SOLUTION 8 FOR 6.013

Oct.29,2002

Solution 8.1

a) Ignore the magnetic field outside the coil,

$$\int \overline{H}d\bar{l} = NI, H \cdot D = NI,\ H = \frac{NI}{D} = \frac{1000I_o}{0.04} = 2.5 \times 10^4 I_o$$

b) According to energy conservation

$$eV = \frac{1}{2}mv^2,\ v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times2.0\times10^4}{9.1\times10^{-31}}} = 8.386 \times 10^7 = 0.28c$$

c)Apply Lorentz force law,

$$\overline{f} = q\overline{v} \times \overline{B} = qv\mu H\hat{f} = 4.2 \times 10^{-13} I_0 \hat{f}$$

The direction of force is vertical to the velocity.

d)The centrifugal force

$$F = \frac{mv^2}{R} = qvB = qv\mu\frac{NI}{D},\ R = \frac{d/2}{\sin 5^0}$$

Plug the second equation into the first,

$$I = \frac{2mD\sin 5^0}{Ndq\mu}v = \frac{2mD\sin 5^0}{Ndq\mu}\sqrt{\frac{2qV}{m}} = 0.066[\text{A}]$$

Solution 8.2

(a) Use integral form of the Faraday's law

$$V = \int \overline{E}d\bar{l} = -\frac{\partial}{\partial t}\int \overline{B}d\overline{S} = -\frac{\partial}{\partial t}Bxd = -Bd\frac{\partial x}{\partial t}$$

$$= -Bdv = -10^{-4} \times 4.5 \times 10^4 \times 3.3 \times 10^6 = -1.485 \times 10^7[\text{v}]$$

The positive charges will accumulate on the back face.

b)Lorentz force law,

$$\overline{f} = \overline{I}d \times \overline{B} = 3.3 \times 10^6 \times \frac{1.485\times10^7}{1} \times 10^{-4}\hat{f} = 4.9 \times 10^9\overline{f}$$

The direction of force points to East. It will accelarate Io.

Solution 8.3

a) The magnetice strength in the gap

$H = \frac{Ni}{g-x}$, where x is the change of gap seperation

The energy in the gap

$$W = \frac{1}{2}\mu H^2 \cdot A(d-x) = \frac{1}{2}\mu(\frac{Ni}{d-x})^2 \cdot A(d-x) = \frac{1}{2}\mu\frac{(Ni)^2}{d-x}A$$

Then the force

$$f = -\frac{\partial W}{\partial x} = -\frac{1}{2}\mu A(\frac{Ni}{d-x})^2$$

b)We know the inductance of the system:

$$L = \frac{\mu AN^2}{d} = \frac{\mu\sqrt{A}(\sqrt{A}-z)N^2}{d}$$

The energy in the gap

$$W = \frac{1}{2}Li^2 = \frac{1}{2}i^2\frac{\mu\sqrt{A}(\sqrt{A}-z)N^2}{d}$$

The the force

$$f = -\frac{\partial W}{\partial z} = \frac{\mu\sqrt{A}(Ni)^2}{2d}$$

c) $V = \frac{d\Lambda}{dt} = \frac{dLi}{dt} = i\frac{dL}{dt} = i\frac{d}{dt}(\frac{\mu AN^2}{d}) = i\mu AN^2\frac{d}{dt}(\frac{1}{d(1+\cos wt)}) = \frac{i\mu AN^2 w\sin wt}{d(1+\cos wt)^2}$

Here i(t) is assumed as constant in terms of time.

Solution 8.4

$$f = ma = \frac{dP}{dt} = n\frac{hf}{c},\ a = \frac{nhf}{mc} = 6.62 \times 10^{17}[\text{m/s}^2]$$