Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.013 -- Electromagnetics and Applications

	Issued:	November 26, 2002
Problem Set 13	Due in Recitation:	December 6, 2002

Note: *Final examination is Thursday, December 19, at 1:30 pm in Walker Memorial, top floor. Closed book, no calculators; an equation sheet will be provided.*

Suggested Reading: *Textbook Sections 10.7-9*
Lectures 22 and 23 Supplementary notes

Problem 13.1

A rectangular acoustic waveguide measures 5×5 millimeters, similar to the ear canal that runs between the outer ear and the eardrum.

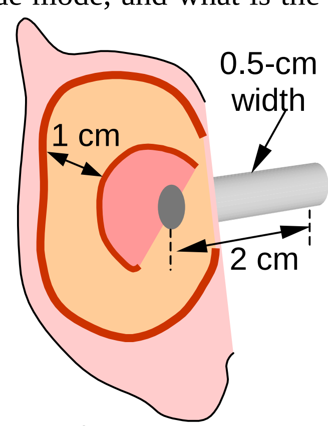
a) What are the cutoff frequencies f_{0-3} for the lowest four acoustic modes for this waveguide? Assume $c_s = 340\ ms^{-1}$.

b) What are the two lowest resonant frequencies f_{01} and f_{01} for the resonator bounded by the eardrum and outer ear for an ear canal that is 2-cm long.

c) Discuss briefly (guess if necessary) how and why the frequencies found in parts (a) and (b) may relate to the nominal cutoff for human hearing, which is typically 12-20 kHz.

d) There is a substantial acoustic impedance discontinuity at the junction of the ear canal and the outer ear. If we guess and model the behavior of the outer ear as similar to that of an acoustic waveguide 5×5 cm, what are the two relative impedances Z_o and Z_{air} for the dominant waveguide mode, and what is the resulting power reflection coefficient $|\underline{\Gamma}|^2$ at the junction?

e) The outer human ear has two strong ridges that tend to focus on the ear canal those sounds arriving from the direction in which that person is facing. If these two ridges are approximately one centimeter apart, as illustrated, what frequencies are favored for this forward focusing effect? What audible frequencies are partially nulled?



f) Note that at some frequencies the acoustic reflections from the two ridges cancel, and at others they add. With respect to your own ears, are the (partially) nulled frequencies the same for all directions of arrival? Explain briefly how this could help our cavemen ancestors determine the direction of an unseen predator.

g) Referring to Part (d), what is the approximate external Q_E of the lowest frequency mode for this ear-canal acoustic resonator in terms of $|\underline{\Gamma}|^2$, which we associate with the power P_{dissE} lost externally that came from the energy w_T stored in the resonator? One approach to this problem is to find the acoustic R_L that has the same $|\underline{\Gamma}|^2$, and then to treat R_L as a lossy perturbation.

Problem 13.2

We can model a certain optical fiber in one dimension as a dielectric slab waveguide 6 microns thick with a core dielectric constant $\varepsilon/\varepsilon_o$ of 1.5, and having a thick cladding with dielectric constant 1.502. The fiber operates near 1.2-microns wavelength (wavelength in air).

a) What is the critical angle for waves inside the core?

b) If the field decay length α^{-1} [m] inside the cladding (where $|\ \overline{E}\ | \propto e^{-\alpha z}$ and z is distance away from the core/cladding boundary) is 1 micron, what is the waveguide wavelength λ_g in the core?

Problem 13.3 (Optional; grades won't count)

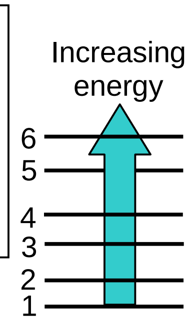
A certain solid-state laser system is characterized by 6 differently spaced energy levels, as illustrated. The relative values of A_{ij} for each pair of levels are listed in the table below. Assume that the frequencies of the laser and the pump are not critical parameters for this set of levels; that is, any pair of frequencies could be accommodated equally well.

a) What is the most stable state i (least likely to decay)?

b) Into what state i is it best to pump? Why this state?

c) What is the best transition i,j to use for laser amplification? Why? If more than one, please explain.

$A_{i,j}$	i = 6	5	4	3	2
j = 5	1				
j = 4	100	100			
j = 3	1	0	0		
j = 2	100	0	1	1	
j = 1	100	0	0	10	1000



Problem 13.4 (Optional; grades won't count)

a) A TEM resonator is indented quite near one of its short-circuited ends. Is the resonant frequency perturbed upwards or downwards? Explain briefly.

b) A TEM parallel-plate resonator open-circuited at both ends is indented in its middle without changing the resonant frequency. Briefly explain for which resonances f_x (if any) this is possible.

c) The same TEM parallel-plate resonator excited at a particular resonance f_y is indented in its middle and the resonant frequency drops a certain maximum amount. The same indentation produces the same maximum frequency drop at two other locations. If we designate the lowest non-zero resonance as f_1, what is this resonance f_y? Explain briefly.