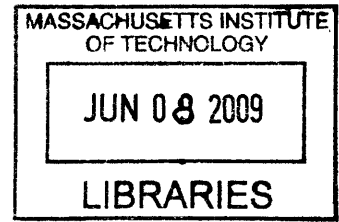


Essays on Sovereign Debt and International Capital Flows

by

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B.A. Economics
Trinity College
University of Cambridge, 2003



Submitted to the Department of Economics
in partial fulfillment of the requirements for the degree of

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Abstract

This dissertation is a collection of three essays on public and private borrowing on international capital markets, with a focus on optimal policy for the government and international financial institutions.

Chapter 1 focuses on sovereign debt and default. Recent sovereign default episodes have been associated with substantial output costs, and the sovereign should take into account that any default decision may exacerbate such costs. I construct a two-period model where sovereign debt is held by both foreign creditors and domestic residents. Default on foreign lenders benefits domestic consumption, but default on domestic residents generates an output cost that increases with the extent of the default. I present two sets of optimal policy results. Firstly, I characterize the optimal default decision and show that full repudiation of debt is not optimal when domestic output costs are sufficiently high. A corollary is that the sovereign can issue debt even in the absence of reputational mechanisms. Secondly, I show that it is optimal for the government to render the domestic economy vulnerable to the adverse effects of default, in order to raise funds cheaply from abroad. Economic fragility is an optimal response to the lack of commitment of the sovereign.

Chapter 2 extends the results to an infinite horizon specification. If the default decision does not lead to reduced capital market access in the future, the results from the two-period model remain valid in the infinite horizon. I expand the framework to incorporate persistent productivity shocks. For this case, an adverse productivity shock leads to a reduction in the feasible set of debt levels today. I show that optimal borrowing may now be increasing, rather than decreasing, in the productivity shock. Finally, I examine whether the government chooses to issue debt in the long run. If the government is allowed to save abroad and simultaneously issue government debt, then it is optimal for the government to have a positive gross debt position even in the long run, irrespective of the discount factor. The results of chapter 1 are therefore operative in the infinite horizon.

Chapter 3 concentrates on private rather than public borrowing. This chapter characterizes optimal IMF policy in an environment with moral hazard followed by adverse selection. In my framework, government actions to improve domestic productivity are not always effective, and the government learns of the success of its actions before foreign investors. Without the IMF, it is not possible for foreign investors to discern the quality of the domestic production sector.

There only exists a pooling equilibrium ex post, which leads to low government effort ex ante. Optimal IMF intervention is the solution to a mechanism design problem in the presence of imperfectly informed competitive markets. Optimal IMF policy is structured so as to reveal the government's private information to foreign investors in a separating equilibrium. Government effort ex ante is high. Countries with weak fundamentals ex post accept IMF transfers and face high interest rates on private capital markets. Countries with strong fundamentals make contributions to the IMF and receive low interest rates from foreign investors.

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sky when I walk home at night. My mother Karabi is the firm foundation of my family, and all of the achievements we call our own are actually reflections of her effort. When I was preparing for my general exams at MIT, I used the same strategies that she taught me in primary school. When events seem to be moving too fast to control, I remember her soothing advice and collect my thoughts. On my travels I remember her sense of wonder and try to convey to her what the Lincoln Memorial feels like at dawn, or what you can see from the top of a cliff in San Francisco, and that is some consolation for the great physical distance between us.

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Cambridge, Massachusetts, USA

May 15, 2009

For my parents Bijay and Karabi Basu

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Chapter 1

Sovereign Debt and Domestic Economic Fragility

1.1 Introduction

When a sovereign government decides to default, it recognizes that such an action may have adverse consequences for the domestic economy, specifically for the domestic financial sector. On the other hand, default may improve consumption by reducing repayments to foreign lenders. The optimal decision of the government balances the costs of default against its benefits. This chapter focuses on the effect of domestic economic costs of default on optimal government policy. Firstly, the consideration of such costs is important for determining whether or not the government should default, and for deciding the scale of debt repudiation in the event of default. The existence of output costs enables the government to credibly assure foreign lenders that it will repay at least a portion of its debt, and this will enable the government to borrow ex ante. Secondly, the government's ex ante debt issuance decision is shaped by its expectations about the costs of default in future periods. If the government can structure its debt issuance policy so as to manipulate the domestic economic costs of default in future periods, it may optimally choose a high level of exposure of the economy to these costs. This enables the government to borrow more ex ante, or to borrow the same amount at lower interest rates.

Evidence from recent default episodes suggests that sovereign default affects the domestic economy. Sturzenegger and Zettelmeyer (2005) report that both domestic and foreign creditors

to the government suffer losses on their holdings of government debt in the event of default. De Paoli et al. (2006) record that sovereign default is often associated with substantial output costs for the domestic economy, especially when the default episode is mired in concurrent banking and/or currency crises.

The survey of defaults and debt restructurings in Sturzenegger and Zettelmeyer (2006) is instructive. In the run up to the Russian debt crisis of 1998, domestic banks had increased their exposure to government debt, so that in the first quarter of 1998 income from government securities amounted to 30 percent of total bank income. The default by the government on domestically held public debt was roughly equal to the economy's aggregate banking capital. In the aftermath of the default, there were runs on some banks. Interbank transactions ground to a halt and the payments system became non-functional. Real GDP fell by 5.3 percent that year. In the Argentinian debt crisis of 2001-2005, 60 percent of the defaulted debt was held by domestic residents. Forced pesification of dollar-denominated assets and liabilities of the financial sector transferred resources from the banks to the government. The banking system became insolvent. Output fell by 3.4 percent in 1999. After the default, it fell by 4.4 percent in 2001 and 10.9 percent in 2002. Clearly, not all the output costs in these default episodes arose from the default decision – in both cases, the default decision was influenced by a prior negative shock to the domestic economy. Nevertheless, the decision by the government to default on its debt contributed to a worsening of the initial crisis, in particular through a disruption to the financial system.

In this chapter, the government cannot contractually commit to repaying its debt in future periods. In addition, we depart from much of the existing literature by considering a framework where default does not lead to reduced access to international capital markets. On the contrary, the government is not sanctioned by foreign creditors even in the period of default. However, when the government defaults on its debt it is forced to default on both domestic and foreign holders of government debt. Default on foreign lenders improves the asset position of the country, but default on domestic lenders generates an output cost that increases with the extent of the default. In a model with a benevolent government which borrows from abroad on behalf of all of its citizens, the default decision trades off these benefits and costs. This chapter produces two sets of results. Firstly, we explore the effect of domestic output costs

on the optimal government default decision. In particular, we show that the contemporaneous output cost of default prevents full default, and therefore supports debt. More generally, the fear of economic crisis is the mechanism that sustains sovereign debt in our model.

The second contribution of the chapter is to endogenize the vulnerability of the domestic economy to crisis. Vulnerability of the financial system is a recurring phenomenon in emerging markets (de Bolle et al. [2006]). The above argument suggests that foreign lenders are willing to lend more to the country if they believe that default will have severe effects on the domestic economy. Suppose that the government can manipulate the output cost resulting from default, for example by influencing the exposure of domestic lenders to government-issued defaultable debt. This chapter proposes that the government may well choose to increase this exposure in order to raise more debt from abroad, or to raise the same amount of debt at lower cost. Therefore, vulnerability of the economy is an optimal response to the underlying economic problem (and market failure) of the sovereign's lack of commitment. In this case, advice to reduce financial system vulnerability may have the counterintuitive side-effect of a reduction in the ability of the government to borrow.

In our model, the event of default entails no punishment by foreign lenders. It is true that recent defaulters have experienced a period of cutoff from international capital markets in the wake of their default decision. However, the absence of any such denial of market access in our model emphasizes that the result of the existence of debt does not depend at all on reputation effects. To make this result even clearer, this chapter is devoted to the analysis of an economy with a finite horizon, where there is by construction no possibility of future sanctions after default.

Sovereign default generates a domestic output cost because default reduces the resources available in the domestic economy for investment and production. This liquidity effect of government default is captured in our framework as follows. At the end of the first period, consumers decide on consumption and savings decisions. Savings are deposited in the banks, which invest these funds and return the proceeds to the consumers at the end of the subsequent period. In order to transfer resources between periods, banks must purchase government debt because there is no storable good between periods (as in Woodford [1990]). Banks enter the second period with holdings of both cash and defaultable government debt. Sovereign default

causes a deterioration in their asset position. By assumption, the banks cannot receive transfers from the government (except via repayment of debt) or from consumers, which means that default results in less resources in the banking system. This means that banks must restrict lending to the domestic productive sector. The outcome is less production. The stark liquidity constraint is an extreme assumption. It is true that most economies have instruments for limiting the economic fallout from default crises. For example, the Argentinian government stepped in to attempt a bailout of the banking system after the default decision and pesification of 2001-2002 had rendered it insolvent. Nevertheless, such bailouts and insurance mechanisms are rarely sufficient to insulate the domestic economy entirely. To the extent that the insurance is imperfect, the mechanism in this chapter will be active. What is more, the logic of the chapter suggests that in order to be able to sustain more debt, the government would want to commit in advance to an institutional setup which provides poor insurance of the domestic production sector, if indeed it can make such a commitment.

