DESIGN OF TUNABLE RESONANT CAVITIES WITH CONSTANT BANDWIDTH

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TECHNICAL REPORT NO. 106

APRIL 5, 1949

RESEARCH LABORATORY OF ELECTRONICS
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

The research reported in this document was made possible through support extended the Massachusetts Institute of Technology, Research Laboratory of Electronics, jointly by the Army Signal Corps, the Navy Department (Office of Naval Research) and the Air Force (Air Materiel Command), under Signal Corps Contract No. W36-039-sc-32037, Project No. 102B; Department of the Army Project No. 3-99-10-022.

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Abstract

The design of tunable waveguide cavities is examined to determine the type of coupling susceptance required to give constant Q_L or constant bandwidth. It is shown that the inductive iris commonly used causes the bandwidth to vary radically as the cavity is tuned. A capacitive iris, however, produces a reasonably constant bandwidth.

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The design of resonant cavities and waveguide filters to have specified bandwidths or transmission characteristics has been dealt with by several authorities (1)(2)(3). These analyses have been concerned with the design about a specified center frequency. One finds, however, that it is often required to build a cavity or a filter which is tunable over a frequency range of 10 per cent or so, and whose bandwidth shall remain essentially constant over this range. This latter requirement is not met by most practical engineering designs with which the author is familiar. The reason for this will be pointed out, and alternative designs suggested in the following sections.

Two-Terminal Resonant Cavities

A resonant cavity formed by shorting one end of a length of waveguide, and coupling to the external circuit by a hole in the opposite end, is simple to analyze and the conclusions may be extended directly to more complicated resonator structures.

We will consider the cavity of Figure 1, consisting of a length of waveguide of characteristic admittance Y_o of length ℓ, shorted by a plunger at one end, and connected by the shunt susceptance B to the external transmission line.

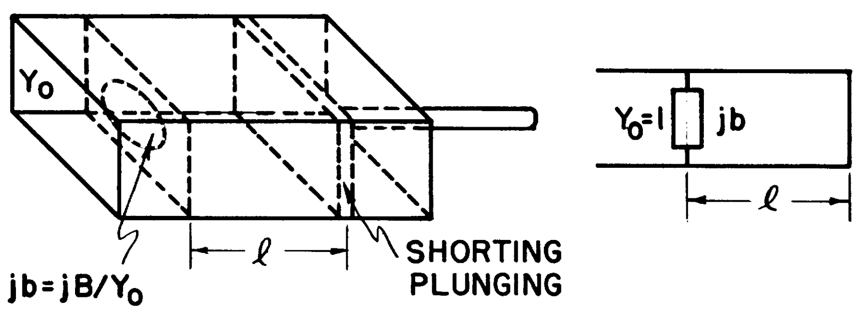

Fig. 1 Two-terminal resonant cavity.

We will confine our attention to a single waveguide mode. The resonant frequency of the cavity will be defined as that at which the input admittance (looking into the coupling susceptance) is pure real. This will occur when

$$Y_o \cot \frac{2\pi\ell}{\lambda_g} = B \quad ;$$

that is,

$$\frac{2\pi\ell}{\lambda_g} = \cot^{-1} \frac{B}{Y_o} + n\pi \qquad (n = 1,2,\ldots) \quad .$$

If $b = B/Y_o >> 1$,

$$\frac{2\pi \ell}{\lambda_g} \simeq \frac{1}{b} + n\pi \quad ,$$

where λ_g is the guide wavelength corresponding to the free-space wavelength λ.

The loaded Q of such a circuit may be written as

$$Q_L = \left(\frac{1}{2G}\right)\omega \frac{dB'}{d\omega} \quad ,$$

where $B' = Y_o(b - \cot 2\pi\ell/\lambda_g)$, the total loading of the cavity is due to the conductance $G = Y_o(1 + b^2 g_c)$, and $Y_o g_c$ is the equivalent conductance, due to cavity losses, which appears at the center of the cavity. From this, the loaded Q of a "half-wave" cavity is (4)

$$Q_{L1} = \frac{\frac{n\pi}{2}\left[\frac{\lambda_g}{\lambda}\right]^2}{\left[\frac{1}{b^2} + g_c\right]} \quad . \qquad (1)$$

