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A UTILITY MODEL FOR PRODUCT POSITIONING

Ralph L. Keeney
International Institute for Applied Systems Analysis
Laxenburg, Austria

Gary L. Lilien
Massachusetts Institute of Technology
Cambridge, Massachusetts

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MASSACHUSETTS
INSTITUTE OF TECHNOLOGY
50 MEMORIAL DRIVE
CAMBRIDGE, MASSACHUSETTS 02139

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ABSTRACT

A model and procedure are proposed to help design and position consumer durable and industrial products. These products are characterized on the one hand by little data of repeat-purchase variety but on the other by a high level of consistency between product preference and purchase behavior. The procedure is based on utility theoretic concepts for assessing preference and inferring probable behavior. Several numerical examples are included.

Firms continually face positioning and design issues related to their products. The product design issue is essentially: "What physical and psychological characteristics would I like my (new or revamped) product to have? [to maximize profit, market share, or more generally, the firm's expected utility]." This product design question is relevant to both new and existing products. The characteristics that can be controlled include price, several dimensions of use or quality, packaging, flavor (perhaps), etc. The firm generally has explicit control over physical quantities (price, taste, objective performance) and implicit control, through message and communications design, over the psychological quantities (a young-swinging beer, e.g.).

How should products be designed and how can those designs be changed during a product's life cycle to meet a firm's objectives? There have been two modes of analysis to help answer these questions. One approach is that of the psychometricians. In particular, making use of similarity data, Stefflre [13] developed "perceptual maps" of market structures. He suggested introducing brands in areas of the map which were relatively vacant. Other work has followed; a summary of the multidimensional scaling literature is available in Green and Carmone [4] and Green and Rao [5].

Another approach to the design question is provided by marketing model builders. Many attempts have been made to establish functional forms which relate product attributes, such as price, to output measures such as market share. Kotler [10] provides a review of much of the literature.

A more recent, promising approach is one taken by Urban [14]. Urban uses psychometric techniques to map the space of consumers' attitudes toward existing brands as well as an "ideal" brand. He postulates that the farther a brand is from the ideal point, the lower its market share should be. The

procedure, which models the trial and repeat-purchase processes separately, assumes that a brand's long run market share is a parameterized quadratic function of its distance from the ideal point. That function is then calibrated using actual market share data from the market. The model was shown to predict ultimate market share for a number of new brands quite well. But, due to data requirements, the model's use would seem to be limited to frequently purchased products. Virtually all work in these areas has been limited to frequently purchased products, mainly due to the greater availability of data. Thus, the problem of analytically positioning consumer durable products and most industrial products has been largely ignored.

A main difference between consumers purchasing packaged foods, and companies purchasing minicomputers is that industrial purchasing behavior conforms more closely to prescribed criteria (see Webster and Wind [15]). Thus one may "prefer" Krispy Krakers to "Whole Oats" and purchase Whole Oats anyway due to "variety seeking," availability, or other random behavior. The same inconsistency is not likely to occur in the purchase of a new minicomputer for one's firm or in the selection by parents of a medical treatment for a baby's congenital defect.

The methodology suggested is designed to be used for precisely those situations in which little data (of a repeat-purchase variety) are likely to be available, but where the customers (individuals or firms) purchase consistently with stated or inferred preferences. The approach is based on utility theory (see Raiffa [11] for a discussion of the basic concepts of utility theory). It assumes customers have a von Neumann-Morgenstern utility function defined over the product variables -- that is, customers are expected utility maximizers and "act as they should." Utility theory has been used in the past mainly in a normative or prescriptive sense -- telling decision

makers what they should do in given circumstances. The market situations considered here are, by definition, those in which individuals do what they should.

Hauser [6] is currently developing a structure for models of choice between finite alternatives. His structure of the analytical process of choice includes: (1) observation of behavior and measurement; (2) reduction and abstraction -- reducing the number of product dimensions to a few, "independent" ones and labelling them; (3) compaction, developing brand preference measures; (4) probability of choice, relating preference to purchase behavior, and (5) aggregation, transforming probability of purchase measures to market share measures.

