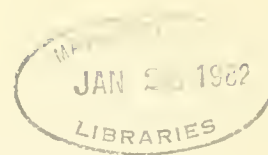


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THE VALUATION OF ASSETS UNDER MORAL HAZARD

by

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WP 1248-81

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ABSTRACT

We explore the valuation of assets in an asymmetric information world where the separation of ownership and control creates moral hazard. If the managers of assets can exercise some influence over the distribution of asset cash flows, the prices of these assets will depend crucially on the characteristics of the information structure associated with agency relationships (between owners and managers of capital) in the economy. When examined rigorously in a capital market setting, this simple observation leads to a number of interesting results about the design of managerial incentive contracts and the role of accounting in the economy.

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THE VALUATION OF ASSETS UNDER MORAL HAZARD

I. INTRODUCTION AND SUMMARY:

The problem of investment in capital projects (assets) has been analyzed in considerable detail in finance. In a nutshell it amounts to estimating the distribution of the future earnings stream that will be generated by an asset and then employing an appropriate valuation rule to determine the value of the asset contingent on the estimated earnings distribution. Most approaches to capital budgeting fit this description and differ only in the rule by which they value assets. A particularly noticeable feature of these approaches is that the probability distribution of asset cash flows is treated as exogenous to the valuation problem, i.e., all firms take the cash flow distributions associated with assets as given. However, such an assumption implicitly treats the firm as a completely passive recipient of its environmental inputs and ignores the managerial decision making process as well as the intricate web of agency relationships within organizations. The purpose of this paper is to explore some of the implications of relaxing this assumption by allowing the manager of an asset to influence the distribution of cash flows generated by the asset. This relaxation appears to accord well with our intuition in practice. If managers could not affect the returns from the assets they were managing, one would be compelled to accept the implausible conjecture that the lucrative compensation packages offered to corporate managers constitute systematic irrationality on the part of firms.

Owners of capital often lack the information and expertise to profitably exploit investment opportunities. This necessitates the contracting of a manager's services. However, since the manager is only an agent of the owner

(principal) of the asset, his interests will, in general, diverge from those of the owner. This gives rise to the now familiar agency problem (cf Jensen and Meckling (1976)). The main theme of agency theory is that the agent can take actions that may not be in the principal's best interests. In our context this implies that the manager, unless properly induced, will not exploit a given investment opportunity to its fullest and thereby will not maximize the returns accruing to the owner. However, the extent of this moral hazard problem depends on the characteristics of the information structure that defines the agency relationship. For example, if information is symmetrically available to the principal and the agent and both can costlessly observe and verify ex-post all the variables of interest (like the agent's actions, the output, etc.), the moral hazard problem can be trivially avoided. On the other hand, the problem is the severest when none of the variables can be observed ex-post. Thus, managerial actions will be different under different information structures. This further implies that asset returns, and consequently asset prices, will vary depending on the characteristics of information structures which define agency relationships.

The preceding paragraph summarizes the principal contention of this paper --if the distribution of the cash flows generated by an asset is partially endogenized by assuming that managerial actions can affect these cash flows, then the market prices of assets will depend crucially on the nature of the information available to principals and agents in the economy. In the sections that follow, this idea is developed rigorously and its implications for the role of accounting as well as the design of managerial incentive contracts are explored.

The economy we consider is a fairly simple one. Throughout all potential managers of assets are assumed to be identical, risk averse and in elastic

supply.¹ In this framework we develop two models. In the first model principals (prospective owners of assets) are assumed to be risk neutral and thus the price of any asset is simply the discounted present value of the net expected terminal cash flow accruing to its owner. This net flow is the expected value of the gross end-of-period revenue yielded by the asset less the contractually agreed upon compensation for the manager. The predetermined managerial compensation formula directly influences managerial actions which in turn (partially) determine asset returns. We use this model to prove a theorem which establishes sufficient and necessary conditions under which changes in the existing accounting system can affect the distribution of asset prices in the economy. An example with specific distributional and managerial risk preference assumptions is used to illustrate the theorem. A significant fact that emerges from this analysis is that advances in information systems, particularly as they relate to the design of performance evaluation mechanisms, can have a substantive impact on asset prices through their effect on managerial productivity. In valuation models this appears to be a neglected source of influence--the focus is usually on changes induced by purely technological innovation.

In the second model we assume that asset returns are generated by a single factor linear model of the type considered by Ross (1976) in his development of the arbitrage theory of capital asset pricing. Specifically, an asset's return is given by

$$R = g(\alpha) + \beta \tilde{m} + (1-\beta)\tilde{\epsilon} ,$$

where α is the action of manager, $g(\alpha)$ is a positive, strictly increasing concave function, β is a non-negative scalar, \tilde{m} is the return on the market (or some zero-residual-risk portfolio), and $\tilde{\epsilon}$ is the firm specific return. The assumption that principals are risk neutral is relaxed and instead, asset prices are assumed to be determined in accordance with the single period

capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965) and Mossin (1966). Thus, every principal's objective is to maximize market value. We use this framework to explore the nature of optimal managerial incentive contracts in a capital market setting familiar to finance theorists. In particular, we seek answers to the following major questions: Should corporate managers be allowed to trade (portions of) their compensation packages in the market? How important is the systematic risk (as opposed to idiosyncratic risk) of a firm in the determination of managerial compensation? If a firm can choose its own systematic risk (by varying the mix of heterogeneous assets which comprise the firm), how will this choice be affected by the presence of moral hazard (agency costs)?

Despite its simplicity, our analysis provides a number of positive and normative implications. The main ones are summarized below.

- (1) With moral hazard, information producing disciplines like accounting may not be independent of the distribution of asset prices. In fact, in an efficient and competitive capital market there may exist a spectrum of potential asset price distributions, with the specific equilibrium distribution determined by the extant 'state of the art' in the accounting system.
- (2) Ceteris paribus, asset values will be higher when managers are not allowed to trade their compensation packages in the capital market. This result holds both with and without moral hazard.
- (3) If the manager is not allowed to trade in the market, the optimal incentive contract does not completely shield him from market risk when moral hazard is present. However, the greater the correlation between asset and market returns the greater is the negative dependence of the optimal contract on the market return. In the absence of moral hazard, the optimal contract is equivalent to a "forcing contract" that provides the manager complete insurance against idiosyncratic risk, contingent on

a promised level of effort being delivered.

(4) If a manager (who already has in his possession a well diversified portfolio of investments) is allowed to trade in the market, the optimal incentive contract depends only on the asset's total return and does not explicitly include the market return observed ex-post if there is moral hazard. That is, the manager does bear the idiosyncratic risk associated with the asset being managed. This is in sharp contrast to the prescriptions of the CAPM, namely to ignore idiosyncratic risks and be concerned only with systematic risk in making production and investment decisions. Further, if there is no moral hazard, the manager receives a constant, predicated upon the expenditure of a desired level of effort. This observation differs from the proven result (see Shavell (1979)) that when both contracting parties are risk averse the agent's optimal compensation cannot be a constant.

(5) Suppose $g(\alpha)$, \tilde{m} and $\tilde{\epsilon}$ are exogenous but β can be chosen by the principal. In this case, the higher the optimal α desired by the principal, the greater will be the optimal β . This means firms with higher 'managerial-action-induced' returns will choose to adopt more systematic risk. This result is interesting because it suggests that moral hazard tends to reinforce the positive relationship between expected return and systematic risk that is implied by both the CAPM and the arbitrage pricing theory (APT).

Our analysis is organized in five remaining sections. In Section II we develop the basic model for valuing assets in the presence of moral hazard. This is the first model in which principals are risk neutral. The model is followed by a derivation of the conditions under which changes in the accounting system can influence asset prices and an illustration that clearly highlights the role of each condition. In Section III we introduce the single

factor linear returns model in conjunction with the CAPM and obtain optimal incentive contracts under the assumptions that contracts are constrained to be linear and managers are not allowed to trade. In Section IV we still restrict our attention to linear contracts but explore the implications of allowing managers to trade. Section V is a rather brief analysis of how principals would select the magnitude of systematic risk, if they could. Our concluding remarks are presented in Section VI.

II THE BASIC MODEL:

In a competitive capital market the value (or price) of an asset can be expressed as

$$P = L[Q(x)] , \quad (1)$$

where $L[\cdot]$ is a positive valuation operator and $Q(x)$ is the cumulative distribution function of the (possibly intertemporal) net cash flows yielded by the asset. This is a general valuation expression that subsumes all the asset pricing models developed in finance as special cases. The specific form of the valuation operator will depend on the assumed market structure and could be influenced by investor preferences. For example, if investors are risk neutral, P will simply be the discounted present value of the mean cash flow or if the capital market is complete and a unique vector of state prices exists, P will be a state-price-weighted sum of the asset cash flows across different states of nature.

Irrespective of the properties of $L[\cdot]$, $Q(x)$ is invariably assumed to be exogenous to the valuation process itself and beyond the control of either the owners or the managers of the asset. While it is reasonable to assume that mutual fund managers assemble portfolios of stocks and bonds without exercising any influence over the return distributions of these securities, the plausibility of extending the assumption to the case of those who manage

productive assets is questionable. Billions of dollars are spent annually to compensate managers for the use of their technical and administrative expertise. In part, such an expenditure reflects the belief that differences in management quality can induce significant differences in the benefits that can be extracted from assets. There is a variety of ways in which managers can affect the distribution of asset cash flows--perhaps the most obvious being a variation in the effort expended in managing the asset. Clearly, a manager who spends all his time playing golf should not expect to get the same distribution of cash flows as another manager who is responsible for an identical project but devotes twelve hours a day to the diligent management of the asset. There are, of course, other ways too and Jensen and Meckling (1976) have provided an extensive discussion of these--indulging in excessive consumption of corporate prerequisites, practicing nepotism, hiring attractive but inefficient secretaries, etc.

Armed with these arguments we will allow the distribution of cash flows of a given asset to be partially endogenized by assuming that managerial actions can change this distribution. The single period cash flow, x , yielded by the asset is therefore a function, $X(\alpha, \theta)$, of the manager's choice of action (effort) α as well as the realization of some exogenous uncertainty θ . It is assumed, for analytical tractability, that α is a scalar, but it should be understood that it subsumes all managerial actions that can affect x . No restrictions are placed on $X(\alpha, \theta)$ except that $\partial X / \partial \alpha > 0$ for each $\theta \in \Theta$ (a possibly nondenumerable set of states). This implies that in any given state a higher level of effort by the manager results in a higher cash flow. No comparison is made across states--a high level of effort in a bad state could lead to a lower payoff than a low level of effort in a good state. Further, the manager's choice of action is made before the realization of θ .