We will limit the argument to nearly lossless cavities ($g_c << 1/b^2$). Then the coupling susceptance required to produce a given value of Q_{L1} is

$$b \simeq \pm\sqrt{\frac{2Q_{L1}}{n\pi}}\ \frac{\lambda}{\lambda_g} \quad . \qquad (2)$$

Substituting

$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left[\frac{\lambda}{\lambda_c}\right]^2}} \quad ,$$

where λ_c is the cutoff wavelength of the guide, we get

$$b \simeq \pm\sqrt{\frac{2Q_{L1}}{n\pi}}\ \sqrt{1 - \left[\frac{\lambda}{\lambda_c}\right]^2} \quad . \qquad (2a)$$

This is the functional form of the coupling susceptance, b, required to maintain a constant value of Q_{L1} (actually a constant value of external Q).

If the requirement is that the bandwidth have a constant value of Δf cycles/sec, where $\Delta f = f/Q_{L1}$, then we find

$$b \simeq \pm\sqrt{\frac{2c}{n\pi\Delta f}}\ \sqrt{\frac{1 - \left[\frac{\lambda}{\lambda_c}\right]^2}{\lambda}} \qquad (3)$$

for $b^2 g_c << 1$.

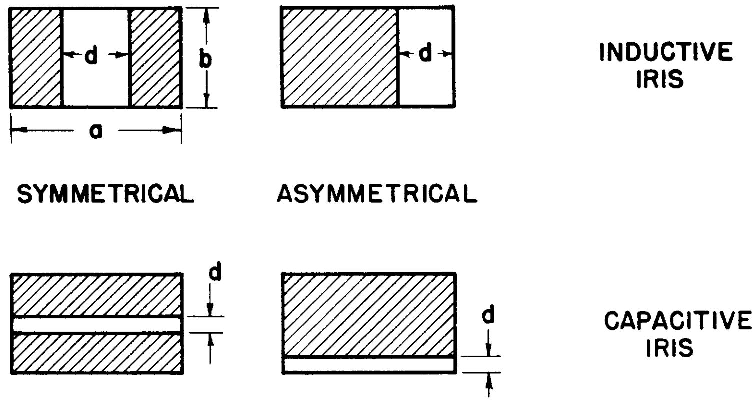

Fig. 2 Waveguide irises.

Figure 2 shows some typical inductive and capacitive irises. The normalized susceptances ($b = B/Y_o$) of thin irises across a waveguide are given by the following approximate expressions (5):

Symmetrical inductive iris:

$$b_L \simeq -\frac{\lambda_g}{a} \cot^2 \frac{\pi d}{2a} \quad ; \tag{4}$$

Symmetrical capacitive iris:

$$b_c \simeq \frac{4b}{\lambda_g} \ln \csc \frac{\pi d}{2b} \quad ; \tag{5}$$

Asymetrical capacitive iris:

$$b_c = \frac{8b}{\lambda_g} \ln \csc \frac{\pi d}{2b} \quad . \tag{5a}$$

Equations (2a), (3), (4) and (5) are plotted in Figure 3. It is evident that the frequency dependence of the inductive iris is just the opposite of what is required, whereas the capacitive iris is approximately correct for constant bandwidth over the range of λ/λ_c ordinarily used in waveguides ($0.6 < \lambda/\lambda_c < 0.8$).

The question arises as to why capacitive irises are so seldom used for coupling to resonant cavities. If high values of loaded Q are required ($Q_L \simeq 5000$), coupling susceptances of the order of $b = B/Y_o = 40$ are required. Examination of Eq.(5) indicates that a really microscopic gap, d, would be required. On the other hand, if one is concerned with Q values of the order of 100 or 200, then we find that we require $b \simeq 6$. With an unsymmetrical iris at $\lambda = 10$ cm in a 1 1/2 × 3-inch waveguide, this would require a gap $d \simeq 0.040$ inch. Although this is a small gap, the tolerances are not severe, since the capacitance varies only as the logarithm of the gap. In practice, since the iris would not have zero thickness, a somewhat larger gap would be required to produce this value of susceptance.