The methods developed here suggest augmenting the observation step, by measuring attitudes in face-to-face interviews, through methods of direct utility function assessment as in [1], [8], [9]. The compaction operation then uses utility theory and the assumption of a von Neumann-Morgenstern utility function obviates the probability-of-choice step. Aggregation is performed by taking explicit account of consumer heterogeneity throughout the procedure.

The paper is organized as follows: Section 1 presents the formal structure of the model and introduces notation. Section 2 develops a general framework of analysis which is applied to a simple, hypothetical example in Section 3.

1. The Model and Notation

Consider a well-defined product class with N firms $F_1, \ldots, F_N$ in a single, specific market. Let m_i be the market share of F_i, and assume that each firm makes a single product.

Let the set of attributes $X_1, X_2, \ldots$ completely characterize a product, where X_1 could be price, X_2 could be reliability ratings, etc. These attributes would be attained by factor analysis of a series of well-defined product ratings

or perhaps non-metric scaling procedures, given a set of similarity judgments. Both methods have proven effective though neither has established "superiority." (Green, [3]) The output of these procedures, then, would be a reduced set of product characteristics, $X_1, \ldots X_J$. A specific level of X_j is x_j so a product is completely described by $\underline{x} = (x_1, \ldots x_J)$. The product of firm F_i will be denoted by $\underline{x}^i = (x_1^i, \ldots, x_J^i)$. A no-product purchase $\underline{x}^o = (x_1^o, \ldots x_J^o)$ could be included for completeness.

Customers will be designated by $C_1 \ldots, C_K$. It will be assumed that each customer C_k has a von Neumann-Morgenstern utility function $u_k(\underline{x}/\lambda)$, where C_k's utility function is specified by the set of parameters $\underline{\lambda} = (\lambda_1, \ldots, \lambda_R)$. Assuming each customer buys a product, utility theory suggests he should buy the product of firm F_i such that his utility is maximized.

Since viewing the problem from the firm's point of view will require the same methodology regardless of the firm, let us take the viewpoint of firm F_1. Firm F_1 has certain objectives which could include maximizing market share, maximizing profit, and so on. We postulate that the objective function of F_1 is also specified by a von Neumann-Morgenstern utility function v_1 over market share m_1, profit and/or other variables.

There are uncertainties here for both firms and customers. The firm wants to know utility functions for all customers in the market. This information about customer heterogeneity will be expressed in the form of a probability distribution $P_\lambda(\underline{\lambda})$ over the parameters $\underline{\lambda}$ describing a random customer's utility function. Thus, customers' utility functions are likely to differ so $\underline{\lambda}$ does not take on a single value, but, rather, is expressed as a probability distribution. Any firm will not have perfect knowledge about the "true" $P_\lambda(\underline{\lambda})$ and will, in general, attempt to estimate the distribution, entailing some error. Thus, we might con-

sider parameterizing the distribution to give $P_{\lambda/\theta}(\underline{\lambda}/\underline{\theta})$ where $\underline{\theta} = (\theta_1, \ldots, \theta_T)$ indicates the uncertainties of F_1 about the true $\underline{\lambda}$. We quantify that uncertainty by the probability distribution $P_\theta(\underline{\theta})$.

Consumers in general will differ in their knowledge or attitude about the characteristics $\underline{X}^i$ of firm F_i's product. From F_1's perspective, this heterogeneity can be described by the parameterized probability distributions $P_X^i(\underline{x}^1/\underline{\phi}^i)$, i=1,...,N, where $\underline{\phi}^i$ indicates a set of parameters for firm F_i. Again F_1 may be unable to estimate these distributions without error, so uncertainty will exist which we may quantify as $P_{\underline{\phi}}(\underline{\phi})$, where $\underline{\phi} = (\underline{\phi}^1, \underline{\phi}^2, \ldots, \underline{\phi}^N)$.

To summarize here, our model contains (from F_1's view):

1. An objective function v_1, which is a utility function, known with certainty.

2. A distribution of utility functions $u_k(x/\lambda)$ which vary across the heterogeneous customer population quantified by $P_{\lambda/\theta}(\underline{\lambda}/\underline{\theta})$. The probability distribution $P_\theta(\underline{\theta})$ quantifies this uncertainty.

