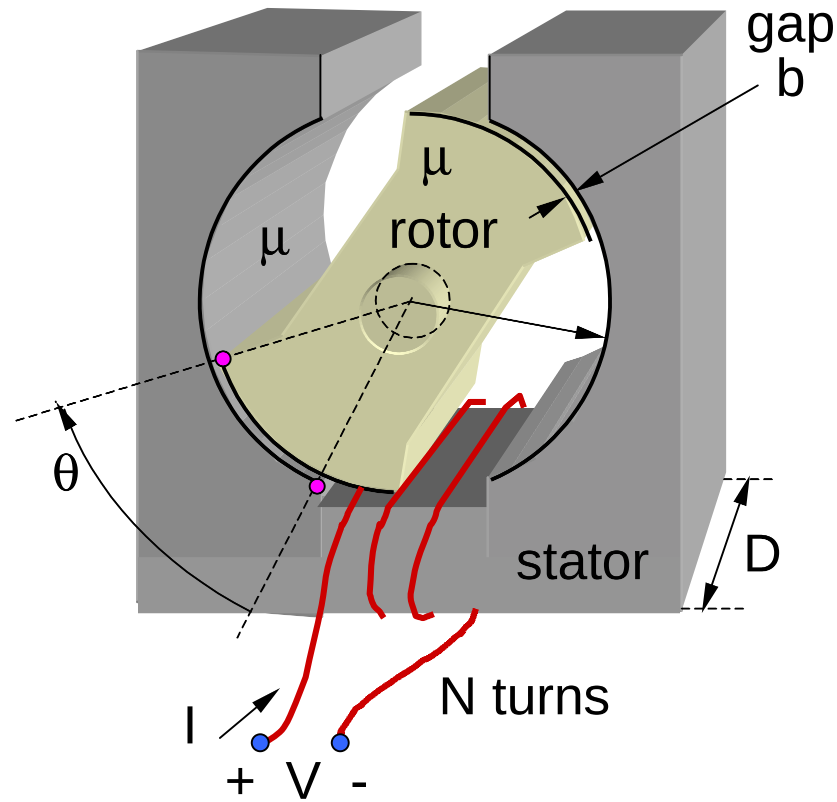
VARIABLE RELUCTANCE MOTORS

Basic 2-Pole Reluctance Motor:

What is the torque on the rotor?



First find B and H:

Since $\nabla \bullet \overline{B} = 0$, then $\overline{B}_{stator} \cong \overline{B}_{gap}$

and $\overline{H}_{stator} \cong (\mu_o/\mu)\overline{H}_{gap} << \overline{H}_{gap}$

$$\left(\mu \cong 10^4 \mu_o\right)$$

$\nabla \times \overline{H} = \overline{J} \Rightarrow$

$$NI = \int_c \left(\overline{H}_{gap} + \overline{H}_{stator}\right) \bullet d\overline{s} \cong 2bH_{gap}$$

Therefore $H_{gap} \cong NI/2b$

(independent of rotor cross-sectional area)

VARIABLE RELUCTANCE MOTORS (2)

Find Torque T[Nm] by Differentiating Energy Storage:

Since: $W_m \propto \mu |H|^2/2 \left[Jm^{-3}\right]$ and $\overline{H}_{stator} \cong (\mu_o/\mu)\overline{H}_{gap}$

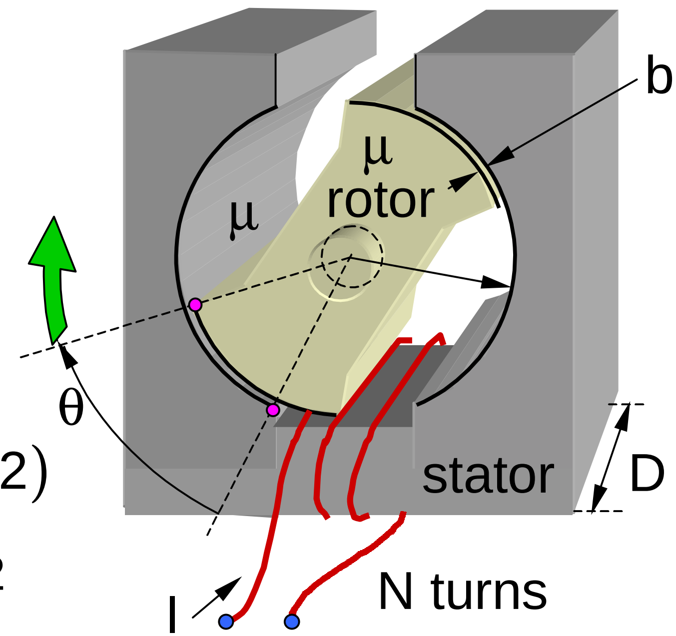
Therefore: $W_{gap} \cong (\mu/\mu_o) W_{stator} \left[Jm^{-3}\right]$

