) with 
\[
\lim_{x? \to \infty} W_2 = -\infty, \quad W_{11} < 0 \quad \text{and} \quad W_{12} \leq 0.
\]
Thus the incumbent's outside value conditional on losing his current position is increasing concave in the takeover premium he is able to extract from the acquirer; decreasing -- ultimately, at an ever-increasing rate -- in any efforts to secure anticipatory defenses in the first period; and the cross-effects of this latter on the former are non-increasing. These assumptions imply that the manager's utility for alternate employment is, for large enough approved requests \( x = x? \), strictly decreasing in \( x \). This does not exclude the possibility that \( W \) is everywhere decreasing in approved requests. To make the problem nontrivial, we suppose \( V(m) > W(0, 0 | m) \).

We argued above that, if \( x \) and \( y \) are fixed and known, only bids which can succeed will be made. At the beginning of the second period, the anticipatory defense \( x \) is indeed given. However, since defense \( y \) is responsive, the incumbent's selection of \( y \) depends, inter alia, on the particular managerial type \( n \) drawn by Nature in the second period. The incumbent's utility and cost schedules, \( W, V, \) and \( c \), are common knowledge, as is his type \( m \). Likewise, once Nature has made her draw, the type \( n \) of the potential bidder in the second period is common knowledge. Therefore, the potential acquirer is capable of calculating the incumbent manager's credible response \( y \) to any bid. A response \( y \) to a bid \( n \) is credible if, when \( n \) is bid, the manager's utility-maximizing response is \( y \). If this utility-maximizing response is sufficient to beat the bidder's best offer (that is, if \( m+x+y > n \)), then -- because of the bidding cost -- the potential acquirer makes no offer at all. If the potential acquirer is capable of topping the incumbent's best response \( (m+x+y < n) \), then she makes the smallest offer necessary to win control of the firm. Though the incumbent loses surely, by definition of credibility he prefers to bear the costs of fighting, \( c(y) \), and the winner pays the premium \([x+y]\). Hence the earlier argument goes through. It remains to determine the set of credible responsive defenses.

Let \( x? \) and \( x \) be given, \( x \in \{0, x?\} \), and define:
\[(2) \quad \gamma(x | x?) = \arg \max_y \in \mathbb{R}_+ \omega(x, y | x?).\]

From (1), we obtain:

\[(3) \quad \frac{\partial \omega}{\partial y} = W_1 - c';\]

\[(4) \quad \frac{\partial^2 \omega}{\partial y^2} = W_{11} - c'' < 0, \forall y.\]

Setting \(\frac{\partial \omega}{\partial y} = 0\) implicitly defines \(\gamma(x | x?)\). By (4), \(\gamma(\cdot | \cdot)\) is unique for all \(x?, x \geq 0\). Since \(W_1 > 0\) and \(\lim_{y \to 0} c'(y) = 0\) by assumption, \(\gamma(\cdot | \cdot) > 0\).

We now define, \(\forall n \geq x + m:\)

\[(5) \quad v(n, x | m) = V(m) - c(n-x-m).\]

The schedule \(v(\cdot)\) describes the maximum second period utility the incumbent manager can obtain, given he fights and defeats a bidder of type \(n\). Define, for \(x? \geq 0\) and \(x \in \{0, x?\}::\)

\[(6) \quad \eta(x | x?) = \begin{cases} \inf n \in V(n, x | \cdot) = \omega(x, \gamma(x | x?) | x?), & \text{if } V(m) > W(x+y*(x | x?), x? | m); \\ \inf n \in V(m) = W(n-m, x? | m), & \text{otherwise.} \end{cases}\]

As described below, \(\eta\) defines the minimum bid which can succeed against the incumbent. Notice that, because \(V(m) > W(0, 0 | m)\) by assumption, \(\eta(x | x?) > m + x\) for all \(x, x? \geq 0\). Inspection of (6) reveals that, for bids \(n \leq \eta(x | x?)\), it is credible that the incumbent will fight and win. On the other hand, any potential acquirer who can bid \(n > \eta(x | x?)\) will be successful, and the premium she will pay depends on \(n\) and on the incumbent's utility function.

To see how premia paid by successful bidders vary, define, for \(x? \geq 0\) and \(x \in \{0, x?\}:\)

\[(7) \quad n^*(x | x?) = x + \gamma(x | x?) + m,\]

where \(\gamma^*\) is defined as in (2), to be the responsive defense that maximizes the incumbent's utility in employment outside the firm. There are two cases: (a) \(n^*(x | x?) > \eta(x | x?)\), which occurs if and only if \(V(m) < W(x+y*(x | x?), x? | \cdot)\), and (b) \(n^*(x | x?) \leq \eta(x | x?)\). Figure 1 illustrates these under the assumption that \(x = x? \geq 0\).

[FIGURE 1 ABOUT HERE]

Consider Figure 1(a) and first let \(n \in (\eta(x | x?), n^*(x | x?)].\) The incumbent's best response is to
Figure 1(a)

Figure 1(b)
fight to extract (virtually) all the bidder's willingness-to-pay for the firm, and then to leave the firm to collect his net outside value, \( \omega(x, n-m-x | x?) \): this is clearly credible. Now let the bidder-type \( n > n^*(x | x?) \). Again the incumbent optimizes by fighting to extract a premium, in this case equal to \([n^*(x | x?) - m]\), and then leaving the firm and obtaining his outside value, \( \omega(x, n^*(x | x?) - m - x | x?) \).

Notice in this last case that bidders do not pay all that they are willing to pay for the firm. In this event, although the firm may attain a value \( n \) in the second period by virtue of the new manager's type, the new manager is assumed to consume the surplus, \( n - n^*(x | x?) \). Since the life of the firm is only two periods, this is reasonable. If one thinks of the new manager as another firm, then this surplus may be viewed as a gain to the owners of the acquiring firm.

