SOLUTION4 FOR 6.013

Sept.30,2002

Solution 4.1

a) One the surface of perfect conductor, $E_v = 0$, $E_h \neq 0$, where E_v is the electric field vertical to terrestrial surface, E_h is horizontal to surface. The receiving antenna is parallel to terrestrial surface. So there is generally a null.

b) The electric field is similar to a): $E_v = 0$, $E_h \neq 0$. But the receiving antenna is perpendicular to surface. So there is generally a peak. But when the horizontal distance between transmitting and receiving antenna is 0, it is null.

c) See the fig.1, for the first non-zero altitude null, distance $D = \lambda/2 = 2h_1 \cos\theta = 2h_1(h_1 + h_2)/\sqrt{r^2 + (h_1 + h_2)^2}$, we can get $h_2 = \lambda r/\sqrt{16h_1 - \lambda^2} - h_1$

d) First we find the first and second non-zero altidude nulls for fixed geometry.

$$D = \lambda_1/2 = 2h_1 \cos\theta = 2h_1(h_1 + h_2)/\sqrt{r^2 + (h_1 + h_2)^2}(first\ null)$$

$$D = 3\lambda_2/2(second\ null)$$

$$\lambda_2 = \lambda_1/3$$

$$\triangle\lambda = \lambda_2 - \lambda_1 = -2\lambda_1/3$$

$$f = c/\lambda,$$

$$\triangle f = -\frac{c}{\lambda_1^2}\triangle\lambda = 2c/3/\lambda_1 = c\sqrt{r^2 + (h_1 + h_2)^2}/(6h_1(h_1 + h_2))$$

solution 4.2

(a) $P_{rec} = (\frac{P_{tr}G\sigma}{4\pi r^2})(\frac{A}{4\pi r^2}) = (\frac{A}{r^2\lambda})^2(\frac{P_{tr}\sigma}{4\pi})$

Choose $\mathrm{\lambda = 8mm, P_{tr} = 10w, r = 100km,}$

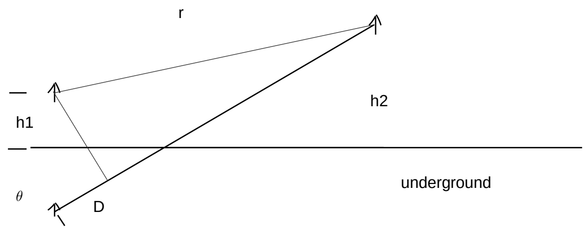

Fig 1

$P_{rec} = (\frac{A}{r^2\lambda})^2(\frac{P_{tr}\sigma}{4\pi}) = 7.7 \times 10^{-17}$
It requires energy for one pulse:
$E = P_{rec} \times T_{pulse} = 7.7 \times 10^{-17} \times 10^{-6} = 7.7 \times 10^{-23}\text{J} < 4 \times 10^{-20}\text{J}$
So it can't satisfy the requirement.

(b) If we choose $P_{tr} = 10\text{w}$, $\lambda = 0.008\text{mm}$, $r = 100\text{km}$

$$P_{rec} = (\frac{A}{r^2\lambda})^2(\frac{P_{tr}\sigma}{4\pi}) = 7.7 \times 10^{-13}[\text{w}]$$

It requires energy for one pulse:
$E = P_{rec} \times T_{pulse} = 7.7 \times 10^{-13} \times 10^{-6} = 7.7 \times 10^{-19}\text{J} > 4 \times 10^{-20}\text{J}$
So we can see this system has signal margin.

(c) Because all radars are apart in a hexagonal grid. Every radar only needs to check the area with radius $r \leq \sqrt{(15^2 + 50^2)}km = 52.2\text{km}$.The beamwidth for parabolic dishes is $\theta_w = \lambda/D = 0.008/1 = 0.008$. It will sweep times $2\pi/\theta_w = 196$.The scanning time each is about $r/c = 52200/3 \times 10^8 = 1.74 \times 10^{-4}$[s]. Scanning time: $196 \times 1.74 \times 10^{-4} = 0.03$[s].

solution 4.3

a) The following two answers are based on $N = 6 \times 10^{23}$ and $N = 6 \times 10^{28}$
$f_p = \sqrt{\frac{Nq^2}{m\epsilon}} = \sqrt{\frac{6\times10^{23}\times(1.6\times10^{-19})^2}{9.1\times10^{-31}\times8.85\times10^{-12}}} = 6.95 \times 10^{12}[\text{Hz}]$
or
$f_p = \sqrt{\frac{Nq^2}{m\epsilon}} = \sqrt{\frac{6\times10^{28}\times(1.6\times10^{-19})^2}{9.1\times10^{-31}\times8.85\times10^{-12}}} = 2.2 \times 10^{15}[\text{Hz}]$

b) $f = 10^{12},\ \delta_1 = \sqrt{\frac{2}{\sigma w\mu}} = \sqrt{\frac{2}{5\times10^7\times2\pi\times10^{12}\times4\pi\times10^{-7}}} = 7.1 \times 10^{-8}[\text{m}]$
$f = 60,\ \delta_1 = \sqrt{\frac{2}{\sigma w\mu}} = \sqrt{\frac{2}{5\times10^7\times2\pi\times60\times4\pi\times10^{-7}}} = 9.1 \times 10^{-3}[\text{m}]$

c) (optional)

Solution 4.4

a) $\triangle\phi = (n + 1/2)\pi = (k^o - k^e)d$

$$k^0 = w\sqrt{\epsilon\mu} = w\sqrt{4.004\epsilon_0\mu}$$

$$k^e = w\sqrt{\epsilon\mu} = w\sqrt{4.\epsilon_0\mu}$$

So $(n + 1/2)\pi = (k^o - k^e)d = (\sqrt{4.004} - \sqrt{4})\frac{2\pi}{\lambda}d = \pi/2$, $d = 0.125$[mm]

b) 1/2 of field will pass the polaroid, reflect from the mirror, it has the same direction linear polarization, pass through the polaroid completely. So totally, 1/2 will be reflected. The quarter-wave plate has no effect on the fraction in this case.

c) 1/2 of field will pass the polaroid, the quarter-wave plate transfers it to circular polarization. After reflecting and passing the quarter-wave plate again, the wave become linear polarization again, but the direction is vertical to polarization direction in the polaroid. So no wave can pass.