So if: $b/2R << \mu_o/\mu$

Then: $w_{gap} >> w_{stator} [J]$ where

$$w_{gap} \cong 2bR\theta D\mu_o \left|H_{gap}\right|^2/2$$

Where: Gap area $= R\theta D\left[m^2\right]$ (for $0 < \theta < \pi/2$)



Therefore(?): $T = -\partial w_{gap}/\partial\theta = -bRD\mu_o \left|H_{gap}\right|^2$

WRONG SIGN!

To find force we must always differentiate <u>total</u> energy, including the energy in the power supply!

To Find Torque Correctly:

Can include the changing source energy, but easier to reformulate the energy expression

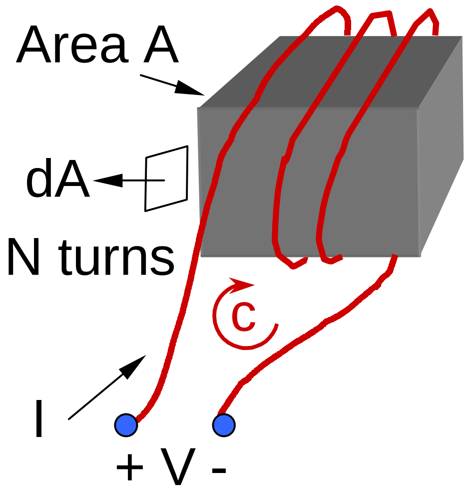
FLUX LINKAGE Λ AND INDUCTANCE L

Definition of Flux Linkage Λ:

$$\text{Flux Linkage } \Lambda = N\int_A \overline{B} \bullet d\overline{a}$$

$$-\nabla \times \overline{E} = \partial\overline{B}/\partial t = N\int_A \left(d\overline{B}/dt\right) \bullet d\overline{a} = d\Lambda/dt$$

$$-\int_{c\ \text{coil}} \overline{E} \bullet d\overline{s} = V = \frac{d\Lambda}{dt} = L\,di/dt$$



Therefore: $L = \Lambda/i$

Reluctance-Motor Flux Linkage Λ:

V = $d\Lambda/dt = 0$ if motor short-circuited; therefore

Λ = constant $\neq f(\theta)$ (key step)

Λ = $N\mu_o H_{gap} A_{gap} = N^2\mu_o I A_{gap}/2b$, where $H_{gap} = NI/2b$ [see L22-1]

$A_{gap} = RD\theta$ here [L22-2]

L = $\Lambda/I = N^2\mu_o A_{gap}/2b$

RELUCTANCE MOTOR TORQUE

Energy Storage and Torque with the Source Short-Circuited:

$w_m = LI^2/2 = \Lambda^2/2L \quad (I = \Lambda/L)$

$T = -\partial w_m/\partial\theta = -\Lambda^2\partial(1/2L)/\partial\theta = -\Lambda^2\partial(b/N^2\mu_o A_{gap})/\partial\theta \quad [A_{gap} = R\theta D]$

$= -(\Lambda^2 b/N^2\mu_o RD)\partial\theta^{-1}/\partial\theta = \Lambda^2 b/N^2\mu_o RD\theta^2$

Recall $\Lambda = N^2\mu_o I A_{gap}/2b$ [see L22-3]

$T = (N^2\mu_o IR\theta D/2b)^2 b/N^2\mu_o RD\theta^2$

Therefore:

$T = N^2\mu_o I^2 RD/4b$ [Nm] $= (\mu_o H_{gap}{}^2/2)(2bRD)$ $\neq f(\theta)$ (e.g. for $0 < \theta < \pi/2$)

$T = W_{mgap}(dV_{olume}/d\theta)$ [Nm]

