SOLUTION 6 FOR 6.013

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Solution 6.1

a) $\int \overline{D} d\overline{S} = Q$, $\epsilon E d2\pi r = Q$, $E = \frac{Q}{\epsilon d2\pi r}$, $V = \int \overline{E} d\overline{l} = \frac{Q}{\epsilon d2\pi}(\log b - \log a)$, $Q = \frac{V\epsilon d2\pi}{\log b - \log a}$,
$\overline{E} = \hat{r}\frac{V}{r(\log b - \log a)}$

b) $\sigma_s = \epsilon E = \frac{9\epsilon_0 V}{a(\log b - \log a)}$

c) $Q = S \cdot \sigma_s = 2\pi a d\sigma_s = \frac{18\pi\epsilon_0 V}{\log b - \log a}$

d) $C = Q/V = \frac{18\pi\epsilon_0 d}{\log b - \log a}$

e) $w_e = \int \frac{1}{2}\epsilon|E|^2 dV = \frac{1}{2}\epsilon(\frac{V}{\log b - \log a})^2 \int \frac{1}{r^2} dV = \frac{1}{2}\epsilon(\frac{V}{\log b - \log a})^2 \int_0^{2\pi} d\theta \int_a^b \frac{1}{r^2} r dr$

$= \frac{9\pi\epsilon_0 V^2}{\log b - \log a} = \frac{CV^2}{2}$

f) $\overline{I} = 2\pi r d\overline{J} = 2\pi r d\sigma \overline{E} = 2\pi r d\sigma \frac{V}{r(\log b - \log a)} = \frac{2\pi\sigma}{\log b - \log a}V$, $R = \frac{\log b - \log a}{2\pi\sigma d}$

Solution 6.2

(a) $\int \overline{H} d\overline{l} = I$, $\overline{H}(\overline{r}) \cdot 2\pi r = I$, $\overline{H}(\overline{r}) = \hat{\phi}\frac{I}{2\pi r}$

(b) $\Lambda = \int_A \overline{B} \cdot \hat{n} da = \mu \int \frac{Id}{2\pi r} dr = \frac{\mu I d(\log b - \log a)}{2\pi}$

(c) $L = \frac{\Lambda}{I} = \frac{\mu d(\log b - \log a)}{2\pi}$

d) $W_e = \int \frac{1}{2}\mu|\overline{H}|^2 dV = \frac{1}{2}\mu d \int (\frac{I}{2\pi r})^2 dr\theta dr = \frac{\mu I^2 d(\log b - \log a)}{4\pi} = \frac{LI^2}{2}$

Solution 6.3

a) $R = 2$, $C = 1/2$

b) $\tau = RC = 1[s]$

Solution 6.4

a) $\int \overline{H} \cdot d\bar{l} = NI$, $H2\pi R = NI$, $\overline{H} = -\hat{\theta}\frac{NI}{2\pi R}$

b) $\Lambda = N\mu H\pi r^2 = \frac{I\mu N^2 r^2}{2R}$

c) $L = \Lambda/I = \frac{\mu N^2 r^2}{2R}$

d) $V_i = L\frac{dI}{dt} = \frac{N^2 r^2}{2R}\frac{dI}{dt}$, $V_0 = 2V_i = \frac{N^2 r^2}{R}\frac{dI}{dt}$

e) For the input coils, $\frac{d\Lambda}{dt} = 0$ $V_i = 0$