MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.013 (New) Electromagnetics and Applications

Issued: October 8, 2002

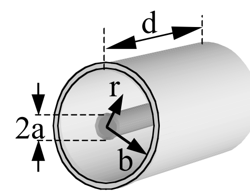
Problem Set 6 Due in Recitation: October 18[1], 2002

Suggested Reading: *Text: Sections 3.4<p88; and pp95-6.*
Notes for Lectures 10 and 11, and Recitation 11.

Quiz 1: *Thursday, October 17 in lecture. Closed book quiz without calculators (equation sheet provided). Since there is another class the following hour, we shall start and end very promptly. Arriving several minutes early would be an excellent idea. The quiz covers the first five weeks and homework sets through October 4.*

Problem 6.1

The cylindrical device illustrated below consists of two perfectly conducting cylinders, one centered inside the other, with a medium between them that is characterized by permittivity $\varepsilon = 9\varepsilon_o$ and conductivity σ [Sm^{-1}]. The radii of the two cylinders are a and b (b>a); they have length d.



a) A DC voltage V is placed across the two cylinders, with the inner cylinder being positive with respect to the outer cylinder. What is $\overline{E}(\overline{r})$ between the two cylinders? Ignore fringing effects near the ends of the device.

b) In terms of $\overline{E}(r = a)$, what is the surface charge density σ_s on the inner cylinder?

c) In terms of σ_s, what is the total charge Q on this capacitor?

d) What is the capacitance C of this capacitor? Please express your answer in terms of a, b, d, and ε.

e) Using the results in (a), alternatively calculate the capacitance C by integrating fields to find the total stored electric energy, and then using $w_e = CV^2/2$ [J]; does your answer agree with (d)?

f) What is the resistance R of this capacitor due to leakage arising from σ?

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[1] Note: Because of the quiz, the homework can be submitted in recitation either Wednesday (10/6) or Friday (Wednesday preferred). Note that some of these problems can be done quite quickly.

Problem 6.2

The same dual-cylinder geometry of Problem 6.1 is used to make an inductor by shorting one end and filling the interior with a medium having $\mu = 10^4\mu_o$. The DC current I flows into the inner conductor and out of the outer conductor.

a) What is $\overline{H}(r)$ between the cylinders?

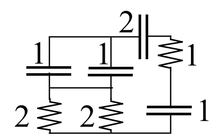
b) What is the flux linkage Λ?

c) What is the inductance L of this structure?

d) Use (a) find the total magnetic energy storage w_m [J] in this device, and then find L using $w_m = LI^2/2$. Do your answers to (c) and (d) agree?

Problem 6.3.

Consider the illustrated circuit in which all elements are either one or two ohms or farads:

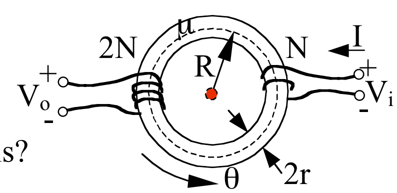


a) Determine an equivalent simplified RC circuit (one R, one C) that has the same time constant τ [s].

b) What is this τ?

Problem 6.4.

Consider the illustrated toroidal inductor of major radius R and minor radius r; the toroid is characterized by ε_o, $\mu = 10^4\mu^o$. An N-turn coil is wound around one side of the toroid and fed with I amperes, DC.



a) What is $\overline{H}$ inside the toroid?

b) What is the flux linkage Λ for the N-turn coil terminals?

c) What is the inductance L of this toroid as seen at the N-turn terminals?

d) A second coil of 2N turns is wound elsewhere on the toroid, as illustrated. Note the direction of the winding and polarity of the output voltage V_o. What is the open-circuit voltage V_o across this winding if the input current is I(t)?

e) Let the second winding be short-circuited so that $d\Lambda/dt = 0$, regardless of the input voltage. If the input current $I(t) = I_o\cos\omega t$, what is the resulting input voltage $V_i(t)$ of this toroid?