6.013-ELECTROMAGNETICS AND APPLICATIONS

General Content:
Electromagnetics and Applications
• Maxwell’s equations and solutions; statics and dynamics, wave phenomena
• Applications: wireless, circuits, media, forces and generators, computer speed, microwaves, optical communications, acoustics, etc.

Mathematical Methods
• Partial differential equations, difference equations, phasors, vector calculus

Problem Solving Techniques
• Perturbation methods, boundary value problems, energy methods, duality

Academic Review
• Mechanics, quantum phenomena, circuits, devices signals, linear systems

Capstone Subject—Professional Preparation

Follow-on Subjects:
Electromagnetic waves: 6.632
Quasistatics: 6.641
PERTURBATIONS OF RESONATORS

Perturbation of LC Resonator Frequency:

\[ \omega_o = (LC)^{-0.5} \]

If C increases, \( \omega_o \) decreases \( \Rightarrow \) indenting top center of cavity is equivalent

Calculation of \( \Delta f \) [Hz] due to Shape Perturbations:

Energy method: Assume closed resonator, no energy escape
Therefore total energy \( w_T = nhf, \ n = \# \) photons at \( f[\text{Hz}] \)
\( n = \) constant, for slow changes in resonator shape
\[ \Delta f = \Delta w_T/nh \ [\text{Hz}] \]

Assume: EM energy increases \( \Delta w_T \) (±) as work (mechanical) is done on the fields when the walls are forced into their new shape
\[ \Delta w_T = -f_{\text{force}} \Delta z \ [\text{J}] \] (mechanical work done on EM \( f_{\text{force}} \))

Approach: Find force density \( F[\text{Nm}^{-2}] \), then \( \Delta w_T \), then \( \Delta f \)
**PERTURBATIONS OF RESONATORS (2)**

**Electromagnetic Forces on Conducting Walls:**

Recall:  \[ F_{\text{magnetic}} = \mu_0 \frac{|\overline{H}|^2}{4}[\text{Nm}^{-2}][\text{Jm}^{-3}] = W_m \]  
Repulsive force density

\[ F_{\text{electric}} = \varepsilon_0 \frac{|\overline{E}|^2}{4} = W_e \]  
Attractive force density

**Electromagnetic Energy Increase**  
is \( \Delta w_T \) due to Wall Movement:

\[-\Delta w_T = \int dz \int F_{EM}(x,y)dx\,dy \text{ (work done on moving walls)}\]
\[ = \int_V F(x,y)dv = \int_V (W_m - W_e)dv \text{ [J]}\]
\[-\Delta w_T = \Delta (w_m - w_e) \text{ [Joules in } V \text{ added by change]}\]

\( V \) is the volume enclosed by the original and deformed surface contours

\( V \) is sufficiently small that \( \overline{E} \) and \( \overline{H} \) are constant within it

**Calculation of \( \Delta f \) [Hz]:**

Recall:  \[ \Delta f = \frac{\Delta w_T}{nh} = \frac{f(\Delta w_T)}{w_T} \text{ [Hz]} \]  
(where \( w_T = nhf \))

Therefore:  \[ \frac{\Delta f}{f} = \frac{\Delta (w_e - w_m)}{w_T} \text{ where } w_m \text{ and } w_e \text{ are values within } V\]

Note:  \( \Delta w \) is the increase in \( w \) in protrusion; \( w \leq 0 \) in an indentation
Example—Cavity Resonator (Recall $\Delta f/f = \Delta (w_e - w_m)/w_T$):

Question: Where does indentation lower frequency $f_0$?

Answer: Within contours where $\Delta w_e$ is negative (removed) and $\Delta (w_e - w_m)$ is also negative.

Note: $w_e \propto \sin^2(\pi x/a) \sin^2(\pi y/a)$

$w_m \propto \cos^2(\pi x/a) \cos^2(\pi y/a)$

Question: Where does indentation raise frequency $f_0$?

Answer: Everywhere else (for this mode, TM$_{101}$).

Question: Same as above, for TEM$_{210}$ mode?
Example—RLC Resonator:

Problem: If \( d \to d + \Delta d \), then \( f \to \Delta f \); what is \( \Delta f \) [Hz]?

Recall: \( \Delta w_e = \left( \frac{\varepsilon |\vec{E}|^2}{4} \right) (A \Delta d) \) if \( d \) increased by \( \Delta d \) (all else remaining constant)

Therefore: \( \Delta f/f_o = \Delta (w_e - w_m)/w_T = \left( \frac{\varepsilon |\vec{E}|^2}{4} \right) (A \Delta d)/\left( \frac{2\varepsilon |\vec{E}|^2}{4} \right) (A d) \)

\( = \Delta d/2d \)

Check Result:

Recall: \( \omega_o = (LC)^{-0.5} = [L\varepsilon A/d]^{-0.5} \to \omega_o' = (LC')^{-0.5} = [L\varepsilon A/(d + \Delta d)]^{-0.5} \)

\( C = \varepsilon A/d \to C' = \varepsilon A/d' \)

Therefore: \( \omega_o' = [L\varepsilon A/d(1 + \Delta d/d)]^{-0.5} \equiv [L\varepsilon A/d]^{-0.5}(1 - \Delta d/2d) \)

\( \Delta \omega = \omega_o - \omega_o' \equiv \omega_o \Delta d/2d \)

\( \Delta f/f_o = \Delta \omega/\omega_o = \Delta d/2d \), agrees with the above.

Note: The energy perturbation method is approximate; here \( \Delta d/d << 1 \)
HUMAN ACOUSTIC RESONATORS

Human Vocal Tract:
\[ f_1 = \frac{\omega_1}{2\pi} = \frac{c_s}{4d} \]
\[ = \frac{340}{(4 \times 0.16)} \]
\[ = 531 \text{ Hz} \]

Higher Resonances:
\[ f_2 = 3f_1 = 1594 \text{ Hz} \]
\[ f_3 = 5f_1 = 2655 \text{ Hz} \]

Energy Densities at Location “ ”
At \( f_1 \):
\[ w_p \approx w_u \]
At \( f_2 \):
\[ w_u \gg w_p \]
At \( f_3 \):
\[ w_p \gg w_u \]
RESONANCE PERTURBATIONS IN HUMAN VOICES

Human Vocal Tract:

Average forces:
- Outward at maximum $|p|$
- Inward at maximum $|u|$
(Bernoulli force)

Resonator Total Energy $w_T = \sum^{n}_{\text{phonons}}$

\[ w_T = nhf_o = w_p + w_u \] (potential + kinetic energy)

If resonator shape presses inward at $p$ maximum then both $w_T$ and $f_o$ increase

Resonance Perturbations:

\[ \Delta f/f = \Delta (w_p - w_u)/w_T \]

$w_p \approx w_u$ at $f_1$, $w_u >> w_p$ at $f_2$, $w_p >> w_u$ at $f_3$

$f_o \propto c_s \propto (\gamma P_o/\rho_o)^{0.5}$