Consider now a simple economy which consists of two types of economic entities--principals and agents. All principals are risk neutral and are assumed to be endowed with capital. They are therefore, existing and prospective owners of assets in the economy. The principals however, do not possess the expertise to profitably manage any asset. This necessitates the contracting of the services of managers. Managers, who are risk averse, are precluded from investing in any assets,² and consequently they can satisfy their consumption needs only by selling their services. This restriction on managerial activities means that the implications of the manager reducing his risk through diversification are ignored. In a state preference framework, it means that the manager is prevented from setting his personal marginal rates of substitution for wealth in different states equal to state prices, even in a complete capital market. The assumption of principal risk neutrality and the inability of the manager to invest in securities is consistent with the assumption that the states that affect the firm's output are all firm specific and do not affect the aggregate output of the economy. Thus, even if principals are risk averse, they will contract as if they were risk neutral if the firm specific risk is diversifiable. This feature of the model makes it distinctly different from the linear returns model of the next section in which market dependencies are allowed. For simplicity, all managers are assumed to have identical preferences and skills. Principals as well as managers are in elastic supply, which means a competitive equilibrium will result.

Since in an agency theoretic framework it makes little sense to discuss firms as separate from the individuals who contract to create them, the usual microeconomic distinction between firms and their employees will not be made. Firms are merely shells and their existence as legal entities is of no particular interest here. Instead, we will focus on assets in the economy

and explore the implications, for their valuation, of principals and managers entering into contractual relationships to share the economic rents accruing from these assets. Associated with each asset is a (possibly unique) technology and thus the cash flow function for the i th asset can be expressed as

$$x_i = X_i(\alpha, \theta) \equiv X(\alpha, \theta, T_i) \quad (2)$$

where T_i is the "technology" related to the i th asset. So when purchasing an asset, the principal (or group of principals) also buys the technology that comes with it. Throughout it will be assumed that principals and managers have homogeneous beliefs about θ . To manage any asset the principal must hire a manager who, for taking an action $\alpha \in A$ (the feasible action space of the manager), will be compensated by the principal according to some pre-determined fee schedule or incentive contract $\langle \phi \rangle$, i.e., the manager is paid an amount ϕ and the principal gets $x - \phi$. Every manager is assumed to be an expected utility maximizer who possesses a von-Neumann-Morgenstern utility function $U(\alpha, \phi)$. It is assumed that $\partial U / \partial \alpha < 0$, $\partial U / \partial \phi > 0$ and $\partial^2 U / \partial \phi^2 < 0$ -- every manager is risk averse and has a positive marginal utility for his share of the payoff. In addition, the manager has an aversion toward higher effort. This introduces moral hazard in the model because the principal is interested in maximizing the expected returns accruing to him from the asset, while the manager simply maximizes his personal expected utility. This kind of moral hazard is similar, but not identical, to that discussed by Jensen and Meckling (1976). In their model the problem is created due to the manager's propensity to consume prerequisites from firm resources. In either case, unless certain special conditions are satisfied, separation of ownership and control will inevitably engender moral hazard.

Actions taken by managers, and consequently the efficiency with which assets are managed, may not be observable or verifiable by principals ex-post.

This will generally create the need for some system under which information can be generated about the activities of managers and measures for evaluating their performance can be constructed. We shall call this the accounting system and represent it by Ω . Mathematically, Ω is assumed to a space of (bounded and measurable) functions and when the action taken by the manager is unobservable ex-post, principals may choose a function $\omega(\alpha)$ ($\in \Omega$) which we shall call a monitor of the manager's effort, i.e. $\omega(\alpha)$ conveys information about managerial activities that could be used as a basis for computing the manager's compensation.³ The information conveyed by $\omega(\alpha)$ could be noisy or imperfect and will depend on the properties of Ω .

In a competitive capital market the price of the i th asset, P_i^* , will be⁴

$$P_i^* = \text{Max}_{\langle \phi_i \rangle \in \Phi} \iint (x - \phi_i(v_i) - \xi_i) q_i(x, \omega_i; \alpha_i) dx d\omega_i \quad (3)$$

$$\langle \phi_i \rangle \in \Phi$$

$$\alpha_i \in A$$

$$\omega_i(\alpha_i) \in \Omega$$

$$\text{subject to } \iint U(\alpha_i, \phi_i(v_i)) q_i(x, \omega_i; \alpha_i) dx d\omega_i = \bar{C} \quad (4)$$

$$\alpha_i \in \arg \max_{\alpha_i \in A} \iint U(\alpha_i, \phi_i(v_i)) q_i(x, \omega_i; \alpha_i) dx d\omega_i \quad (5)$$

where $q_i(\cdot, \cdot; \cdot)$ is the joint density function of the output and the monitor for the i th asset, conditional upon the effort α_i chosen by the i th asset's manager⁵ (the above formulation explicitly recognizes the fact that the difference between the α_i revealed ex-post by $\omega_i(\cdot)$ and the true α_i is, in general, stochastic ex-ante or in other words, the monitoring could be noisy); ξ_i is the cost of utilizing the monitor; Φ is the space of bounded and measurable feasible fee schedules⁶; v_i is a vector representing the variables on which the manager's compensation depends⁷; and \bar{C} is the manager's reservation utility level.

Essentially, this mathematical formulation implies that in a competitive market the price of any asset will be the expected value of the net cash flow accruing to the (prospective) owner or owners of the asset, assuming that the asset will be optimally managed.⁸ By optimal asset management we mean that the principal will select a (cost effective) monitor and a fee schedule that will induce the manager to take an action that will maximize the principal's welfare subject to the constraint that the manager's expected utility (given his optimal choice of action in response to the fee schedule chosen) is equal to a certain minimum reservation level \bar{C} . Note that \bar{C} will be the outcome of an equilibrium in the managerial labor market and the use of the constraint represented by (4) is to ensure that the manager will agree to work for the principal.⁹ It is clear that the formal statement of the valuation problem is considerably simplified by the assumed risk neutrality of principals--with risk averse principals and an incomplete market, at least one serious problem that would have to be resolved is that of a possible lack of unanimity between the (prospective) owners of the assets about its value (see Baron (1979)). In general, the monitoring functions contained in Ω can be classified as informative, non-informative, efficient and inefficient. These classifications are defined below. Since in each definition the asset is fixed, the subscript i is dropped.¹⁰

Definition 1: Consider a specific asset. A monitor ω of α is called informative with respect to this asset if there exist at least two sets M^+ and M^- in the range of ω such that

$$\frac{q_{\alpha}(x, M^+; \alpha)}{q(x, M^+; \alpha)} \neq \frac{q_{\alpha}(x, M^-; \alpha)}{q(x, M^-; \alpha)} \quad (6)$$

where $q(x, M^+; \alpha) = \int_{M^+} q(x, \omega; \alpha) d\omega$, $q(x, M^-; \alpha) = \int_{M^-} q(x, \omega; \alpha) d\omega$,

$$q_\alpha(x, M^+; \alpha) = \int_{M^+} q_\alpha(x, \omega; \alpha) d\omega$$

and $q_\alpha(x, M^-; \alpha) = \int_{M^-} q_\alpha(x, \omega; \alpha) d\omega$.

ω is called non-informative if $\frac{q_\alpha(x, \omega; \alpha)}{q(x, \omega; \alpha)}$ is constant for all ω . Subscripts

denote partial derivatives.

Definition 2: Consider a specific asset. A monitor ω of α is called efficient with respect to this asset if $P^* > P^0$ where P^* is the price of the asset if the monitor is used and P^0 is its price if the monitor is not used. The monitor is called inefficient if $P^* \leq P^0$.

A point to be noted is that the informativeness and efficiency of any monitor are defined for a fixed asset. Thus even though the cost, ξ , of using a particular monitor is unaffected by the asset under consideration, a monitor could be efficient with respect to one asset but inefficient with respect to another. These definitions are now used, in the theorem given below, to establish the contention that when moral hazard is explicitly considered in asset valuation, changes in the accounting system can affect the probability distribution of asset cash flows and consequently, asset prices. The key to this result is the ex-post unobservability of managerial actions¹¹--with perfect observability, forcing contracts can be employed to eliminate the moral hazard problem, as demonstrated by Harris and Raviv (1979).

THEOREM 1: Consider some specific asset. Assume that managerial actions cannot be observed without error ex-post, and that the current accounting system, Ω , contains only non-informative and inefficient monitors.¹² Then,

the (minimum) necessary conditions for a change in Ω^0 to cause an increase in the price of the asset are

- (i) managers are risk averse,
- (ii) the probability density function of the asset's cash flow does not have a compact support that moves with α , and ¹³
- (iii) the change in Ω^0 is effected through the addition of at least one informative monitor

The above conditions are sufficient if the informative monitor added is also efficient.

PROOF: See Appendix.

The proof demonstrates how a change in the accounting system can alter the manager's choice of action (in the proof the monitor induces the manager to change his effort) and thereby, the price of the asset. Although it is assumed that the initial starting point is an accounting system that contains only inefficient monitors, the theorem can be easily extended to the case where an accounting system with efficient monitors is augmented by more efficient monitors. The following brief argument not only supports this assertion but also provides a (heuristic) condensation of the proof of the theorem.

Let X be the possible set of values x can take and assume X is not compact. From the maximization problem in (3), (4) and (5) it is clear that when the manager's action is unobservable with perfect accuracy ex-post,

$$\alpha: \Phi \rightarrow A \tag{7}$$

and $\phi: X \times \Omega \rightarrow \Phi \tag{8}$

According to (8) changes in Ω will generally induce a change in the optimal fee schedule, $\phi^*(v)$, via a change in the monitor, ω^* , chosen and (7) implies that this will cause the manager's optimal choice of action, α^* , to be different. This, in turn, will perturb $q(x, \omega^*; \alpha^*)$ and consequently, the

price of the asset, P^* . Moral hazard will therefore cause the price of an asset to be different under different accounting regimes. This difference is not due to any variation in the intrinsic value of the asset, but merely reflects different in the efficiency with which it is managed. This notion is now illustrated through an example.

PROBLEM: Consider an economy in which all principals are risk neutral and all managers have utility functions of the form $U(\alpha, \phi(x)) = \sqrt{\phi(x)} - \alpha$ and $\bar{C} = 0$. Consider two assets--B and D. Asset B's cash flow is generated by the production function $x_B = \alpha\theta_B$, where θ_B is lognormal with $E(\log \theta_B) = 0$ and $\text{var}(\log \theta_B) = \sigma^2$. Asset D's cash flow is generated by the production function $x_D = \alpha\theta_D$, where θ_D is uniformly distributed between $\exp(\sigma^2/2)-k$ and $\exp(\sigma^2/2)+k$; $k(\epsilon(0, \exp(\sigma^2/2)))$ is a fixed scalar. We want to compute the prices of these two assets under a variety of information structures.

