SOLUTION2

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Solution 2.1

$a)$ $wt + kx = 10^9 t + 10x$, $w = 2\pi f = 10^9$, $f = \frac{10^9}{2\pi} = 1.59 \times 10^9\ (Hz)$

$b)$ -x direction

$c)$ $v = \frac{w}{k} = \frac{10^9}{10} = 10^8\ (ms^{-1})$

$d)$ $\lambda = \frac{v}{f} = \frac{10^8}{1.59\times 10^8} = 0.629\ (m)$

$e)$ $k = w\sqrt{\mu\epsilon_0}$, $\mu = \frac{k^2}{w^2\epsilon_0} = \frac{10^2}{(10^9)^2\times 8.85\times 10^{-12}} = 1.13 \times 10^{-5}\ (Hm^{-1})$

$f)$ $\overline{E} = -\eta\hat{k} \times \overline{H} = \hat{z}\eta \sin(10^9 t + 10x) = \hat{z}1129.97 \sin(10^9 t + 10x)(Vm^{-1})$

Solution 2.2

$\overline{H} = -\hat{z}\cos(wt - ky)/\eta_0$

a) $W_e(\overline{r}, t) = \epsilon\frac{|\overline{E}(\overline{r},t)|^2}{2} = \frac{\epsilon}{2}\cos^2(wt - ky)\ (Jm^{-3})$

b) $W_m(\overline{r}, t) = \mu\frac{|\overline{H}(\overline{r},t)|^2}{2} = \frac{\mu}{2\eta_0^2}\cos^2(wt - ky) = \frac{\epsilon}{2}\cos^2(wt - ky)\ (Jm^{-3})$

c) $\overline{S}(\overline{r}, t) = \overline{E} \times \overline{H} = \hat{y}\frac{1}{\eta_0}\cos^2(wt - ky)\ (Wm^{-2})$

d) $I = <\overline{S}(\overline{r}, t)> = \hat{y}\frac{1}{\eta_0}\frac{1}{2\pi}\int_0^{2\pi}\cos^2(wt - ky)\ d(wt) = \hat{y}\frac{1}{2\eta_0}\ (Wm^{-2})$

e) From a) and b), we can see $W_m(\overline{r}, t) = W_e(\overline{r}, t)$

f) $W_e(\overline{r}, t) = W_m(\overline{r}, t) = \frac{\epsilon}{2}\cos^2(wt - ky) = \frac{\epsilon\mu}{2\mu}\cos^2(wt - ky) = \frac{1}{2\mu c^2}\cos^2(wt - ky)\ (Jm^{-3})$

g) $\overline{H} = \hat{z}\frac{1}{\eta_0}[-2\cos(wt - ky) + \cos(wt + ky)]$

$\overline{W}_e(\overline{r}, t) = W_m(\overline{r}, t) = \frac{\epsilon}{2}[2\cos(wt - ky) + \cos(wt + ky)]^2$

$$\overline{S}(\overline{r},t) = \overline{E} \times \overline{H} = -\hat{y}\frac{1}{\eta_0}[-4\cos^2(wt-ky) + \cos^2(wt+ky)]$$

h) $\underline{\overline{S}} = \underline{\overline{E}} \times \underline{\overline{H}}^* = \hat{x}[2e^{-jky} + e^{jky}] \times \hat{z}\frac{1}{\eta_0}[-2e^{jky} + e^{-jky}] = \hat{y}\frac{1}{\eta_0}(-2e^{-2jky} + 2e^{2jky} + 3)$

i) $\overline{H} = -\hat{z}\frac{1}{\eta_0}\cos(wt-ky) + \hat{x}\frac{1}{\eta_0}\sin(wt-ky)$

$$\overline{S} = \overline{E} \times \overline{H} = \frac{1}{\eta_0}[\hat{x}\cos(wt-ky) + \hat{z}\sin(wt-ky)] \times [-\hat{z}\frac{1}{\eta_0}\cos(wt-ky) + \hat{x}\frac{1}{\eta_0}\sin(wt-ky)]$$

$$= \frac{1}{\eta_0}[\hat{y}\cos^2(wt-ky) + \hat{y}\sin^2(wt-ky)]$$

Solution 2.3

a) $\nabla \cdot \overline{D} = \rho,\ \int_v \nabla \cdot \overline{D} dv = \int_v \rho dv,\ \ \int_s \overline{D} d\overline{s} = \int_s \rho dv,\ \ E_0 4\pi r_0^2 = Q,$

$$E_0 = \frac{Q}{4\pi\epsilon \times 0.01^2} = \frac{Q}{4\pi\epsilon} \times 10^4\ [Vm^{-1}]$$

b) $V_0 = \phi_0 = \int_\infty^{r_0} \overline{E}(r) d\overline{r} = -\int_\infty^{r_0} E(r) dr = -\int_\infty^{r_0} \frac{Q}{4\pi\epsilon r^2} dr = \frac{Q}{4\pi\epsilon r_0}$

c) $C = \frac{Q}{V} = \frac{Q}{Q/4\pi\epsilon r_0} = 4\pi\epsilon r_0\ (Farads)$

Solution 2.4

a) $\nabla \times \underline{\overline{H}} = \underline{\overline{J}} + jw\underline{\overline{D}} = jw\underline{\overline{D}}$

At left-hand-side use operater $\nabla\times$:

$$\nabla \times (\nabla \times \underline{\overline{H}}) = \nabla(\nabla \cdot \underline{\overline{H}}) - \nabla^2 \underline{\overline{H}}$$

Because $\underline{\overline{H}} = \frac{1}{-jw\mu}\nabla \times \underline{\overline{E}}$, so $\nabla(\nabla \cdot \underline{\overline{H}}) = 0$

At right-hand-side use operater $\nabla\times$:

$$jw\nabla \times \underline{\overline{D}} = jw\epsilon\nabla \times \underline{\overline{E}} = jw\epsilon(-jw\mu)\underline{\overline{H}} = w^2\mu\epsilon\underline{\overline{H}} = k^2\underline{\overline{H}}$$

So, $-\nabla^2\underline{\overline{H}} = k^2\underline{\overline{H}},\ \ \nabla^2\underline{\overline{H}} + k^2\underline{\overline{H}} = 0$

b) $\nabla \times \underline{\overline{H}} = \underline{\overline{J}} + jw\underline{\overline{D}}$

$$\nabla \cdot (\nabla \times \underline{\overline{H}}) = \nabla \cdot \underline{\overline{J}} + jw\nabla \cdot \underline{\overline{D}} = 0$$

So $\nabla \cdot \overline{\underline{J}} = -jw\nabla \cdot \overline{\underline{D}} = -jw\rho$

Solution 2.5

a) $A\cos(wt - \pi/2) \leftrightarrow Ae^{-j\pi/2} = -jA$

b) $(1+j)e^{jkz} = e^{jkz} + e^{jkz+j\pi/2} \leftrightarrow \cos(wt+kz) + \cos(wt+kz+\pi/2)$

c) $A\cos(wt - kz + \pi/2) = A\cos(wt - (kz - \pi/2)) \leftrightarrow Ae^{-j(kz-\pi/2)} = jAe^{-jkz}$