In Figure 1(b), the incumbent prefers employment in the firm until \( n(x | x?) \), where the fight becomes too costly. Thus, only types \( n > n(x | x?) \) will make takeover bids. All successful bidders pay a premium of \([n^*(x | x?) - m]\), however, regardless of type. This is true because the incumbent, observing a bidder of type \( n > n(x | x?) \), recognizes defeat, but fights to \( n^* \) to maximize his outside value.

For future reference, for all \( n > n(x | x?) \) and for any \( x \geq 0 \), let \( \pi(n, x | x?) \) denote the premium actually paid by the successful bidder of type \( n \), above the value \( m \) contributed by the incumbent. For any successful bid, the premium \( \pi \) is:

\[
\pi(n, x | x?) = \min\{n - m, n^*(x | x?) - m\}.
\]

We are now in a position to specify the incumbent's expected second period payoff, viewed from the first period, as a function of anticipatory defenses \( x \):

\[
U(x | x?) = F(n(x | x?)) \cdot V(m) + \delta \cdot [F(n^*(x | x?)) - F(n(x | x?))] \int_n^{n^*} \omega(x, n-m-x | x?) \cdot f(n) \, dn + \{\delta [1-F(n^*(x | x?))] + [1-\delta] [1-F(n(x | x?))]\} \cdot \omega(x, n^*(x | x?) - m - x | x?),
\]

where \( \delta = 1 \) iff \( n(x | x?) < n^*(x | x?) \), and \( \delta = 0 \) otherwise. The first period expected value of the firm under management \( m \) is given by:
(10)  \[ S(x | x?) = m + \delta \left[ F(n^*(x | x?)) - F(n(x | x?)) \right] \int_{n-m}^{n} f(n) \, dn \\
+ \{ \delta [1-F(n^*(x | x?))] + [1-\delta] [1-F(n(x | x?))] \} \cdot [n^*(x | x?) - m] \]

with \( \delta \) defined as above.

The responsive defense \( y \) is chosen optimally by the management, independent of shareholder approval and given the bidder type that appears in the second period. Anticipatory defenses \( x \) require shareholder approval and have to be set in the first period. The manager's first period optimization problem is therefore to choose a request, \( x? \geq 0 \), to:

(11)  \[
\text{max. } U(x | x?) \\
\text{subject to: } S(x | x?) \geq S(0 | x?).
\]

Clearly, if the manager asks \( x? = 0 \), then the constraint trivially binds. However, if \( x? > 0 \) then the constraint says that the expected value of the firm conditional on the request being approved must be no less than its value if the request is rejected, given \( x? > 0 \). Because the incumbent manager's decision environment in the second period depends, inter alia, on his request, \( x? \), in the first period -- even if it is not granted -- we have in general that \( S(0 | 0) \neq S(0 | x?) \) for any \( x? > 0 \). In other words, the alternative that rational shareholders compare against the manager's request is not \( S(0 | 0) \), their situation prior to any request, but is \( S(0 | x?) \), the result if they reject the request. This is the central idea of the model.

If the constraint binds at some \( x? > 0 \), then shareholders are indifferent between accepting and rejecting the request. Since managers are not indifferent, they can insure acceptance in such circumstances by slightly perturbing the request, \( x? \), to induce strict preference on the part of shareholders: as Appendix A shows, such a perturbation is always available. We assume that if the constraint binds, then the manager's request is approved. Alternatively, we can adopt the convention that in cases of shareholder indifference, shareholders always "vote with the management".\(^{10}\)

We turn now to some results.
4. Results

There are three possible conditions at the beginning of the second period: (1) the manager requested no anticipatory defenses, (2) the manager made a request which was voted down by shareholders, and (3) the manager's request was approved. Proposition 1 puts an ordering on the second period response to a takeover bid which maximizes the manager's outside value, conditional on the first-period outcome.

Proposition 1: Let \( x = x^? > 0 \). Then \( y^*(0 \mid 0) \geq y^*(0 \mid x^?) > y^*(x \mid x^?) > 0 \).

Corollary 1: Let \( x = x^? > 0 \). Then \( x + y^*(x \mid x^?) > y^*(0 \mid x^?) \).

If a first-period request is rejected by shareholders, the second-period maximizing response is strictly larger than if the request is approved. However, from Corollary 1, total defenses (anticipatory plus responsive) are smaller if the request is rejected. Since total defenses determine the premium received in a successful bid, this implies that shareholders can expect a smaller premium if they reject the manager's request and a successful bidder emerges.

An interesting feature evident from Proposition 1 is that the second-period maximizing response is never larger after a request, even if the request is denied, than if no request is made. Thus, the request carries no implicit threat of "scorched earth" responses if shareholders deny it. Rather, as we show below, the request alters the manager's incentives to fight for a higher premium if a successful bidder appears.

Anticipatory and responsive defenses are substitutes in their effects on \( n \), the minimum bid required to acquire the firm. Generally, the higher the level of anticipatory defense approved by shareholders in the first period, the lower will be the manager's choice of responsive defenses.
Comparative statics on $y^*(x \mid x?)$, the maximizing second-period response (Appendix A, (a3) - (a7)) show that this response is non-increasing in the size of the first-period request.

In evaluating how to vote, value-maximizing shareholders consider both the size of the premium they can expect from a successful bid, and the probability that such a bid will emerge. Proposition 2 describes the probability of a successful takeover bid, as a function of requested anticipatory defenses and shareholder approval of the defenses.

Proposition 2:

(a) $\pi(x \mid x?)$ is strictly increasing in $x?$, $x \in \{0, x?\}$;

(b) $\forall x = x? > 0, \pi(x \mid x?) \geq \pi(0 \mid x?) > \pi(0 \mid 0)$.

