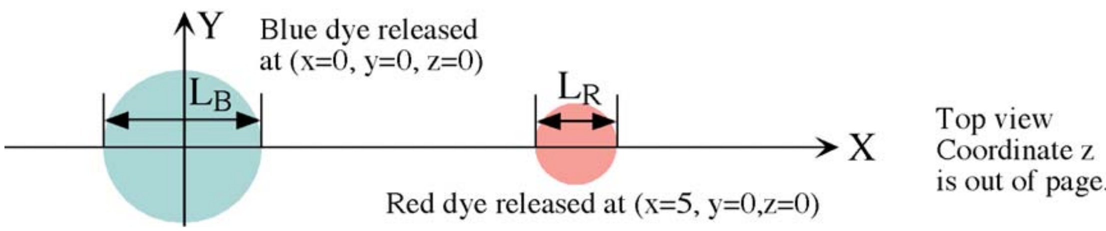
Answer 3.3
Hint 1: Make sketch that defines the diameter of each cloud



a. While both clouds are fully visible (C > 10-g l^{-1}), which cloud will appear larger, and by how much?

Since the size of the cloud increases in proportion to the diffusion coefficient, the blue cloud will grow more rapidly, and thus appear bigger, than the red cloud. Specifically, the length scale of each cloud, as defined in chapter 3, equation 26, is

$L_B = 4\sigma_B = 4\sqrt{2D_B t}$ and $L_R = 4\sigma_R = 4\sqrt{2D_R t}$.
The ratio of dye cloud diameters is then, $L_B / L_R = \sqrt{D_B / D_R} = 2$.

b. At what time and at what location will the two dye clouds first appear to touch?
Hint 2: Simplify governing equation with assumptions.
Begin with the full transport equation that governs the evolution of both dye drops.

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = \frac{\partial}{\partial x}D_x\frac{\partial C}{\partial x} + \frac{\partial}{\partial y}D_y\frac{\partial C}{\partial y} + \frac{\partial}{\partial z}D_z\frac{\partial C}{\partial z} \pm S$$

(1) (2) (3) (4) (5) (6) (7) (8)

Simplifying assumptions:
- The dyes do not interact, so we can drop the source/sink term 8.
- The fluid is stagnant, thus u=v=w=0, and we can drop terms 2,3,4
- Molecular diffusion is homogeneous and isotropic, $D_x=D_y=D_z=D$, and $D \neq f(x,y,z)$.
  This reduces the diffusion terms to: $D\,(\partial^2C/dx^2 + \partial^2C/dy^2 + \partial^2C/dz^2)$.
- The dyes mix rapidly between the plates, such that $\partial C/\partial z = 0$, and we can drop term 7.

Simplified governing equation: $$\frac{\partial C}{\partial t} = D\left[\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right].$$

Initial conditions:
Blue Dye: $C_B(x,y,t=0) = M\,\delta(x)\,\delta(y)$
Red Dye: $C_R(x,y,t=0) = M\,\delta(x-5)\,\delta(y)$

The diffusion of each dye is thus described by the two-dimensional, instantaneous, point release solution. See equation 23 in Chapter 3, with $D_x=D_y=D$. Note, here L_Z is the plate gap, 5mm.

$$C_B(x,y,t) = \frac{M}{L_z\, 4\pi\, D_B\, t} \exp\left(-\frac{x^2 + y^2}{4 D_B t} \right)$$

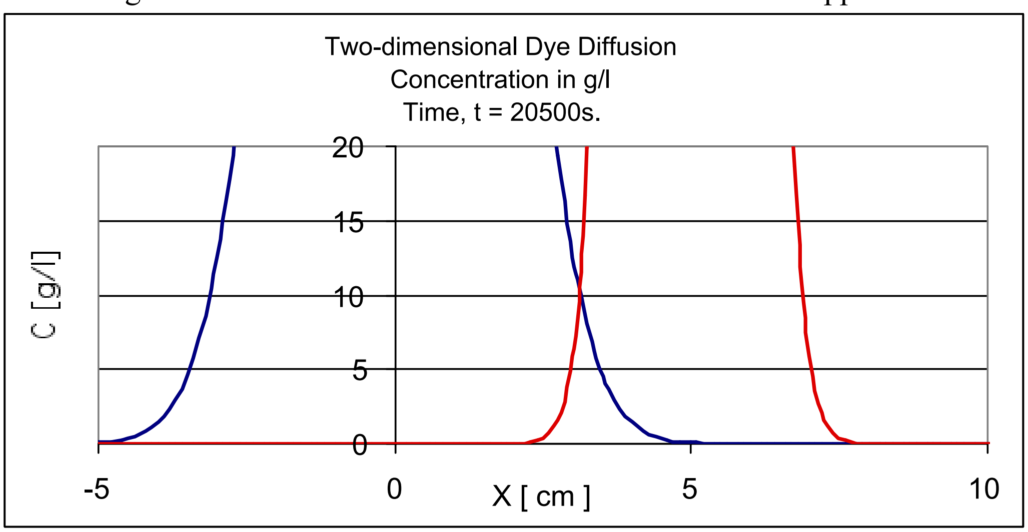
$$C_R(x,y,t) = \frac{M}{L_z\, 4\pi\, D_R\, t} \exp\left(-\frac{(x-5)^2 + y^2}{4 D_R t} \right)$$

Note that the exponential is written to be one at the center of the red cloud, i.e. at (x=10,y=0), with the position given in units of centimeters.

Hint 3: Write mathematical criteria for condition when clouds first touch

From geometry, the two clouds will first touch along the line y = 0. We wish to find the time, t, and location, x, for which $C_B(x, y=0, t) = C_R(x, y=0, t) = 10$ g/l. This could be tackled analyticaly by first finding the position $x_{BR}(t)$ at which $C_B=C_R$. Then solve for t using this position to constrain x, e.g. solve $C_B(x = x_{BR}, y=0, t)$. However, a simpler approach is to graph C_B and C_R in an interactive graphing package, such as Excel, and then vary time until the intersection of the two concentration curves lies at C = 10 g/l

Solution: - At t = 20500s, the intersection of the blue and red concentration curves corresponds to C = 10 g/l and is located at x = 3.1 cm. The clouds will first appear to touch and x = 3.1 cm.



Make a rough estimate of the location using your result from part a?

Based on the definition used in a. and the definition sketch shown above, the two clouds first appear to touch when, $(L_B/2)+(L_R/2) = 5$. Additionally, $L_R = (L_B/2)$, so the edge of the blue cloud will be at $x = (L_B/2) = 5/1.5=3.3$cm, when it first touches the red cloud.

c. At what time will the line connecting the release points be completely purple?
Hint 4: - Define a mathematical criteria for this to occur?
This condition requires that C_B and $C_R > 10$ g/l in the region x = 0 to 10 cm. Use the spreadsheet created in part b to interrogate the concentration field over a range of time.

Solution - Graph C_B and C_R in an interactive graphing package, such as Excel and vary time until the above criteria is met. You will find that the criteria is never met. Between $0<x<5$ cm, when $C_R \geq 10$g/l then $C_B < 10$ g/l, and when $C_B \geq 10$g/l, $C_R < 10$ g/l.