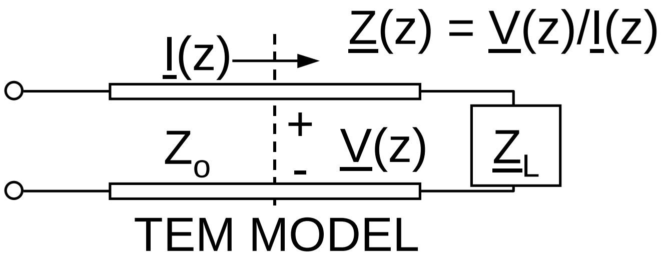
CIRCUIT TRANSFORMATION MAGIC

Printed Microwave Circuits (TEM lines):

Can contain R's (resistive rectangles)
Can yield L's and C's (near ω_o)
Can be resonators for tuning or transformer use
Can match impedances to maximize power transfer
Can perform nearly all circuit functions with transistors only
Has size scale ~ $\lambda/4 \Rightarrow f \gtrsim 1$ GHz



Waveguide Circuits and Systems:

Can be made lossy
Can have inductive or capacitive obstacles (L's and C's)
Can resonate for tuning or transforming
Can match impedances to maximize power transfer
Can make circulators (non-reciprocal devices)

Optical Circuits:

Similar to waveguide circuits, but in fibers or free space

GAMMA PLANE ⇔ SMITH CHART

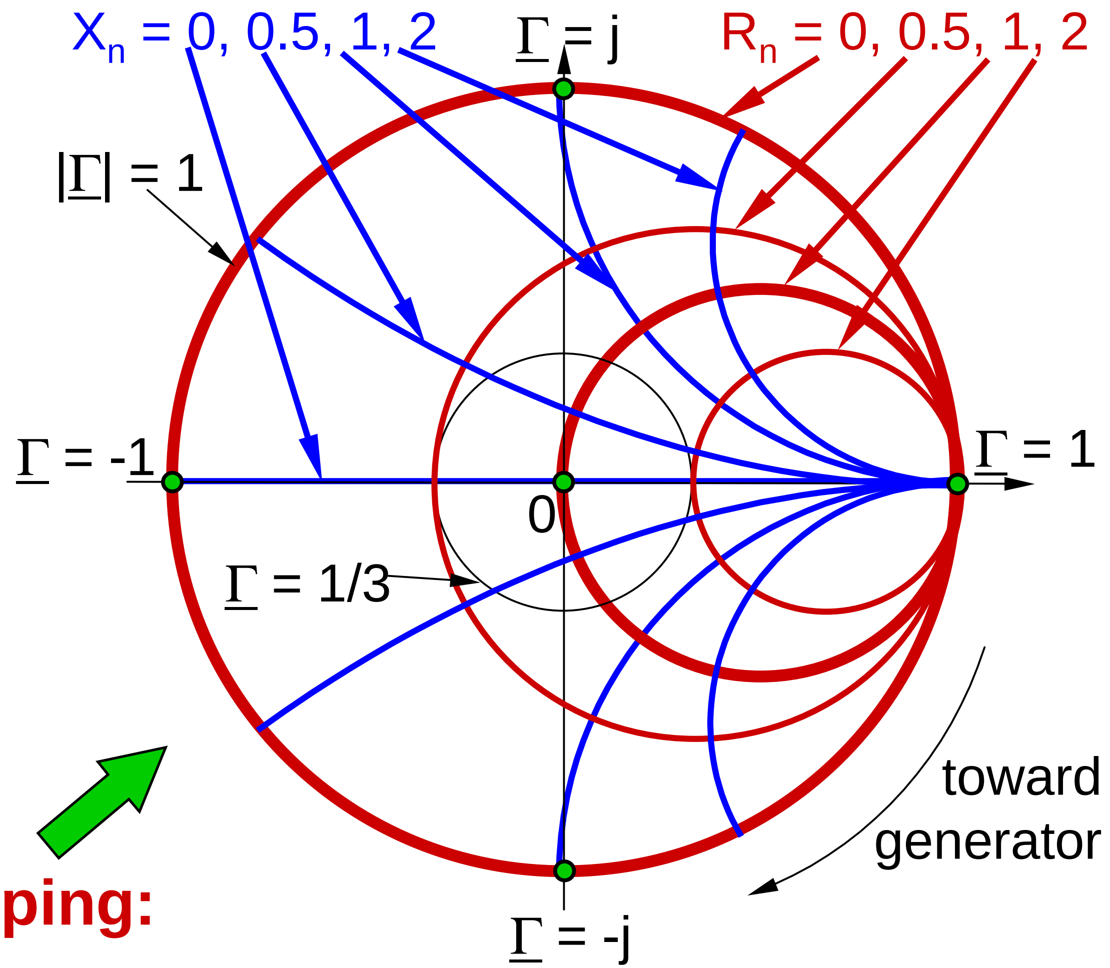
TEM Lines:

$$\underline{V}(z) = \underline{V}_+ e^{-jkz} + \underline{V}_- e^{+jkz}$$

$$\underline{I}(z) = Y_o \left[\underline{V}_+ e^{-jkz} - \underline{V}_- e^{+jkz} \right]$$

$$\underline{Z}(z) = Z_o \left(1 + \underline{\Gamma}(z)\right) / \left(1 - \underline{\Gamma}(z)\right)$$

$$\underline{\Gamma}(z) \triangleq \left(\underline{V}_- / \underline{V}_+\right) e^{2jkz} = \underline{\Gamma}_L e^{2jkz}$$



$Z_n = \underline{Z}(z)/Z_o = R_n + jX_n$:

Gamma plane is useful because $\underline{\Gamma}(z) = \underline{\Gamma}_L e^{2jkz}$, which is simply rotation on the plane.

One-to-One Mapping:

$$\underline{Z}_n = \left(1 + \underline{\Gamma}\right) / \left(1 - \underline{\Gamma}\right)$$

$$\underline{\Gamma} = \left(\underline{Z}_n - 1\right) / \left(\underline{Z}_n + 1\right)$$

SPECIAL PROPERTIES OF THE SMITH CHART

Admittance ⇔ Impedance:

If $\underline{Z}_n \rightarrow \underline{Z}_n^{-1} = \underline{Y}_n$, then $\underline{\Gamma} \rightarrow \underline{\Gamma}^*$

Proof: $\underline{\Gamma} = (\underline{Z}_n - 1)/(\underline{Z}_n + 1)$, so if $\underline{Z}_n \rightarrow \underline{Z}_n^{-1}$, then

$$\left(\underline{Z}_n^{-1} - 1\right)/\left(\underline{Z}_n^{-1} + 1\right) = (1 - \underline{Z}_n)/(\underline{Z}_n + 1) = -\underline{\Gamma} \quad \text{Q.E.D.}$$

Voltage Standing Wave Ratio (VSWR):

$$\text{VSWR} = |\underline{V}_{max}|/|\underline{V}_{min}| = \left(\left|\underline{V}_+ e^{-jkz}\right| + \left|\underline{V}_- e^{+jkz}\right|\right)/\left(\left|\underline{V}_+ e^{-jkz}\right| - \left|\underline{V}_- e^{+jkz}\right|\right)$$

$$= (1 + |\underline{\Gamma}|)/(1 - |\underline{\Gamma}|) = R_{n\ max}$$ (evident on Chart, on x axis)

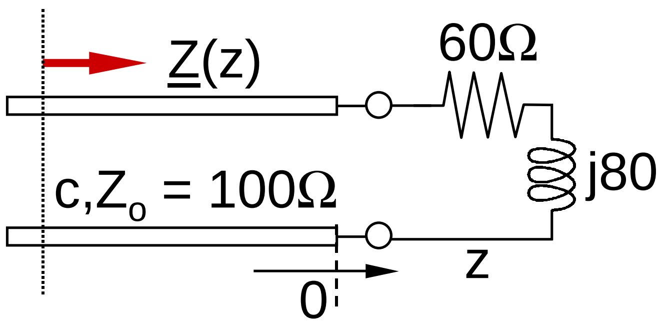
Rotation Around Chart:

$\lambda/2$ corresponds to one full rotation around chart

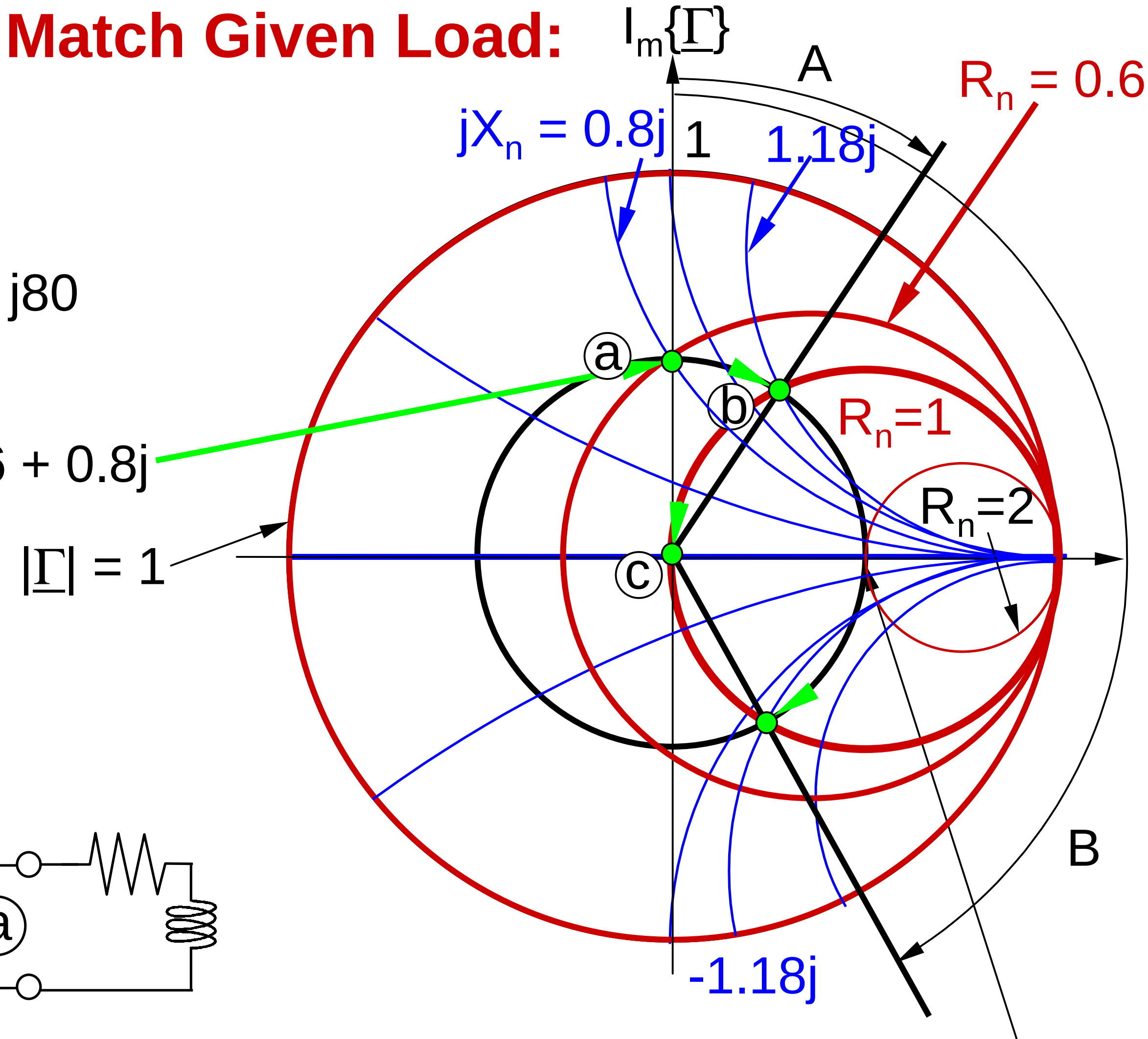
$$\left(e^{2jkz} \Rightarrow e^{2j(2\pi/\lambda)(\lambda/2)} = e^{j2\pi} = 1\right)$$