~Same as for electric motors!
T is limited by maximum energy density

Maximize T: maximize $(NI)^2$ and RD (~weight), minimize b (tolerances)

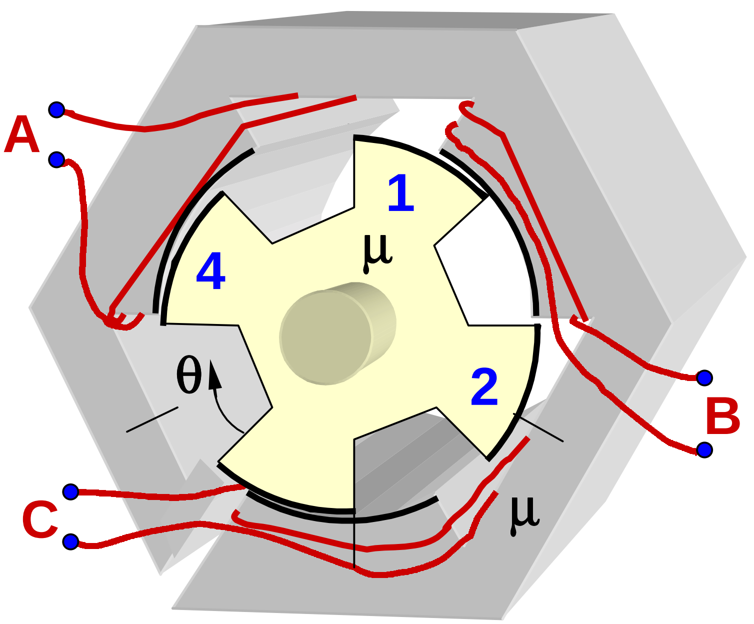
Motor Drive Circuit:

I is turned on when A_{gap} is minimum; torque then increases A_{gap} as the rotor is pulled by the stator. When A_{gap} = maximum, I is set to zero and the rotor coasts until A_{gap} = minimum, and the cycle repeats. (<50% duty cycle here)

¾-POLE VARIABLE RELUCTANCE MOTOR

Winding Excitation Plan:

Start by exciting windings A,B; this pulls pole 1 into pole B; for winding A, the pole area is temporarily constant. When $\Delta\theta = \pi/3$, the currents are switched to B,C; when $\Delta\theta = 2\pi/3$ we excite C,A. Repeating this cycle results in nearly constant clockwise torque.



To go counter-clockwise, excite BC, then AB, then CA.

Torque:

Only one pole is being pulled in; the other winding has either one rotor pole fully in, or one entering and one leaving that cancel. Many pole combinations are used (more poles, more torque).

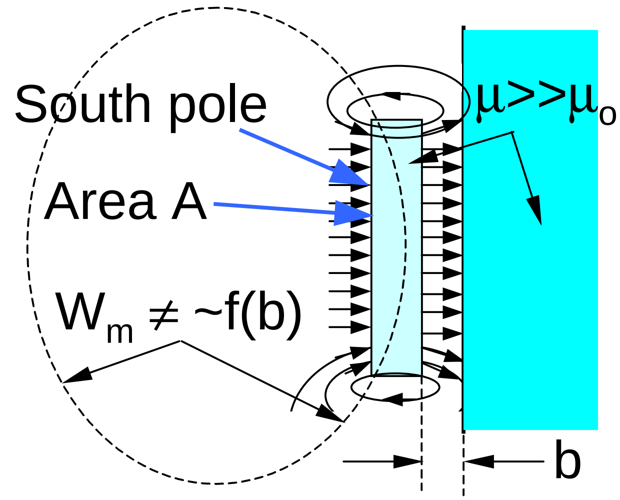
PERMANENT MAGNET SYSTEMS

Refrigerator Magnets:

Force: $f = -dw_m/db$

Variable energy: $w_m \cong bA\mu_o H_{gap}^2/2$ [J]

$= bAB_{gap}^2/2\mu_o$



Force density: $f/A = B_{gap}^2/2\mu_o$ [Nm^{-2}]

Example: Let B = 1 Tesla (10,000 gauss), A = 10 cm^2

Then $f = 0.001 \times 1^2/2\times 4\pi 10^{-7} \cong 400$ N $\cong$ 100-lb force