3. A set of distributions of product-perceptions, $P_X^i(\underline{x}^i/\underline{\phi}^i)$, i=1,...,N, also varying across the heterogeneous customer-population. The probability distribution $P_\phi(\underline{\phi})$ quantifies this uncertainty.

2. General Model Structure

In this section, we first consider the decision an individual consumer must make and how his decisions are inputs to the decision-making processes of the firm. Then we focus on how firms can use the model for product positioning decisions.

2.1 Consumer Decisions

The consumer must decide which if any of the products in the market to buy given that he will buy at most one. Thus we explicitly consider the case of a consumer not purchasing any product. Another possibility would have been to define consumers as those who will buy one product and then formally include uncertainty about the number of consumers in the market.

Our model does not explicitly include individual consumer uncertainty about product characteristics. Rather, F_i is uncertain both about the set of consumer utility functions and the set of consumer product perceptions. Explicit inclusion of consumer uncertainty would needlessly complicate the problem. We will assume that C_k has a utility function $u_k(\underline{x})$ and his choices are not to buy a product and receive $\underline{x}^o$ or to buy the product of F_i and receive $\underline{x}^i$, i=1,2,...N. He will choose the option $\underline{x}^*$ giving him the highest utility where x* is defined by

$$(1) \qquad u_k(\underline{x}^*) = \max_{i=0,1,\ldots N} \{u_k(\underline{x}^i)\}$$

In the case of uncertainty, the consumer should choose the product of firm F_i which maximizes his expected utility. If $P_X^{ik}(\underline{x})$ represents the judgment of consumer C_k about firm F_i's product, the option $\underline{x}^*$ should be chosen such that

$$E[u_k(\underline{x}^*)] = \max_{i=0,1,\ldots,N} \int u_k(x)\, P_X^{ik}(\underline{x}^i)\, dx^i,$$

where $E[u_k(\underline{x}^*)]$ is the expected utility of the product of firm F_i (where F_o designates the no product option) for individual C_k.

2.2 Firm Decisions Under Certainty

Under certainty, firm F_1 should maximize its market utility v_1. Here we assume the distribution of utility functions is known and that customers do not vary in their perception of brand characteristics (that is $\underline{\theta}$ and $\underline{t}^i$, i=1, 2,...,N, are known). The condition of certainty could be used as a first cut, as less information is required here to attain a product design decision. Firm F_1 has a product with characteristics $\underline{x}^1$ and the population utility functions are represented by $u_k(\underline{x}/\underline{\lambda})$ where $P_\lambda(\underline{\lambda})$ represents the population heterogeneity. From (1) it follows that a consumer with parameters $\underline{\lambda}$ will choose firm F_1's product if

$$(2) \qquad u_k(\underline{x}^1/\underline{\lambda}) > u_k(\underline{x}^h/\underline{\lambda}),\ h=0,1,\ldots,\ N,\ \ h\neq 1,$$

where we neglect ties in terms of equally desireable alternatives. The proportion of consumers m_1 who choose x^1 is

$$(3) \qquad m_1 = \int_{\underline{\lambda}\varepsilon\Lambda_1} P_\lambda(\underline{\lambda})d\underline{\lambda},$$

where Λ_1 is that set of λ's such that (2) holds. Neglecting ties, $\Lambda_o, \Lambda_1, \ldots, \Lambda_N$ are mutually exclusive and collectively exhaustive so $\sum_{i=0}^{N} m_i = 1$ as required.

Suppose F_1 is considering changing its product position from $\underline{x}^1$ to $\hat{\underline{x}}^1$. Then a new $\hat{m}_1$ can be determined exactly as m_1 was using (2) and (3).

Suppose a new firm with utility function v is trying to enter a volume inelastic market with a maximum profit product $(\underline{x})$; that is, say, $v = m \cdot G \cdot (s - C(x)) - d(\underline{x})$ where s is unit product selling price, C is the unit cost of the product $(\underline{x})$, G is the market volume, m is its market

share, and $d(\underline{x})$ is the fixed development cost (plant, R&D, etc.) associated with $(\underline{x})$. Then v is the profit associated with the new product. There will likely be a set Q of alternate product positions, generically denoted by $\underline{x}$, that the new firm could attain. Given the existing products in the market, for each possible $\underline{x}$ there is a set $\Lambda(\underline{x})$ (perhaps null) defined by

$$(4) \qquad \Lambda(\underline{x}) \equiv \{\lambda:\ u_k(\underline{x}/\lambda) > u_k(\underline{x}^i/\underline{\lambda}),\ i=0,\ 1,\ \ldots,\ N\}$$