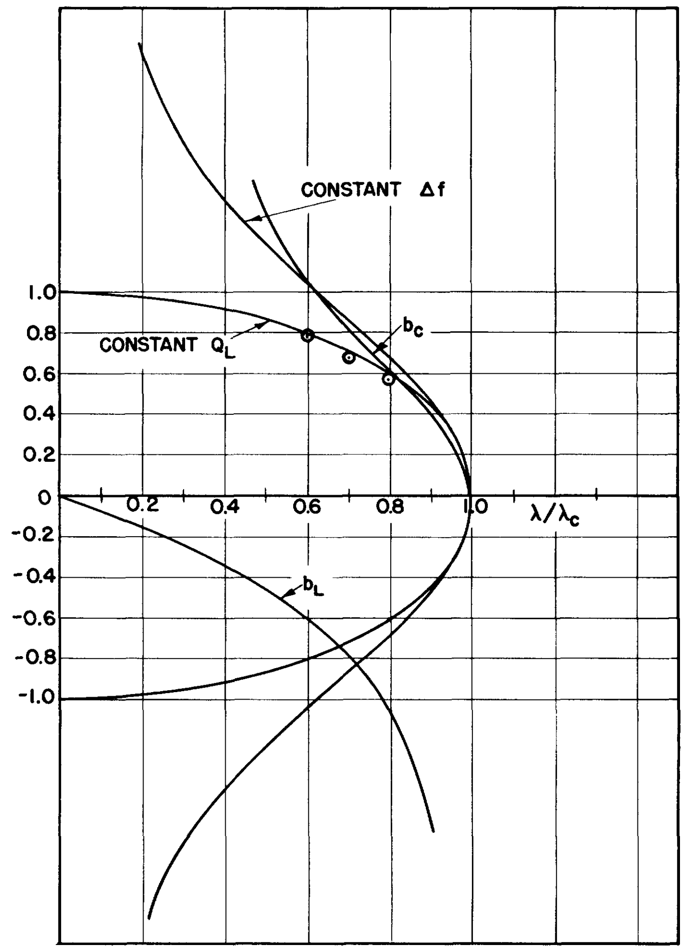

Fig. 3 Coupling susceptances necessary to give constant Q_L and constant Δf.

An inductive iris with the same value of susceptance ($b = 6$) would have an opening $d \simeq 0.88$ inch. Thus, there are obvious mechanical advantages in favor of the inductive iris. On the other hand, one can attain a nearly constant bandwidth by the use of a capacitive iris, and this may justify the extra difficulty of construction.

Four-Terminal (Transmission) Cavities

The two-terminal resonant cavity is seldom used in microwave filters except as a spot-frequency-rejection filter. Double- and triple-tuned filters (2), consisting of tandem resonant elements connected by lengths of transmission lines, have found extensive applications in microwave-communication apparatus. The basic element of such filters is a four-terminal resonant cavity. Figure 4 illustrates the usual form of construction of such cavities. It is a section of waveguide of length 2ℓ, coupled by input and output susceptances b_1 and b_2, and tuned by a screw at the center, whose susceptance is $2b_0$. The loaded Q of such a cavity, when coupled to a matched load and matched generator, is

$$Q_{L2} = \frac{\left[b_o^2 + 1\right] \frac{2\pi\ell}{\lambda_g} \left[\frac{\lambda_g}{\lambda}\right]^2 + b_o}{\left[\frac{1}{b_1^2} + \frac{1}{b_2^2} + g_c\right]} .$$

However, the condition for resonance is that

$$2b_o = \frac{b_1 + \tan\theta}{b_1 \tan\theta} + \frac{b_2 + \tan\theta}{b_2 \tan\theta} , \qquad (7)$$

where $\theta = 2\pi\ell/\lambda_g$. If θ is not too different from $\frac{\pi}{2}$, we can write

$$\tan\theta \simeq \frac{1}{\left[\frac{\pi}{2} - \frac{2\pi}{\lambda_g}\right]} .$$

