Answer 2.2

a) Estimate the following time scales:

$$T_{sink} = 3/k_{sink} = 3/(0.15 \text{ d}^{-1}) = 20 \text{ d}$$

$$T_{air} = 3/k_{air} = 3/(0.9d^{-1}) = 3.3 d$$

$$T_D = h^2/D = (10m)^2/(0.5 \text{ m}^2 \text{ d}^{-1}) = 200 \text{ d}$$

b) At some time t > 0 profiles of O_2 show that the bottom of the pond is depleted of oxygen, but the surface is not. Using the above time scales explain why this is so.

The delivery of O_2 from the atmosphere is fast enough to balance the sink $[T_{air} < T_{sink}]$. However, the atmospheric flux of oxygen occurs at the water surface. The transport of oxygen to lower waters depends on vertical diffusion. The time-scale for the delivery of new oxygen to the lower waters by diffusion, T_D , is much longer than the time-scale for consumption $[T_D >> T_{sink}]$, so oxygen will be depleted there.

c) Find the near-surface depth, Δz , for which diffusion can restore oxygen fast enough to keep up with the consumption.

The time-scale for diffusion is proportional to the length scale squared, so for depths less than h, T_D will be smaller than the value given above. Consider a distance $\Delta z < h$ from the water surface. The time scale to mix over this depth is $\Delta z^2/D$. Depths for which $\Delta z^2/D < T_{sink}$ will not be depleted of oxygen, because oxygen is restored by diffusion faster than it is consumed. Thus, we expect that depths less than

$$\Delta z = (T_{\text{sink}} D)^{1/2} = 3.2 \text{ m},$$

should not be depleted of oxygen.