LASERS

Applications:

Amplification: Broad-band communications links
(e.g. EFDA; avoids down-conversion)

Oscillator: Frequency/distance reference, local oscillators, illuminators,
sources for fiber communications, CD/DVD players

Focused power: Laser machining, weapons, laser fusion (pellet compression).
Peak > 10^{15} W (10-\mu m spot ⇒ \bar{E} \approx 10^{14} Vm^{-1} vs 10^6 in H atom)
Average > 1kw; high intensity because I ∝ |\sum_i E_i|^2

Basic Principles:

Quantum states characterize atoms and molecules in gases, impurities in solids,
and electrons and holes in semiconductors

A transition to a lower state emits a photon coherent with the triggering photon
⇒ amplification (or, with internal reflection, oscillation)

Amplification/lasing requires upper state population to exceed lower state
Basic Amplification Process:

Pump (repopulates level 2)

Optical fiber

Input

amplification, exponential growth

Intensity-limited amplification

n_2_ replacement-rate limited amplification

[Each ● is a separate atom or molecule; need n_2 > n_1 for amplification]

Amplification frequency f [Hz]:

\[ E_2 - E_1 = hf \] [J]

\[ h = 6.625 \times 10^{-34} \] [Js]
ENERGY STATES AND POPULATIONS

Energy States:
- Ionization, 0 e.v.
- Free electron
- Ground state
- -13.6 e.v.
- 1420 MHz

Hydrogen atom (Galactic arms)

States:
- Electronic (visible, UV)
- Vibrational (visible)
- Bending (IR)
- Rotational (microwave)

Water vapor H$_2$O
(e.g. water vapor masers around stars)
(electric dipole transitions)

Also: Chromium atoms in lattice (e.g. ruby), Erbium atoms in glass

Level Populations—Kinetic Temperature $T_k$:
Thermal equilibrium dominated by collisions

⇒ Boltzmann distribution:
$$\frac{n_i}{n_j} = e^{-(E_i-E_j)/kT_k}$$

If thermal equilibrium dominated by radiation, $T_{i,j}$

⇒ Boltzmann distribution:
$$\frac{n_i}{n_j} = e^{-(E_i-E_j)/kT_{rad}}$$

$n_2 > n_1$ if $T_{rad} < 0$

State energy $E_i$ [J]

$n_2 \propto e^{E/kT}$

$n_1 

\Rightarrow n_i \rightarrow n_j$ if $T_{rad} \rightarrow \infty$
**“A” AND “B” COEFFICIENTS**

**Rate Equation:**

Assume: Two-level system, \( E_2 > E_1 \), and \( n_i = \text{atoms m}^{-1} \text{ in state } i \)

Then: \[
\frac{dn_2}{dt} = -An_2 - B(n_2 - n_1) \quad \text{[m}^{-1}\text{s}^{-1}] \quad \text{(collisionless system)}
\]

Spontaneous emission  Induced emission

**Spontaneous emission between states i and j:**

\[
A_{ij} = \omega^3 |D_{ij}|^2 (2/3h\epsilon c^3) \quad \text{[s}^{-1}] \]

\( D_{ij} \) [Cm] is a quantum mechanical dipole moment (electric or magnetic)

Decay time \( \tau_A = A^{-1} \)

Note: \( \tau_A \propto \omega^{-3} \), so “visible” \( \tau \)'s are very short, microwave \( \tau \)'s are long

**Stimulated emission and absorption:**

B coefficient:

\[
B_{ij} = F\sigma_{ij} \propto F g_{ij}(f) A_{ij}/\omega^3 \quad \int_{-\infty}^{\infty} g_{ij}(f) df = 1
\]

Photon flux density \( F \):

\[
F = |\mathbf{E}|^2/2\eta_0hf \quad \text{[photons m}^{-2}\text{ s}^{-1}] \]

Lorentzian line shape \( g_{ij}(f) \):

\[
g_{ij}(f) = [2/\pi(\Delta f)]/[1 + 4(f - f_o)^2/(\Delta f)^2]
\]
PUMPING LASERS

Two-Level Lasers:

No degree of pumping can yield $n_2 > n_1$
(early 2-level lasers spatially isolated $n_2$ group)

Three-Level Lasers:

Pump levels 1,3 so $n_1 \cong n_3$
Large $A_{32}$ populates 2 so $n_2 >> n_1 \cong n_3 \cong 0$

More levels sometimes used, e.g. to utilize quantum states with larger $A$’s

Laser Power Efficiency ($P_{\text{out}}/P_{\text{in}}$):