SOLUTION:

First note that for asset B

$$\begin{aligned} E(\log x_B) &= \log \alpha, \text{var}(\log x_B) = \text{var}(\log \theta) = \sigma^2, \\ q(x_B|\alpha) &= \{\sqrt{2\pi} \sigma x_B\}^{-1} \exp\{-[(\sqrt{2\pi}\sigma)^{-1} \log(x_B|\alpha)]^2\}, \\ q_\alpha(x_B|\alpha) &= q(x_B|\alpha) \cdot (\alpha\sigma^2)^{-1} \log(x_B|\alpha), \end{aligned}$$

$$\text{and } E(x_B) = \exp[\log \alpha + \sigma^2/2] = \alpha \psi, \text{ where } \psi \equiv \exp(\sigma^2/2).$$

Similarly, for asset D

$$E(x_D) = \alpha \psi \text{ and } \text{var}(x_D) = k^2/3.$$

Now let λ be the multiplier for (4) and μ the multiplier for (5). If only x is observable ex-post, (5) will have to be used. Assuming for the moment that Ω is empty and optimizing the Lagrangian pointwise gives us the following characterization for $\phi_i^*(x)$ ($i = B, D$):

$$\{\partial U(\phi_i(x_i))/\partial \phi_i(x_i)\}^{-1} = \lambda_i + \mu_i \{\partial q(x_i; \alpha_i) / \partial \alpha_i\} \{q(x_i; \alpha_i)\}^{-1} \quad (9)$$

where U is assumed to be separable ($U(\alpha, \phi(x)) \equiv U(\phi(x)) - V(\alpha)$) as in the proof of Theorem 1 and α_i is obtained from (5). Note that λ_i is obtained as a solution to (4) for a given \bar{C} and μ_i is obtained as a solution to

$$\int (x_i - \phi_i(x_i)) (\partial q(x_i; \alpha_i) / \partial \alpha_i) dx_i + \mu_i \left\{ \int U(\phi_i(x_i)) (\partial^2 q(x_i; \alpha_i) / \partial \alpha_i^2) dx_i - \partial V^2 / \partial \alpha_i^2 \right\} = 0. \quad (10)$$

However, if both x and α are observable ex-post, (5) can be dropped and the optimal contract will have the form

$$\phi_i^*(\alpha) = \begin{cases} t & \text{if } \alpha = \alpha_i^* \\ 0 & \text{otherwise} \end{cases}$$

where t (a constant) is obtained as a solution to

$$\{\partial U(t) / \partial t\}^{-1} = \lambda \quad (11)$$

Initially assume that both x and α are observable without error ex-post.

Solving for the optimal contracts gives

$$\phi_B^*(\alpha) = \begin{cases} (\psi/2)^2 & \text{if } \alpha = \psi/2 \\ 0 & \text{otherwise} \end{cases}$$

and

$$\phi_D^*(\alpha) = \begin{cases} (\psi/2)^2 & \text{if } \alpha = \psi/2 \\ 0 & \text{otherwise} \end{cases}$$

where $\phi_B^*(.)$ and $\phi_D^*(.)$ are the optimal contracts for assets B and D respectively. It is clear that $\alpha_B^* \equiv \alpha(\langle \phi_B^* \rangle) = \psi/2$ and $\alpha_D^* \equiv \alpha(\langle \phi_D^* \rangle) = \psi/2$ and the prices of the two assets, in a competitive capital market, should be the same, namely $(\psi/2)^2$.

Next, assume that only x is observable ex-post and Ω is empty. From the Euler equation we see that the optimal contract for asset B is of the form

$$\{\lambda/2 + (2\alpha\sigma^2)^{-1} \mu \log(x_B/\alpha)\}^2.$$

Straightforward calculations yield $\mu = 2\alpha^2\sigma^2$ and $\lambda = 2\alpha$. Consequently, the expected value of the payoff accruing to the principal is

$$\alpha\psi - \alpha^2 - \alpha^2\sigma^2.$$

From the usual first order condition, it follows that the principal's expected payoff is maximized with

$$\hat{\alpha}_B \equiv \alpha(\langle \hat{\phi}_B \rangle) = [2(1 + \sigma^2)]^{-1} \psi.$$

To ensure this choice of action by the manager, the principal must offer the contract

$$\hat{\phi}_B(x_B) = \{([2(1 + \sigma^2)]^{-1} \psi) (1 + \log[2x_B(1 + \sigma^2)\psi^{-1}])\}^2.$$

Lack of observability of managerial actions reduces the price of the asset to $(\psi/2)^2(1 + \sigma^2)^{-1}$. Thus, only if $\sigma^2=0$ will the price of asset B be unaffected by the principal's inability to costlessly verify the manager's actions. This of course simply confirms a well known result--in a world of certainty, information about α is of no value because it can be directly inferred from x anyway.

For asset D, however, the first best solution can be obtained even under uncertainty if $3k < \psi$. This is possible because the distribution of this asset's cash flow has a compact moving support. To see this note that with the action $\psi/2$, the lowest possible output is $(\psi/2) (\psi-k)$. Thus, a fee schedule of the form

$$\hat{\phi}_D(x_D) = \begin{cases} \psi/2 & \text{if } x \geq (\psi/2) (\psi-k) \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

will do the trick if $\hat{\alpha}_D \equiv \alpha(\langle \hat{\phi}_D \rangle) = \psi/2$. Suppose the manager picks an action $\alpha_D^0 \neq \hat{\alpha}_D$. In this case, $\mathcal{P}(x_D > (\psi/2) (\psi-k)) = \{2\alpha_D^0 k\}^{-1} [\alpha_D^0 (\psi+k) - (\psi/2) (\psi-k)]$ and the manager's expected utility is

$$[\psi/2] \mathcal{P}(x_D \geq (\psi/2) (\psi-k)) - \alpha_D^0.$$

(Here \mathcal{P} is a probability measure).

Note that this term can be positive (and thus greater than $\bar{C} = 0$) only if $\alpha_D^0 < \psi/2$. This means that if the manager decides to take an action other than $\psi/2$, he must necessarily choose a lower action. Further, for the manager

to have a non-zero probability of escaping detection (that his action differs from α_D^0), we must have $\alpha_D^0(\psi+k) > (\psi/2) (\psi-k)$, which implies $\alpha_D^0 > (\psi/2) (\psi-k) (\psi+k)^{-1}$. Thus, if $\alpha_D^0 \neq \hat{\alpha}_D$, it must be true that $\alpha_D^0 \in ((\psi/2) (\psi-k) (\psi+k)^{-1}, \psi/2)$.

Now if α_D^0 is the manager's optimal choice, it must satisfy the first order condition

$$(\psi/2) \{ [4(\alpha_D^0)^2 k]^{-1} [\psi(\psi-k)] \} - 1 = 0,$$

which means $\alpha_D^0 = \sqrt{\psi(\psi-k) (8k)^{-1}}$. It is easy to see that if $\psi > 3k$, then $\alpha_D^0 > \psi/2$ and this violates (13). Obviously then, as long as the distribution of θ_D satisfies the condition $\psi > 3k$, the manager's optimal choice of action, α_D^0 , will be $\psi/2$ and the fee schedule described in (12) yields a first-best solution.¹⁴

The Role of Monitoring: It is clear that if the θ_D distribution satisfies the stipulated restriction, the principal has no incentive to acquire costly monitoring for asset D. On the other hand, monitoring could prove valuable for asset B. Suppose a monitor ω is available and its relationship to α can be expressed as $\omega = \alpha e$, where e is a lognormal random variable with $\text{cov}(e, \theta) = 0$, $E(\log e) = 0$, and $\text{var}(\log e) = \sigma_\omega^2$. Assume initially that the monitor is costless. It is obvious then that it is efficient.

For the moment, let us drop the subscript B for notational convenience. Since $q_\alpha(x, \omega; \alpha) = q(x, \omega; \alpha) \cdot \Delta$, where $\Delta \equiv (\alpha \sigma_\omega^2)^{-1} \log(x/\alpha) + (\alpha \sigma_\omega^2)^{-1} \log(\omega/x)$, the Euler equation can be used to show that the optimal contract is of the form

$$[\lambda/2 + \mu\Delta/2]^2.$$

Simple calculations give $\mu = 2\alpha^2 [\sigma^2 + \sigma_\omega^2]^{-1}$, and with $\bar{C} = 0$, we get $\lambda = 2\alpha$.

Making these substitutions, the optimal contract can be written as

$$\alpha^2 [1 + \{1 + \sigma^2 \sigma_\omega^2\}^{-1} \{ \log(x/\alpha) + [\sigma^2 \sigma_\omega^2] \log(\omega/\alpha) \}]^2,$$

and the expected value of the principal's share of the payoff is

$$\alpha\psi - \alpha^2 [1 + \sigma^2 \sigma_\omega^2 (\sigma^2 + \sigma_\omega^2)^{-1}].$$

The principal's welfare is therefore maximized by offering the optimal incentive contract

$$\phi^*(x, \omega) = [\psi \zeta_1^{-1} \{1 + \zeta_2 + \zeta_3 + \zeta_4\} \zeta_4 \{1 + \zeta_2\}^{-1}]^2 \quad (14)$$

where $\zeta_1 \equiv 2 + 2\sigma_\omega^2 (\sigma^2 + \sigma_\omega^2)^{-1}$

$$\zeta_2 \equiv \sigma^2 \sigma_\omega^{-2}$$

$$\zeta_3 \equiv \log x - \log(\psi \zeta_1^{-1})$$

$$\zeta_4 \equiv \log \omega - \log(\psi \zeta_1^{-1}).$$

With this contract the manager's optimal choice of action is $\alpha^* = \psi \zeta_1^{-1}$, and the price of the asset is

$$P^* = \psi^2 \{4[1 + \sigma^2 \sigma_\omega^{-2} + \sigma^2] \{1 + \sigma^2 \sigma_\omega^{-2}\}^{-1}\} \quad (15)$$

An examination of (14) reveals an intuitively appealing property--the weight assigned to the monitor in the optimal contract is inversely proportional to the variance of the monitor. In the limit, as this variance goes to infinity, the weight attached to the monitor goes to zero. Of course, this observation is predicated upon the assumption that $\xi = 0$. Otherwise, if ξ and σ_ω^2 are also inversely related, the monotonicity of the (inverse) relationship between the variance of the monitor and the extent of its use in the fee schedule may be violated. Also note that $\lim_{\sigma_\omega^2 \uparrow \infty} P^* = \psi^2 \{4(1 + \sigma^2)\}^{-1}$, which is the same as the second best solution with no monitoring, and $\lim_{\sigma_\omega^2 \downarrow 0} P^* = \psi^2/4$, which is identical to the first-best solution when α is observable. Therefore, if the monitoring technology has a positive cost ξ , using the monitor with asset B would be of value if $P^* - \xi > \psi^2 \{4(1 + \sigma^2)\}^{-1}$, where P^* is given by (15).