Throughout this chapter, we assume that the government defaults equally on domestic and foreign lenders. Clearly, there are incentives for it not to do so, since it is benevolent and cares about the utility of domestic agents. The appendix considers possible justifications for this setup. Equal haircuts¹ may result from an inability on the part of the government to distinguish between holders of the debt in the period of repayment, or from a legal obligation to repay all debtholders within an asset class equally. Even if the government can in principle choose to make different repayments to different categories of debtholders, the existence of secondary markets for debt may constrain its ability to do so. Broner et al. (2006) explore the effects of introducing secondary markets on the ability of the government to issue debt in a finite horizon model. They also consider a version of their model where the government can make short term commitments within periods. It can credibly announce the haircut decision in the final period a moment prior to executing the haircuts, and secondary markets are open in the time interval between these actions. In the appendix, we present an extension of our baseline model that draws upon the insights in Broner et al., and we show that equal haircuts on foreign and domestic debt are an equilibrium outcome. The mechanism of the chapter operates under the weaker condition that the haircuts on foreign and domestic lenders are positively related.

¹The haircut on sovereign debt is the proportion of the debt that is not repaid.

Within our framework, the optimal haircut decision in a particular period is a function of inherited debt variables and the current productivity shock. Inherited debt variables include two key indicators: the level of exposure of the domestic economy to defaultable debt, and the ratio of the defaultable debt held by foreigners to that held by domestic agents. Domestic exposure of the economy is captured by the fraction of defaultable debt relative to cash in the assets of the banking system. This exposure generates an output cost of default and prevents full repudiation. The higher is the ratio of the debt held by foreigners to that held by domestic agents, the lower the absolute volume of repayments. The optimal haircut is larger, the lower is the productivity shock. In the low productivity state, the default decision then reduces output further, amplifying the effect of the productivity shock.

The theoretical literature on sovereign default has long noted that the penalty of autarky following default (the default event triggers loss of reputation) can induce the repayment of debt (Eaton and Gersovitz [1981]). However, governments typically have access to savings technologies abroad even if borrowing from international capital markets is no longer feasible. Bulow and Rogoff (1989) show that if this savings technology takes the form of cash-in-advance contracts that can be indexed to the same variables as the implicit reputational contract, no debt can be sustained. There have been a number of subsequent models that have proposed mechanisms by which debt can be supported.

One class of papers has explored the direct sanctions available to creditors, such as interference with the borrower nation's trade (empirical evidence in Rose [2005]). Official trade embargoes are not common, but governments would be wary of provoking such retaliation. Another group of papers has expanded the scope of reputation. Cole and Kehoe (1995), Eaton (1996) and Kletzer and Wright (2000) examine models where the default decision does adversely affect the economy's future consumption possibilities. Amador (2003) considers how political economy considerations may induce a government to repay when the event of default changes the set of future feasible allocations. In these models, the event of default saves on repayments in the current period, but brings with it costs for the domestic economy in the future. This chapter abstracts from creditor punishments altogether, and instead focuses on domestic output costs. We believe such costs are an important ingredient of a more general model of sovereign borrowing.

Arellano (2008) presents a model and quantitative analysis with noncontingent sovereign debt and endowment shocks, where default leads to temporary autarky. In related work, Mendoza and Yue (2008) develop a model with output costs in the period of default and they obtain that the scale of default is negatively related to a measure of the productivity shock in that period. A theoretical framework incorporating output costs of default is presented by Dooley (2000), who explores the role of output costs as a mechanism for sustaining debt. Our work builds on these contributions, in a framework where the government is allowed to manipulate the output cost.

The literature on financial systems has recorded that imprudent regulation of the banking system can increase the vulnerability of the financial infrastructure to shocks such as default. Burnside et al. (2001) show that government guarantees to banks and their foreign creditors diminish the incentive of the banking system to hedge against exchange rate collapse, which results in a fragile banking industry. Livshits (2007) presents a framework where the government may wish to increase financial system exposure to its debt. The incentive to increase exposure in our model derives from the need for the government to assure foreign creditors that default will harm the domestic economy. Exposure helps to align the interests of the government and foreign creditors more closely. A similar motive is present in spirit in the analysis of Tirole (2003). He considers a different economic environment, where the government's actions have an effect on the relationship between domestic firms and foreign financiers, and concludes that the promotion of "safer" forms of finance may be insufficient to achieve the required match of interests between stakeholders and the government.

Finally, this chapter is related methodologically to the optimal policy literature. In our problem, the government chooses the most preferred rational expectations equilibrium out of the set of feasible equilibria. The government's problem can be decomposed into two interdependent subproblems. On the one hand, it must decide on the level of borrowing. On the other hand, it must choose the optimal composition of debt and domestic economic exposure that supports the level of debt chosen. The analytical decomposition draws upon insights in Werning (2003) regarding the solution of the noncontingent debt problem of Aiyagari et al. (2002).

The remainder of the chapter is structured as follows. Section 1.2 summarizes the model. There are two specifications of interest. In the first specification, defaultable government debt

is not tradable between domestic and foreign agents in the period of issue. In the second specification, it is tradable. Section 1.3 solves some benchmark cases of the model. Section 1.4 summarizes the construction of the program, the theoretical results and numerical simulations for the first specification. Section 1.5 does the same for the second. Section 1.6 considers policy implications arising from the model. Section 1.7 concludes.

1.2 Model

1.2.1 Preferences and Technology

The model has two periods, $t = 1$ and 2. There are five categories of actors in our framework: consumers, firms, banks, the government and foreign creditors. There is a continuum of consumers and firms, both of measure 1, and a continuum of banks. There is also a continuum of foreign creditors.

Preferences Each consumer is identical, with preferences over consumption streams $\{c_1, c_2\}$ given by the expression

$$u(c_1) + \beta \mathbb{E}u(c_2).$$

$\beta \in (0, 1)$ is the discount factor and the period utility function is continuously differentiable and strictly increasing: $u'(c) > 0$.

The government is benevolent and maximizes the utility of the representative consumer. Firms, banks and foreign creditors are risk neutral and maximize expected profits.

Technology In the first period, each consumer receives an endowment y_1 . In addition, it is possible for the economy as a whole to borrow resources z from foreign creditors. There is no domestic storable good between periods. Accordingly, the resource constraint for the economy in this period is written:

$$c_1 \leq y_1 + z$$

At the beginning of the second period, each consumer receives an endowment y_2 . Then the economy has access to a production technology, operated by firms. Specifically, the economy can invest x units of its endowment income in the production sector, which produces $F(x, \tilde{R})$

units of output:

$$F(x, \tilde{R}) = x + \tilde{R}f(x).$$

\tilde{R} is a stochastic productivity variable. Its value is realized at the beginning of the second period. We assume $\tilde{R} \geq 0$, with highest and lowest values \bar{R} and \underline{R} respectively. The production function $f(x)$ is strictly increasing and strictly concave up to an input level \bar{x} , and is flat for input levels beyond this:

$$f'(x) \geq 0, f''(x) < 0 \quad \forall x \in [0, \bar{x}]$$

$$f(x) = f(\bar{x}) \quad \forall x \geq \bar{x}.$$

$f(x)$ is twice differentiable. We impose $\lim_{x \rightarrow 0} f'(x) = \infty$ and $f'(\bar{x}) = 0$. The output of the production sector cannot be reinvested in the same sector.

In the second period, the economy makes repayments v to foreign creditors. The resource constraint is derived:

$$c_2 \leq y_2 + \tilde{R}f(x) - v$$

Foreign creditors maximize profits from their lending to the domestic economy, and they have access to an international riskless asset which yields the interest rate r between periods. This imposes the following rational expectations restriction across periods:

$$z = \frac{1}{1+r} \mathbb{E}v$$

1.2.2 Market Structure

Figure 1.1 illustrates the order of events and actions in periods $t = 1$ and 2. This section uses the timeline to describe the market structure we impose in our framework.

Figure 1.1: Model Timeline

Period 1

- Endowment y_1 realized.
- Government issues debt A_d, B_d, B_f and transfers proceeds T_1 to consumers.

Consumers consume c_1 goods and save s_1 in banks.

Banks invest in government debt A_d, B_d .

Foreigners purchase government debt B_f .

Period 2

- Productivity shock \tilde{R} realized.
- Government imposes lump sum taxes T_2 and applies haircut h on debt B_d, B_f .
- Banks lend x to firms.

Firms borrow and produce $F(x, \tilde{R}) = x + \tilde{R}f(x)$.

- Consumers consume c_2 goods.

Consumers Each consumer solves the following maximization problem:

$$\max_{\{c_1, s_1, c_2\}} u(c_1) + \beta \mathbb{E}u(c_2) \quad (1.1)$$

subject to

$$c_1 \leq y_1 + T_1 - s_1 \quad (1.2)$$

$$c_2 \leq y_2 + T_2 + \Pi_B + \Pi_F + S(s_1, \tilde{R}) \quad (1.3)$$

$$c_1, c_2 \geq 0 \quad (1.4)$$

In the first period, each consumer decides on its consumption and savings $\{c_1, s_1\}$, taking transfers from the government T_1 and T_2 as given. Savings are deposited in the banks, and yield a gross investment return of $S(s_1, \tilde{R})$ in the subsequent period. Each consumer owns an equal share in all the banks and all the firms that exist in the second period. Π_B and Π_F denote bank and firm profits respectively.

Consumers, firms and banks cannot borrow from or save abroad.

Bank Deposit Contracts Banks compete for savings of the consumers in period 1. They can offer contracts χ to consumers of the form:

$$\chi : s_1 \longrightarrow S(s_1, \tilde{R})$$

The contract takes s_1 from consumers in period 1 and returns $S(s_1, \tilde{R})$ to consumers in period 2. No other transfers between consumers and banks are allowed. Consumers observe the set of contracts available $\{\chi(s_1)\}$ and choose the contract that maximizes their expected utility. In equilibrium, the banks will make zero profits ($\Pi_B = 0$) and they will invest in assets so as to maximize consumer utility. Since there is no storable good between periods, the only means by which the banking system can transfer resources between periods is via the purchase of government-issued debt. The set of available assets is described next.