Intrinsic efficiency: $\eta_i = f_L/f_p (P \propto nhf [W]) < 1$

B/A efficiency: $\eta_B = B_{21}/(A_{21} + B_{21}) < 1$

A/A efficiency: $\eta_A = A_{32}/(A_{31} + A_{32}) < 1$

Total efficiency: $\eta = \eta_i \eta_B \eta_A$

Recall $A \propto \omega^3$; if $B >> A \propto \omega^3$, then x-ray lasers need very high $B$ (pump values) ($B \neq \sim f(\omega)$)
**LINE SHAPE**

Lorentzian Line Shape and Broadening Mechanisms:

Lorentzian line shape:

\[ g_{ij}(f) = \frac{2}{\pi(\Delta f)} \frac{1}{1 + 4(f - f_o)^2/(\Delta f)^2} \]

\( g_{ij}(f) \) has unity integral:

\[ \int_{-\infty}^{\infty} g_{ij}(f) df = 1 \]

\( g_{ij}(f)/g_o = 0.5 \) for \( |f - f_o| = \Delta f/2 \)

Broadening mechanisms:

- Homogeneous broadening: \( \Delta f_o > 1/\tau_A = A \approx 10 \text{ MHz} \) (minimum linewidth); collisions, lattice interactions, Fields \((E, B)\), & Doppler all broaden \( \Delta f \)
- Inhomogeneous broadening: Each narrow-band atom shifted differently, e.g. HeNe; \( \Rightarrow \) hole burning

E.g., each atom has \( \Delta f \approx 4 \text{ THz} \); a single frequency can drain G;
E.g. EDFA’s, most solid-state and semiconductor lasers

Collisions \( \Rightarrow \) phase changes, Fourier transform \( \Rightarrow \) \( \Delta f \)
LASER OSCILLATORS

Laser Oscillation:

Oscillator: Assume length L, perfect mirrors at both ends; Closed lossless amplifier must oscillate and saturate

Threshold: Gain $m^{-1}$ must exceed loss (threshold condition) Gain $\propto$ pump power $P_p$, therefore $P_p >$ threshold too

Mirrors: Assume one mirror has power transmission coefficient $T > 0$
Gain $\equiv$ Loss: $P_+(1 - T)e^{2(g-\alpha)L} \geq P_+$
$\Rightarrow$ Two-pass gain $e^{2(g-\alpha)L} \geq 1/(1 - T) > 1$ for oscillation ($g > \alpha$)

Output Limit: $P_{out} = \eta P_{pump}$, $P_+ = P_{out}/T$, so $P_+/P_{out} \rightarrow \infty$ as $T \rightarrow 0$

Q-switching: Set $T \equiv 0$ until $P_+$ peaks, then set $T \equiv 1$; yields very large “Q-switched pulse”
LASER RESONANCES

Oscillator Resonant Frequencies f:

Resonances when \( m\lambda_m/2 = L \) (mirrors approx. short circuits)

\[ \Rightarrow \lambda_m = \frac{2L}{m}, \quad f_m = \frac{c}{2LN} \quad (N = \text{index of refraction}) \]

\[ f_{i+1} - f_i = \frac{c}{2LN} \cong 10^8 \text{ Hz (100 MHz)} \text{ for 1-meter fiber;} \]

\[ \cong 50 \text{ GHz line spacing for 0.5-mm diodes} \]

Laser Output Spectrum:

In saturation, Gain(f) \( \cong \) frequency independent

If laser linewidth \( \Delta f > \) line spacing,

dominant line wins if exponential growth

If linewidth \( \Delta f < \) line spacing,

must tune cavity length to \( f_o \)
Examples of Lasers

Electrically Pumped Solid-State Lasers:

Forward biased GaAs p-n junction injects carriers into conduction band. Compact (grain of sand), ~50 percent efficiency, >100 W/cm² for arrays, 1 mW/micron² for diodes (1-1000 mW typical).

\[ \Delta E = hf \]

Forward-biased p-n junction with mirrors at ends of transparent region.
Astrophysical Masers:

- Stellar pumped: Water vapor, OH, CO, etc.
- Interstellar collisions: OH, etc.

Gas Lasers:

- Ammonia (23 GHz): “Pumped” by diverting molecules in ground state
- CO₂, HeNe: Pumped by electrical discharges that form energetic plasmas
- Chemical: Chemical combustion yields upper-state excess

Externally Pumped Solid-State Lasers

- Ruby: Pumped by flash lamps, etc.
- EDFA: Pumped by semiconductor lasers