Proposition 2 states that the minimum bid which can succeed against incumbent management increases with the amount of anticipatory defense requested. Since the probability that a successful bid will emerge falls as the minimum successful bid rises, part (a) of Proposition 2 implies that the probability of takeover declines with increases in the level of requested anticipatory defenses, whether or not these are approved. Part (b) implies that the probability of takeover is largest if no anticipatory defenses are requested, declines if there is a request, and may decline more if the request is approved. Notice that part (b) is not an immediate consequence of part (a): the change in state from $[x = x? \mid x? > 0]$ to $[x = 0 \mid x? > 0]$ is not incremental because of the "take it or leave it" nature of the shareholders' decision.

The value to shareholders from anticipatory takeover defense depends on both the premium if a successful bid is made, which generally increases with the level of anticipatory defense, and on the probability of a successful bid, which generally decreases. Proposition 3 shows that shareholders are unambiguously worse off if they reject management's request for anticipatory defenses than they were before the request.
Proposition 3: For \( x^? > 0 \), \( S(0 \mid 0) > S(0 \mid x^?) \).

The voting (incentive compatibility) constraint in the model requires that approval of a request leave shareholders at least as well off as rejection. Proposition 3 states that shareholders are worse off if they reject the request than they would have been if no request had been made. (Note that this latter circumstance, no request, is not available to shareholders deciding how to vote on a request.) To complete our description of the apparent paradox that shareholders vote for defenses which leave them worse off, we require a comparison of shareholder value if the request is approved with shareholder value if no request is made: \( S(x \mid x^?) \) versus \( S(0 \mid 0) \). Proposition 4 gives a sufficient condition for the apparent paradox to occur.

Proposition 4: \( V(m) < W(y^*(0 \mid 0), 0 \mid m) \)

\[ \Rightarrow \exists x^? \in (0, \infty): S(0 \mid 0) > S(x \mid x^?) \geq S(0 \mid x^?), \ x = x^?. \]

According to Proposition 4, if the manager's utility from employment with the firm is less than his maximum gross outside utility before any request for anticipatory defenses, then requests exist which shareholders will approve, but which leave them worse off than they were before. It is important to observe that this maximum gross outside utility is available only if, in the second period, Nature draws a bidder of type \( n \geq n^*(0 \mid 0) \). Recall that we assumed the incumbent's utility from employment in the firm was larger than his initial gross outside utility: that is, his utility before defenses \( x \) or \( y \) are considered. Proposition 4 concerns the incumbent's maximum gross outside utility: that is, considering his optimal choice of \( y \) at \( x^? = 0 \). The situation is pictured in Figure 1(a), by setting \( x = x^? = 0 \).

With Proposition 4, we have shown the existence of requested levels of anticipatory defense
which result in the voting behavior we hoped to describe. That is, rational, fully informed, value-maximizing shareholders will approve the defenses, but are made worse off by them. The sufficient condition for the result depends on the manager's utility in current and alternative employment. We have not yet demonstrated, however, that these levels of anticipatory defense would in fact be requested by managers with utilities satisfying the sufficient condition. There is no inconsistency between utilities satisfying this condition and utilities generating the level of anticipatory defense, identified in the Proposition, as a best choice. In Appendix B, we provide an example of a manager with utility satisfying the condition of Proposition 4, whose optimizing choice of anticipatory defense will be approved by shareholders, and will leave them worse off than if no request had been made. The example is in no sense pathological.

It is worth noting that under some circumstances, shareholders can be made better off by implementing some anticipatory defense. By Proposition 3, a necessary condition for this to occur is that the manager's request, x?, be strictly interior to the constraint set; i.e. \( S(x \mid x?) > S(0 \mid x?) \). It is not, however, a sufficient condition. In our example in Appendix B, shareholders would like some anticipatory defense x, but the manager prefers more and, despite the constraint not binding, the request results in a decline in shareholder wealth.

The primary implication of our model for interpreting the empirical work which has preceded it is that inferences about whether shareholders vote rationally cannot be made from a comparison of shareholder wealth before and after the vote. We describe the alternative to voting with management as an unambiguous drop in shareholder value, not as a return to the pre-proposal status quo. Therefore, pre-proposal shareholder wealth is not the correct benchmark for determining shareholder rationality. Our model suggests that empirical investigation of shareholder voting and takeover defenses should consider the manager's outside employment opportunities, and the manager's skill relative to potential bidders.
5. Conclusion

We have provided a rational choice model of shareholder voting on anticipatory takeover defenses. In our model, it is feasible for informed, value-maximizing shareholders to approve measures which leave them worse off than they were before the measures were requested. What drives this result is that a manager, in requesting such measures, lowers his outside market value. Consequently, the manager's optimal response to an actual takeover bid is different if the request is rejected than if he had made no request. Shareholders recognize this in evaluating how to vote.

In the model, the manager's type and utility schedule are common knowledge. This raises the question of why any manager would be hired who will wish to implement anticipatory defenses which decrease the value of the firm to shareholders. To this extent, our model is incomplete. We have in mind a signalling game which precedes the two periods we study. At this earlier stage, perhaps the hiring stage, the manager's type is not known with certainty. The request \( x \) functions as a signal, and we are implicitly assuming the signal fully reveals the manager's type (and utility). In other words, our model is predicated on the existence of a separating equilibrium to this earlier signalling game.

Finally, we offer two remarks. First, it is frequently asserted (cf. Easterbrook and Fischel (1983)) that the alienability of ownership claims protects shareholders from detrimental management entrenchment tactics. However, unless shareholders anticipate the proposal of takeover defenses by management, they cannot sell a voting claim in response to the proposal. SEC proxy mailing requirements demand that the record date for shareholder voting precede the proposal date. Once the proposal is announced, the constituency is fixed. Our model suggests that any drop in share value should occur with the announcement of the proposal, not with the vote.