Government Debt In the first period, the government can issue two types of debt, cash A_d and defaultable debt B . Of the defaultable debt, B_d is purchased by domestic banks and B_f is purchased by foreign creditors. There is no government expenditure in this model. The government may transfer to the consumers any resources raised from debt issuance:

$$T_1 \leq p_A A_d + p_B B_d + q B_f, \tag{1.5}$$

where positive quantities are used to denote debt. p_A is the price of cash in terms of output. p_B and q are the prices of defaultable debt held by domestic banks and foreign creditors respectively. If defaultable debt is not tradable between domestic and foreign agents in the period of issue, these prices may differ. If the defaultable debt is tradable in the period of issue, then:

$$p_B = q.$$

In the second period, the government observes the productivity shock and then decides on its repayments to holders of the defaultable debt. In our model, the government cannot default on cash, and it must default on all holders of defaultable debt by an equal haircut h . The haircut is the proportion of the face value of debt that is not repaid. The government imposes

lump sum transfers on consumers in order to make repayments on its debt:

$$-T_2 \geq A_d + (1 - h) [B_d + B_f] \quad (1.6)$$

The key feature of the government is that it cannot commit in period 1 to the level of the haircut h in period 2.

Bank Holdings of Government Debt Cash and defaultable debt are issued by the government in the first period. Banks choose their holdings of these categories of debt in order to maximize their profits, taking the prices $\{p_A, p_B, q\}$ as given.

Foreign Creditors The rational expectations restriction imposed in the previous subsection may now be rewritten in terms of the debt variables:

$$\begin{aligned} & \max_{B_f} \left\{ \frac{1}{1+r} \mathbb{E}(1-h)B_f - qB_f \right\} \\ \implies & q = \frac{1}{1+r} \mathbb{E}(1-h) \end{aligned} \quad (1.7)$$

This equation determines the price of foreign-held defaultable debt.

Loans Market in Period 2 Since the banks enter the second period with holdings of government-issued cash and defaultable debt, the government's haircut decision affects the asset position of the banks. Let

$$X = A_d + (1 - h)B_d$$

denote the resources in the banking system after the default decision. We assume that the government cannot transfer resources from consumers to banks except through repayment of cash and defaultable debt, and that banks have no other means of raising funds from consumers in period 2.

Banks can choose to either hold these resources X until the end of the second period, or to lend these resources to firms in a competitive market for loanable funds. In the latter case, firms use the loaned funds as inputs in production and repay the banks with interest before the end of the period. The supply of loanable funds by banks takes the shape illustrated in figure

1.2. At the end of the second period, banks transfer the promised units of output $S(s_1, \tilde{R})$ to consumers.

Firms take the loan rate for funds ρ as given and choose to borrow x units of input in order to maximize profits:

$$\begin{aligned} & \max_x \{ x + \tilde{R}f(x) - \rho x \} & (1.8) \\ \implies & 1 + \tilde{R}f'(x) = \rho \end{aligned}$$

The resulting demand curve is shown in figure 1.2. By inspection, the equilibrium loan rate is given by

$$\rho = 1 + \tilde{R}f'(X)$$

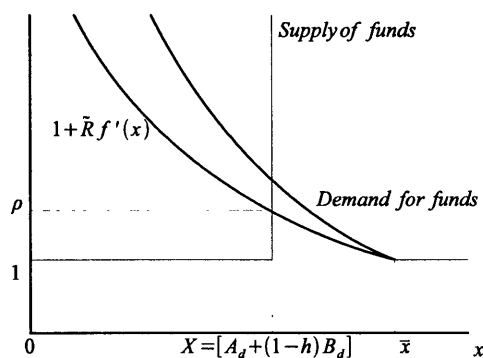


Figure 1.2: Loans Market in Period 2

The key constraint that summarizes the market imperfection on the production side of the economy is

$$x \leq A_d + (1 - h)B_d$$

In the final period, inputs into the domestic production sector are less than or equal to the total value of repaid government bonds. Effectively, we have the following structure. In period 1, consumer savings are invested in government bonds. In period 2, inputs into production are constrained by the gross return on these investments.

1.2.3 Equilibrium Definition

We use the following equilibrium definition in this chapter.

Definition 1.1 A *Rational Expectations Equilibrium* for this economy comprises sequences for allocation rules $\{c_1, s_1, c_2, \{\chi\}, x\}$, prices $\{p_A, p_B, q, \rho\}$ and policies $\{A_d, B_d, B_f, h, T_1, T_2\}$ that satisfy:

(a) Consumers choose $\{c_1, s_1, c_2\}$ to maximize utility (1.1) subject to the budget constraints (1.2), (1.3) and the nonnegativity constraints on consumption (1.4), taking prices, bank contract offers, government policies and the endowment as given.

(b) Banks offer contract schedules $\chi : s_1 \rightarrow S(s_1, \tilde{R})$ in period 1 to maximize expected profits, taking prices and government policies in period 2 as given.

Banks choose lending quantity x in period 2 to maximize profits, taking the loan rate ρ as given.

(c) Firms choose borrowing level x to maximize profits (1.8), taking the loan rate ρ as given.

(d) Government chooses $\{h, T_2\}$ in period 2 to satisfy the government budget constraint (1.6) in that period, taking $\{A_d, B_d, B_f\}$ and the shock \tilde{R} as given.

Government chooses $\{A_d, B_d, B_f, T_1\}$ in period 1 to satisfy the government budget constraint (1.5) in that period, taking the price functions $\{p_A(A_d, B_d, B_f), p_B(A_d, B_d, B_f), q(A_d, B_d, B_f), \rho(x)\}$ and government policies in period 2, both $h(A_d, B_d, B_f, \tilde{R})$ and $T_2(A_d, B_d, B_f, \tilde{R})$, as given.

(e) All markets clear for the economy. In particular, the markets for cash, defaultable debt, goods and loans clear.

(f) Bond prices for foreign debt follow rational expectations: $q(A_d, B_d, B_f) = \frac{1}{1+r} \mathbb{E}\{1 - h\}$, taking the government policy $h(A_d, B_d, B_f, \tilde{R})$ in period 2 as given.

Now we turn to the optimal policy problem for the government. In the second period, the government observes the shock to productivity \tilde{R} and then makes a haircut decision. The government lacks commitment: it cannot credibly commit in period 1 to the haircut it will impose in period 2.

Definition 1.2 *The Government Problem* is to maximize utility (1.1) over time consistent rational expectations equilibria. In particular, we must satisfy not only the equilibrium conditions above but also the additional optimization decisions:

(g) Government chooses $\{h, T_2\}$ in period 2 to maximize $u(c_2)$ given $\{A_d, B_d, B_f\}$ and the shock \tilde{R} .

Government chooses $\{A_d, B_d, B_f, T_1\}$ in period 1 to maximize $u(c_1) + \beta \mathbb{E}u(c_2)$, taking the price functions $\{p_A(A_d, B_d, B_f), p_B(A_d, B_d, B_f), q(A_d, B_d, B_f), \rho(x)\}$ and government policies in period 2, $h(A_d, B_d, B_f, \tilde{R})$ and $T_2(A_d, B_d, B_f, \tilde{R})$, as given.

In this chapter, we consider two different scenarios. In the first specification, defaultable debt is not tradable between domestic banks and foreign creditors in the period of issue. In the second specification, defaultable debt is tradable in the period of issue. In the latter case, we impose the additional restriction:

$$p_B = q.$$

1.2.4 Discussion of the Environment

Our model has by construction ruled out the possibility of sanctions by foreign lenders in the event of default. This helps to emphasize that the results regarding the feasibility of sovereign debt obtained in the remainder of the chapter do not rely upon reputation effects.

In order to capture the fact that governments are typically unable to completely insure the domestic productive sector in the event of default, we have utilized a model construction that imposes a sharp liquidity constraint on the domestic production sector. After a default event, we assume that the government cannot transfer resources from consumers to banks except through repayment of cash and defaultable debt, and that banks have no other means of raising funds from consumers in period 2. Our model mechanism will be operative whenever the productive sector is adversely affected by the default event and its consequences.

All domestic debt is issued by the government in our model. It can issue both cash and defaultable debt – the proportion of the latter in the portfolio of domestic banks captures the exposure of the economy to sovereign default. Cash can only be held by domestic agents. We do not consider debt types that can only be held by foreign creditors. In our model, the

government will obviously default fully on all such debt. In section 1.8.B of the appendix, this result is derived formally.

Why does the government treat all holders of debt type B in the same manner? Section 1.8.D of the appendix considers possible justifications for equal haircuts for domestic and foreign debtholders. One possible justification is that the government cannot observe who holds its debt. Alternatively, even if the government can in principle choose to make different repayments to different categories of debtholders, the existence of secondary markets for debt may constrain its ability to do so. Broner et al. (2006) argue that the government would like to treat domestic and foreign lenders differently in a setup where the government has no commitment power. In the appendix we consider a model where the government can distinguish the residence of debtholders, but where it still cannot effect transfers to the domestic productive sector except by repaying debt. The government cannot commit in period 1 to make particular repayments in period 2. But in period 2, just prior to default, it makes an announcement of the haircuts for both domestic and foreign lenders. Following this announcement there is an opportunity for domestic and foreign lenders to trade the debt with each other on secondary markets. Then the government executes the haircuts for period 2, but it must fulfil the announcements that it made earlier in the (same) period, i.e., there is “short-term (within-period) commitment” in the terminology of Broner et al. We show that it is an equilibrium for the government to choose the same haircut for domestic and foreign lenders.

