SOLUTION 9 FOR 6.013

Nov.21,2002

Solution 9.1

a) $L = \mu_0 d/W = 4\pi \times 10^{-7} \times 10^{-5}/4.0 \times 10^{-4} = 3.14 \times 10^{-8}$H/m

$$C = 4\epsilon_0 W/d = 4 \times 8.85 \times 10^{-12} \times 4.0 \times 10^{-4}/10^{-5} = 1.4 \times 10^{-9}\text{F/m}$$

$J = \sigma E$, $G = \frac{I}{V} = \frac{JA}{V} = \frac{\sigma EA}{Ed} = \frac{\sigma Wl}{d} = 0.016$S/m, where $l = 1$m.

$$Z = \sqrt{\frac{jwL+R}{jwC+G}} = \sqrt{\frac{jwL}{jwC+\frac{\sigma W}{d}}} = \sqrt{\frac{jwd}{jwCd+\sigma W}}$$

b) $G = 0.016$S/m

c) 10dB means that $\frac{P(z=l)}{P(z=0)} = 10^{-1}$

$P \propto |e^{-jk'z-k"z}|^2 = e^{-2k"z} = 10^{-1}$, where $2k" = G\sqrt{L/C} = 0.00757$

$$z = -\frac{\ln 10}{2k"} = 30.3\text{m}$$

Solution 9.2

(a) $i(z,t) = Y_0(f_+(t - z/v) - f_-(t + z/v))$

$$P = vi = Y_0(f_+^2(t - z/v) - f_-^2(t + z/v)) = P_+ - P_-$$

b) $v(z,t) = f_1(t - z/v) + f_2(t - z/v)$

$$i(z,t) = Y_0(f_1(t - z/v) + f_2(t - z/v))$$

$$P = vi = Y_0(f_1^2(t - z/v) + f_2^2(t - z/v) + 2f_1(t - z/v)f_2(t - z/v))$$

$$\neq P_1 + P_2$$

Solution 9.3

a) The velocitiy of wave: $v = 1/\sqrt{\mu\epsilon} = c/2 = 1.5 \times 10^8$[m/s]. For one trip, it needs time: $\tau = l/v = 0.2/1.5 \times 10^8 = 1.3 \times 10^{-9} = 1.3$[ns].

When $t = 1$ns, the pulse reaches $z = t \cdot v = 10^{-9} \times 1.5 \times 10^8 = 15$cm. The forwarding voltage: $v_+ = \frac{Z_0}{Z_s + Z_0} v = \frac{4}{5} \times 10 = 8$, $i_+ = v_+/Z_0 = 8/200 = 0.04$A. Please see fig 1.

b) At $t = 2$ns, the reflected pulse reaches $z = v \cdot (2.6 - 2)$ns $= 9$cm. The reflected coefficient is $\Gamma = \frac{Z_0 - Z_l}{Z_0 + Z_l} = -0.6$. The backwarding voltage: $v_- = \Gamma v_+ = -4.8$v, $i_- = v_-/Z_0 = 4.8/200 = 0.024$A. Please see fig 2.

c) The transmited coefficient on the right end: $T = 1 + \Gamma = 0.4$ The voltage on the load $v_l = v_+ T U(t - \tau) + v_+ T \Gamma^2 U(t - 3\tau) + ... + v_+ T \Gamma^{2n} U(t - (2n+1)\tau) + ...$ So $v_l = 3.2$ $(n = 0)$, $v_l = 4.352$ $(n = 1)$. Please fig.3.

d) When $t \to \infty$, $v_l = 10 \times 1/2 = 5$v

e) Because of match, the will be no reflected voltage. There is no excessive delay.

f) $v_+ = 8 \times U(t - z/v)$, $0 < t < 1$ns, where $v = 1.5 \times 10^8$[m/s].

Solution 9.4

This is an initial values problems. When $t = 0$, the lightning strikes the line. First find $v_+(z, t = 0)$, and $v_-(z, t = 0)$.

$$v(z,t) = v_+(z, t = 0) + v_-(z, t = 0) = V_0$$

$$i(z,t) = Y_0(v_+(z, t = 0) - v_-(z, t = 0)) = I_0 = P/V_0$$

We can get: $v_+ = \frac{1}{2}(V_0 + I_0 Z_0) = 200000$v, $v_- = \frac{1}{2}(V_0 - I_0 Z_0) = -100000$v. At the center point of the line, the transmited and reflected coefficients are $\Gamma_1 = \frac{R_1 || Z_0 - Z_0}{R_1 || Z_0 + Z_0} = -0.9375$, $T_1 = 1 + \Gamma_1 = 0.0625$. At the right end point, $\Gamma_2 = \frac{R_2 - Z_0}{R_2 + Z_0} = -0.5$, $T_2 = 1 + \Gamma_2 = 0.5$, where $R_1 = 10$ohm, $R_2 = V_0/I_0 = 100$ohm.

Refer to fig 4.
At the left point, we don't have to consider because it always satifies the boundary. The backword and forward voltage are always the same.
At the center,$v_1 = \Gamma_1 v_- + T_1 v_+ = 106250$, $v_2 = \Gamma_1 v_+ + T_1 v_- = -193750$,$i_1 = -v_1/Z0 = 645.8$, $i_2 = v_2/Z0 = 354.2$,
At the right end, $v_3 = \Gamma_2 v_+ = -100000$, $i_3 = v_3/Z0 = 333.3$

Solution 9.5
Please see fig. 5.