Second, our model implies that changes in shareholder wealth associated with voting on takeover defenses may be positive or negative, depending on the manager's type and utility
schedule. In cases where shareholders' wealth is reduced, the reduction should occur at the date of the manager's request. If there is any detectable change at the date of the vote, it should be positive. This follows from Proposition 3 and the voting constraint: the first states that shareholders' wealth is unambiguously reduced if they reject a request; the second ensures that shareholders' wealth under rejection is no larger than their wealth under approval. On this interpretation, empirical results which find shareholders' wealth declines following implementation of anticipatory defenses are not evidence of shareholder irrationality or ignorance, but reflect an informed choice of the lesser of two declines in value.
Appendix A: Proofs

**Proposition 1**: Let \( x = x^* > 0 \). Then: \( y^*(0 | 0) \geq y^*(0 | x^*) > y^*(x^* | x^*) > 0 \).

**Proof**:

(i) \( y^*(x^* | x^*) > 0 \) follows from assuming \( W_1 > 0 \) everywhere, and \( \lim_{y \to 0} c'(y) = 0 \).

(ii) \( y^*(0 | 0) \geq y^*(0 | x^*) \)

Suppose \( y^*(0 | 0) < y^*(0 | x^*) \). Then, by \( W_{11} < 0, W_{12} \leq 0 \) and \( x^* > 0 \):

\[ W_1(y^*(0 | 0), 0 | -) - W_1(y^*(0 | x^*), x^* | -) > 0. \]

Use (3) to obtain:

\[ (a.1) \quad W_1(y^*(0 | 0), 0 | -) - W_1(y^*(0 | x^*), x^* | -) = c'(y^*(0 | 0)) - c'(y^*(0 | x^*)). \]

But \( c'' > 0 \) implies the RHS(a.1) \( < 0 \): contradiction.

(iii) \( y^*(0 | x^*) > y^*(x^* | x^*) \)

Suppose \( y^*(0 | x^*) \leq y^*(x^* | x^*) \), and again use the first order condition (3) to get:

\[ (a.2) \quad W_1(x + y^*(x^* | x^*), x^* | -) - W_1(y^*(0 | x^*), x^* | -) = c'(y^*(x^* | x^*)) - c'(y^*(0 | x^*)). \]

Then \( c'' > 0 \) implies RHS(a.2) \( \geq 0 \).

But \( x = x^* > 0 \), so that \( W_{11} < 0 \) implies LHS(a.2) \( < 0 \): contradiction. ||

**Remark 1**: \( y^*(0 | 0) = y^*(0 | x^*) \) iff \( W_{12} \equiv 0 \). Hence, \( x = x^* > 0 \) and \( W_{12} \equiv 0 \) imply:

\[ [x + y^*(x^* | x^*)] > y^*(0 | 0). \]

**Corollary 1**: Let \( x = x^* > 0 \). Then: \( [x + y^*(x^* | x^*)] > y^*(0 | x^*) \).

**Proof**: Use (3) and Proposition 1 with \( W_{11} < 0 \) and \( c'' > 0 \). ||

**Comparative Statics**:

\[ (a.3) \quad x = x^* \Rightarrow dy^*(x^* | x^*)/dx^* = -[W_{11} + W_{12}]/[W_{11} - c''] < 0. \]
(a.4) \[ dy*(0 \mid x?) / dx? = -W_12 / [W_{11} - c'] \leq 0, \text{ with the inequality strict iff } W_{12} < 0. \]

(a.5) \[ x = x? \Rightarrow d\omega(x, y \mid x?) / dx? = W_1 + W_2, \text{ which a priori has ambiguous sign.} \]

(a.6) \[ d\omega(0, y \mid x?) / dx? = W_2 < 0. \]

Note that at any \( y, |(a.5)| < |(a.6)|. \)

Since \( n*(x \mid x?) = x + y*(x \mid x?) + m, \) and \( y*(x \mid x?) \) is differentiable in \( x? \), \( n*(x \mid x?) \) is differentiable in \( x? \). If \( x = 0 \), then \( \partial n*(0 \mid x?) / \partial x? \) is given by (a.4). If \( x = x? \), then \( \partial n*(x \mid x?) / \partial x? = 1 + dy*(x \mid x?) / \partial x? \). Substituting from (a.3) and collecting terms, we obtain:

\[ (a.7) \quad \partial n*(x \mid x?) / \partial x? \geq (<) 0 \text{ as } c"(y*(x \mid x?)) \geq (<) -W_{12}(x+y*(x \mid x?), x? |). \]

Lemma 1: Let \( x? \geq 0, x \in \{0, x?\} \). Then: \( n(x \mid x?) \) is differentiable in \( x? \); and \( \partial n(x \mid x?) / \partial x? > 0. \)

Proof:

(i) \( n(x \mid x?) \) is differentiable.

First, recall from section 3 that \( n(x \mid x?) > x + m, \forall x, x? \geq 0 \). Let \( x = x? \geq 0 \), and define:

\[ \Delta* = [v(n*(x \mid x?), x \mid;) - \omega(x, y*(x \mid x?) \mid x?)]. \]

Differentiating \( \Delta* \) with respect to \( x? \) at \( x = x? \), and using (3) gives:

\[ \partial \Delta*/\partial x? = -W_2(x+y*(x \mid x?), x? \mid;) > 0. \]

Therefore, \( \Delta* \) can change sign at most once as \( x \) increases, and only from negative to positive.