For clarity of exposition, the model environment dictates that the government cannot concurrently purchase debt issued by foreign institutions and issue defaultable debt. This assumption rules out scenarios where the government both saves abroad and issues defaultable debt to domestic and foreign lenders. Section 1.8.C of the appendix considers an environment where this assumption is relaxed. The feasible set of debt levels is unchanged from the model considered in this chapter.

For the remainder of this chapter, we consider two different specifications regarding the tradability of debt in the period of issue. The aim of this exercise is to clarify the mechanisms operating in our model. In both specifications, the exposure of the domestic economy to sovereign default is the underlying mechanism that makes debt issuance feasible. In the case with nontradable debt, this exposure channel for sustaining debt is the only mechanism in our model,

and can be analyzed in isolation. In the case with tradable debt, an additional restriction is added – namely, that the valuation of the debt by domestic and foreign bondholders must be equal. This reduces the size of the feasible set. In particular, unlike in the nontradable debt case, the discount factor and risk aversion of the representative consumer are now relevant for the characterization of the feasible set of debt values. Analysis of the latter case provides us with an understanding of the overall model when the exposure mechanism and the equal valuation restriction are combined.

1.3 Benchmark Cases

For the purposes of comparison with the setup developed in this chapter, in this section we solve two benchmark cases of the model. Proofs are relegated to the appendix.

1.3.1 First Best Case

Suppose that the government can both (i) contractually commit in period 1 to the haircut schedule in period 2 (full commitment), and (ii) save abroad and issue debt at the same time. Then the first best is achieved.

The full commitment case is a major difference from the model with lack of commitment studied in the subsequent sections. The requirement that the country also be able to save and borrow at the same time allows the sovereign to make its debt repayment in period 2 fully contingent, so that it may actually make net repayments in high productivity states and receive net transfers from abroad in low productivity states. This configuration is not possible if the government must either save or borrow.

Proposition 1.1 (First Best Case) *Assume that y_2 is sufficiently high. The optimal consumption schedule (c_1, c_2) is the same whether debt is tradable or not. It has the properties:*

1. *Production by domestic firms is equal to $\bar{x} + \tilde{R}f(\bar{x})$ when the productivity shock is \tilde{R} .*

The optimal allocation solves:

$$\max_{B_f, \{h\}} \left\{ u \left(y_1 + \frac{B_f}{1+r} \mathbb{E}(1-h) \right) + \beta \mathbb{E} u \left(y_2 - (1-h) B_f + \tilde{R}f(\bar{x}) \right) \right\}$$

2. Consumption c_1 and borrowing in period 1 are chosen to satisfy the representative consumer's Euler equation.
3. Consumption c_2 is equalized across states of nature \tilde{R} in period 2 (by appropriate selection of haircuts in period 2).

At the first best, the total output of domestic firms is at the maximum level in every state of nature \tilde{R} in period 2. The output of this sector does vary due to the fluctuation in the productivity shock value. The government fully insures the consumption of domestic consumers against the productivity shock, via state-contingent transfers to and from foreigners. The corollary is that repayments to foreigners in period 2 vary across different states of nature.

To achieve the allocation described, the government can issue $A_d \geq \bar{x}$. B_f solves the expression above. B_d is set arbitrarily in the nontradable debt case. In the tradable debt case, B_d is equal to desired debtholdings by domestic banks at the optimal allocation.

1.3.2 Nondefaultable Debt

Consider the case where the government is not able to default at all on its debt issuance, whether to foreign or domestic agents. In effect, $h = 0$ for all values of the productivity shock \tilde{R} . The following proposition applies for this case.

Proposition 1.2 (Nondefaultable Debt) *Assume that y_2 is sufficiently high. The optimal consumption schedule (c_1, c_2) is the same whether debt is tradable or not. It has the properties:*

1. Production by domestic firms is equal to $\bar{x} + \tilde{R}f(\bar{x})$ when the productivity shock is \tilde{R} .

The optimal allocation solves:

$$\max_{B_f \in (-\infty, y_2]} \left\{ u \left(y_1 + \frac{B_f}{1+r} \right) + \beta \mathbb{E} u \left(y_2 - B_f + \tilde{R}f(\bar{x}) \right) \right\}$$

2. Consumption c_1 and borrowing in period 1 are chosen to satisfy the representative consumer's Euler equation for noncontingent and nondefaultable debt.
3. Consumption c_2 is increasing in the value of the productivity shock \tilde{R} .

Again, the total output of domestic firms is at the maximum level in every state of nature \tilde{R} in period 2. The maximum output level varies with \tilde{R} . However, in this case the government is not able to fully insure domestic consumers against the productivity shock, because the repayments to foreign creditors are not state contingent in the final period. Therefore, consumption in period 2 is increasing in the value of the productivity shock.

The government can choose $A_d \geq \bar{x}$, with B_f as given above. B_d is set arbitrarily in the nontradable debt case. For tradable debt, it is equal to desired debtholdings by domestic banks at the optimal allocation.

1.4 Nontradable Debt

Formulation of the government problem follows directly from the equilibrium concept and the assumption of lack of commitment on the part of the sovereign. In this section, we characterize and solve the government problem. The crucial element of the analysis is the reduction of the number of state variables to just one variable, the total level of real resources raised through foreign borrowing in that period. On the theoretical front, the resulting program can be broken up into two parts: an intratemporal component, which calculates the optimal combinations of debt and exposure for any given level of borrowing from abroad; and an intertemporal component, which determines the optimal level of borrowing in period 1. Both of these subproblems will be analyzed. On the numerical side, the reduction of the number of state variables renders the model more tractable for simulations.

The proofs for the results in this and subsequent sections are contained in section 1.8.A of the appendix.

For the purposes of the remainder of the chapter, we make a variable transformation that enables us to visualize more clearly the exposure of the domestic economy to government debt. We may rewrite any combination of government debt issuance (A_d, B_d, B_f) as a combination

(α, D, B_f) such that:

$$\begin{aligned} D &= A_d + B_d \\ \text{where } A_d &= (1 - \alpha) D \\ B_d &= \alpha D \end{aligned}$$

D is a measure of total face value of government-issued debt held by the banks at the beginning of period 2, including cash and defaultable debt. α is the fraction of defaultable debt in total bank assets.

The next subsection constructs the program for the government problem. Subsection 1.4.2 characterizes the optimal government default policy in period 2. A corollary of this result is that it is feasible for the government to issue debt in our model (in the absence of reputation effects). Subsections 1.4.4 to 1.4.7 analyze optimal government debt issuance policy in period 1.

1.4.1 Construction of Program

Let us apply Definitions 1.1 and 1.2 to derive the program for the government problem. In period 1:

$$U_1 = \max_{c_1, \alpha, D, B_f} \left\{ u(c_1) + \beta \mathbb{E} U_2 \left(\alpha, D, B_f, \tilde{R} \right) \right\}$$

subject to

$$c_1 = y_1 + qB_f$$

$$c_1 \geq 0$$

$$q = \frac{1}{1+r} \mathbb{E} \left\{ 1 - h \left(\alpha, D, B_f, \tilde{R} \right) \right\}$$

$$B_f < 0 \Rightarrow \alpha = 0,$$

where the expression for the period utility in period 2 is given by

$$U_2 \left(\alpha, D, B_f, \tilde{R} \right) = \max_{c_2, h} u(c_2)$$

subject to

$$\begin{aligned}
c_2 &= y_2 - (1 - h)B_f + \tilde{R}f([(1 - \alpha) + (1 - h)\alpha]D) \\
c_2 &\geq 0 \\
y_2 &\geq (1 - \alpha)D + (1 - h)[\alpha D + B_f] \\
0 &\leq h \leq 1.
\end{aligned} \tag{1.9}$$

In period 1, the government may borrow or save abroad. Each combination (α, D, B_f) corresponds to a default schedule across states $h(\alpha, D, B_f, \tilde{R})$ in the next period, and hence to the bond price function $q = Q(\alpha, D, B_f)$. This function is calculated using rational expectations over the default schedule in period 2, and is taken as given by the government in period 1.

Expression (1.9) states that government debt repayments in period 2 must be less than or equal to the consumer endowment in that period. For the remainder of this chapter, we assume that y_2 is large enough so that this constraint is never binding. A sufficient condition on the production function to ensure that this approach is valid is: $\lim_{x \rightarrow 0} x f'(x) = 0$. We assume that this condition is satisfied throughout.

An important observation to make from the program above is that the haircut decision in the last period can be analytically derived. Simply apply the first order condition with respect to the haircut for interior values of h , and apply the boundary condition as required for values of h that are not interior. In the next subsection, we examine the expression for the haircut. For the purposes of the analysis in this subsection, it suffices to note that the expression for the haircut may be written:

$$h = H(\alpha, D, B_f, \tilde{R}).$$

In turn, this means that we can also derive the expression for the bond price schedule:

$$Q(\alpha, D, B_f) = \frac{1}{1+r} \mathbb{E} \left\{ 1 - H(\alpha, D, B_f, \tilde{R}) \right\}.$$

Observe also that consumption in period 1 depends upon the combination (α, D, B_f) to the extent that it affects the total real resources raised by the government from foreign creditors $z = qB_f$. Therefore, we can rewrite the problem as one in which the government chooses how

much to raise from abroad z , and then decides the optimal combination (α, D, B_f) that achieves this level of borrowing. The optimal combination is decided before the state of nature in period 2 is realized, therefore we may rewrite the government problem as follows.

$$V_1 = \max_{c_1, z} \mathbb{E} \{u(c_1) + \beta V_2(z)\}$$

subject to

$$c_1 = y_1 + z$$

$$c_1 \geq 0$$

$$z \in G,$$

where we define $V_2(z)$ as follows:

$$V_2(z) = \max_{c_2, \alpha, D, B_f} \mathbb{E} \{u(c_2)\}$$

subject to

$$c_2 = y_2 - (1 - h)B_f + \tilde{R}f([(1 - \alpha) + (1 - h)\alpha]D)$$

$$h = H(\alpha, D, B_f, \tilde{R})$$

$$z = Q(\alpha, D, B_f) \cdot B_f$$

$$z < 0 \Rightarrow \alpha = 0$$

for some set G . Our notation suppresses the dependence of h on $(\alpha, D, B_f, \tilde{R})$ in the consumption equation. Note that the combination (α, D, B_f) is still chosen before the productivity shock in period 2 is realized.