MATCHING IMPEDANCES

Problem: Losslessly Match Given Load:

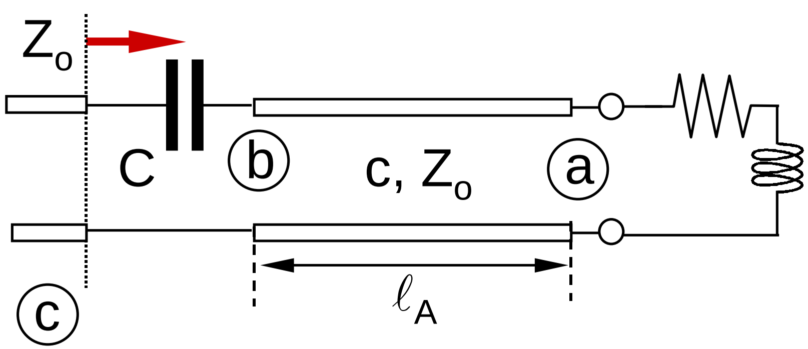


$Z_{Ln} = 0.6 + 0.8j$



$jX = -j1.18Z_o = 1/j\omega C$

So $C = 1/118\omega$

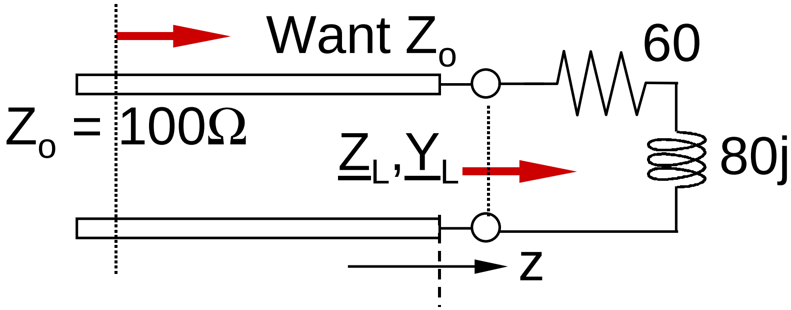


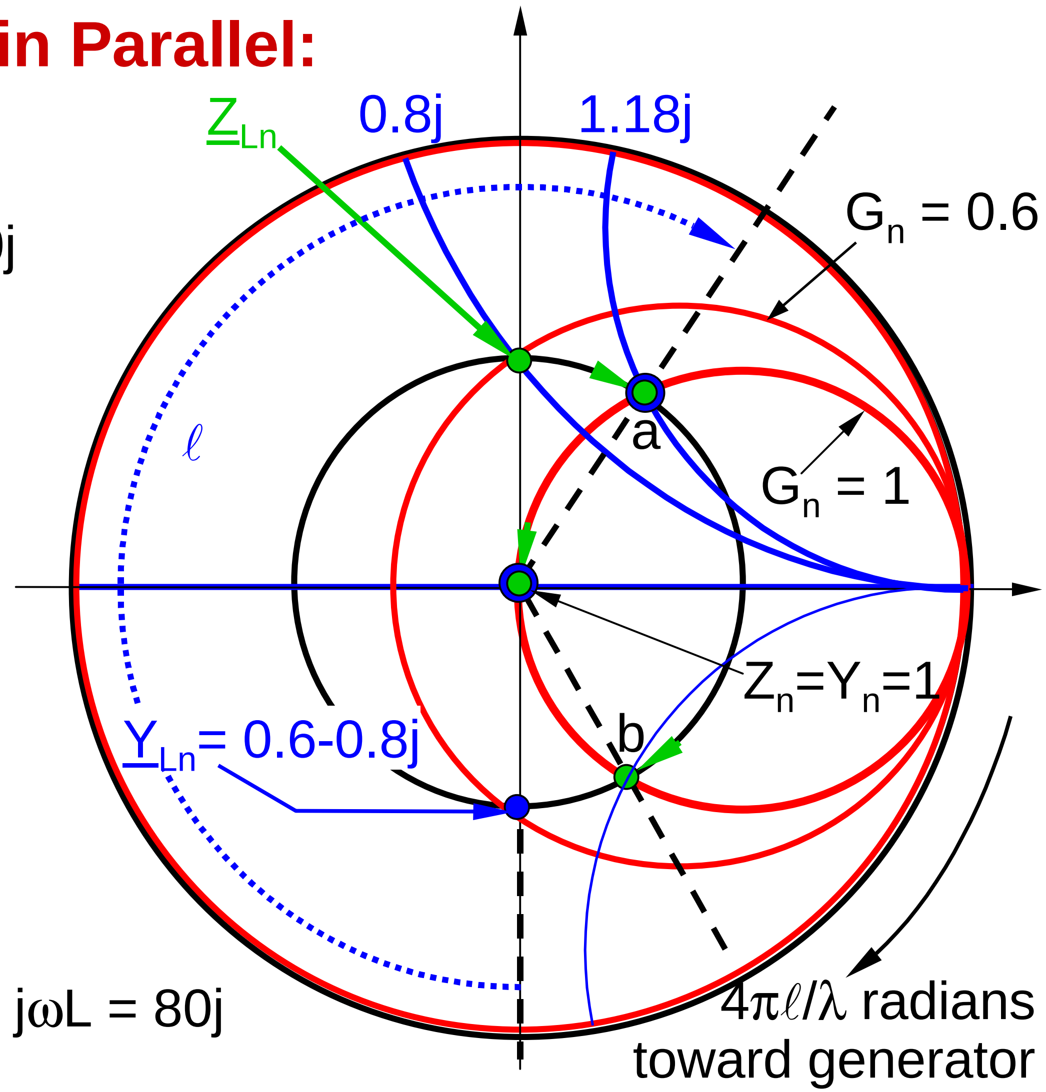
(Or use $j\omega L = j1.18Z_o$ and ℓ_B)

(Since $R_{n\,max} = 2$, so is the VSWR for the mismatched load)

MATCHING ADMITTANCES