Suppose there exists an \( x = x? \) such that \( \Delta* = 0 \). Then \( \Delta* \) is unique and:

\[ V(m) \leq (>) W(x+y*(x \mid x?), x? \mid;) \text{ iff } x \leq (>) x. \]

So by (6): \( n(x \mid x?) \leq (>) n*(x \mid x?) \text{ iff } x \leq (>) x. \)

Case \( a \): Consider \( x < x \).

By (6), \( n(x \mid x?) \) is implicitly defined by: \( V(m) - W(n(x \mid x?) - m, x? \mid;) = 0. \)

In this case, \( n(x \mid x?) \) is differentiable because \( W \) is differentiable. In particular, \( \forall x < x \).

\[ (a.8) \quad \partial n(x \mid x?) / \partial x? = -W_2(n(x \mid x?) - m, x? \mid;) / W_1(n(x \mid x?) - m, x? \mid;); \]

and,
\[ \lim_{x \to x^-} \frac{\partial \Omega(\cdot)}{\partial x^?} = -W_2(\n(x | x?)-m, x? \cdot) / W_1(\n(x | x?)-m, x? \cdot). \]

**Case β:** Consider \( x > x \).

By (6), \( \Omega(x | x?) \) is implicitly defined by: \( V(m) - c(\n(x | x?)-x-m) - \omega(x, y^*(x | x?) | x?) = 0. \) Differentiability of \( \Omega(x | x?) \) follows from differentiability of \( c \) and \( W \). In particular, using (3):

\[ \frac{\partial \Omega(x | x?)}{\partial x^?} = \frac{c'(y^*(x | x?)) - W_1^*(x) - W_2^*(x)}{c'(\n(x | x?) - x-m)}, \]

where \( W_i^*(x) \equiv W_i(\n(x | x?) - m, x? \cdot), i = 1, 2. \) Hence,

\[ \lim_{x \to x^+} \frac{\partial \Omega(\cdot)}{\partial x^?} = \frac{c'(\n(x | x?)) - W_1^*(x) - W_2^*(x)}{c'(\n(x | x?) - x-m)} = -W_2(\n(x | x?) - m, x? \cdot) / W_1(\n(x | x?) - m, x? \cdot); \]

the second equality follows from another application of (3). Together, (a.9) and (a.11) complete the argument that \( \Omega(x | x?) \) is differentiable everywhere when \( x = x? \).

(ii) \( \frac{\partial \Omega(x | x?)}{\partial x^?} > 0 \)

**Case α:** \( x = x? \leq x \)

The proof of this case is immediate from (a.8) and (a.9).

**Case β:** \( x = x? > x \)

Consider (a.10). By (3), \( W_1^*(x) = c'(y^*(x | x?)) = c'(n(x | x?) - x-m). \) Since \( \Delta^* > 0, n(x | x?) < n(x | x?). \) Therefore, \( c'' > 0 \forall y \) implies,

\[ \frac{\partial \Omega(x | x?) - x-m)}{W_1^*(x)} > 0. \]

Moreover, \( W_1 + W_2 < W_1. \) Therefore, the numerator of (a.10) is strictly positive. Since \( c' > 0, \)
this completes the proof of the Lemma for \( x = x? \).

Now suppose \( x = 0 \) and \( x? > 0. \) Similar reasoning as before gives \( \Omega(0 | x?) \) differentiable in \( x?, \) and \( \frac{\partial \Omega(0 | x?)}{\partial x^?} > 0 \) follows on implicit differentiation of (6) for \( x? \leq x \) and for \( x? > x. \)

Remark 2: (a.7) and Lemma 1 imply that \( U(x | x?) \) and \( S(x | x?) \) are differentiable in \( x? \).

**Lemma 2:** Let \( x = x? > 0. \) Then: \( \Omega(x | x?) \geq \Omega(0 | x?). \)
Proof:

**case α:** \( V(m) < W(x+y^*(x \mid x?), x? \mid) \)

By (6) and the premise, \( V(m) = W(\Pi(x \mid x?) - m, x? \mid) \). There are then two possibilities:

(α.1) \( V(m) \leq W(\Pi(0 \mid x?) - m, x? \mid) \)

\[ \Rightarrow \, V(m) = W(\Pi(0 \mid x?) - m, x? \mid), \text{ by (6)} \]

\[ \Rightarrow \, W(\Pi(x \mid x?) - m, x? \mid) = W(\Pi(0 \mid x?) - m, x? \mid) \]

\[ \Rightarrow \, \Pi(x \mid x?) = \Pi(0 \mid x?). \]

(α.2) \( V(m) > W(\Pi(0 \mid x?) - m, x? \mid) \)

\[ \Rightarrow \, V(m) = [\omega(0, y^*(0 \mid x?) \mid x?) + c(\Pi(0 \mid x?) - m)], \text{ by (6)} \]

\[ \Rightarrow \, W(\Pi(x \mid x?) - m, x? \mid) - c(\Pi(0 \mid x?) - m) = \omega(0, y^*(0 \mid x?) \mid x?). \]

By definition of \( y^*(0 \mid x?) \), \( \omega(0, y^*(0 \mid x?) \mid x?) > W(\Pi(0 \mid x?) - m, x? \mid) - c(\Pi(0 \mid x?) - m). \)

Therefore, \( W(\Pi(x \mid x?) - m, x? \mid) - c(\Pi(0 \mid x?) - m) > W(\Pi(0 \mid x?) - m, x? \mid) - c(\Pi(0 \mid x?) - m) \)

\[ \Rightarrow \, n(x \mid x?) > n(0 \mid x?). \]

This proves the proposition for case (α).