The best decision for this firm is to choose $\underline{x}$ in Q to maximize

$$(5)\ v(\underline{x}) = \left[\int_{\underline{\lambda}\varepsilon\Lambda(\underline{x})} P_{\lambda}(\underline{\lambda})\,d\lambda\right] \cdot \left[(G \cdot (s-C(\underline{x})) - d(\underline{x})\right]$$

where the first term in brackets in (5) represents the market share of product $(\underline{x})$ and $v(\underline{x})$ is the profit associated with the product. Under "nice" conditions it may be possible to simply differentiate $v(\underline{x})$ with respect to $\underline{x}$ to determine the product position to maximize profit.

2.3 Firm Decisions Uncer Uncertainty

The problems under uncertainty are parallel but more complicated than those with certainty. The two sources of uncertainty are the firms' imperfect knowledge about preference (or utility) heterogeneity, characterized by $\underline{\theta}$, and imperfect knowledge about perceptual heterogeneity, characterized by $\underline{\phi}^i$, i=0,1,...,N.

We will assume here, for simplicity, that perceptual heterogeneity and utility heterogeneity are independent within individuals. Fix the utility heterogeneity parameters as $\underline{\lambda}$ and select a customer with utility function $u(\underline{x}/\underline{\lambda})$ at random. That customer's selection will be from the set of product

characteristic distributions $P_X^i(\underline{x}^i/\underline{\phi}^i)$, i=0,1,...,N. The product purchased should be $\underline{x}^*$ such that

$$E(u(\underline{x}^*/\underline{\lambda},\underline{\phi}) = \max_{i=0,1,...N} E(u(x^i)/\underline{\lambda},\underline{\phi}^i)$$

where $E[u(x^i)/\underline{\lambda},\underline{\phi}^i]$ indicates the expected utility using $u(\underline{x}/\underline{\lambda})$ of $\underline{x}^i$ given $\underline{\lambda}$ and $\underline{\phi}^i$. That is,

$$(6) \qquad E[u(x^i)/\underline{\lambda},\underline{\phi}^i] = \int_{\underline{x}^i} u(x^i/\underline{\lambda}) \, P_X^i(\underline{x}^i/\underline{\phi}^i)d\underline{x}$$

As before, define $\Lambda_i(\phi)$ as the set of λ for which $\underline{x}^* = x^i$, where Λ is now dependent on ϕ. The market share of firm F_i, given $\underline{\theta}$ and $\underline{\phi}^i$, i=1,2,...,N, are known, is

$$(7) \qquad m_i(\underline{\theta},\underline{\phi}) = \int_{\lambda\varepsilon\Lambda_i(\phi)} P_{\lambda/\theta}(\underline{\lambda}/\underline{\theta})d\lambda.$$

If F_1's utility function v_1 has market share as its argument, it should choose $\underline{x}^i$ to maximize its expected utility, given by

$$(8) \qquad E(v_1) = \int_{\underline{\theta}^i} \int_{\underline{\phi}} V_1 P_\theta(\underline{\theta}) P_\phi(\underline{\phi}) d\underline{\phi} d\underline{\theta}$$

where P_θ, P_ϕ describe the uncertainty in F_1's measure of consumer's preferences and perceptions respectively.

3. A Simple Example (Price Imputed Quality)

A number of authors (see Rao and Shakun [12], Gabor and Granger [2], Kamen and Toman [7]) have made considerable use out of the concept that, in certain product classes which offer nearly indistinguishable products, (such as gasoline, packaged soaps, etc.) price, as an indicator of quality (and perhaps value) is the most important, if not sole determinant of purchase behavior. Without delving into this subject we offer the following simple example in which a single product-characteristic (say, price) distinguishes market products. Thus, the product characteristics are described by the single attribute X (price) and λ, θ, ϕ^i, all i, are univariate. The general problem is tractable with aid of a computer, perhaps through simulation, if necessary, but a more complex example here would obscure the basic ideas of the method.

