MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.013 Electromagnetics and Applications

Final Examination | Closed book, no calculators | December 19, 2002

Please note the two pages of formulas provided on a separate sheet. There are 8 problems on three pages. Please simplify all expressions, circle and dimension your answers, and present numerical answers to the extent practical without a calculator or tedious computation. You may leave natural constants in symbolic form (π, ε_o, μ_o, η_o, h, e, etc.).

Problem 1. (30/200 points)

The complex representation of the electric field for a certain electromagnetic wave in vacuum is: $\underline{E} = (\hat{y} - \hat{z})e^{+jx\,-2jy\,-2jz}$.

a) What is the polarization for this wave (linear, circular, elliptical)?

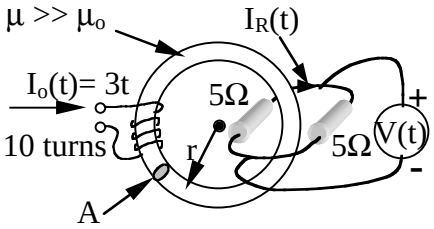
b) What is an equivalent time-domain expression $\overline{E}(t, x, y, z)$?

c) What is the time-average wave intensity I [Wm^{-2}]?

d) What is the frequency f [Hz] for this wave?

Problem 2. (30/200 points)

A high-permeability toroid ($\mu >> \mu_o$) has a major radius r and cross-sectional area A, as illustrated. It is excited by a ten-turn coil carrying a time-varying current $I_o(t) = 3t$ amperes, where t is time in seconds. Encircling the toroid are two five-ohm resistors R in series, as shown; the resistor voltage is being measured by an ideal voltmeter V.



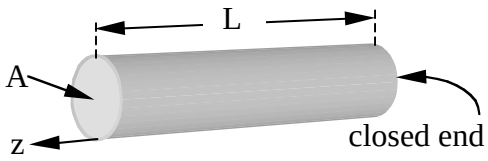
a) In terms of $I_o(t)$, what is the approximate magnitude of the magnetic field H(t) inside the toroid?

b) In terms of H(t), what is the magnitude of the current $|I_R(t)|$ flowing through the resistors?

c) In terms of $|I_R(t)|$, what are the magnitude and sign of the voltage V(t) measured by the voltmeter?

Problem 3. (20/200 points)

An acoustic musical instrument consists of a narrow pipe of length L closed at one end and open at the other; its internal area is A. The speed of sound is c_s and the acoustic wave impedance of air is η_s.

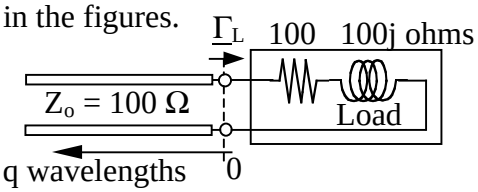


a) What are the two lowest frequency resonances f_n of this pipe [Hz]?

b) What is the time-average total stored acoustic energy w_T [J] in terms of the maximum instantaneous acoustic pressure p_o? In answering this part, assume both ends of the pipe are perfectly reflecting (that is, $\underline{\Gamma} = \pm 1$).

Problem 4. (35/200 points)

A 100-ohm TEM transmission line operating at frequency f is terminated with a load consisting of a 100-ohm resistor in series with an inductor having a reactance of 100j, as illustrated. Additional details and values are shown in the figures.