**case β:** \( V(m) \geq W(x+y^*(x \mid x?), x? \mid) \)

\[ \Rightarrow \, V(m) = \omega(x, y^*(x \mid x?) \mid x?) + c(\Pi(x \mid x?) - m), \text{ by (6).} \]

By Corollary 1 and \( x? > 0 \), \( W(x+y^*(x \mid x?), x? \mid) > W(y^*(0 \mid x?), x? \mid) \).

Hence, \( V(m) > W(\Pi(0 \mid x?) - m, x? \mid). \) So by (6),

(a.13) \( V(m) = \omega(0, y^*(0 \mid x?) \mid x?) + c(\Pi(0 \mid x?) - m). \)

Therefore,

(a.14) \( \omega(x, y^*(x \mid x?) \mid x?) - \omega(0, y^*(0 \mid x?) \mid x?) = c(\Pi(0 \mid x?) - m) - c(\Pi(x \mid x?) - m). \)

By Proposition 1 and Corollary 1, LHS(a.14) is strictly positive; hence, the RHS(a.14) must likewise be strictly positive. Since \( c' > 0 \) and \( x = x? > 0 \), this implies,

(a.15) \( n(0 \mid x?) > n(x \mid x?) - x. \)

Now implicitly differentiating (a.13), we obtain \( \forall x? \geq 0: \)
\[ \frac{\partial \Pi}{\partial x} = -W_2(y*(0|x?), x? |/)c'(n(0|x?)-m) > 0. \]

Using (a.10), we have \( \forall x? \geq 0: \)

(a.16) \[ [\frac{\partial \Pi}{\partial x}? - \frac{\partial \Pi}{\partial x?}] = \{1 - [W_1(x+y*(x|x?), x? |/)c'(n(x|x?)-x-m)] \}
+ \{[W_2(y*(0|x?), x? |/)c'(n(0|x?)-m)] - [W_2(x+y*(x|x?), x? |/)c'(n(x|x?)-x-m)]\}. \]

By (a.12), the first term of (a.16) in \( \{ \cdot \} \) is nonnegative \( \forall x? > 0. \) Consider the second term in \( \{ \cdot \} \).

By Corollary 1, \( W_2 < 0, \) and \( W_{12} \leq 0, \)

(a.17) \[ 0 > W_2(y*(0|x?), x? |/) \geq W_2(x+y*(x|x?), x? |/). \]

By (a.15) and \( c'' > 0, \)

(a.18) \[ 0 < c'(n(x|x?)-x-m) < c'(n(0|x?)-m). \]

Together, (a.17) and (a.18) imply that the second term of (a.16) in \( \{ \cdot \} \) is also positive \( \forall x? > 0. \)

When \( x? = 0, \) both terms in \( \{ \cdot \} \) vanish. Therefore, \( \forall x? \geq 0: \)

\[ [n(x | x?) - n(0 | x?)] = \int_{0}^{x?} [\frac{\partial \Pi}{\partial r} | r | dr - \frac{\partial \Pi}{\partial n}(0 | r) \partial r] dr \geq 0, \]

as required (with equality iff \( x? = 0). \)

Remark 3: Notice that Lemma 2 is not implied by Lemma 1: this is because the change in state from \( [x = x? | x? > 0] \) to \( [x = 0 | x? > 0] \) is not incremental.

Remark 4: Together, Remark 2, footnote 8 and Lemma 2 justify the claim made in section 3 that, if \( x = x? \) and \( S(x | x?) = S(0 | x?), \) then there exists a perturbation in \( x? \) -- say, \( x?$ -- such that \( S(x~ | x?) > S(0 | x?). \)

Lemma 3: For any \( x? > 0, n(0 | x?) > n(0 | 0). \)

Proof: Given \( x = 0, v(n, x | m) = V(m) - c(n-m). \) Since \( c'' > 0, v(n, x | m) \) is strictly decreasing in \( n. \) By \( W_2 < 0, W(y, x? |/) < W(y, 0 |/), \forall y \geq 0. \) So by (6), \( n(0 | x?) > n(0 | 0), \forall x? > 0. \)
Proposition 2: (a) $n(x | x?)$ is strictly increasing in $x?$, $x \in \{0, x?\}$; and,

(b) $\forall x = x? > 0$, $n(x | x?) \geq n(0 | x?) > n(0 | 0)$.

Proof: The proposition follows from Lemmas 1, 2 and 3. ||

Proposition 3: For $x? > 0$, $S(0 | 0) > S(0 | x?)$.

Proof: By Lemma 3, and the fact that $F$ is a c.d.f. defined on a continuous variable $n$:

(a.19) $[1-F(n(0 | x?))] < [1-F(n(0 | 0))]$.

By Proposition 1, $n^*(0 | x?) \leq n^*(0 | 0)$, with the inequality strict iff $W_{12} < 0$ at any $y \leq y^*(0 | 0)$.

**case α:** $n \geq n(0 | x?)$.

By (8):

(a.20) $\pi(n, 0 | 0) \geq \pi(n, 0 | x?)$.

If $n^*(0 | x?) < n^*(0 | 0)$, the inequality in (a.20) is strict $\forall n > n^*(0 | x?)$.

**case β:** $n \in (n(0 | 0), n(0 | x?))$.

By an argument in section 3, only types $n > n(\cdot | \cdot)$ will make takeover bids. Therefore:

(a.21) $\pi(n, 0 | 0) > \pi(n, 0 | x?) \equiv 0$.

**case γ:** $n \leq n(0 | 0)$.

By the same argument as case β:

(a.22) $\pi(n, 0 | 0) = \pi(n, 0 | x?) \equiv 0$.

Together, (10) and (a.19) - (a.22) yield the desired result. ||

Lemma 4: Let $x = x?$. Then: $\lim_{x? \to \infty} S(x | x?) = m$.