This formulation separates the problem into two subproblems. The intertemporal component of the problem concerns how much to borrow in the initial period, z , in order to smooth consumption between periods. The intratemporal component takes the default decision $h(\alpha, D, B_f, \tilde{R})$ in the final period as given, and uses this information in order to calculate the optimal combination (α, D, B_f) for the chosen z value. Section 1.8.A of the appendix explains the generation of the set of feasible debt values G . For sufficiently high y_2 , the set can be

characterized using only the relation $Q(\cdot)$.

1.4.2 Haircut Decision

In this subsection, we characterize the optimal haircut in period 2 as a function of the productivity shock and the inherited debt variables. Let us assume that the government has issued debt to foreigners, i.e., $B_f > 0$. To begin, we state a simple lemma that allows us to focus on a restricted subset of D values.

Lemma 1.1 *Define $\bar{D} = \bar{x}$. For any combination $C = (\alpha, D, B_f)$ such that $D > \bar{D}$, there exists some other combination $C' = (\alpha', D', B'_f)$ where $D' = \bar{D}$, such that C' raises the same revenues as C in period 1 and is equivalent to C in terms of repayments abroad, output and hence consumption for all values of the productivity shock \tilde{R} in period 2.*

Corollary 1.1 *We can restrict our attention to combinations $C = (\alpha, D, B_f)$ such that $D \in [0, \bar{D}]$.*

For intuition, let us consider the case with $\alpha = 1$. In this case, if the quantity of total domestic debt exceeds \bar{D} in magnitude, the government can reduce its debt by defaulting on the portion $(D - \bar{D})$ for every realization of the productivity shock \tilde{R} in period 2 without any adverse output effect. Indeed, it will exercise this option. Any issuance of domestic debt in excess of the output-maximizing value \bar{D} merely increases the haircut on debt for every shock realization \tilde{R} , and therefore depresses the price of the foreign debt in period 1. The same total revenues may be raised by issuing the output-maximizing level of domestic debt and a lower volume of foreign debt. The appendix formalizes this intuition and shows that an amended argument can be applied for any possible configuration (α, D, B_f) . Therefore, we can restrict our attention to the set $\{(\alpha, D, B_f) : D \in [0, \bar{D}]\}$.

Proposition 1.3 (Haircut Decision) *The haircut decision $h = H(\alpha, D, B_f, \tilde{R})$ satisfies the following formulation:*

$$h = \max\{0, \min\{1, \theta\}\}$$

where θ satisfies

$$\frac{B_f}{\tilde{R}\alpha D} = f'([(1 - \alpha) + (1 - \theta)\alpha] D). \quad (1.10)$$

1. The haircut is (weakly) increasing in the volume of foreign debt issued B_f .
2. The haircut is (weakly) decreasing in the productivity shock \tilde{R} .

The haircut is always selected to maximize consumption in the final period. An increase in the haircut benefits consumption by reducing repayments abroad. However, for $D \in [0, \bar{D}]$, it also reduces the output of the production sector. This output cost is increasing and convex in the scale of default. For an interior haircut, the marginal benefit is set equal to the marginal cost. The haircut is either 0 or 1 when one of these marginal effects on consumption exceeds the other for all $h \in [0, 1]$. The model predicts that default will be highest when foreign holdings of defaultable debt are very high relative to domestically held defaultable debt, and/or when the productivity shock is low. The higher is the ratio of foreign to domestic defaultable debt $\frac{B_f}{\alpha D}$, the higher is the marginal benefit of default, since an increase in the haircut of 1 percent corresponds to a larger absolute reduction in debt repayment. When the productivity shock is lower, the marginal output cost of default is lower. Therefore, the marginal costs of default are lower and the optimal scale of default higher. According to the model, default occurs after a poor productivity realization, and the act of default results in a further reduction in output. This matches recent default episodes.

Figure 1.3 illustrates the haircut decision in this model. It may help provide some intuition for the results that follow in this section and others, and is inspired by the first order condition with respect to h , equation (1.10).

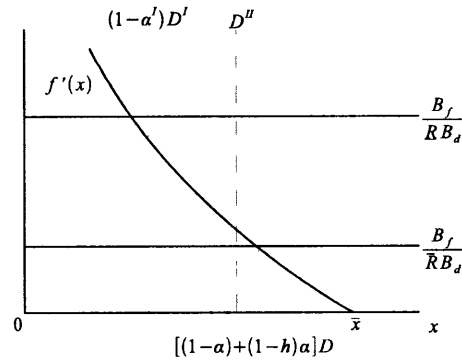


Figure 1.3: Haircut Decision

Fix the ratio $\frac{B_f}{\alpha D}$ at some value. The sequence of horizontal lines on the figure captures the values of $\frac{B_f}{\tilde{R}\alpha D}$ for different values of the productivity shock \tilde{R} , with lower lines corresponding to states with higher productivity. The haircut decision in any state of nature is marked by the point of intersection of the $f'(x)$ function with the horizontal line corresponding to that state. Assume first that the haircut decision is always interior, and ignore the vertical lines on the diagram. Clearly, for higher productivity the intersection occurs for a higher value of $x = [(1 - \alpha) + (1 - h)\alpha] D$, which corresponds to a lower haircut. Note that a higher value of x also corresponds to higher output $f(x)$. A higher volume of foreign debt issuance B_f shifts all the horizontal lines upwards, leading to higher haircuts across all states in period 2. Now consider cases where the haircut decision is not interior. If the level of cash is equal to $(1 - \alpha^I)D^I$, then $h = 1$ will be binding in the low productivity state. If the total debt level is equal to D^{II} , then $h = 0$ will be binding in the high productivity state.

In the specification in this chapter, default has no reputation effects. The optimal default decision does not depend upon risk aversion. It does depend on the production function.

A corollary to this result is the ability of the government to issue debt in period 1. The government does not always set $h = 1$ in period 2. Therefore, the government is able to borrow from abroad in period 1.

Proposition 1.4 (Feasibility of Debt) *It is feasible for the sovereign to issue debt in period 1.*

The set G and the implied maximum level of debt depends on the production function in our model framework. The above result states that the upper bound of the aforementioned feasible set G is strictly positive.

1.4.3 Special Cases

The government does not default fully on all of its debt in period 2 if the domestic economy is sufficiently exposed to the adverse consequences of default. To illustrate this exposure mechanism more clearly, we analyze some special cases of the model.

Case 1: $\alpha = 0$ Domestic banks only hold cash, and all defaultable debt is held by foreigners. Consumption in period 2 is given by

$$c_2 = y_2 - (1 - h)B_f + \tilde{R}f(D)$$

The optimal haircut is $h = 1$ for all realizations of the productivity shock \tilde{R} . The price of debt in period 1 is given by rational expectations:

$$\begin{aligned} q &= \frac{1}{1+r} \mathbb{E} \left\{ 1 - h \left(\alpha, D, B_f, \tilde{R} \right) \right\} \\ &= 0, \end{aligned}$$

which immediately yields the result that $z = 0$. No debt can be sustained.

Case 2: $\alpha = 1, D = \bar{D}, B_f > 0$ Cash does not exist, so domestic banks must invest solely in defaultable debt. Consumption in period 2 is given by

$$c_2 = y_2 - (1 - h)B_f + \tilde{R}f((1 - h)\bar{D})$$

The optimal haircut is $h \in (0, 1)$ for all realizations of the productivity shock \tilde{R} . The price of debt in period 1 is given by rational expectations:

$$\begin{aligned} q &= \frac{1}{1+r} \mathbb{E} \left\{ 1 - h \left(\alpha, D, B_f, \tilde{R} \right) \right\} \\ &> 0, \end{aligned}$$

For $B_f > 0$, this means that $z > 0$. The sovereign can raise resources from abroad.

The special cases above illustrate that the government's debt issuance decision in period 1 affects default decisions in period 2. In the following subsections, we analyze the optimal government debt issuance decision in period 1.

1.4.4 Feasible Debt Levels as a Function of the Domestic Exposure Level

In this and the next two subsections, we focus on the intratemporal dimension of the problem. In other words, we take the level of z as given and find the optimal combination $C = (\alpha, D, B_f)$

that raises this level of resources from abroad in period 1. The haircut decision in period 2 is also taken as given. First, we characterize the relationship between domestic exposure of the economy and the ability of the government to issue debt abroad.

The following proposition applies when the government chooses to save rather than borrow.

Proposition 1.5 (Saving) *For $z < 0$, the government chooses: (i) $\alpha = 0$; (ii) $D = \bar{D}$; (iii) $B_f = (1 + r)z$.*

Foreigners can credibly commit not to default on the government's savings. The government has no default decision of its own in the final period since domestic exposure is set to zero. It chooses the total quantity of domestic debt to maximize domestic production. In other words, the case where $z < 0$ is a version of the standard model with noncontingent debt and no sovereign default.