The above illustration reemphasizes the potential dependence of asset values on the observability of contract variables ex-post and helps to clarify an apparent misunderstanding about the role of accounting. Numerous empirical studies of the efficient markets hypothesis have found that a significant portion of the information contained in accounting statements is impounded in

security prices even before these statements are released. This appears to have led to the conclusion that at any instant in time the distribution of future security prices is independent of the accounting system and somehow determined by other factors in the economy. For example, Beaver (1972) argues that accounting information is valuable because it helps the individual investor to assess the systematic risk of securities. Gonedes (1976) claims that newly generated accounting information provides signals on the true underlying distribution of returns. If the basic premise of this paper (that managers can affect asset cash flows) is accepted, then it is clear that such theorizing, which ignores the stewardship role of accounting, is at best incomplete. The reason is that principals can very rarely observe managerial actions ex-post and therefore, in the absence of any monitoring technology, incentive contracts would depend only on x . However, accountants (particularly auditors) provide principals with information about managerial actions in addition to that provided by the realized output of the firm. This permits the use of monitors and more efficient contracts and consequently, creates a variation in the distribution of asset cash flows via a change in managerial actions. For example, in the numerical illustration the open interval $(\psi^2\{4(1+\sigma^2)\}^{-1}, \psi^2/4)$ corresponds to the possible distribution of asset (B) prices associated with varying degrees of efficiency of Ω . In fact, assuming for the moment that ξ is independent of σ_ω^2 , one can parameterize the efficiency of Ω by σ_ω^2 . Since $\partial P^*/\partial \sigma_\omega^2 < 0$, we can say that as the accounting system becomes more efficient (that is, as σ_ω^2 declines), the price of asset B increases. In other words, associated with each degree of efficiency in Ω is a different competitive equilibrium price for asset B. This implies that the distribution of asset prices is not independent of the accounting system, as is so often claimed, but is at least partially determined

by it. Note that this observation is not inconsistent with market efficiency as defined in the finance literature. As long as asset prices fully reflect the decisions of principals and managers (in the sense that asset prices are the solution to (3), (4) and (5) in equilibrium), the capital market will always be efficient (and no empirical studies will be able to detect any serious deviations) regardless of the characteristics of the accounting system. The effect of changing the accounting system will be merely to move the economy from one efficient markets equilibrium to another. In general, associated with any given accounting system is an information structure that defines the feasible set of fee schedules that can be employed to motivate and compensate managers. This feasible set then implies a set of optimal managerial actions which in turn determine the probability distributions of cash flows for various assets. If, as in our illustration, the capital market is informationally efficient, the equilibrium prices of assets will correctly reflect the efficiency with which these assets are managed. In other words, the state of the accounting system will be accurately impounded in asset prices. However, if the accounting system itself changes, the probability distributions of cash flows of these assets will change and a new (efficient markets) equilibrium will be established with new set of security prices.

III. THE LINEAR RETURNS MODEL WITH NO MANAGERIAL TRADING

In this section we alter the characteristics of the problem by assuming that (i) principals are risk averse, (ii) managers have mean-variance utility functions, (iii) incentive contracts are constrained to be linear, and (iv) asset returns are generated by a linear process. The assumption that principals are risk averse becomes important if a particular asset's marginal product is correlated with the income principals get from other assets, because the asset's profitability will then not be a diversifiable risk. The mean-variance

utility function assumption has been commonly employed in portfolio theory. Constraining managerial incentive contracts to be linear is admittedly a major simplification. However, apart from the analytical convenience they afford, such contracts are appealing because they are ubiquitous in practice. (For other analyses that use linear contracts, see Ross (1974, 1977).) Finally, the linearity of asset returns is consistent with the burgeoning literature on the APT.

Suppose the return R_i on the i th asset in the economy is generated by a linear process of the form

$$R_i = g_i(\alpha) + \theta_i = g_i(\alpha) + \beta_i m + (1-\beta_i)\epsilon_i, \quad (16)$$

where $E(m) = \bar{m}$, $E(\epsilon_i) = \bar{\epsilon}_i$, $\text{cov}(\epsilon_i, m) = 0 \forall i$, $\text{var}(m) = \sigma_m^2$, $\text{var}(\epsilon_i) = \sigma_{\epsilon_i}^2$ and $E(R_i) = \bar{R}$, and $\text{cov}(\epsilon_i, \epsilon_j) = 0 \forall i, j$, where $E(\cdot)$, $\text{var}(\cdot)$, and $\text{cov}(\cdot, \cdot)$ are the expectation, variance and covariance operators respectively.

These specifications imply that

$$E(\theta_i) = \beta_i \bar{m} + (1-\beta_i)\bar{\epsilon}_i, \text{cov}(\theta_i, m) = \beta_i \sigma_m^2, \text{ and}$$

$$\text{var}(\theta_i) = \beta_i^2 \sigma_m^2 + (1-\beta_i)^2 \sigma_{\epsilon_i}^2.$$

A comparison of (16) with the single factor model used by Ross (1976) indicates three differences. First, the β_i here obviously has a slightly different interpretation. Second, the idiosyncratic risk ϵ_i is not constrained to have an expected value of zero, although this variable can be easily decomposed into a constant and a random variable with expected value zero. Finally, the first term on the right hand side of (16) is a function of α rather than a constant. This property of the returns process allows the manager discretionary power over the end-of-period wealth generated by the asset and represents the most significant departure from the usual valuation models.

Assume now that every manager has a mean-variance utility function described by

$$U(\alpha, \phi) = E(\phi) - \tau \sigma^2(\phi) - V(\alpha) (1+r) , \quad (17)$$

where $E(\phi)$ and $\sigma^2(\phi)$ are the expected value and variance of ϕ respectively, τ is a risk-aversion parameter, r is the riskless rate of interest and $V(\alpha)$, the effort disutility function, is increasing and strictly convex. For simplicity ϕ is restricted to be linear and for notational convenience we drop the subscript i henceforth. Assuming that the principal can observe R and m but not α or ε , the general linear fee schedule can be expressed as

$$\phi(R, m) = \eta_0 + \eta_1 R - \eta_2 m , \quad (18)$$

where η_0 , η_1 and η_2 are positive scalars.

We assume that the market values of all assets are determined in accordance with the single period CAPM¹⁵. In contrast to the risk neutrality assumption for principals employed in the previous section we now assume that the principal's objective is to maximize market value. Thus, the current price of an asset, P_0 , is

$$P_0 = [1+r]^{-1} [E(P_1) - \{m-r\} \{cov(P_1, P_m)\} \{\sigma_m \sigma(P_m)\}^{-1}] , \quad (19)$$

where P_1 = the net end-of-period wealth generated by the asset,

P_m = the end-of-period value of the market portfolio ,

$\sigma(P_m)$ = standard deviation of P_m .

With these preliminaries we explore the properties of the optimal managerial incentive contracts when managerial actions (i) are unobservable, and (ii) can be observed without error ex-post.

Case 1: Only R and m observable ex-post:

THEOREM 2: When the manager is not allowed to trade in the market, the optimal managerial incentive contract will not completely shield the manager from market risk. However, the dependence of the optimal contract on the market return is inversely related to the correlation between the asset and market returns.

PROOF: The manager's compensation can be written as

$$\phi = \eta_0 + \eta_1(g(\alpha) + \beta m + (1-\beta)\epsilon) - \eta_2 m,$$

and the manager's expected utility as

$$\begin{aligned} & \eta_0 + \eta_1(g(\alpha) + \beta \bar{m} + (1-\beta)\bar{\epsilon}) - \eta_2 \bar{m} \\ & - \tau [\eta_1^2 (1-\beta)^2 \sigma_\epsilon^2 + (\beta \eta_1 - \eta_2)^2 \sigma_m^2] - V(\alpha) (1+r). \end{aligned} \quad (20)$$

Using (20) we obtain the manager's optimal choice of action $\bar{\alpha}$ from the first order condition

$$\begin{aligned} & \eta_1 g_\alpha(\bar{\alpha}) - V_\alpha(\bar{\alpha}) (1+r) = 0 \\ \text{or } & \eta_1(\bar{\alpha}) = [V_\alpha(\bar{\alpha}) (1+r)] [g_\alpha(\bar{\alpha})]^{-1}. \end{aligned} \quad (21)$$

Thus, the principal's choice of η_1 depends on what α he desires the manager to take--given the functions $V(\alpha)$ and $g(\alpha)$, the manager's optimal choice of action depends only on η_1 . In other words, $\bar{\alpha}$ can be fixed by fixing η_1 . Now without loss of generality let the managerial reservation utility level $\bar{C} = 0$. Then (20) implies

$$\begin{aligned} & \eta_0 = -\eta_1(g(\bar{\alpha}) + \beta \bar{m} + (1-\beta)\bar{\epsilon}) + \eta_2 \bar{m} \\ & + \tau [\eta_1^2 (1-\beta)^2 \sigma_\epsilon^2 + (\beta \eta_1 - \eta_2)^2 \sigma_m^2] + V(\bar{\alpha}) (1+r). \end{aligned} \quad (22)$$

If the level of investment in the asset is taken to be a fixed amount I , the net end-of-period wealth accruing to the principal is

$$\begin{aligned} P_1 &= IR - \eta_1 R + \eta_2 m - \eta_0 \\ &= R(I - \eta_1) + \eta_2 m - \eta_0, \end{aligned} \quad (23)$$

$$\text{which means } \text{cov}(P_1, m) = \sigma_m^2 [(I - \eta_1)\beta + \eta_2] \quad (24)$$

$$\text{and } E(P_1) = \bar{R}(I - \eta_1) + \eta_2 \bar{m} - \eta_0, \quad (25)$$

Substituting above for η_0 from (22) we get

$$E(P_1) = \bar{I}\bar{R} - V(\bar{\alpha}) (1+r) - \tau [(1-\beta)^2 \eta_1^2 \sigma_\epsilon^2 + (\beta \eta_1 - \eta_2)^2 \sigma_m^2]. \quad (26)$$

Using (19) and making the required substitutions from (24) and (26) gives us the current market value of the asset as

$$P_0 = \left[\begin{array}{l} \bar{R} - V(\bar{\alpha}) (1+r) - \tau [(1-\beta)^2 \eta_1^2 \sigma_\epsilon^2 + (\beta \eta_1 - \eta_2)^2 \sigma_m^2] \\ - (\bar{m}-r) [1\beta - (\beta \eta_1 - \eta_2)] \end{array} \right] (1+r)^{-1} \quad (27)$$

In determining the optimal contract to offer the manager, we can view the principal as initially choosing $(\beta \eta_1 - \eta_2)$. The first order condition is

$$\partial P_0 / \partial (\beta \eta_1 - \eta_2) = (1+r)^{-1} [\bar{m}-r - 2\tau (\beta \eta_1 - \eta_2) \sigma_m^2] = 0, \quad (28)$$

which means $(\beta \eta_1 - \eta_2) = (\bar{m}-r) (2\tau \sigma_m^2)^{-1}$. (29)

Note that since $\bar{m} > r$, (29) implies that $\beta \eta_1 > \eta_2$. It is also interesting that the optimal $(\beta \eta_1 - \eta_2)$ chosen is invariant to the β of the asset--for a fixed η_1, η_2 goes up as β goes up. Now substituting (29) in (27) and rearranging gives us

$$P_0 = \left[\begin{array}{l} 1\{\bar{R} - \beta(\bar{m}-r)\} - V(\bar{\alpha}) (1+r) \\ - \tau \eta_1^2 (1-\beta)^2 \sigma_\epsilon^2 + (\bar{m}-r)^2 (4\tau \sigma_m^2)^{-1} \end{array} \right] [(1+r)^{-1}] \quad (30)$$

The optimal η_1 (and thus $\bar{\alpha}$) can be obtained by setting $\partial P_0 / \partial \eta_1 = 0$.