Proof:

By assumption, $W_{11} < 0$ and $\lim_{x? \to \infty} W_2 = -\infty$. Therefore, for sufficiently large $x?$, say $x?$ $> x_1?$, $V(m) > W(x+y^*(x | x?), x? | \cdot)$. Hence, by (10), $\forall x? \geq x_1?$,

$S(x | x?) = m + [1-F(n(x | x?))]\cdot [n^*(x | x?) - m]$. 
By Remark 2, $S(\cdot | \cdot)$ is differentiable in $x?$. In particular, $\forall x? \geq x_1?,$

$$(a.23) \quad \partial S(x | x?)/\partial x? =$$

$$[1-F(n(x | x?))]|\partial n^{*}(x | x?)|/\partial x? - \{f(n(x | x?)) \cdot \partial n(x | x?)/\partial x? : [n^{*}(x | x?)-m}\}.$$ 

We prove the Lemma by first showing $\partial S(x | x?)/\partial x? < 0$ for all finite $x? \geq x_2?$, $x_2?$ sufficiently large. By Lemma 1 and the assumption that $V(m) > W(0, 0 | m)$, the term in $\{\cdot\}$ on the RHS(a.23) is strictly positive for all finite $x?$. By (a.7), the first term on the RHS(a.23) is of ambiguous sign.

**Case α:** $\partial n^{*}(\cdot | \cdot)/\partial x? \leq 0$ for all $x? \geq x_2?$.

Then, $\partial S(x | x?)/\partial x? < 0$ for all finite $x? \geq x_2?$.

**Case β:** $\partial n^{*}(x | x?)/\partial x? > 0$ for all $x? \geq x_2?$.

By (7) and (a.3), $\sup [\partial n^{*}(x | x?)/\partial x?] = 1$. For sufficiently large $x?$, $\partial S(x | x?)/\partial x? < 0$ if:

$$(a.24) \quad [1-F(n(x | x?))/f(n(x | x?))] < \partial n(x | x?)/\partial x? : [n^{*}(x | x?) - m].$$

By assumption, $\lim_{t \to \infty} [1-F(t)/f(t)] < \infty$. Therefore, by Lemma 1 and $x = x?$, $\lim_{x? \to \infty} \text{LHS(a.24)} < \infty$. Also by assumption, $W_{11} < 0$ and $\lim_{x? \to \infty} W_2 = -\infty$. From (a.10), therefore, $\partial n(x | x?)/\partial x? > 1$ for sufficiently large $x?$. By hypothesis, $\partial n^{*}(x | x?)/\partial x? > 0$, $\forall x? \geq x_2?$.

Hence, (7) and $y \geq 0$ imply that $\lim_{x? \to \infty} \text{RHS(a.24)} = \infty$. Therefore, there exists a sufficiently large value of $x?$, $x_2?$, such that $\partial S(x | x?)/\partial x? < 0$ for all $x? \geq x_2?$. Since $\lim_{t \to \infty} [1-F(t)] = 0$, the Lemma follows from (10).

**Proposition 4:** $V(m) < W(y^{*}(0 | 0), 0 | m)$

$$\Rightarrow \exists x? \in (0, \infty) : S(0 | 0) > S(x | x?) \geq S(0 | x?), x = x?.$$ 

**Proof:** By Corollary 1, $\exists x'' = x'? > 0$ such that both $V(m) < W(x'' + y^{*}(x'' | x?''), x?' | \cdot)$, and $V(m) \leq W(y^{*}(0 | x?''), x?' | \cdot)$ obtain. Then by case (α.1) of Lemma 2, $n(x'' | x?'') = n(0 | x?'') = n$. Hence:

$$(a.25) \quad [1-F(n(x'' | x?''))] = [1-F(n(0 | x?''))] = [1-F(n)].$$

By Corollary 1, $n^{*}(x'' | x?'') \geq n^{*}(0 | x?'')$. From the premise of the Proposition and the choice
of $x''$, $n^*(0 \mid x'') > n$. By an argument of section 3, only types $n > n$ will make a takeover bid.

Using this and (8):

(a.26) \[ \forall n \leq n, \pi(n, x'' \mid x''') = \pi(n, 0 \mid x''') \equiv 0; \]
\[ \forall n \in (n, n^*(0 \mid x''')], \pi(n, x'' \mid x''') = \pi(n, 0 \mid x''') = n - m; \]
\[ \forall n > n^*(0 \mid x'''), \pi(n, x'' \mid x''') \geq \pi(n, 0 \mid x'''). \]

Given (10), (a.25) and (a.26) imply that $S(x'' \mid x''') \geq S(0 \mid x''')$. Therefore,
\[ C = \{ x'' > 0 \mid S(x'' \mid x?) \geq S(0 \mid x?) \}, x = x? \} \neq \emptyset. \]

Let $x = x?$. There are now two cases:

(i) \[ \exists x'' \in C : x'' < \infty \land S(x'' \mid x?) = S(0 \mid x?) \]
\[ \Rightarrow [S(0 \mid 0) > S(x'' \mid x?)], \text{by Proposition 3}; \]

(ii) \[ \forall x'' \in C : x'' < \infty, \ S(x'' \mid x?) > S(0 \mid x?) \]
\[ \Rightarrow [\exists x'' \in C : x'' < \infty \land S(0 \mid 0) > S(x'' \mid x?)], \text{by Lemma 4 and } S(0 \mid 0) > m. \text{ To check this last inequality, recall } V(m) < W(y*(0 \mid 0), 0 \mid \cdot) \text{ by hypothesis. Hence, by (6) and } y*(0 \mid 0) > 0 \text{ (Proposition 1), } m < n(0 \mid 0) < \infty \text{ and } n^*(0 \mid 0) > m. \]
Appendix B: Example

We claim in section 4 of the text that there exist managerial utilities satisfying the sufficient condition of Proposition 4, under which the manager would ask for, and shareholders approve, a level of anticipatory defense, x?, such that \( S(0 \mid 0) > S(x \mid x?) \). In this appendix, we justify our claim with an example.