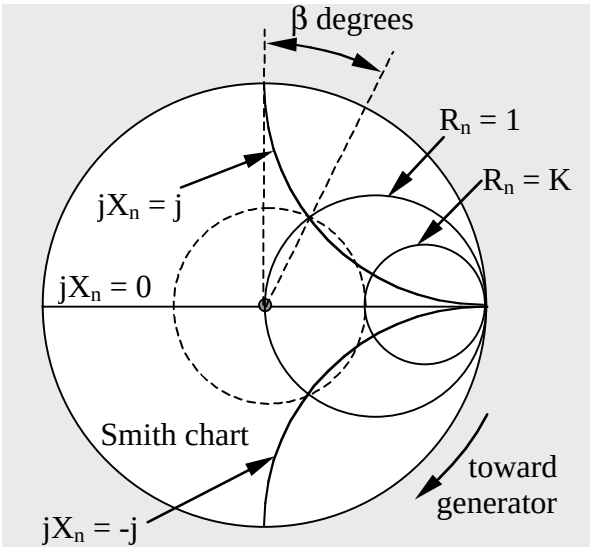


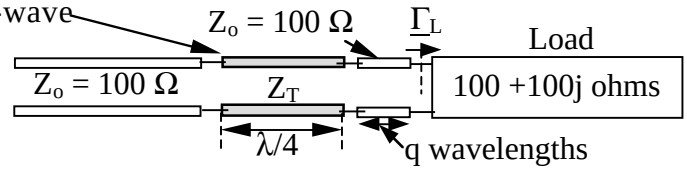
a) In terms of the complex reflection coefficient $\underline{\Gamma}_L$ of the load, what fraction A of the power incident upon the load is reflected?

b) For this load what is the numerical value of the complex reflection coefficient $\underline{\Gamma}_L = a + jb$?

c) At what fraction of a wavelength $q = D/\lambda$ (and in terms of β, see figure), is the distance D from the load of the first point where $\underline{Z}(z)$ is purely real?

d) To match this load a quarter-wave transformer is inserted at the first point where $\underline{Z}(z)$ is purely real. In terms of K, what should be the characteristic impedance Z_T of the quarter-wave transformer (see figure)?





Problem 5. (15/200 points)

What is the maximum fraction of the active atoms in an ideal laser-pumped three-level laser that can be in the proper state for stimulated emission at the laser frequency if that proper state is (a) the topmost state, and (b) the middle state? Please explain briefly.

Problem 6. (20/200 points)

The waves propagating in a particular air-filled parallel-plate waveguide are characterized by the electric field:

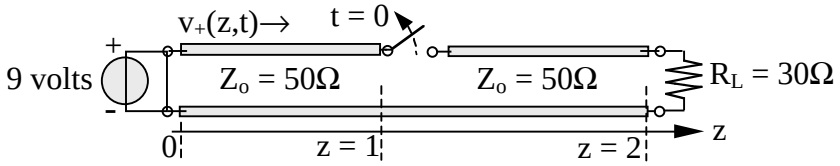
$$\bar{E} = \hat{x}\, E_o \cos(\omega t - z)\sin y$$

a) What is the separation d [meters] between the two conducting plates if this exact field corresponds to the TE_1 mode? Please give a numerical answer.

b) What is the cutoff frequency ω_{co} (radians s^{-1}) for this mode in this waveguide?

Problem 7. (25/200 points)

A certain 9-volt DC power source is connected to a 30-ohm load via a lossless 50-ohm air-filled TEM transmission line 2-meters long, as illustrated, with the switch closed.

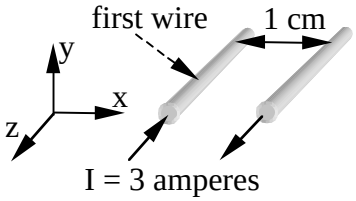


a) In steady state for t < 0, what are the amplitudes [volts] of the forward- and backward-propagating waves $v_+(z,t)$ and $v_-(z,t)$? Briefly explain your method.

b) At time t = 0 the switch in the middle of the line (at z = 1 meter) is opened. Sketch and dimension the amplitude of the voltage v(z,t) on the line to the left of the switch ($0 < z < 1$) at $t = 10^{-9}$ seconds.

Problem 8. (25/200 points)

Two parallel wires are spaced 1 cm apart in air and carry I = 3 amperes in opposite directions, as illustrated.



a) What is the vector magnetic field $\bar{H}$ at the first wire produced by the second wire? Note the given xyz coordinate system.

b) What is the magnetic force density vector $\bar{F}$ [Nm^{-1}] acting on the first wire?

c) The 3-amp current drives an inductance $L = Ap^2$ Henries, where p is the position along the x axis of a slideable slug, as illustrated. What is the force vector $\bar{f}$ [N] acting on the slug as a function of p?