Finally, the optimal managerial incentive contract is

$$\phi(R, m) = \eta_0 + \eta_1 R - \eta_2 m \quad (31)$$

$$= \eta_0 + \eta_1 (R - \beta m) + (\bar{m}-r) (2\tau \sigma_m^2)^{-1} m. \quad (32)$$

(upon substitution from (29) and rearranging).

Note that $(R - \beta m)$ is independent of m . Now from (32) it is clear that the manager's effective consumption must depend on the market return observed ex-post. Moreover, since η_2 goes up as β goes up, (31) indicates that the greater the correlation between the asset's return and the market return, the greater is the negative dependence of $\phi(R, m)$ on m .

The above theorem is appealing because of its obvious implications for the design of incentive contracts in practice. At first blush it might appear that the intuition behind the theorem lies in the observation that when managerial actions cannot be verified ex-post, basing the contract on the observable market return is of value for monitoring reasons--information about the performance of an aggregate index like the market could reveal something about the effort expended by the manager. However, a closer scrutiny of the proof of the theorem indicates that such reasoning is not accurate. The main function of the market return in the optimal incentive contract here is to facilitate efficient risk sharing. The asset's return has two sources of risk--systematic and unsystematic. If the contract is based solely on the asset's return, the manager may be faced with excessive systematic risk and since he is risk averse, this may be undesirable. In a competitive managerial labor market the reservation expected utility constraint has to be satisfied at all times, which means that the principal may be forced to compensate the manager for the increased systematic risk by raising η_0 , the fixed component of the latter's fee. Our analysis, however, shows that this mechanism for sharing risk is inefficient. In (31) the term $\eta_2 m$ appears with a negative sign (and $\eta_2 > 0$), which implies we are effectively removing some of the systematic risk from the manager's compensation package. Note, however, that since $(\beta\eta_1 - \eta_2) \neq 0$ the manager is not completely insured against systematic variations in the asset's return. This is because he may otherwise bear excessive unsystematic risk. This also explains the logic behind the other claim of the theorem that a higher correlation between R and m increases the negative weight assigned to m in the contract.

The model also lends itself to some interesting comparative statics. But we first need to derive the optimal η_1 . To do this, substitute (21) in (30)

and use the first order condition ($\partial P_o / \partial \eta_1 = 0$) to obtain

$$\eta_1 = \{2(1+r)\tau(1-\beta)^2 \sigma_\epsilon^2 [g_\alpha V_{\alpha\alpha} - V_\alpha g_{\alpha\alpha}]\}^{-1} \{I(g_\alpha)^3 - (1+r)V_\alpha (g_\alpha)^2\}. \quad (33)$$

The three most interesting and intuitively appealing properties of $\phi(R,m)$ are discussed below:

- (i) An increase in \bar{m} increases the weight assigned to m . To see this note that (33) implies η_1 is unaffected, but by (29) we know $(\beta\eta_1 - \eta_2)$ goes up. So, η_2 must decline. Moreover, (22) shows that η_0 also increases.
- (ii) As σ_m^2 rises, the dependence of the optimal contract on m decreases. This follows from the fact that η_1 is unaffected by σ_m^2 , but $(\beta\eta_1 - \eta_2)$ is inversely related to σ_m^2 . Also, the increase in η_2 is partially offset by a concomitant increase in η_0 .
- (iii) A higher σ_ϵ^2 implies lower η_1 and η_2 but a lower η_0 . This is because of the inverse relationship between η_1 and σ_ϵ^2 and the independence of $(\beta\eta_1 - \eta_2)$ from σ_ϵ^2 . The decline in η_0 is implied by (22).

Case 2: R, m and α observable ex-post:

THEOREM 3: When the manager is not allowed to trade in the market and managerial actions can be freely observed ex-post, the optimal incentive contract completely shields the manager from the asset's idiosyncratic risk, and is equivalent to a contract of the following form

$$\phi(\alpha, m) = \begin{cases} \eta'_0 + \eta'_2 m & \text{if } \alpha = \alpha^* \\ 0 & \text{otherwise,} \end{cases}$$

where, α^* is the action desired by the principal and η'_0 and η'_2 are positive constants.

PROOF: It is well known that when all the relevant contract variables can be freely observed ex-post, the optimal managerial compensation formula is a forcing contract contingent on the effort. Such a contract can be expressed as

$$\phi(\alpha, R, m) = \begin{cases} \eta_0 + \eta_1 R - \eta_2 m & \text{if } \alpha = \alpha^* \\ 0 & \text{otherwise.} \end{cases}$$

With such a contract, if the manager takes the desired action his compensation is

$$\eta'_0 + \eta_1 (1-\beta)\epsilon + \eta'_2 m,$$

where $\eta'_0 \equiv \eta_0 + \eta_1 g(\alpha)$ and $\eta'_2 \equiv \eta_1 \beta - \eta_2$. Thus, all that needs to be proved is that $\eta_1 \equiv 0$ in the optimal contract.

Now, the manager's expected utility is

$$\eta'_0 + \eta_1 (1-\beta)\bar{\epsilon} + \eta'_2 \bar{m} - \tau[\eta_1^2 (1-\beta)^2 \sigma_\epsilon^2 + (\eta'_2 \sigma_m)^2] - V(\alpha^*) (1+r)$$

if he takes the action α^* . Assuming once again that the managerial reservation expected utility is zero, we get

$$-\eta'_0 = \eta_1 (1-\beta)\bar{\epsilon} + \eta'_2 \bar{m} - \tau[\eta_1^2 (1-\beta)^2 \sigma_\epsilon^2 + (\eta'_2 \sigma_m)^2] - V(\alpha^*) (1+r). \quad (34)$$

With an investment level of I , the end-of-period wealth accruing to the principal is

$$P_1 = IR - \eta'_0 - \eta_1 (1-\beta)\epsilon - \eta'_2 m.$$

Using (34), this implies that

$$E(P_1) = I\bar{R} - \tau[\eta_1^2 (1-\beta)^2 \sigma_\epsilon^2 + (\eta'_2 \sigma_m)^2] - V(\alpha^*) (1+r). \quad (35)$$

$$\text{and } \text{cov}(P_1, m) = \sigma_m^2 [I\beta - \eta'_2]. \quad (36)$$

Thus, the price of the asset is given by

$$P_0 = (1+r)^{-1} \begin{Bmatrix} I\bar{R} - \tau[\eta_1^2 (1-\beta)^2 \sigma_\epsilon^2 + (\eta'_2 \sigma_m)^2] - V(\alpha^*) (1+r) \\ -(\bar{m} - r) (I\beta - \eta'_2) \end{Bmatrix}. \quad (37)$$

From (37) it is clear that for any α^* the principal chooses, P_0 is maximized at $\eta_1 = 0$. Note that because the manager's actions can be completely monitored, the principal is not concerned about the impact of η_1 on the manager's choice of α . The issue here is thus one of pure risk sharing as in Wilson (1968).

Further, by setting $\partial P_o / \partial \eta'_2 = 0$, we can see that the optimal η'_2 is a constant given by

$$(\bar{m} - r) (2\tau\sigma_m^2)^{-1}.$$

Note that the choice of η'_2 is independent of α^* , and since $\bar{m} > r$, it follows that $\eta'_2 > 0$.

The intuition behind this result is straightforward. Since the manager is risk averse, he prefers to bear as little risk as possible. Although principals are also risk averse, they are neutral to the asset's idiosyncratic risk because it can be diversified away. However, since the market risk is not diversifiable, it appears in the optimal contract to provide efficient risk sharing. The amount of market risk optimally borne by the manager depends on his personal risk aversion parameter, τ . Because η_2 is a decreasing convex function of τ , the more risk averse the manager the lower is the market risk imposed on him.

IV. THE LINEAR RETURNS MODEL WITH MANAGERIAL TRADING

In this section we retain exactly the same structure employed in Section III, but relax the assumption that the manager is not allowed to trade (any portion of) his compensation package in the capital market. The most obvious implication (and perhaps advantage) of this is that even though the manager evaluates returns on an expected utility basis, the specific valuation operator he uses to value his incentive contract may not be idiosyncratic to him.¹⁶ Of course, permitting the manager to trade in his own firm's claims raises the usual questions of monitoring the enforcement as well as the issue of the extent to which insider information may be reflected in prices. In fact, we will argue later on that for reasons related to moral hazard, the manager will be able to trade only a portion of his total compensation package. As in the previous section, we will consider both first and second best solutions.

That is, in addition to analyzing the situation in which only R and m are verifiable ex-post we will also explore the implications of being able to costlessly observe α ex-post.

Case 1: Only R and m observable:

The general linear managerial incentive contract is expressed as

$$\begin{aligned}\phi &= \eta_0 + \eta_1 R - \eta_2 m \\ &= \eta_0 + \eta_1 (g(\alpha) + (1-\beta)\epsilon) + (\eta_1\beta - \eta_2)m.\end{aligned}$$

Since α and ϵ cannot be verified ex-post, the only portion of his compensation that the manager can trade (at established market prices) is $\eta_0 + (\eta_1\beta - \eta_2)m$. There can be no market for either the stochastic (ϵ) or the nonstochastic portion ($g(\alpha)$) of the idiosyncratic component of the assets return, because contracting between agents in the marketplace cannot be based on unobservable variables. A major impediment to such contracting is moral hazard and the inability to reach an equilibrium price. Assuming that the manager has in his possession a diversified portfolio of investments, he will use existing market prices to evaluate the market component of his compensation. The CAPM tells us that the current market price of this component is $(\eta_1\beta - \eta_2)r(1+r)^{-1}$, or equivalently, that its end-of-period price is $(\eta_1\beta - \eta_2)r$. The manager will use his (mean-variance) utility function to evaluate the rest of his compensation. It turns out that allowing the manager to trade even a fraction of his total payoff has a substantial impact on the nature of the optimal incentive contract, as the following theorem indicates.

THEOREM 4: When the manager can trade in the capital market, the optimal incentive contract is equivalent to a contract that depends only on the asset's return observed ex-post and does not explicitly rely on the observed market return. Further, the value of the asset managed is lower with managerial trading than without.

