MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.013 (New) Electromagnetics and Applications

Issued: September 10, 2002
Problem Set 2 Due in Recitation: September 18, 2002

Suggested Reading: Text (Staelin, Morgenthaler, and Kong), Sections 1.1-2, 1.6, 2.1-2, and Appendix A. View UPW and dipole radiation movies.

Problem 2.1

Consider a uniform plane wave having $\overline{H}(\overline{r},t) = \hat{y}\sin(10^9t + 10x)$ [Am^{-1}]. For this wave, what is its:

(a) Frequency (Hz)?
(b) Direction of propagation?
(c) Velocity?
(d) Wavelength?
(e) Value of μ? ($\varepsilon = \varepsilon_o$ here)
(f) $\overline{E}(\overline{r},t)$?

Please always indicate the dimensional units of your answers (e.g. ms^{-1}).

Problem 2.2

For a wave in free space having $\overline{E} = \hat{x}\cos(\omega t - ky)$, determine the values for:

(a) Electric energy density $W_e(\overline{r},t)[Jm^{-3}]$
(b) Magnetic energy density $W_m(\overline{r},t)[Jm^{-3}]$
(c) Poynting vector $\overline{S}(\overline{r},t)$ [Wm^{-2}]
(d) Average intensity I [Wm^{-2}]

(e) Find a simple relationship between your answers for (a) and (b) above.
(f) Find a simple relationship between c[ms^{-1}] and your answers for (a) and (d) above.
(g) Repeat (a) - (c) for the wave $\overline{E} = \hat{x}[2\cos(\omega t - ky) + \cos(\omega t + ky)]$
(h) For the wave of (g), evaluate the complex Poynting vector $\underline{\overline{S}}(\overline{r})$
(i) For the wave $\overline{E} = \hat{x}\cos(\omega t - ky) + \hat{z}\sin(\omega t - ky)$ evaluate $\overline{S}(\overline{r},t)$
(j) Is the physical significance of your answers to the above questions clear?

Problem 2.3

A conducting sphere 1 cm in diameter is isolated in space and has a charge Q.

(a) What is the electric field strength E_o [Vm^{-1}] at the surface of the sphere?
(b) What is its electrostatic potential $V_o = \Phi_o$ relative to infinity (where $\Phi = 0$)?
(c) We define capacitance C = Q/V [Farads]; what is the capacitance of this sphere?

Problem 2.4

Using the complex form of Maxwell's equations,

$$\nabla \times \underline{\overline{E}} = -j\omega\ \underline{\overline{B}} \qquad \nabla \times \underline{\overline{H}} = \underline{\overline{J}} + j\omega\ \underline{\overline{D}} \qquad \nabla \bullet \underline{\overline{D}} = \underline{\rho} = 0 \qquad \nabla \bullet \underline{\overline{B}} = 0,$$

(a) Derive for free space ($\rho = \underline{\overline{J}} = 0$) the complex form of the wave equation for $\underline{\overline{H}}$:

$[\nabla^2 + k^2]\ \underline{\overline{H}} = 0$. Recall the identity $\nabla \times (\nabla \times \overline{A}) = \nabla(\nabla \bullet \overline{A}) - \nabla^2 \overline{A}$.

(b) Derive the conservation of charge equation $\nabla \bullet \underline{\overline{J}} = -j\omega\underline{\rho}$ for the case $\underline{\overline{J}} = \sigma\ \underline{\overline{E}}$. Recall the identity $\nabla \bullet (\nabla \times \overline{A}) = 0$.

Problem 2.5

Convert each of the following expressions to its alternate form, i.e., convert time-domain expressions to complex notation and vice versa. For example:

$$A \sin(\omega t - kz) \leftrightarrow jA\ e^{-jkz}$$

(a) $A \cos(\omega t - \pi/2)$ (b) $(1+j)e^{+jkz}$ (c) $A \cos(\omega t - kz + \pi/2)$