Let the set of utility functions of the consumers be

$$(10) \qquad u_k(x/\lambda) = \lambda x - x^2, \ k = 1, \ldots, K,$$

where x is a price for the product and suppose the "true" distribution of λ among consumers is quantified by

$$(11) \qquad P_\lambda(\lambda) = \frac{1}{10} \ , \quad 0 \leq \lambda \leq 10.$$

There are three firms F_1, F_2, and F_3 and the consumers are heterogeneous in their perceptions of prices x^1, x^2, and x^3 as follows:

$$(12) \qquad P_X^i(x^i/\phi^i) = \frac{1}{\sqrt{2\pi}} e^{\frac{-(x^i - \phi^i)^2}{2}} \qquad i = 1, 2, \ldots$$

That is, the P_X^i are normal distributions with means ϕ^1, ϕ^2, and ϕ^3 respectively and unit variances (see Gabor and Granger [2] for empirical justification using x_i = log price).

3.1 Consumer Decisions

Given the uncertainty encoded by (12), the expected utility of product x^i to consumer C_k is

$$(13) \qquad E[u_k(x^i/\lambda)] = \int_X u_k(x/\lambda)\, P_X^i(x/\phi^i)\,dx$$

$$= \int_X (\lambda x - x^2)\, \frac{1}{\sqrt{2\pi}}\, e^{-\frac{(x-\phi^i)^2}{2}}\,dx$$

$$= \lambda\phi^i - (\phi^i)^2 - 1,$$

where clearly ϕ^i is the mean value of x^i, or mean perceived price.

Let us assume that

$$(14) \qquad \phi^1 = 2, \quad \phi^2 = 3, \text{ and } \phi^3 = 5.$$

Now we want to find the values of λ for which x^1, x^2, and x^3 are preferred. From (13), it follows that x^1 is preferred to x^2 on average if and only if

$$\lambda\phi^1 - (\phi^1)^2 - 1 > \lambda\phi^2 - (\phi^2)^2 - 1$$

which holds if $\lambda < \phi^1 + \phi^2$. Similarly x^1 is preferred to x^2 if and only if $\lambda < \phi^1 + \phi^3$ and x^2 is preferred to x^3 if and only if $\lambda < \phi^2 + \phi^3$. From this, since $\phi^1 < \phi^2 < \phi^3$, we have

$$(15) \qquad \left.\begin{matrix} x^1 \\ x^2 \\ x^3 \end{matrix}\right\} \text{ preferred iff } \left\{\begin{array}{l} \lambda < \phi^1 + \phi^2 = 5 \\ \phi^1 + \phi^2 \leq \lambda < \phi^2 + \phi^3 \\ \lambda \geq \phi^2 + \phi^2 = 8 \end{array}\right.$$

The market share of firm F_1 should be

$$(16) \qquad m_1 = \int_{\lambda=0}^{5} P_\lambda(\lambda)d\lambda = \int_{\lambda=0}^{5} \frac{1}{10}\, d\lambda = \frac{1}{2} .$$

In an analogous fashion, $m_2 = \frac{3}{10}$ and $m_3 = \frac{2}{10}$.

3.2 Firm Decision Under Certainty

Let us take a look at one special problem here. Suppose a new firm F_4 were to introduce a new product to compete with x^1, x^2, and x^3. If the consumer distribution of x^4 is to be characterized by ϕ^4 and

$$(17) \qquad P_X^4(x/\phi^4) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\phi^4)^2}{2}} ,$$

what is the best value of ϕ^4 (mean perceived price) to maximize m_4, the market share of firm F_4. (We are assuming firm F_4 accepts the other information in the problem, e.g. the $P^i(x/\phi^i)$). From analysis similar to that leading to (15), it follows that if $\phi^4 < \phi^1$,

$$\left.\begin{matrix} x^4 \\ x^1 \\ x^2 \\ x^3 \end{matrix}\right\} \text{ preferred iff } \left\{\begin{array}{l} \lambda < \phi^4 + \phi^1 \\ \phi^4 + \phi^1 < \lambda \leq \phi^1 + \phi^2 \\ \phi^1 + \phi^2 < \lambda \leq \phi^2 + \phi^3 \\ \lambda \geq \phi^2 + \phi^3 \end{array}\right.$$

so $m_4 = \frac{\phi^4 + \phi^1}{10}$. If $\phi^1 < \phi^4 < \phi^2$, then

$m_4 = \frac{\phi^4 + \phi^2 - (\phi^4 + \phi^1)}{10} = \frac{\phi^2 - \phi^1}{10}$; if $\phi^2 < \phi^4 < \phi^3$, then