PROOF: As stated in the preceding discussion the manager evaluates

$\eta_0 + \eta_1 [g(\alpha) + (1-\beta)\epsilon]$ using his utility function and values the market component of his fee at $(\eta_1\beta - \eta_2)r$ in end-of-period value terms. The manager's objective is to maximize his expected utility

$$EU(\alpha, \phi) = \eta_0 + \eta_1 g(\alpha) + (1-\beta)\eta_1 \bar{\epsilon} + (\eta_1\beta - \eta_2)r - \tau(1-\beta)^2 \eta_1^2 \sigma_\epsilon^2 - V(\alpha) (1+r) \quad (38)$$

and his optimal action, $\bar{\alpha}$, can be derived from the first order condition

$$\eta_1 g'(\bar{\alpha}) - V'_\alpha(\bar{\alpha}) (1+r) = 0$$

$$\text{which implies } \eta_1 \equiv \eta_1(\bar{\alpha}) = [V'_\alpha(\bar{\alpha}) (1+r)] [g'_\alpha(\bar{\alpha})]^{-1}. \quad (39)$$

Inserting $\alpha = \bar{\alpha}$ in (38) and assuming that the managerial reservation expected utility level is zero, we obtain

$$\eta_0 = - \eta_1 g(\bar{\alpha}) - (1-\beta)\eta_1 \bar{\epsilon} - (\beta\eta_1 - \eta_2)r + \tau(1-\beta)^2 \eta_1^2 \sigma_\epsilon^2 + V(\bar{\alpha}) (1+r). \quad (40)$$

With a fixed investment of I in the asset the end-of-period wealth accruing to the principal is

$$P_1 = [I - \eta_1(\bar{\alpha})][g(\bar{\alpha}) + (1-\beta)\epsilon + \beta m] + \eta_2 m - \eta_0. \quad (41)$$

Substituting above for η_0 from (40), taking expected values and rearranging gives

$$E(P_1) = I\bar{R} - (\eta_1\beta - \eta_2)(\bar{m} - r) - \tau(1-\beta)^2 \eta_1^2 \sigma_\epsilon^2 - V(\bar{\alpha}) (1+r). \quad (42)$$

Now note that $\text{cov}(P_1, m) = \sigma_m^2 [(I - \eta_1)\beta + \eta_2]$, and use (19) and (42) to obtain the current market value of the asset as

$$P_0 = [1+r]^{-1} \{I[\bar{R} - \beta(\bar{m} - r)] - \tau(1-\beta)^2 \eta_1^2 \sigma_\epsilon^2 - V(\bar{\alpha}) (1+r)\}. \quad (43)$$

By (39) express η_1 as a function of $\bar{\alpha}$ in (43) above and then use the first order condition, $\partial P_0 / \partial \eta_1 = 0$, to get the optimal η_1 , which in this case is identical to (33).

Since η_2 does not appear in (43) it is an irrelevant parameter and can

be set equal to zero. In other words, the optimal incentive contract is equivalent to

$$\phi(R,m) \equiv \phi(R) = \eta_0 + \eta_1 R . \quad (44)$$

Note that (44) holds for all m (and any \bar{m}, σ_m^2), which means that the actual market return observed ex-post is of no consequence to the managerial compensation, except indirectly through its impact on R . To complete the proof compare (30) and (43). Since $(\bar{m}-r)^2 (4\tau\sigma_m^2)^{-1} > 0$, it is obvious that the P_0 in (30) exceeds the P_0 in (43).

Intuitively, the market return is assigned a zero weight because its valuation is no longer idiosyncratic to the manager--both the principal and the manager use the existing equilibrium market valuation mechanism to price this component. Since the payment of $\eta_2 m$ to the manager is not contingent on observed performance it can essentially be treated as a constant and subsumed in the fixed component, η_0 . What is perhaps less obvious is why the value of the asset falls when the manager is allowed to trade. The preferred explanation for the desirability of proscribing managerial trading relies on the enforcement and monitoring issues mentioned in the introduction to this section. However, the rationale here is quite different. When the manager is prohibited from trading there are two sources of risk (ϵ and m) at the disposal of the principal in designing the incentive contract. If the principal so desires, he can always completely insure the manager against market risk by setting $\beta\eta_1 = \eta_2$ (in which case the P_0 's in (30) and (43) will be equal), but Theorem 2 shows that this strategy is sub-optimal. On the other hand, when managerial trading is permitted, the manager ends up capitalizing his market component at established prices and thus effectively neutralizes one source of risk sharing available to the principal. In either case the satisfaction of the managerial reservation expected utility constraint as an equality implies that the manager is indifferent to the two alternatives.

Thus, managerial trading lowers the price of the asset because it reduces the degrees of freedom the principal has in designing the optimal incentive contract, without improving the manager's lot.

Let us now turn to an examination of (43) which reveals an interesting decomposition of the value of the asset. The first term, \bar{IR} , is just the expected cash flow; the second term, $I\beta(\bar{m}-r)$, is a market risk adjustment factor; the third term, $\tau(1-\beta)^2 \eta_1 \sigma_e^2$, represents an agency cost and the fourth term, $V(\bar{\alpha})(1+r)$, is simply the cost of inducing the manager to expend the necessary effort. The agency cost term is particularly noteworthy because it represents a quantification of the loss in efficiency caused by separation of ownership and control. The extent of this efficiency loss clearly depends on the manager's risk aversion parameter as well as the variability associated with the untraded portion of the incentive contract. In other words, the agency cost here is a consequence of inefficient risk sharing. In the next theorem we show that if managerial actions can be costlessly verified ex-post, the optimal compensation arrangement takes the form of a contingent type contract that completely shields the manager from exogenous risk. Thus, the agency cost in this model is basically engendered by moral hazard.¹⁷

Case 2: R, m and α observable:

THEOREM 5: Suppose the manager is allowed to trade and his actions can be costlessly verified ex-post. The optimal managerial incentive contract is equivalent to a contract of the form

$$\phi(\alpha) = \begin{cases} V(\alpha^*) (1+r) & \text{if } \alpha = \alpha^* \\ 0 & \text{otherwise,} \end{cases} \quad (45)$$

and the value of the asset is given by

$$P_0^* = \{I[g(\alpha^*) + \beta r + (1-\beta)\bar{e}] - V(\alpha^*) (1+r)\} (1+r)^{-1}. \quad (46)$$

where α^* is the optimal action desired by the principal. Further, the value of the asset is lower with managerial trading than without.

PROOF: When managerial actions can be costlessly verified ex-post we know that the optimal contract is equivalent to a contract of the form

$$\phi(\alpha, R, m) = \begin{cases} \eta_0 + \eta_1 R - \eta_2 m & \text{if } \alpha = \alpha^* \\ 0 & \text{otherwise.} \end{cases} \quad (47)$$

Now since investors can observe α ex-post, the manager can sell his entire fee in the market, contingent on the promised level of effort being delivered. He will therefore value his compensation using the existing market valuation mechanism. That is, the manager's expected utility can be expressed as

$$EU(\alpha, \phi) = \eta_0 + \eta_1 g(\alpha) + (\eta_1 \beta - \eta_2) r + \eta_1 (1 - \beta) \bar{\epsilon} - V(\alpha) (1+r). \quad (48)$$

Since the manager's reservation expected utility is zero, (48) implies that

$$-\eta_0 = \eta_1 g(\alpha^*) + (\eta_1 \beta - \eta_2) r + \eta_1 (1 - \beta) \bar{\epsilon} - V(\alpha^*) (1+r). \quad (49)$$

Using (49) to substitute for η_0 , we see that the end-of-period wealth accruing to the principal has the following expected value and covariance with the market return respectively,

$$E(P_1) = I\bar{R} + (\bar{m} - r) (\eta_2 - \eta_1 \beta) - V(\alpha^*) (1+r), \quad (50)$$

and
$$\text{cov}(P_1, m) = \sigma_m^2 [I\beta + \eta_2 - \eta_1 \beta] \quad (51)$$

Thus, the price of the asset, P_0^* , is given by (46).

Note that η_1 and η_2 do not appear in (46). Therefore, they are irrelevant and can be set equal to zero. From (49) it follows then that

$$\eta_0 = V(\alpha^*) (1+r). \quad (52)$$

Making the required substitutions for η_0 , η_1 and η_2 in (47), we see that the optimal incentive contract is equivalent to (45). Finally, note that the value of the asset in Theorem 3 can be expressed as

$$P_0 = (1+r)^{-1} \{I[g(\alpha^*) + \beta r + (1-\beta)\bar{\epsilon}] + (\bar{m}-r)^2 (4\tau\sigma_m^2)^{-1} - V(\alpha^*) (1+r)\} \quad (53)$$

and thus $P_0^* < P_0$.

When both parties are risk-averse and managerial actions can be verified ex-post, it is well known that the optimal contract specifies that the manager receive a predetermined share of the (risky) output if he chooses the stipulated action and nothing otherwise.¹⁸ However, the above theorem says more--the manager receives a fixed compensation for taking the desired action.¹⁹ This result also stands in contrast to Theorem 3 which states that in the absence of moral hazard the manager's compensation depends on the market return.

The key to Theorem 5 is that when managerial trading is allowed, it does not matter whether risk is borne by the owner or the manager of the asset, as long as there is no moral hazard. Since the manager can sell his entire compensation package in the market, contingent on the delivery of a promised effort level, both he and the principal value any risky payoff using the same equilibrium market price. This means that unlike the case in Theorem 3, imposing market risk on the manager does not promote more efficient risk sharing. Consequently, one possible source of risk sharing available to the principal is nullified and the price of the asset is lower with managerial trading than without. This is most clearly seen by comparing (46) and (53). The amount by which P_0 exceeds P_0^* is $(\bar{m}-r)^2 (4\tau\sigma_m^2)^{-1}$, which is identical to the price decline caused by managerial trading in the presence of moral hazard.

In closing, note that the difference between (43) and (46) is that the agency cost term, $(1-\beta)^2 \eta_1^2 \sigma_\epsilon^2$, does not appear in the latter equation. This should be viewed as a confirmation of the fact that free observability of relevant contract variables eliminates the moral hazard problem and permits efficient risk sharing. In the next section we discuss the issue of agency costs further. Specifically, we study the relationship between the optimal α desired by the principal, agency costs and project selection.

V. PROJECT SELECTION UNDER MORAL HAZARD

In the real world the existence of assets with varying degrees of systematic risk implies that the prospective owners of these assets can exercise some choice with respect to the β they desire. The questions we ask is: how will this choice be influenced by the level of effort the principal wants from the manager? The scenario we consider is one in which there is a very large number of assets and cross-sectionally the ϵ 's and m are independent and identically distributed (i.i.d.) random variables. We assume that the ϵ 's and m are i.i.d. because the principal is choosing between idiosyncratic and systematic risk, and such choice problems are usually interesting only when similar quantities are compared. For simplicity, the $g(\alpha)$'s can be assumed identical for all assets and the principal can be viewed as first determining the optimal effort level required and then choosing a mix of assets to get the desired β . This decision making process is formalized in the following theorem.

THEOREM 6: Assume that managerial trading is allowed. Suppose $\bar{m} = \bar{\epsilon}$, $\sigma_m^2 = \sigma_\epsilon^2$, and that the risk attitude of the manager coincides with that of the market. Then if the principal can choose β , his optimal choice will be

- (i) $\beta = 0$ in the absence of moral hazard.
- (ii) $\beta \neq 0$ in the presence of moral hazard.

Moreover, in the latter case the optimal β is an increasing function of the level of effort desired by the principal.