In the interests of computational simplicity, some technical assumptions of the model are violated in the example: viz. the support of F is not the whole real line, and \( W_1 \) is zero at \( x = y = 0 \). The example therefore shows that the main result (Proposition 4) does not depend on these assumptions. Also, there are choices of the distribution function F and the outside utility W, that do satisfy all the assumptions of the text, and which are arbitrarily closely approximated by the functional forms exploited in the example.

Let F be uniform on the closed interval [-2,2], and assume the manager's type is \( m = 0 \).

Let:

\[
V(m) = 1.57,
\]

\[
W(x + y, x? \mid m) = 2 \cdot [x + y]^{1/2} - [x^2]/2 + m,
\]

\[
c(y) = (2/3)y^{3/2}.
\]

For this specification, \( U(\cdot \mid \cdot) \) is strictly concave. Denote by \( x^* \) the point where

\[
\partial U(x \mid x?) / \partial x? \bigg|_{x=x^?} = 0,
\]

that is, the incumbent's unconstrained optimizing choice of x?. The incumbent's utility function \( U(x \mid x?) \) is as defined in equation (9) of the text. Given this specification, we obtain the following results, illustrated in Figure B.1. For manager:

\[
x^* = 0.35
\]

\[
U(0 \mid 0) = 1.3623
\]

\[
U(x \mid x^*) = 1.3766
\]

\[
\partial U(x \mid x?) / \partial x? \bigg|_{x=x^*} = 0.0
\]
For the shareholders:

\[
\begin{align*}
S(0 | 0) &= 0.2592 \\
S(x | x^*) &= 0.2582 \\
S(0 | x^*) &= 0.2570
\end{align*}
\]

so the condition \( S(0 | 0) > S(x | x^*) > S(0 | x^*) \) holds, and \( S(x | x^*) \) is the outcome for shareholders. That is, by the second inequality, shareholders are worse off if they reject the request than they are if they approve it, but by the first inequality, the final outcome \( S(x | x^*) \) is worse than their starting value \( S(0 | 0) \).

In this example, at \( x = x^* = 0 \):

\[
\frac{\partial U(x | x^*)}{\partial x^*}|_{x = x^* = 0} = 0.095 > 0, \text{ and} \\
\frac{\partial S(x | x^*)}{\partial x^*}|_{x = x^* = 0} = 0.024 > 0,
\]

so both the manager and the shareholders prefer a strictly positive \( x \) to \( x = 0 \). However, the manager's most-preferred \( x \) is strictly greater than the shareholders'.

[FIGURE B.1 ABOUT HERE]

Notice that, for this parameterization, the manager's unconstrained optimal choice of anticipatory defense, \( x^* \), is low enough that the constraint, \( S(x | x^*) \geq S(0 | x^*) \), does not bind. If the manager's unconstrained optimum were approximately 0.4 or above in this example, then the constrained optimum \( x^* \) would be approximately 0.39, where \( S(x | x^*) = S(0 | x^*) \). (We can obtain this alternate result by raising \( V(m) \), the incumbent's utility for current employment.) In this circumstance, shareholders are indifferent between approving or rejecting. To ensure that \( x^* \) is accepted we should introduce the convention that, when indifferent, shareholders always vote with management. Alternately, as described at the end of section 3 (and justified more formally in Remark 4 of Appendix A), the manager can reduce his request \( x^* \) by \( \epsilon > 0 \) to induce strict preference by the shareholders.
Figure B.1

U, S

U(x | x?)

S(0 | 0)

S(0 | x?)

S(x | x?)

0

x?*  x, x?
Footnotes.

1. DeAngelo and Rice (1983) provide a thorough discussion of these hypotheses.

2. This type of "take it or leave it" agenda control has been extensively studied in a political context by Romer and Rosenthal (1978, 1979).

3. The use of the term "type" differs somewhat from the usual game-theoretic concept. Here, "type" indicates a skill level, as impounded into the value of the firm. In contrast to conventional usage, managers of the same "type" can have different utilities. Further, allowing infinite m is only a technical convenience; a claim we justify via the worked example of Appendix B.

4. For example, the uniform, normal, gamma and exponential distributions all satisfy this restriction.

5. This structure is similar to that used in Grossman and Hart (1980).

6. To make the bidding-cost explicit, we would have to write \[1-F(m+\epsilon)\]: this adds nothing.

7. The distinction we make between anticipatory and responsive takeover defenses, that the former are voted by shareholders while the latter are not, is an abstraction. It is a useful one for our purposes, and does not do violence to empirical reality.

8. The role of the assumption that \(\lim_{y \to 0} c'(y) = 0\), is to insure that some strictly positive level of \(y\) will always be chosen by the manager, whatever the value of \(x\). Consequently, we can use the calculus in our analysis. Without the assumption, we have to consider the corner case explicitly: this adds very little and does not substantively alter our main results. Similarly, insisting on the cost function being strictly convex in \(y\) is not essential for our results. It does, however, make the arguments cleaner.

9. Although the model is not explicitly game-theoretic, it can be reformulated as an extensive-form game in which the manager, Nature and the shareholders are players. The concept of credibility is simply that of (subgame) perfection (Selten, 1975).

10. In the explicit game-theoretic formulation (see fn.9), in equilibrium shareholders
necessarily accept a take-it-or-leave-it offer when indifferent (see, e.g., Banks & Gasmi, 1987).

Note that both managerial proposals and acquirers' bids are such offers.

11. The derivations of these results are available from the authors upon request.
References