$m_4 = \frac{\phi^3 - \phi^2}{10}$; and if $\phi^4 > \phi^3$, $m_4 = 0$ since x^4 would only be preferred if $\lambda > \phi^4 + \phi^3$ and $\phi^3 = 5$ and $\lambda_{max} = 10$. Thus, it is clear that under certain knowledge on the part of the firm F_4, the optimal ϕ^4 subject to (17) is $\phi^4 + \varepsilon = \phi^1 = 2$ for some small ε. Then $m_4 = \frac{\phi^1 + \phi^4}{10} = \frac{4 - \varepsilon}{10}$.

3.3 Firm Decisions Under Uncertainty

Return now to the three firms with three products described by (12) and suppose the class of consumer utility functions is given by (10). Suppose that Firm F_1, whose point of view we will take, feels

(19) $$f_\lambda(\lambda/\theta) = \frac{1}{\theta}, \ 0 \leq \lambda \leq \theta,$$

but due to the lack of available data, the firm's market research team feels there is some uncertainty about the true θ, which is characterized by

(20) $$P_\theta(\theta) = \frac{1}{3}, \ 9 \leq \theta \leq 12.$$

Suppose that x^2 is the "standard product" in the market that everyone knows well and $\phi^2 = 3$. Our own product x^1 and the competitor's x^3 are newer so they are subject to more consumer variability so let ϕ^1 be uniformly distributed from four to six. If these perceptions are independent, we have

(21) $$P_\phi(\phi^1,\phi^2,\phi^3) = \frac{1}{4}\ ,\ 1 \le \phi^1 \le 3\ ,\ \phi^2 = 3\ ,$$

$$4 \le \phi^3 \le 6\ .$$

It is easy to see that, as before $\phi^1 < \phi^2 < \phi^3$ so analogous to (15),

$$\left.\begin{array}{l} x^1 \\ x^2 \\ x^3 \end{array}\right\} \text{ preferred iff } \left\{\begin{array}{l} \lambda < \phi^1 \quad \phi^2 \\ \phi^1 + \phi^2 < \lambda \le \phi^2 + \phi^3 \\ \lambda \ge \phi^2 + \phi^3 \end{array}\right.$$

Hence to calculate the probability distribution m_1 for F_1, we just calculate the probability $\lambda < \phi^1 + \phi^2$. Note that the distribution of $\phi^1 + \phi^2$ is uniform from four to six so if $\alpha \equiv \phi^1 + \phi^2$;

(23) $$P_\alpha(\alpha) = \frac{1}{2}\ ,\ 4 \le \alpha \le 6.$$

Refer to figure 1 which shows the possible combinations of θ and α. From (20) and (23), it is clear that $p(\theta,\alpha) = \frac{1}{6}$. Note that $m_1 = \frac{\alpha}{\theta}$

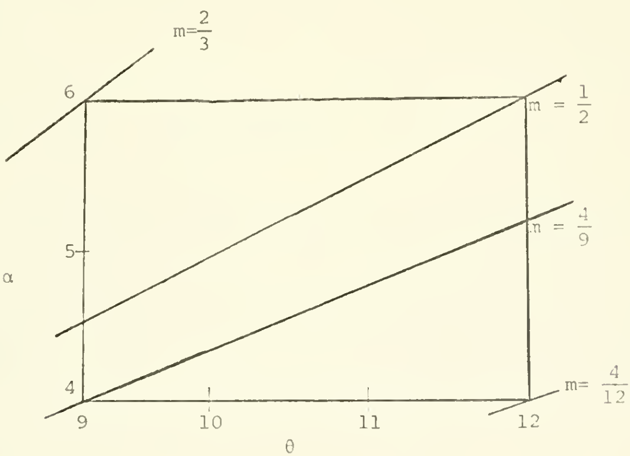

Figure 1

so at worst, $m_1 = \frac{\alpha_{min}}{\theta_{max}} = \frac{1}{3}$ and at best $m_1 = \frac{\alpha_{max}}{\theta_{min}} = \frac{2}{3}$. Hence for $\frac{1}{3} < m_1 < \frac{2}{3}$, the probability $m_1 < m$ equals $\frac{1}{3} + \frac{1}{3} P(\frac{\alpha}{\theta} < m)$. Integrating over the appropriate regions in Figure 1, we get the cumulative probability distribution for m_1 which is differentiated to yield