Reactive Tuning Elements in Parallel:



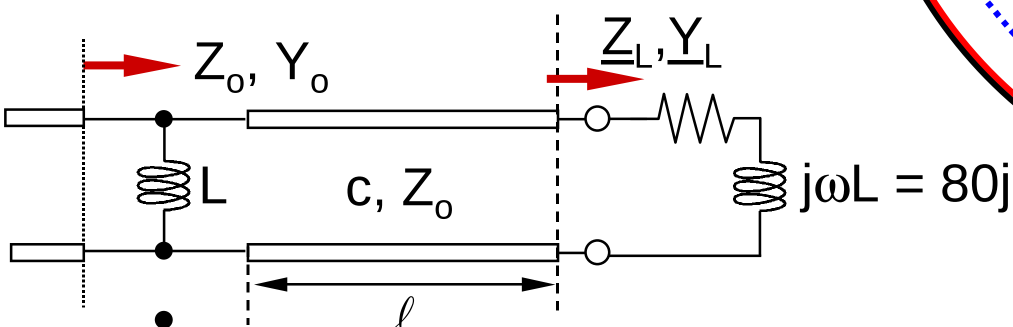


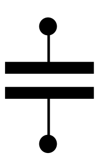
Find $\underline{Z}_{Ln}$ on chart, then $\underline{Y}_{Ln}$ opposite; rotate ℓ to "a"

where $G_n = 1$ and add $-1.18j$ admittance to yield $\underline{Y}_n = Y_o$

$\underline{Y} = -1.18jY_o = 1/j\omega L$

$\Rightarrow L = Z_o/1.18\omega$





Use C with point "b" and +1.18j

Can match any load!