Now let us focus on the case where $z > 0$. The sovereign can raise resources from abroad in the initial period to the extent that it can be relied upon to make repayments in the final period. The key result in this chapter is that the sovereign can issue debt abroad if domestic agents also hold defaultable debt, because in that case the event of default has a concomitant output cost, and this output cost prevents full default on debtholders. What level of domestic exposure is needed in order to raise any given level of debt from abroad? The answer to this question is characterized in the next proposition.

Proposition 1.6 (Minimum Domestic Exposure) *Fix $D = D'$. For any level of borrowing in the set $[0, z_{\max}(D')]$ to be achieved, it is required that the level of domestic exposure is sufficiently high, i.e., $\alpha \in [\underline{\alpha}(z), 1]$. The necessary exposure level has the following properties:*

1. $\underline{\alpha}(0) = 0$.
2. $\underline{\alpha}(z)$ is weakly increasing in z .
3. $\underline{\alpha}(z_{\max}(D')) = 1$.

This is a feasibility result: it is a description of the feasible set of combinations $C = (\alpha, D, B_f)$ available to the government to raise any given level of resources from abroad z . The shape of the function $\underline{\alpha}(z)$ depends on the production function.

For fixed $D = D'$, increasing the exposure of the domestic economy to the adverse effects of default reduces the optimal haircut of the government, by raising the costs of default relative to the benefits. This in turn sustains more debt in the first period. In other words, although the government suffers from a problem of lack of commitment, it can effectively “purchase commitment” by increasing the vulnerability of the domestic economy to a default episode. This vulnerability is what sustains foreign debt issuance, and is a necessary side-effect of the sovereign’s lack of commitment. Recommendations from international financial institutions to reduce financial fragility will also diminish the ability of the country to borrow from abroad.

Numerical characterization of the minimum domestic exposure function $\underline{\alpha}(z)$ utilizes the following production function:

$$f(x) = \begin{cases} x^\theta - \delta x & \text{for } x \leq \bar{x} \\ \bar{x}^\theta - \delta \bar{x} & \text{for } x > \bar{x} \end{cases}$$

where \bar{x} is set to the value that maximizes $f(x)$, i.e., $\bar{x} = \left(\frac{\theta}{\delta}\right)^{\frac{1}{1-\theta}}$. This production function satisfies the assumptions in subsection 1.2.1. In addition, it satisfies the property that $\lim_{x \rightarrow 0} x f'(x) = 0$.

Parameter values are selected as follows. The riskless rate of return is equal to $r = 0.05$ and the production function parameters are $\theta = \delta = 0.5$. The implied \bar{x} (and hence \bar{D}) is therefore equal to unity. In each period, there are ten possible values of the productivity shock, which occur with equal probability. Possible values of the shock are located between $\underline{R} = 8$ and $\bar{R} = 12$, with equal intervals between the possible shock realizations. Figure 1.4 plots the production function for different values of the shock. With the above parametrization, the upper bound of the set G is $z_{\max} = 1.1857$. This is the largest value of debt the economy can support.

Figure 1.5 plots the function $\underline{\alpha}(z)$ for this production function, setting $D = \bar{D}$. It satisfies the properties described in the above proposition.

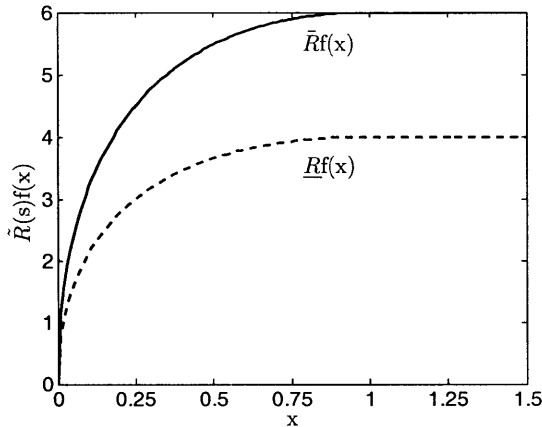


Figure 1.4: Production Function for Numerical Simulations

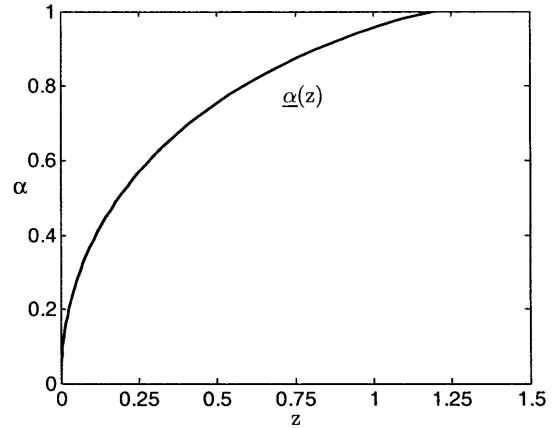


Figure 1.5: Debt Level z and Minimum Domestic Exposure Level $\alpha(z)$

1.4.5 Optimal Levels of Domestic Debt and Domestic Exposure

The optimal level of total domestically held debt (cash and defaultable debt) is given by the following proposition.

Proposition 1.7 (Total Domestic Debt) *It is an optimum to set $D = \bar{D}$.*

Consider figure 1.3. For any value of the productivity shock \tilde{R} , the government may default on a portion of its defaultable debt to domestic and foreign lenders. This limits the level of inputs into the productive sector in that state. In the specification with nontradable debt, the government never wishes to constrain itself to produce less than the level mandated by the intersection of the $f'(x)$ function with the horizontal line corresponding to the productivity shock \tilde{R} in period 2. For $B_f > 0$, it is possible to achieve this using a range of values of D . One of these is the output-maximizing value, \bar{D} . For $B_f = 0$, only the output-maximizing value of D is optimal. So let us set $D = \bar{D}$ irrespective of the value of B_f .

What does this mean for haircuts? When there is any foreign debt, the government will find it optimal to default on some of it in all states of nature. The economic reason is that at $h = 0$, an increase in the haircut has a first order impact on reducing repayments, but only a second order output cost (since $f'(\bar{D}) = 0$). This is true for all values of the productivity

shock \tilde{R} . The effective interest rate facing the country will exceed the riskless rate owing to a default premium.

In the previous subsection, we described the minimum domestic exposure level that is necessary to raise any given level of debt z . This depended on the production function specification. The actual level of domestic exposure chosen depends upon the risk aversion of the representative consumer. The next proposition states this result.

Proposition 1.8 (Optimal Domestic Exposure) *Consider a combination $C = (\alpha, \bar{D}, B_f)$ which raises debt z such that $\alpha > \underline{\alpha}(z)$. At the margin, it is feasible to raise the same level of debt z by reducing α and increasing $\gamma = \frac{B_f}{\alpha \bar{D}}$. Whether this perturbation is optimal depends on the risk aversion of the representative consumer.*

If the domestic exposure level associated with a particular combination $C = (\alpha, \bar{D}, B_f)$ is above the value necessary to raise the level of debt z , it is feasible for the sovereign to reduce the domestic exposure level and still raise the same amount of resources from abroad. For $z > 0$, the perturbation described in the proposition raises the haircuts and reduces consumption for the highest values of the productivity shock \tilde{R} in period 2. The effect on average consumption in period 2, and on consumption for the lowest values of \tilde{R} , depends on the production function. There exist permissible production functions such that the perturbation reduces average consumption but increases consumption for the lowest values of \tilde{R} . The latter effect arises either via an increase in output (if $h = 1$ is binding for this productivity shock), or a decrease in repayments abroad in excess of the reduction in output (if h is interior). Suppose that consumption in the lowest states of nature does indeed increase as a result of the perturbation while average consumption falls. Then the perturbation considered is analogous to purchasing an insurance contract across states of nature in period 2. The desirability of such a perturbation then clearly depends on the risk aversion of the representative consumer.

For the numerical exercise in this subsection, we utilize the same parameterization of the production function that was used to illustrate the minimum domestic exposure result. The following functional form is used for the utility function:

$$u(c) = \frac{-e^{-\psi c}}{\psi}$$

The value of the endowment income in period 2 is set to $y_2 = 9$ and the discount factor is $\beta = 0.8$. Figure 1.6 plots the optimal domestic exposure as a function of the debt level z , for $\psi = 10, 60$ and 100 . As the coefficient of absolute risk aversion increases, the optimal domestic exposure for any given debt level weakly decreases. For $\psi = 10$, the optimal exposure is $\alpha = 1$ for all debt levels. For $\psi = 100$, the optimal domestic exposure is very close to the minimum feasible exposure level.

Consider the special case of $u(c) = c$, so that the representative consumer is risk neutral. From the numerical simulations, it is clear that the optimal domestic exposure is $\alpha = 1$ for all debt levels z .

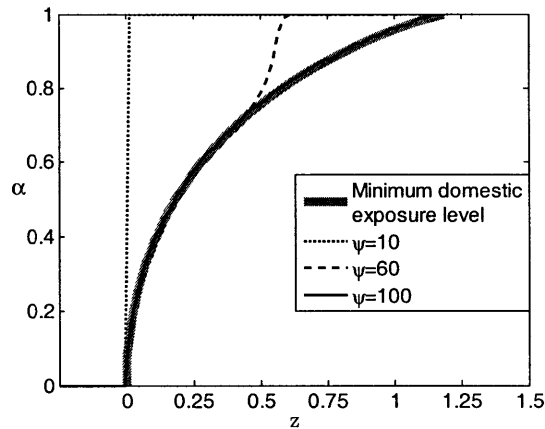


Figure 1.6: Optimal Domestic Exposure Level

1.4.6 Foreign Debt Issuance

In the model with nontradable debt, it remains to determine the volume of foreign debt issuance associated with any level of resources raised from abroad z . In general, the domestic exposure and foreign debt issuance jointly comove as z varies. To help provide intuition for the movements in B_f , the following proposition applies for cases where the optimal domestic exposure level is always given by $\alpha = 1$.