$$(24) \qquad P_{m_1}(m) = \begin{cases} 0 & , \quad m_1 \leq \frac{1}{3} \\ 12 - \frac{4}{3m^2} & , \quad \frac{1}{3} < m \leq \frac{4}{9} \\ \frac{21}{4} & \quad \frac{4}{9} < m \leq \frac{1}{2} \\ \frac{3}{m^2} - \frac{27}{4} & \quad \frac{1}{2} < m \leq \frac{2}{3} \\ 0 & , \quad m > \frac{2}{3} \end{cases}$$

This probability distribution is shown in Figure 2. The expected market share is given by

$$(25) \qquad E[m_1] = \int_{\theta=9}^{12} \int_{\alpha=4}^{6} \frac{\alpha}{\theta}(\frac{1}{6})\, d\alpha d\theta = 0.466$$

If firm F_1's preferences are quantified with the utility function v_1 over various market share levels, then one can simply use the probability distribution $P_{m_1}(m)$ and $v_1(m)$ to calculate the expected utility. Given a choice among options, whose impact is quantified by probability distributions over market share, firm F_1 should calculate the respective expected utilities and choose the option associated with the highest.

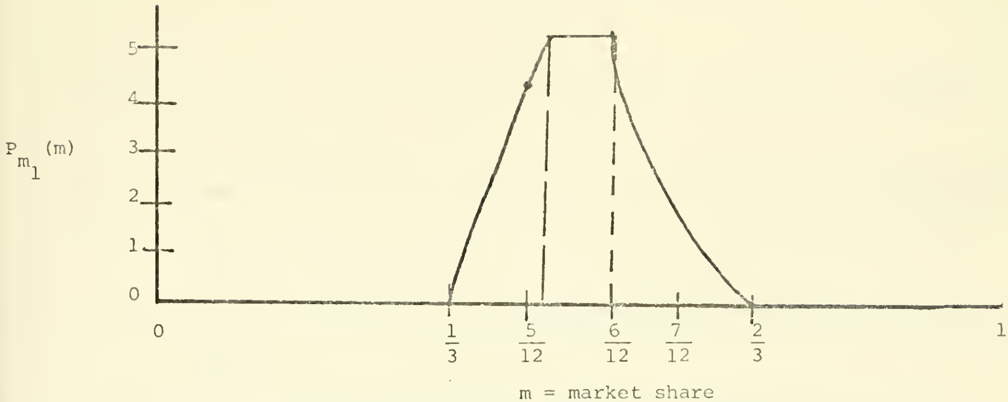

Figure 2

4. Model Status and Use

For a firm to utilize such a model, it would need to assess its utility function v and select parametric models for the population utility functions $u_k(\underline{x}/\underline{\lambda})$, their heterogeneity of preference $P_{\lambda/\theta}(\underline{\lambda}/\underline{\theta})$, perceptions $P_x^i(\underline{x}^i/\underline{\phi}^i)$, i=1,2,...,N, of the products. A market analysis team can model these features, utilizing limited interviews with customers. Then the parameters $\underline{\theta}$ and $\underline{\phi}^i$ i=1,2,...,N, could be quantified in a consumer survey. The relative frequency of responses could be smoothed to model the parameter distributions $P_\theta(\underline{\theta})$ and $P_{\phi^i}(\underline{\phi}^i)$. i=1,2,...,N.

As presented, the procedure is in a conceptual, proposal-for-use state. Empirical work is currently underway, using some concepts developed here, to design and position a prepaid health maintenance organization and other products. It is however too early to pass final judgment on the applicability of this methodology; reports of successful (or failed) application are the subject for other publications.

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