Answer 10.2

Start by estimating the settling velocity of the particles assuming creeping flow.

$$w_P = \frac{gd^2(\rho_P - \rho_F)}{18\mu_F} = \frac{9.8\text{ms}^{-2}(10^{-5}\text{m})^2(2000 - 998\text{ kgm}^{-3})}{18(10^{-3}\text{kgm}^{-1}\text{s}^{-1})} = 5.5 \text{ x } 10^{-5}\text{ms}^{-1}$$

Check assumption of creeping flow. $Re = \rho_F w_P d/\mu_F = 0.0005 < 1$, so creeping flow assumption is OK. Next, find the time-scale at which the center of mass of the particle cloud will settle.

$T_{settle} = h/w_s = 0.05 \text{ m} / (5.5 \text{ x}10^{-5} \text{ ms}^{-1}) = 910 \text{ s}$.

With this time scale and the mean advection, u = 0.1 cm/s (assumed constant over channel depth), the center of the cloud will settle at a distance $x = uT_{settle} = 91$ cm downstream from the source. The footprint of the particle cloud, *i.e.* its length and width, depends on the size of the cloud as it meets the bed. While in suspension, the particles spread in all directions by isotropic diffusion, $D = 10^{-13}\text{ m}^2\text{s}^{-1}$. Neglecting the boundary for a moment, at $t = T_{settle}$ the particle cloud will be spherical with diameter $\approx 4\sigma = 4\sqrt{(2 \text{ x } 10^{-13}\text{m}^2\text{s}^{-1} \text{ x } 910 \text{ s})} = 5.4 \text{ x } 10^{-5}$ m. Because the cloud is distributed vertically over a distance $\approx 4\sigma$, individual particles may settle slightly before or after the mean settling time given above. The range of settling times is $\Delta T = 4\sigma/w_s = 1$ s. With advection u = 0.1cm/s, the range of settling times will spread the particle patch longitudinally over a distance, $\Delta T\, u = 0.1$ cm. In addition, the cloud is already distributed over a longitudinal distance 4σ as it settles. But, the longitudinal spread due to diffusion while in suspension, $4\sigma = 0.005$ cm, is small compared to the longitudinal spread accomplished by the differential settling times (0.1 cm), so we ignore the former. Thus we expect the footprint of the particle cloud to be 0.1 cm long, 0.005 cm wide, and centered 91 cm downstream of the release. Given that the longitudinal distance traveled (91 cm) is much greater than the distribution caused by diffusion (0.1 cm), to first order the particles' fate is determined solely by the settling velocity and current speed, i.e. by advection. We could have predicted this apriori by considering the Peclet number based on the particle settling speed. $Pe = w_P h/D = 3\text{x}10^7 >> 1$, indicating that advective transport dominates in this case.