Proposition 1.9 (Foreign Debt Issuance) *Let $u(c)$ and $f(x)$ be specified such that $\alpha = 1$ is the optimal level of exposure for all levels of debt. Then for $z \in [0, z_{\max}]$:*

1. B_f is increasing in z .
2. The interest rate on government debt is increasing in z and the volume of foreign debt issuance B_f .

The intuition for this result comes from an examination of the effects of an increase in B_f . This pushes up the ratio of foreign-held to domestically-held debt, $\frac{B_f}{D}$. Such an increase reduces output in every state because the optimal haircut on domestic debt rises. It has two opposing effects on repayments. On the one hand, the rise in the ratio increases the optimal haircut, and this pushes repayments down. On the other hand, the higher value of B_f raises repayments for any given haircut. Because the haircut is optimally determined, an Envelope Theorem argument establishes that the first of these effects on the repayment exactly offsets the output effect, in terms of their respective effects on consumption in period 2. Therefore, the effect of an increase in foreign debt issuance is a reduction in consumption c_2 for every value of the productivity shock \tilde{R} . A corollary of this result is that the government chooses the lowest magnitude of B_f that achieves any given level of borrowing z . It is straightforward to prove that this yields an upward-sloping relation between B_f and z .

Therefore, as z rises, so does the ratio of foreign-held to domestically-held debt. The optimal haircut rises for all values of the productivity shock \tilde{R} in the final period. The increased default risk is reflected in a higher default risk premium on government bonds.

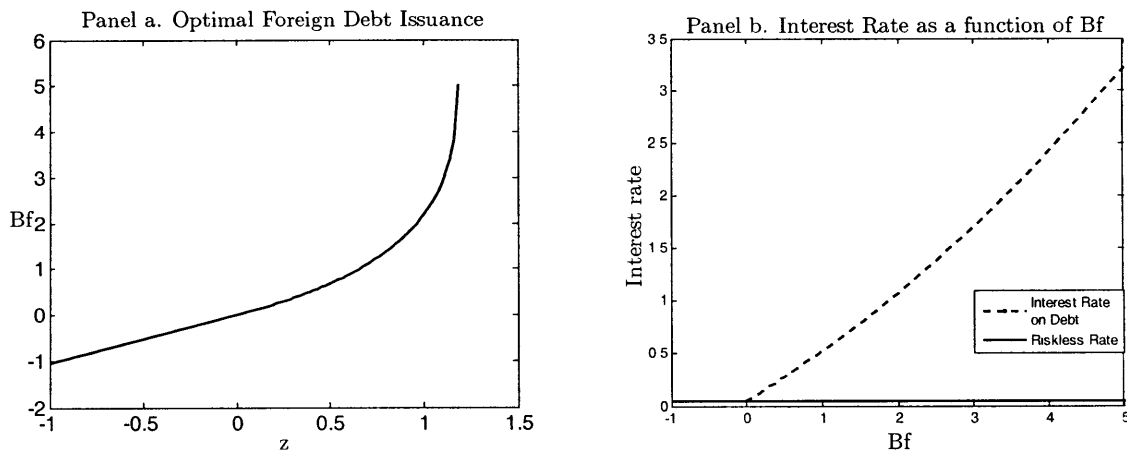


Figure 1.7: Optimal Foreign Debt Issuance and Interest Rates

Numerical implementation of the model utilizes the functional forms for utility and production functions given above, with $\psi = 10$. As mentioned above, the upper bound of the set G is $z_{\max} = 1.1857$. Panel a of figure 1.7 captures the evolution of foreign debt issuance B_f as a function of the total level of resources raised z . There is a positive relation between the two variables. As the level of debt grows, the magnitude of B_f required increases at a faster rate. This is because as foreign debt issuance increases, the optimal haircut in period 2 increases for all values of the productivity shock \tilde{R} , and the price of foreign debt in period 1 declines. This is reflected in the interest rate schedule shown in panel b. The interest rate shown by the blue line is the promised rate to be paid on all debt. The green line is the riskless rate.

1.4.7 Euler Equation

In this subsection, we focus on the intertemporal dimension of the problem. This subproblem determines the optimal choice of z by the sovereign, taking the optimal combination schedule $C = (\alpha, D, B_f)$ for any level of debt z as given.

The government problem may be written as follows.

$$\max_z \mathbb{E} \left\{ u(y_1 + z) + \beta u \left(c_2 \left(z, \tilde{R} \right) \right) \right\}$$

subject to

$$y_1 + z \geq 0$$

$$c_2 \left(z, \tilde{R} \right) \geq 0$$

$$z \in G.$$

We define $c_2 \left(z, \tilde{R} \right)$ to be the optimal schedule of consumption across states of nature \tilde{R} in period 2, for any given debt level z chosen in period 1. In other words, for any given debt level z , we choose the optimal combination $C^* = \left(\alpha^*, D^*, B_f^* \right)$ that achieves this debt level. $c_2 \left(z, \tilde{R} \right)$ is the schedule of consumption across states of nature in period 2 corresponding to the combination C^* .

Let us now focus on the choice of z in the government's intertemporal subproblem, and suppress the dependence of c_2 on $\left(z, \tilde{R} \right)$. The Euler equation is derived for z in the interior of

G :

$$u'(y_1 + z) = \beta \mathbb{E} \left\{ u'(c_2) \cdot \left| \frac{dc_2}{dz} \right| \right\}, \quad (1.11)$$

where the equality is replaced with an inequality \geq for z at the upper boundary of G^2 .

For $z < 0$, Proposition 1.5 applies and the Euler equation reduces to the standard formula for nondefaultable noncontingent assets, with $\left| \frac{dc_2}{dz} \right| = 1 + r$.

In the range of debt $z \geq 0$, $\frac{dc_2}{dz}$ depends on how the optimal combination $C^* = (\alpha^*, D^*, B_f^*)$ evolves as z changes. Proposition 1.7 states that we may restrict attention to optimal combinations where $D^* = \bar{D}$. To help us understand some basic properties of the government's intertemporal problem, let us assume that the functions $u(c)$ and $f(x)$ are defined such that the optimal domestic exposure α is equal to 1 for all levels of debt. We derive the following result.

Proposition 1.10 (Intertemporal Problem) *Let $u(c)$ and $f(x)$ be specified such that $\alpha = 1$ is the optimal level of exposure for all levels of debt. Then for $z \in [0, z_{\max}]$:*

1. $\frac{dc_2}{dz} < 0$ for all values of the productivity shock \tilde{R} .
2. $\left| \mathbb{E} \left[\frac{dc_2}{dz} \right] \right| > 1 + r$.
3. $\left| \frac{dc_2}{dz} \right|$ is increasing in the value of the productivity shock \tilde{R} .

Now to interpret these three findings. The sign of the derivative in the first result indicates that an increase in the magnitude of z in period 1 necessarily involves a reduction in consumption for every value of the productivity shock \tilde{R} in period 2. This result is to be expected. In our environment, an increase in z necessitates an increase in foreign debt issuance B_f , and hence the ratio $\frac{B_f}{D}$. This reduces consumption in all states of nature, as explained in the previous subsection.

If the government has the ability to commit to repay, and only noncontingent debt is available, then the government simply repays the gross return of $1 + r$ on its debt at every state of nature in the final period. In our environment, the sovereign does not have the ability to

²It is possible that a marginal change in the value z causes a jump in the value of c_2 . We abstract from this in the main text. The appendix discusses the more general case.

commit to repay its debt. It must “purchase commitment” by exposing the economy to an output cost in the event of default. The true cost of debt issuance in the first period of this model is the reduction in consumption in the second period. Result 2 of the proposition above states that the expected consumption cost of a marginal unit of debt exceeds the gross riskless rate $1 + r$. So in expected terms, foreign lenders receive the riskless rate on sovereign debt, but the domestic economy pays a cost that exceeds this rate of return. In other words, the cost of “purchasing commitment” is a series of deadweight losses across states of nature in the final period.

Yet of course, the government does not pay the same consumption cost in every state of nature. The third result establishes that although an increase in the magnitude of debt depresses consumption across all states of nature of the final period, the reduction in consumption is largest in the best states of nature and less severe in the worse states. Thus, the consumption cost of debt issuance varies across states of nature in a direction that we would expect from fully contingent debt. Our model supports the contention of Grossman and van Huyck (1988) that default imparts a contingent property to noncontingent debt. However, note that there is no reason why the variation in the consumption cost in our model should have the same magnitude as that observed in the first best case. In particular, for fully contingent debt with government commitment, repayments in a particular state of nature for a unit of debt would depend on the productivity realization \tilde{R} and the maximum value of output $f(\bar{x})$. In our environment, repayments depend on \tilde{R} , and additionally on the ratio of foreign to domestic defaultable debt and the marginal product schedule $f'(x)$. The latter two are not relevant in the first best case.

Consider the special case when the representative consumer has an infinite elasticity of intertemporal substitution between periods. For $\beta(1 + r) > 1$, the government’s decision in the standard case with nondefaultable noncontingent debt is to save the entire endowment: $z = -y_1$. This is still the optimal policy in our framework. However, the optimal policy in our model differs from the standard case for $\beta(1 + r) < 1$, for two reasons. Firstly, in our framework it is not possible to borrow any more than z_{\max} . Secondly, the higher is the government’s chosen value of z , the lower is average consumption in period 2, and the expected consumption cost of borrowing exceeds $1 + r$. This may reduce the optimal level of borrowing z by the government, if the expected consumption cost is large enough.

Numerical implementation of the model utilizes the functional forms for utility and production functions given in the previous subsection. Figure 1.8 presents the optimal level of borrowing as a function of y_1 , the endowment in period 1. For this specification, the government borrows less when the endowment in the initial period is higher.

