Superposition of Waves, Applications:

Multiple waves launched by antenna
   (Antennas designed to yield desired pattern)

Multiple reflections intercepted by receiver
   (Multipath)

Multiple points of interception in receiver antenna
   (Same as transmitting antenna, time reversed)

\[ \cos \omega t + \cos (\omega t + \phi) = A \cos(\omega t + \theta) \]
\[ = 2 \cos (\omega t + \phi/2) \cos (\phi/2) \]

Note: fields add, powers do not
SUPERPOSITION OF PHASORS

Graphical Method:

\[ A(t) = R_e \{ A e^{j\omega t} \} = R_e \{ [R_e \{ A \} + j I_m \{ A \}] [\cos \omega t + j \sin \omega t] \} \]

\[ = R_e \{ A \} \cos \omega t - I_m \{ A \} \sin \omega t \]

Superposition of Phasors:

\[ 2 + j = (1^2 + 2^2)^{0.5} e^{j\beta} \Rightarrow \]

\[ 2 \cos \omega t - \sin \omega t = (1^2 + 2^2)^{0.5} \cos(\omega t + \beta) \]

(“phase lead” of \( \beta \))

\[ B = \sum_i |A_i| e^{j\phi_i} \]

\[ 1 \cdot e^{j\pi/2} \rightarrow |A_i| = 1, \]

\[ \phi_i = \pi/2 \]
Superposition of Waves:

\[ \vec{E}(r, \theta, \phi) = \sum_i a_i \vec{E}_i e^{-jk\vec{r}_i} \]

\[ = \vec{E} \left( \sum_i a_i e^{-jk\vec{r}_i} \right) = (\text{element factor } \vec{E}) (\text{array factor}) \]

\( \vec{E}(r, \theta, \phi) \) can be factored if elements have the same orientation and pattern (so \( \vec{E}_i = \vec{E} \)), but different locations \( \vec{r}_i \), and amplitudes and phases \( a_i \), where \( a_i \) characterizes the currents driving each radiating element.

**Example, horizontal arrays of vertical dipoles:**

In phase:

At 30° relative power = 2
(90° out of phase)

Relative power in x direction = 4

\( \frac{\lambda}{2} \)

Array factor

\( \frac{\lambda}{4} \)

Relative power in y direction = 4

180° out of phase:

90° out of phase, relative power = 2

\( \frac{\lambda}{2} \)

Element factor

\( \frac{\lambda}{4} \)
**TWO-ELEMENT ARRAYS**

**In-Phase, 2\(\lambda\) Separation:**

Nulls at:
\[
\theta = \cos^{-1}\left(\frac{\lambda}{2}\right) \quad \text{null}
\]
\[
\theta = \cos^{-1}\left(\frac{3\lambda}{2}\right) \quad \text{null}
\]

Peaks at:
\[
\theta = 0
\]
\[
\theta = \cos^{-1}\left(\frac{\lambda}{2}\right) = \frac{\pi}{3}
\]

**90° phase, \(\lambda/4\) separation**

180° phase, \(\lambda/2\) separation
unequal sources

2 unequal sources cannot produce a null
LINEAR ANTENNA ARRAYS

Linear uniformly excited array:

Phasor for Uniform 8-Element Array:

Peak power $\propto 8^2$

First null

Second null

$\theta_{\text{first null}} = \sin^{-1}\left[\frac{\lambda/2}{(4D/7)}\right]

\theta_{\text{null}} = \sin \frac{\lambda}{2L} = \sim \frac{\lambda}{D}$
Cell phones:

Frequency allocations are limited. Therefore multiple narrowband signals or other orthogonal signals are used (e.g., TDM). Base station antennas enable frequency reuse.

- All frequencies, $\alpha = 0$, $V_i = V_0 e^{jm\alpha}$
- All frequencies, $\alpha = \pi$

Element factor $\bar{E}(\theta)$

Typical 4-antenna face

Most clients can access non-faded frequencies

Assume $3\lambda$
MULTIPATH EFFECTS

Fading, scintillation: 
\[ aE e^{-j\phi} \]

Causes of fading:
- Moving reflectors (trucks, trees) vary \( \phi \)
- Moving sources and receivers
- Variations in \( c \) due to temperature, humidity
- Polarization rotation or variation

Examples:
- FM radios in moving cars click during nulls below FM threshold
- Snowy TV broadcast stations waiver as planes fly overhead or trucks pass
- Strength of radio stations varies with the weather (also due to refraction)
- Ionosphere (faraday) rotates linear polarization \( \sim 3 \text{ GHz} \), causing fades

Doppler shift \( \Delta f \):
If the path \( L \) to the source increases at \( v = \partial L/\partial t \), we lose
\[ f_D = \frac{\partial L/\partial t}{\lambda} \text{ cycles per second} = \frac{v}{\lambda} = f_o \frac{v}{c} \text{ Hz}, \text{ so } f_D = f_o (1 - \frac{v}{c}) \text{ Hz} \]

Remedies:
- Doppler shift – retune receiver
- Fading – high sensitivity and dynamic range; error-correction codes;
  space, frequency, polarization diversity
**DIFFRACTION**

**Uniformly Illuminated Aperture:**

\[ B = \sum_i |A_i| e^{j\phi_i} \rightarrow \int A(x) e^{j\mathbf{r} \cdot \hat{x}} dx \]

\[ \theta_{\text{first null}} = \sin^{-1}\left[\frac{\lambda/2}{(4D/7)}\right] \]

\[ \theta_B(\text{eye}) \equiv \sin^{-1}(\lambda/D) \approx 0.5 \times 10^{-6}/10^{-3} \approx 1 \text{ arc min} \]

**Planar Aperture:**

\[ \theta_{\text{first null}} = \sin^{-1}(\lambda/D) \]

\[ \theta_B(\text{eye}) \equiv \sin^{-1}(\lambda/D) \approx 0.5 \times 10^{-6}/10^{-3} \approx 1 \text{ arc min} \]

**Uniformly Illuminated Aperture A:**

\[ \mathcal{E}_{ff} \equiv \hat{\theta} \left(jke^{-jkr}/2\pi r\right) \cos \theta \int_{A} \mathcal{E}_x(x',y') e^{jkr \cdot \hat{r}'} \, da' \]

\[ (\sim \text{Fourier transform})(r \text{ is distance from origin to } ff) \]