PROOF: Since $\bar{\epsilon} = \bar{m} > r$, the claim that $\beta = 0$ follows directly from (46). To establish the second claim, hold η_1 constant and partially differentiate (43) with respect to β . That is,

$$\partial P_0 / \partial \beta = [1+r]^{-1} \{ I[r-\bar{m}] + 2\tau(1-\beta)\eta_1^2 \sigma_\epsilon^2 \} = 0,$$

which implies $1-\beta = \{ 2\tau\eta_1^2 \sigma_m^2 \}^{-1} \{ I(\bar{m}-r) \}.$ (54)

Since in general the term on the right hand side of (54) need not be unity, $\beta = 0$ need not be the optimal choice.

Now from the assumption that the manager and the market have identical risk preferences it follows from the CAPM that

$$\tau = \sigma_m^{-2} (\bar{m}-r). \quad (55)$$

That is, in the CAPM framework the manager's personal unit price for variance is the same as that of the market.

Substituting (55) and (39) in (54) gives us

$$1-\beta = I[g_\alpha(\bar{\alpha})]^2 \{2(1+r)^2 [V_\alpha(\bar{\alpha})]^2\}^{-1}. \quad (56)$$

If the principal wants a higher $\bar{\alpha}$, the term $[g_\alpha(\bar{\alpha})/V_\alpha(\bar{\alpha})]^2$ will decrease due to the concavity of $g(\alpha)$ and the convexity of $V(\alpha)$. Thus, $1-\beta$ is a declining function of $\bar{\alpha}$. ■

From a practical standpoint one important implication of the above theorem is that, ceteris paribus, those firms (assets) which enjoy a higher level of 'managerial-effort-induced' returns will also be characterized by a higher level of systematic or market risk. Note that the CAPM and the APT imply the converse of this result, namely that the higher the systematic risk of a firm the greater its expected return. Thus, while the theorem has the intuitive appeal of reinforcing, even with moral hazard, the commonly accepted positive linear relationship between risk and return, it also suggests that the task of empirically testing simple hypotheses like "higher β 's imply higher expected returns" may be far more complicated than one might ordinarily suspect. This is because in the presence of moral hazard we may have a complete loop-- higher systematic risk implies higher expected return which in turn implies higher systematic risk!

To get an intuitive feel for this result note that the term $\tau(V_\alpha(\bar{\alpha}) \times \{g_\alpha(\bar{\alpha})\}^{-1})^2 (1+r)^2 (1-\beta)^2 \sigma_\epsilon^2$ represents the agency cost created by moral hazard.

Thus, if the principal desires a higher $\bar{\alpha}$, he faces a higher agency cost. To counteract this he increases the β of the assets under management. But why is the agency cost positively related to $\bar{\alpha}$ and inversely related to β ? The answer lies again in moral hazard and risk. Remember that the only component of his fee that the manager can trade is the market component. Thus, an increase in β implies an increase in the traded portion of the manager's fee and a decline in the risk borne by the manager. This improves risk sharing and reduces the agency cost. The same logic applies to $\bar{\alpha}$. Since the return derived from the manager's expenditure of effort is untraded, an increase $\bar{\alpha}$ implies a relative decline in the traded portion of the manager's fee and consequently a higher agency cost.

VI. CONCLUDING REMARKS

A distinctive feature of modern capital markets is that management and ownership of assets are often well dichotomized functions. A growing body of literature in agency theory has provided numerous insights into the implications of this dichotomy for the design of multilayered organizations (Mirrlees (1976)), capital structure decisions (Jensen and Meckling (1976)), and the derivation of appropriate managerial incentive contracts to induce optimal financial signalling and activity choice decisions (Ross (1978)). However, the seemingly important question of how the separation of management from ownership affects the values of the assets being managed has received scant attention in the literature.²⁰ Our paper can therefore be viewed as an initial attempt to provoke further discussion on this subject.

Although the models we have used are fairly simple, they have yielded strong conclusions. This suggests that much may be learned by using our framework to study alternative valuation models and perhaps richer time structures. For instance, it would be illuminating to employ (a modified version of) the (discrete time, discrete returns) intertemporal CAPM and examine

the dynamic aspects of managerial incentive contracts. Another interesting extension would be to assume that asset returns are generated by a diffusion process in continuous time. Such a specification could significantly change the complexion of the problem and may give rise to a plethora of important new questions like: how long should be the time horizon at the end of which the manager's fee is computed and what is the impact of this time horizon on asset values? A third issue of some importance is the formal introduction of monitoring (through the auditing function) in the third and the fourth sections and the explicit derivation of market values for auditing services. Hopefully, such topics will occupy the attention of financial economists in the future.

FOOTNOTES

1. This 'identical agents' assumption may appear overly restrictive, especially in light of the fact that we later allow assets to be heterogeneous with respect to the technologies with which they are endowed. However, we can easily allow agents to have variegated skills--if each asset technology requires a different type of skill, all that we need is a very large number of agents associated with each type of skill. A competitive equilibrium will then result. Although the assumption that agents have identical preferences is not quite that inconsequential to the analysis, it is by no means unduly heroic. In a partial equilibrium analysis of the market for agents, Ross (1979) has shown that if there is an elastic supply of two types of agents who differ in their risk attitudes, the less risk averse agents will drive the more risk averse agents out of the market. Although Ross does not consider effort aversion on the part of the agent, his analysis can be easily extended to include the agent's aversion to effort if it is assumed that effort is freely observable ex-post.
2. We have therefore created an impenetrable barrier between principals and agents. Admittedly, this is a significantly abstract version of the real world where managers frequently have ownership claims to assets managed by other managers as well as by themselves. This implies that the theoretically convenient dichotomy between owners and managers is often very fuzzy in practice. However, given our objectives, the simple setting proposed here is not only adequate but also helps to avoid highly complex and seemingly intractable mathematical formulations.
3. Note that Ω could consist of a variety of functions in addition to monitoring functions, and in practice of course, the accounting system is used for considerably more than merely monitoring managers. However, for our purpose it suffices to focus on the 'monitoring subset' of Ω .
4. The notation "argmax" means the set of arguments that maximize the objective function. Since the expectation in (5) need not be concave in α , the use of this notation is necessary. If the set of optimal actions, $\{\alpha_i\}$, is not a singleton, it will be assumed that the manager will choose an $\alpha_i(\in\{\alpha_i\})$ so as to maximize the principal's welfare.
5. We have suppressed the dependence of x on θ and have employed the implied distribution of x . This is done to avoid differentiability problems caused by the assumed boundedness of the fee schedules. A discussion of this issue appear in Mirrlees (1974) and Holmström (1977).
6. If Φ is a family of bounded functions and is equicontinuous or consists of functions of bounded variation, a solution to the maximization problem in (3), (4) and (5) can be guaranteed. However, if Φ is expanded to contain all bounded and measurable functions, no general existence proof is available. In that case, it will be necessary to assume that there exists an optimal solution $(\alpha_i^*, \phi_i^*(v_i), \omega_i^*(\alpha_i))$ such that $\alpha_i^* (\neq 0) \in \text{Int } A$.
7. Thus, if α_i is observable ex-post, $v_i = (\alpha_i)$; if α_i is unobservable and ξ_i and Ω are such that the optimal $\omega_i(\cdot)$ is the null element, $v_i = (x_i)$. In general, when α_i is unobservable, $v_i = (x_i, \omega_i(\alpha_i))$.

8. It is assumed, without loss of generality (for the purposes of this section), that the discount factor (the riskless rate of interest) for principals is zero.
9. In general, a weak inequality (\geq) should be used in (4) to allow for monopolistic elements in the managerial labor market. However, the equality implies that this market is assumed to be competitive.
10. Definition 1 is due to Holmström (1979).
11. Throughout this paper it is assumed that the end-of-period cash flow x is costlessly observed without error ex-post.
12. It will be proven in this theorem that any non-informative monitor, irrespective of its cost, will be inefficient.
13. When the distribution of an asset's cash flow has a compact support that moves with α , it means that there is a positive probability of detecting any deviation from a prespecified action.
14. An alternative means of obtaining a first-best solution is to replace the zero in (12) with a penalty that is large enough to force the agent to take the desired action $\psi/2$. However, a commonly stated objection to penalties is that they may have to be infeasibly large. If there are constraints on managers' wealth, which restrict the extent to which managers may be penalized, it may be impossible to resort to this scheme for achieving first-best efficiency.
15. The weaknesses of the CAPM have been extensively discussed in the literature. We do not wish to divert attention from the issue at hand here by either defending or rejecting it as a viable model of capital market equilibrium--our use of the CAPM is more illustrative than anything else. We feel that the ideas we are conveying have robustness and leave it to the interested reader to explore their sensitivity to the specific valuation model employed.
16. One could argue (as does Ross (1976a)) that this could be true even when the manager is restricted from trading, if the compensation is kept very small relative to the manager's wealth.
17. In the context of financial market equilibrium models, the observation that agency costs are created by moral hazard is well known (See Jensen and Meckling (1976)). However, we feel that the precise quantification of these costs provides useful insights, especially in view of the fact that although the concept of agency costs has been extensively discussed in the financial literature, it still remains somewhat nebulous.
18. This result was first formally proved by Harris and Raviv (1979). However, their proof relies on the assumption that the state of nature is unobservable. That is, they have proved that such contracts are Pareto optimal within the class of contracts that depend only on the output and the effort. Later, Ramakrishnan and Thakor (1980) generalized the result by demonstrating the optimality of these contracts even when the state of nature is observable ex-post.
19. It is well known that the contract specified in (45) is Pareto optimal if the principal is risk neutral. The novelty of Theorem 5, however, lies in the fact that the principal is assumed to be risk averse.

20. Heckerman (1975) is perhaps the only paper that is related to this subject, although the problem analyzed there is somewhat different. Heckerman derives optimal incentive contracts designed to minimize the moral hazard encountered in situations where the returns of firms are completely exogenous (unlike our models) but managers have information about returns that is superior to that of the owners.

APPENDIX

PROOF OF THEOREM 1: We will first prove that if managers are risk neutral or asset cash flows have compact moving supports, no changes in Ω^0 can affect the price of the asset. Throughout the manager's utility function will be assumed to have the separable form $U(\alpha, \phi) \equiv U(\phi) - V(\alpha)$ with $U'(\cdot) > 0$, $U''(\cdot) < 0$, $V'(\alpha) > 0$.

If managers are risk neutral, Harris and Raviv (1979) have shown that optimal fee schedules are of the form $\phi^*(x) = x - \bar{k}$, where \bar{k} is a constant. With such 'pure rental' type contracts (with zero probability of default) the principal is indifferent to the agent's choice of action and thus changes in the accounting system can have no impact on managerial actions. This means that managerial risk aversion is necessary for the accounting system to affect asset prices.