(beware resonances)

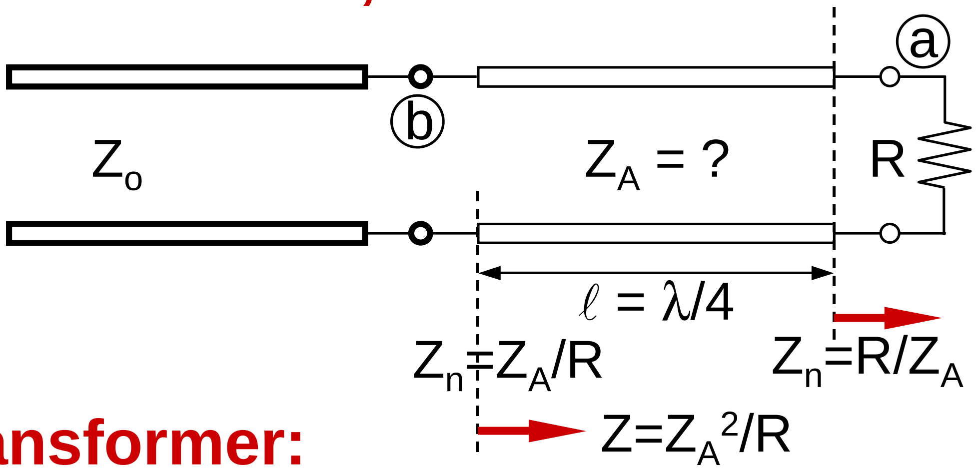
QUARTER-WAVE TRANSFORMER

Matching Real Impedances without L,C:

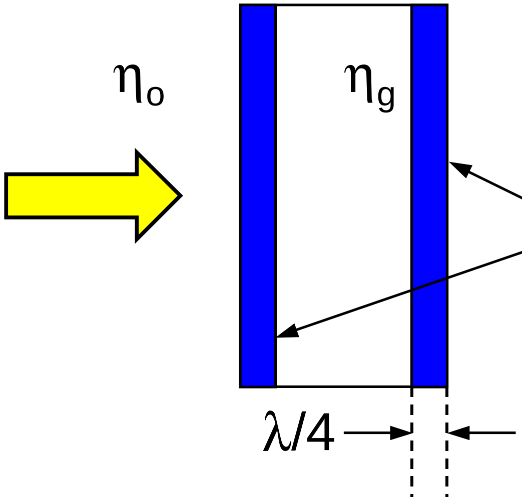
Let: $\ell = \lambda/4$, then

Setting: $Z_o = Z_A^2/R$,

Yields: $Z_A = (Z_oR)^{0.5}$



Optical Quarter-Wave Transformer:



$\eta_o = (\mu_o/\varepsilon_o)^{0.5} = 377\Omega$

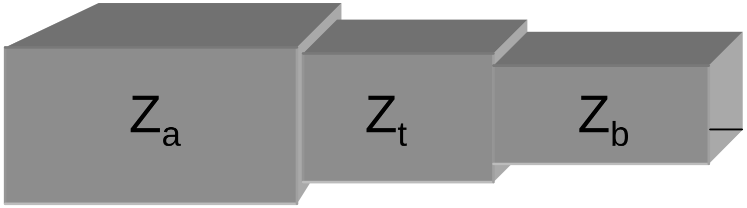
$\eta_t = (\eta_o\eta_g)^{0.5} \Rightarrow \varepsilon_t = (\varepsilon_o\varepsilon_g)^{0.5}$

Applications: coated camera lenses, glasses, optoelectronic components, high-power lasers, etc.

Invented by Prof. Smakula at Leitz

MORE QUARTER-WAVE TRANSFORMERS

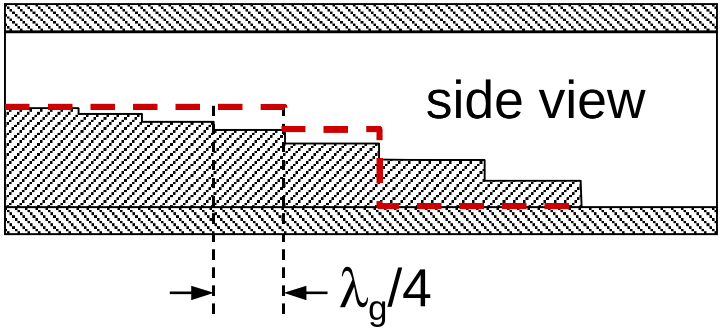
Waveguide Transformers:



Z_o varies with waveguide sizes
$Z_t = (Z_a Z_b)^{0.5}$

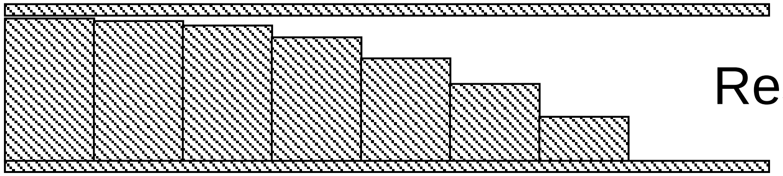
Multi-step Transitions:

Waveguides can have N multiple steps spaced $\lambda/4$ apart



Example, 1:256 Transformer:

For N = 2, Z_a = 1 ohm, Z_b = 256 ohms, and $Z_t = (1 \times 256)^{0.5}$ = 16 ohms
For N = 4, $Z_{t1} = (1 \times 16)^{0.5}$ = 4 ohms, Z_{t2} = 16, $Z_{t3} = (16 \times 256)^{0.5}$ = 64
For N = 8, $Z_{t1} = (1 \times 4)^{0.5}$ = 2, Z_{t2} = 4, (rest are 8, 16, 32, 64, and 128 ohms)



Result is exponential series

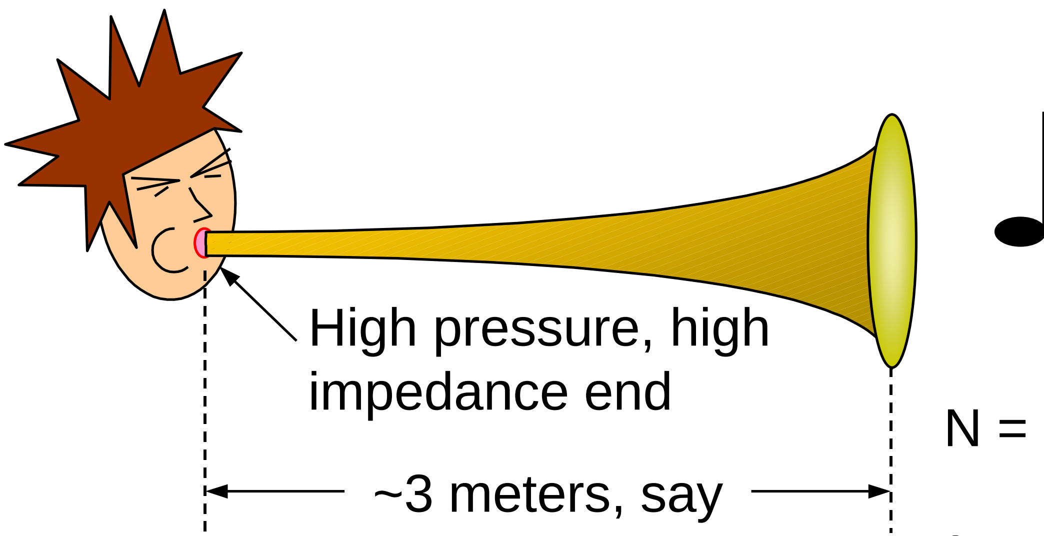
EXPONENTIAL TRANSITIONS AND HORNS

Acoustic Transformers, Exponential Horns:

We use $N \cong 4L/\lambda_g$ sections, where L is the length of the transformer
In the limit we can smooth the steps to yield an exponential shape

Acoustic Examples:

French horn, trumpet, loudspeakers:



low-pressure,
low-impedence end

$N = 8 \Rightarrow \lambda_{max} = 4L/N$
$= 4 \bullet 3/8 = 1.5$ meters
$f_{min} = c_s/\lambda = \sim 300/(1.5) = 200$ Hz

~Acoustic transmission line