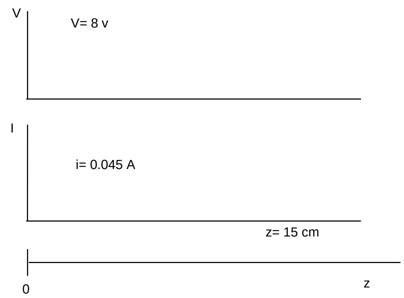

Fig. 1 ps 9.3 a)

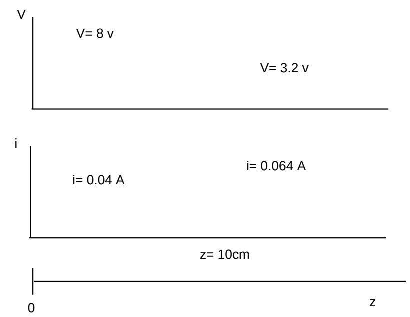

Fig. 2 ps 9.3 b)

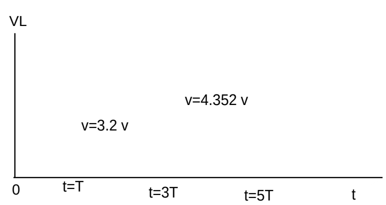

Fig. 3 ps 9.3 c)

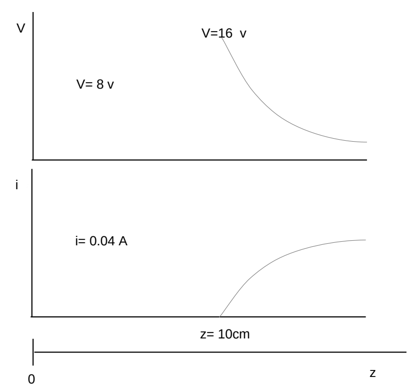

Fig. 5 ps 9.5

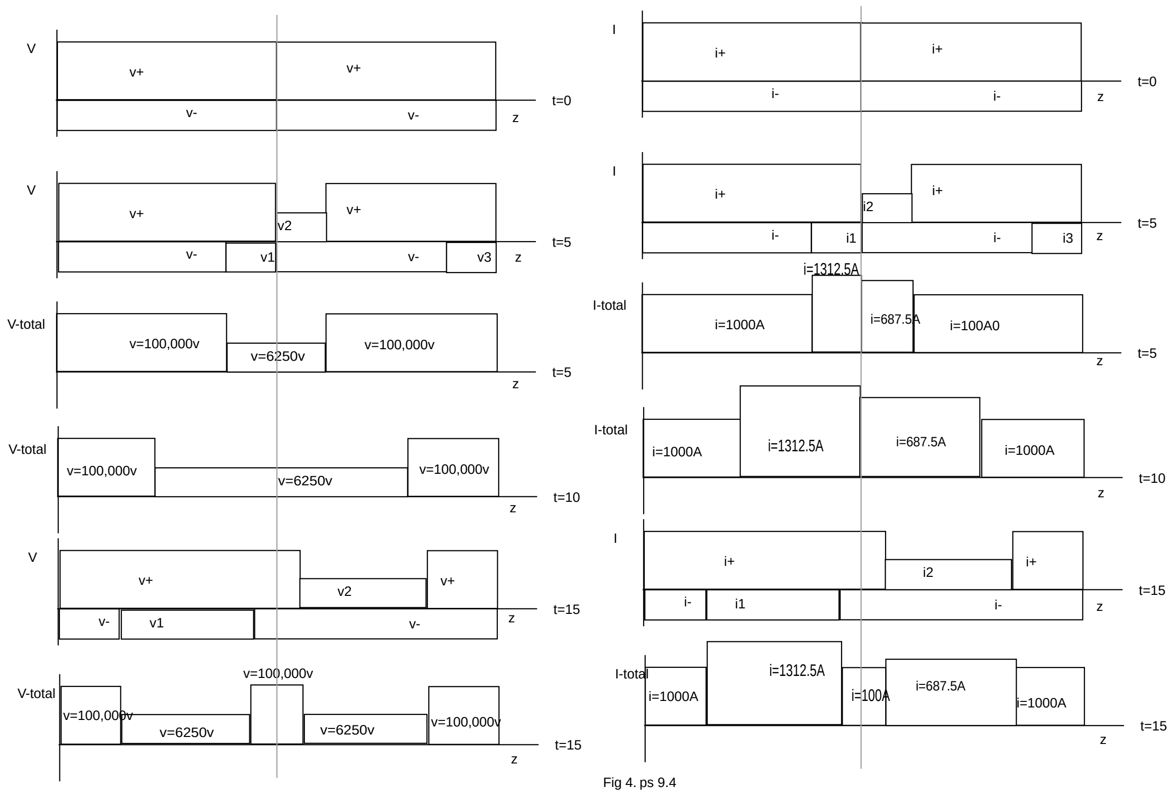
V
v+
v+
v-
v-
t=0
z
V
v+
v2
v+
v-
v1
v-
v3
t=5
z
V-total
v=100,000v
v=6250v
v=100,000v
t=5
z
V-total
v=100,000v
v=6250v
v=100,000v
t=10
z
V
v+
v2
v+
v-
v1
v-
t=15
z
V-total
v=100,000v
v=100,000v
v=6250v
v=6250v
v=100,000v
t=15
z
I
i+
i+
i-
i-
t=0
z
I
i+
i2
i+
i-
i1
i-
i3
t=5
z
i=1312.5A
I-total
i=1000A
i=687.5A
i=100A0
t=5
z
I-total
i=1000A
i=1312.5A
i=687.5A
i=1000A
t=10
z
I
i+
i2
i+
i-
i1
i-
t=15
z
I-total
i=1000A
i=1312.5A
i=100A
i=687.5A
i=1000A
t=15
z

Fig 4. ps 9.4