Now suppose the density function $q(x, \alpha)$ has a compact moving support that shifts with α . Assume $Q_\alpha(\hat{x}, \hat{\alpha}) < 0$, where \hat{x} and $\hat{\alpha}$ are some fixed values of x and α respectively and

$$Q(\hat{x}, \hat{\alpha}) = \int_{x_{\min}(\hat{\alpha})}^{\hat{x}} q(x; \hat{\alpha}) dx$$

with $X(\hat{\alpha}, \hat{\theta}) \in [x_{\min}(\hat{\alpha}), x_{\max}(\hat{\alpha})] \subset (-N_-, N_+)$,

where N_- and N_+ are positive finite scalars. In this case, if managerial actions are freely verifiable ex-post the solution to the maximization problem in (3), (4) and (5) will be

$$\phi^*(\alpha) = \begin{cases} t(\in R_+) & \text{if } \alpha = \alpha^* \\ 0 & \text{otherwise} \end{cases}$$

Suppose now the manager's choice of action is unobservable. Do we need to try to infer, through the use of monitors, what action the manager has taken? The answer is in the negative, because a fee schedule of the form

$$\phi_0^*(x) = \begin{cases} t & \text{if } x \geq x_{\min}(\alpha^*) \\ -y(y \geq 0) & \text{otherwise} \end{cases}$$

will induce the manager to set $\alpha = \alpha^*$ if the density function, $q(x, \alpha)$, and the maximum feasible penalty ($-y$) are such that

$$\int_{x_{\min}(\alpha)}^{x_{\min}(\alpha^*)} U(-y)q(x; \alpha) dx + \int_{x_{\min}(\alpha^*)}^{x_{\max}(\alpha)} U(t)q(x; \alpha) dx - V(\alpha) \leq U(t) - V(\alpha^*) \quad \forall \alpha \in A$$

If constraints on managerial wealth impose no restrictions on the size of y , the above inequality can be satisfied for any $q(x, \alpha)$ with a compact moving support and thus changes in Ω^0 can have no impact on asset prices. But, if y is restricted, our claim can be true only for those distributions which satisfy the inequality for the maximum feasible y . Although this fact is fairly well known in the agency literature we have presented the proof here for completeness.

Next, assume that managers are risk averse and that the asset cash flow is unbounded. In this case, Holmström (1979) has shown that the addition of a non-informative monitor to Ω^0 cannot affect the price of the asset even if the cost, ξ , of using this monitor is zero. That is, a non-informative monitor cannot be efficient.

We have therefore established that if any of the three conditions

mentioned in the theorem is violated a change in Ω^0 will not affect the price of the asset. We will now show that when all three conditions are satisfied, the price of the asset will increase if the cost of the (informative) monitor is sufficiently low. This claim is essentially similar to Proposition 5 in Holmström (1979), but its proof is given here because Holmström's proof is flawed, as we pointed out in Ramakrishnan and Thakor (1980).

Consider an informative monitor, ω , of α , and let $\phi(x)$ be Pareto optimal within the class of fee schedules depending only on x . Let $\bar{\alpha}$ be such that

$$\iint U(\phi(x))q_{\alpha}(x;\bar{\alpha}) dx - V_{\alpha}(\bar{\alpha}) = 0 \quad (\text{A-1})$$

$$\text{and } \iint U(\phi(x))q_{\alpha\alpha}(x;\bar{\alpha}) dx - V_{\alpha\alpha}(\bar{\alpha}) < 0 \quad (\text{A-2})$$

Since ω is informative, with positive scalars b and c a variation $b\delta\phi(x,\omega) + bc$ can be constructed to satisfy

$$\delta\phi(x,M^-)q(x, M^-; \bar{\alpha}) + \delta\phi(x,M^+)q(x, M^+; \bar{\alpha}) = 0 \quad (\text{A-3})$$

$$\delta\phi(x,M^-)q_{\alpha}(x, M^-; \bar{\alpha}) + \delta\phi(x,M^+)q_{\alpha}(x, M^+; \bar{\alpha}) = 1 \quad (\text{A-4})$$

where M^- and M^+ satisfy (6). This type of variation was first used by Shavell (1979).

$$\text{Let } Z(\alpha, b, c) = \iint U(\phi(x) + b\delta\phi(x,\omega) + bc)q(x, \omega; \alpha)d\omega dx - V(\alpha(b,c)) \quad (\text{A-5})$$

For a given b and c , let $\alpha(b,c)$ be the solution to the manager's maximization problem, i.e.

$$\begin{aligned} Z_{\alpha}(\alpha(b,c), b, c) &= \iint U(\phi(x) + b\delta\phi(x,\omega) + bc)q_{\alpha}(x,\omega; \alpha(b,c)) d\omega dx - V_{\alpha}(\alpha(b,c)) \\ &= 0 \end{aligned} \quad (\text{A-6})$$

and the second order condition

$$Z_{\alpha\alpha}(\alpha(b,c), b, c) < 0 \quad (A-7)$$

also holds. Note that $\alpha(0,c) = \bar{\alpha}$.

To find the effect of positive variations in b , note that

$$\begin{aligned} Z_b(\alpha(b,c), b, c) &= Z_b(\alpha, b, c) + \alpha_b(b, c) Z_\alpha(\alpha, b, c) \\ &= Z_b(\alpha, b, c) \text{ since from (A-6), } Z_\alpha(\alpha, b, c) = 0. \end{aligned}$$

At $b=0$,

$$\begin{aligned} Z_b(\bar{\alpha}, 0, c) &= \int U'(\phi(x)) \int \delta\phi(x, \omega) q(x, \omega; \bar{\alpha}) \, d\omega dx \\ &\quad + c \int U'(\phi(x)) q(x, \omega; \bar{\alpha}) \, d\omega dx \\ &= c \int U'(\phi(x)) q(x, \omega; \bar{\alpha}) \, d\omega dx \quad \text{from (A-3)} \end{aligned}$$

So for any $c > 0$, $Z_b(\bar{\alpha}, 0, c)$ is positive.

Differentiating (A-6) with respect to b we get

$$Z_{\alpha\alpha}(\alpha(b,c), b, c) \cdot \alpha_b(b,c) + Z_{\alpha b}(\alpha(b,c), b, c) = 0$$

$$\text{or } \alpha_b(b,c) = - \frac{Z_{\alpha b}(\alpha(b,c), b, c)}{Z_{\alpha\alpha}(\alpha(b,c), b, c)} \quad (A-8)$$

Moreover,

$$Z_{\alpha b}(\alpha(b,c), b, c) = \int \int (\delta\phi(x, \omega) + c) U'(\phi(x) + b\delta\phi(x, \omega) + bc) q_\alpha(x, \omega; \alpha(b,c)) \, dx d\omega$$

At $b=0$,

$$Z_{\alpha b}(\bar{\alpha}, 0, c) = \int U'(\phi(x)) \, dx + c \int U'(\phi(x)) q_\alpha(x, \omega; \bar{\alpha}) \, d\omega dx \text{ (using (A-4))}$$

Irrespective of the sign of $\int q_\alpha(x, \omega; \bar{\alpha}) d\omega$, $Z_{\alpha b}(\bar{\alpha}, 0, c)$ is positive if c is sufficiently small, since the first term, $\int U'(\phi(x)) dx$ is positive. Further, since $Z_{\alpha\alpha}(\bar{\alpha}, 0, c) < 0$, (A-8) implies that $\alpha_b(0, c) > 0$. This means that for a sufficiently small c , introduction of the proposed variation will induce the manager to increase his effort, at least in the (positive) neighborhood of $b=0$. Since for any $c>0$, $Z_b(\bar{\alpha}, 0, c) > 0$, the manager's expected utility also goes up with the new fee schedule.

For the principal, define

$$F(b, c) = \iint (x - \phi(x) - b\delta\phi(x, \omega) - bc - \xi) q(x, \omega; \alpha(b, c)) d\omega dx$$

$$F_b(b, c) = -\iint (\delta\phi(x, \omega) + c) q(x, \omega; \alpha(b, c)) d\omega dx$$

$$+ \iint (x - \phi(x) - b\delta\phi(x, \omega) - bc - \xi) q_\alpha(x, \omega; \alpha(b, c)) \cdot \alpha_b(b, c) d\omega dx$$

At $b=0$, using (A-3) and the fact that $\iint q(x, \omega; \bar{\alpha}) d\omega dx = 1$ and $\iint q_\alpha(x, \omega; \bar{\alpha}) d\omega dx = 0$ we have

$$F_b(0, c) = -c + [\iint (x - \phi(x)) q_\alpha(x, \omega; \bar{\alpha}) d\omega dx] (\alpha_b(0, c)) \quad (A-9)$$

Note that $\alpha_b(0, c) > 0$ and $\iint (x - \phi(x)) q_\alpha(x, \omega; \bar{\alpha}) d\omega dx$ represents the effect of an increase in α on the principal's welfare. If this double integral is positive, the second term on the right hand side (RHS) of (A-9) will also be positive. With a small enough c we can then make $F_b(0, c) > 0$. On the other hand, if $\iint (x - \phi(x)) q_\alpha(x, \omega; \bar{\alpha}) d\omega dx < 0$, we can go back and let the new fee schedule be $\phi(x) - b\delta\phi(x, \omega) + bc$. The first term on the RHS of (A-4) then becomes $-\int U'(\phi(x)) dx$ and by making c small enough, we can ensure $Z_{\alpha b}(\bar{\alpha}, 0, c) < 0$ irrespective of the sign of $\int q_\alpha(x, \omega; \bar{\alpha}) d\omega$. This means $\alpha_b(0, c) < 0$, which in turn implies that the second term on the RHS of (A-9) can once again be guaranteed to be positive. Therefore, $F_b(0, c)$ can be made positive

in either case, by appropriately adjusting c . It is fairly straightforward to verify that $\int \int f'(x-\phi(x))q_\alpha(x, \omega; \bar{\alpha}) d\omega dx = 0$ is impossible, for if it were true, we could perturb $\phi(x)$ with some function $r(x)$ and make the agent strictly better off without worsening the principal's lot. This would violate the presumed Pareto optimality of $\phi(x)$. To ensure that the principal will be better off in spite of the cost ξ , we must have

$$\int \int (x-\phi(x) - b^* \delta\phi(x, \omega) - b^* c^* - \xi) q(x, \omega; \alpha(b^*, c^*)) d\omega dx > \int (x-\phi(x)) q(x; \bar{\alpha}) dx$$

or

$$\xi < \left[\int \int (x-\phi(x) - b^* \delta\phi(x, \omega)) q(x, \omega; \alpha(b^*, c^*)) d\omega dx \right. \\ \left. - \int (x-\phi(x)) q(x; \bar{\alpha}) dx - b^* c^* \right] \quad (A-10)$$

where b^* and c^* are the optimal choices in the variation. In other words, the informative monitor, ω , should be efficient. Since in the above proof, the manager was assumed risk averse, asset cash flows were not constrained to have a compact moving support, and the monitor employed was informative, the sufficiency of (i), (ii) and (iii) is established. ■

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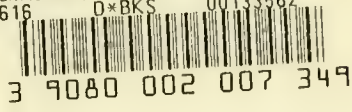
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