Now, for simplicity, let $b_1 = b_2$ and $g_c = 0$. Then, solving for b_1 to give a constant value of loaded Q,

$$b_1 \simeq \pm\sqrt{\frac{4Q_{L2}}{\pi}} \left\{ \frac{4\ell}{\lambda_g}\left[\frac{\lambda_g}{\lambda}\right]^2 \left[1 + \frac{\pi^2}{4}\left(1 - \frac{4\ell}{\lambda_g}\right)^2 + \frac{\pi}{b_1}\left(1 - \frac{4\ell}{\lambda_g}\right)\right] + \left(1 - \frac{4\ell}{\lambda_g}\right) + \frac{2}{\pi b_1} \right\}^{-1/2} . \qquad (8)$$

The terms on the right-hand side containing $1/b_1$ are small if b_1 is of the order of 10, and a nominal constant value of b_1 may be used in computations.

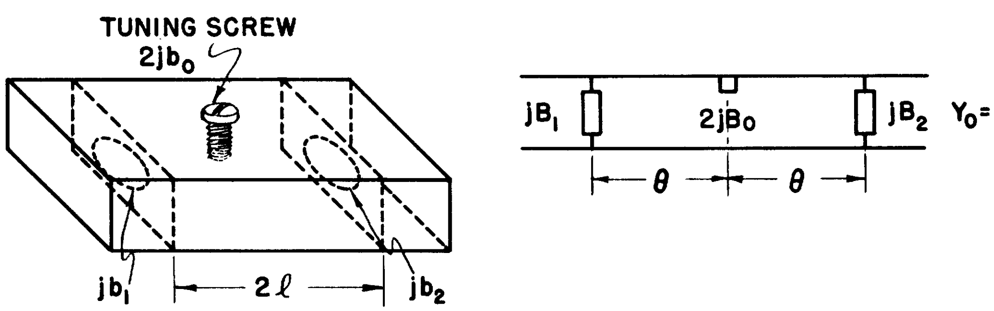

Fig. 4 Four-terminal (transmission) resonant cavity.

The points indicated by circles on Figure 3 are computed from Eq.(8), for b = 10. Again, a capacitive iris is a far better approximation to the required susceptance than is the inductive iris.

Numerical Results

The accuracy of these computations, based upon equivalent circuit concepts, may be gauged by a comparison between the computed behavior and the experiment. The experimental values were furnished by W. Sichak, of

Federal Telecommunications Laboratories, who had been working independently. His measurements were made on the cavity shown in Figure 5. Values of b_1

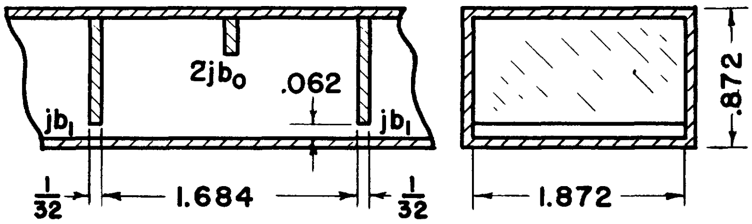

Fig. 5 Test cavity (dimensions in inches).

were not given explicitly. The computed values were normalized with respect to his measured value for $2b_o = 0$. It was assumed that $b_1 \sim 1/\lambda_g$.

f	$2b_o$		b_1	Q_{L2}		Δf (Mc/sec)	
Mc/sec	meas.	calc.	calc.	meas.	calc.	meas.	calc.
4960	0	0	7.0	72	72.0	68.8	68.8
4640	0.28	0.208	6.2	62	61.0	74.8	76.0
4457	0.48	0.331	5.73	60	57.3	74.3	77.8

If an inductive iris had been used, and the computations normalized at f = 4960 Mc/sec, at f = 4457 Mc/sec we would find Q_{L2} = 120 and Δf = 37.2 Mc/sec.

Conclusion

Two-terminal and four-terminal tunable waveguide cavities have been examined to determine what form of coupling susceptance would be required to give constant loaded Q or constant bandwidth. Capacitive irises have a frequency dependence which is approximately that required to produce constant bandwidth. The frequency dependence of inductive irises, on the other hand, is such as to produce a large variation of bandwidth over the tuning range of the cavity.

In the approximate range of $50 < Q_L < 300$, it seems practical to build capacitive irises to be used across rectangular waveguides.

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