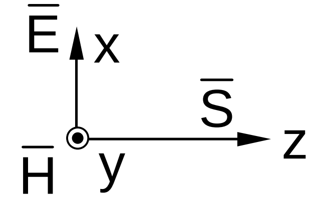
Permanent Magnet Properties:

Strength: Typical strong magnets ~0.2T; can $\rightarrow$ 1Tesla

Temperature : Above ~200C they fail; some fail at low temperatures

FORCES FROM ELECTROMAGNETIC WAVES

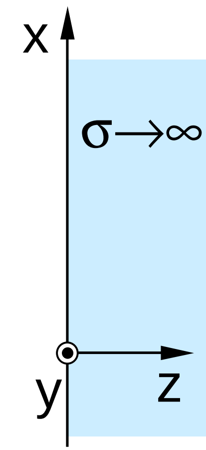
Waves Impacting Conductors:



$$\overline{E}_x(z=0) = 0 = \left[\overline{E}_+ \cos(\omega t - kz) + \overline{E}_- \cos(\omega t + kz)\right]$$

$$H(z=0) = \hat{y}\left(2E_+/\eta_o\right)\cos\omega t \Rightarrow \overline{J}_s = \hat{x}\left(2E_+/\eta_o\right)\cos\omega t$$

Forces on Conductor:



$$\overline{f} = q\left(\overline{E} + \overline{v} \times \mu_o \overline{H}\right) \Rightarrow \text{quasistatic pressure } P\left[Nm^{-2}\right];$$

$$F\left[Nm^{-3}\right] = nqv\mu_o H = J\mu_o H \text{ where } n = \#/m^3 \text{ charge carriers}$$

$$\overline{P} = \int_0^\infty \overline{J}(z) \times \mu_o \overline{H}(z)dz \text{ where } \overline{J}(z) = \nabla \times \overline{H}(z) \text{ for } \sigma \rightarrow \infty$$

$$\overline{J}(z) \cong \nabla \times \overline{H} = \det\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x \rightarrow 0 & \partial/\partial y \rightarrow 0 & \partial/\partial z \\ H_x \rightarrow 0 & H_y & H_z \rightarrow 0 \end{vmatrix} = -\hat{x}\partial H_y/\partial z$$

$$\overline{P}(t) = -\hat{z}\mu_o \int_0^\infty \left(\partial H_y/\partial z\right)H_y dz = -\hat{z}\mu_o \int_{H_y(z=0)=H_{y_o}}^{H_y=0} H_y dH_y = \hat{z}\mu_o H_{y_o}{}^2(t)/2$$

[same as L22-6]

$$\left\langle \overline{P}(t)\right\rangle = \hat{z}\mu_o \left\langle H_{y0}{}^2(t)\right\rangle/4 = 2\left\langle \overline{S}(t)\right\rangle/c$$

where $H_{y0} = 2H_+$ and

$$\left\langle S(t)\right\rangle = \eta_o H_+{}^2/2 = \mu_o c H_+{}^2/2$$

PHOTON PRESSURE

Photon Energy:

hf [J] = "m"c^2

h = Planck's constant = 6.625×10^{-34} [Js], f = frequency [Hz]

Photon Momentum:

"m"c = hf/c [Nms^{-1}]

Transferred to mirror upon reflection: Δ momentum = 2hf/c

Photon Pressure P [Nm^{-2}]:

Pressure P is change of momentum $s^{-1}m^{-2}$ [Mechanics: f = d(mv)/dt]

$$\langle P \rangle = n2hf/c = 2\langle S(t) \rangle / c \quad [Nm^{-2}]$$

where n = # photons reflected $s^{-1}m^{-2}$

If a photon is absorbed, then Δ momentum = hf/c, and $P = \langle S(t) \rangle / c$

In general, if the incident and reflected fluxes are S_1 and S_2 [Wm^{-2}],

$$\langle P \rangle = \langle S_1 - S_2 \rangle / c \quad [Nm^{-2}]$$

Solar Sailing Across Solar System:

Say 1 km^2 at 1.4 kW/m^2 $\Rightarrow A\langle P \rangle = A2\langle S \rangle / c = 10^6 \times 2800 \big/ 3 \times 10^8 = 9N$

(say 10^{-6} thick, $\rho = 1g/cm^3$)

v = at = (f/m)t $\cong (9/1000)3\times10^7 \cong 3\times10^5$ ms^{-1} in 1 year = 10^{-3}c