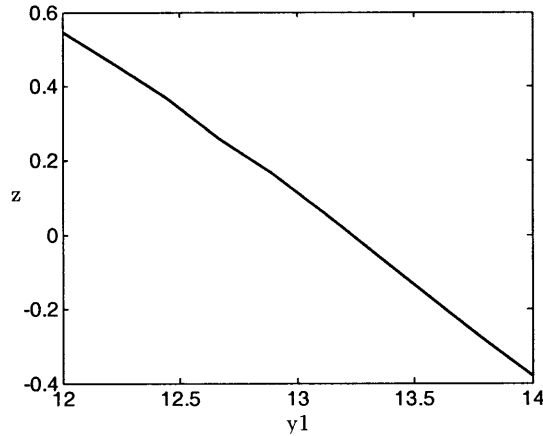


Figure 1.8: Optimal Debt Level as a Function of the Initial Endowment

How can we use the above findings to compare different economies? One simple comparison between an emerging market and a developed country in the context of our model framework proceeds as follows. For any given productivity shock distribution in period 2, the emerging market economy has a lower endowment in period 1. In other words, its expected output growth between periods is higher. Holding the discount factor β constant, the optimal response is for the emerging market economy to borrow more in the initial period. In order to implement this borrowing level, the emerging market economy finds it optimal to issue a greater fraction of its debt to foreign as opposed to domestic debtholders – which means that it has a higher $\frac{B_f}{B_d}$ ratio.

The emerging market economy enters period 2 with a higher $\frac{B_f}{B_d}$ ratio than the developed economy. For any value of the productivity shock \tilde{R} in period 2, the emerging market economy exhibits higher haircuts and lower output. In fact, because the haircut is higher for lower values of the productivity shock, realized output in period 2 will be an amplification of the productivity

shock. Therefore, even if the shock process in period 2 is the same for all economies, the output of the emerging market economy will appear to be more volatile.

1.5 Tradable Debt

When defaultable debt is not tradable in the period of issue, the government determines directly the proportions of cash and defaultable debt in the portfolio of the banking system. This enables us to focus exclusively on the exposure channel for sustaining debt. Foreign creditors are willing to lend to the sovereign in period 1 because if the output cost of default in period 2 is sufficiently large, some debt repayments will be made. In this section, we consider a model where defaultable debt is tradable between foreign creditors and domestic banks in the period of debt issuance. This adds an additional restriction on the set of feasible debt values – namely, that the valuation of the marginal unit of debt by domestic and foreign bondholders must be equal.

The specification with tradable debt quickly becomes intractable. Therefore, the approach we take is as follows. First, we construct the program for the government problem. Then we provide some analytical results for a special case of the model: the linear utility case. Clearly, the choice of the debt level z in this case is trivial, so we do not explore this. We focus instead on understanding how the optimal combination $C = (\alpha, D, B_f)$ that raises any given level of debt z differs from the nontradable debt case. The final subsection returns to the general case with a concave utility function. In this section, we provide numerical simulations to illustrate the results for the model with tradable debt.

1.5.1 Construction of Program

Definitions 1.1 and 1.2 may again be used to derive the program for the government problem, under the additional restriction:

$$p_B = q.$$

The government problem may be written as follows. In period 1:

$$U_1 = \max_{c_1, \alpha, D, B_f} \left\{ u(c_1) + \beta \mathbb{E} U_2 \left(\alpha, D, B_f, \tilde{R} \right) \right\}$$

subject to

$$c_1 = y_1 + qB_f$$

$$c_1 \geq 0$$

$$\begin{aligned} q &= \frac{1}{1+r} \mathbb{E} \left\{ 1 - h \left(\alpha, D, B_f, \tilde{R} \right) \right\} && \text{for } B_f > 0 \\ &\geq && \text{"} && \text{for } B_f = 0 \end{aligned}$$

$$\begin{aligned} qu'(c_1) &= \beta \mathbb{E} \left\{ u'(c_2) \cdot (1-h) \cdot \left[1 + \tilde{R}f' \left([(1-\alpha) + (1-h)\alpha] D \right) \right] \right\} && \text{for } \alpha D > 0 \\ &\geq && \text{"} && \text{for } \alpha D = 0 \end{aligned}$$

$$B_f < 0 \Rightarrow \alpha = 0$$

where the expression for the period utility in period 2 is given by

$$U_2 \left(\alpha, D, B_f, \tilde{R} \right) = \max_{c_2, h} u(c_2)$$

subject to

$$c_2 = y_2 - (1-h)B_f + \tilde{R}f' \left([(1-\alpha) + (1-h)\alpha] D \right)$$

$$c_2 \geq 0$$

$$y_2 \geq (1-\alpha)D + (1-h)[\alpha D + B_f] \tag{1.12}$$

$$0 \leq h \leq 1.$$

We again assume that y_2 is large enough so that the constraint (1.12) is never binding (we impose $\lim_{x \rightarrow 0} xf'(x) = 0$).

Since there is a continuum of domestic banks, all the gains from investments in period 1 accrue in the end to the consumers in period 2. The willingness of domestic banks to purchase defaultable debt depends on the discount factor of the representative consumer and the haircuts on defaultable debt in period 2. The Euler equation of the representative consumer for

defaultable debt must be satisfied.

In the specification with nontradable debt, it is necessary to satisfy the Euler equation for domestic consumers regarding their holdings of the cash and defaultable debt. The prices of these debt categories, p_A and p_B , do not appear in any other equation, and therefore the Euler equations are redundant constraints for the government problem. In particular, it is possible to solve the government problem and then derive the prices p_A and p_B as residuals of the exercise. With tradable defaultable debt, the two Euler equations corresponding to the two types of debt must also be satisfied for the representative consumer. The price of cash p_A can again be derived as a residual of the government problem. However, since the defaultable debt is tradable in the period of issue, the same price q must be faced by domestic and foreign lenders. Therefore, the Euler equation for defaultable debt is not redundant.

We continue using the (α, D, B_f) notation to characterize the problem. Of course, the government can no longer select α , D and B_f arbitrarily. The government issues a quantity B of defaultable debt, which is divided between $B_d = \alpha D$ and B_f according to the portfolio decisions of domestic banks and foreign creditors. But since the government understands the portfolio decisions and equilibrium conditions, it is equivalent to consider a problem where it directly selects a combination $C = (\alpha, D, B_f)$, subject to the condition that this combination is realized in a rational expectations equilibrium.

The problem in period 2 is unchanged. The haircut function $h = H(\alpha, D, B_f, \tilde{R})$ will be exactly the same as in the case with nontradable debt. This can be used to derive the bond price schedule $Q(\alpha, D, B_f)$. Using similar techniques to those applied for the nontradable debt case, we derive the following representation of the government problem.

$$V_1 = \max_{c_1, z} \mathbb{E} \{u(c_1) + \beta V_2(z, \lambda)\}$$

subject to

$$c_1 = y_1 + z$$

$$c_1 \geq 0$$

$$\lambda = u'(c_1)$$

$$(z, \lambda) \in \tilde{G}$$

for some set \tilde{G} .

$t = 2'$:

$$V_2(z, \lambda) = \max_{c_2, \alpha, D, B_f} \mathbb{E} \{u(c_2)\}$$

subject to

$$c_2 = y_2 - (1 - h)B_f + \tilde{R}f([(1 - \alpha) + (1 - h)\alpha]D)$$

$$h = H(\alpha, D, B_f, \tilde{R})$$

For $z < 0$, combinations $C = (\alpha, D, B_f)$ satisfy: $\alpha = 0, B_f < 0$.

For $z = 0$, combinations $C = (\alpha, D, B_f)$ satisfy: $B_f = 0, \alpha D = 0$.

For $z > 0$, combinations $C = (\alpha, D, B_f)$ satisfy:

$$\frac{1}{1+r} \mathbb{E}(1-h) = \beta \mathbb{E} \left\{ \frac{u'(c_2)}{\lambda} \cdot (1-h) \cdot \left[1 + \tilde{R}f'([(1-\alpha) + (1-h)\alpha]D) \right] \right\}. \quad (1.13)$$

Our notation suppresses the dependence of h on $(\alpha, D, B_f, \tilde{R})$.

There are now two state variables for the problem, z and λ , the marginal utility of consumption in period 1. Relative to the nontradable debt case, we have added the constraint that foreign creditors and the domestic representative consumer value the marginal unit of debt equally. Equation (1.13) is the equal marginal valuation constraint.

For debt to exist, i.e., $z > 0$, both domestic banks and foreign creditors must hold defaultable debt. If domestic banks hold all the debt, no resources are raised from abroad so $z = 0$. If foreign creditors hold all the debt, the government defaults fully on the debt in period 2. Defaultable debt will have price zero in period 1, so again $z = 0$. We can ignore combinations $C = (\alpha, D, B_f)$ where defaultable debt is held solely by either foreigners or domestic banks: they are allocationally equivalent to a configuration $C' = (\alpha', D', B'_f)$ where no defaultable debt is issued.

With tradable debt, the shape of the utility function and the discount factor β are relevant for the characterization of the feasible set. The intuition for this result is as follows. The valuation of defaultable debt by the representative consumer depends on the utility function

and the discount factor β , from the Euler equation. This may differ from the valuation of the foreign creditors for two reasons. Firstly, the expected return from holding defaultable debt is different for domestic banks and foreign creditors, because domestic banks have the extra option of lending their total post-haircut assets to the firms, and they receive a loan rate from this market. Therefore, domestic banks receive a higher gross return from holding defaultable debt than the foreign creditors. Secondly, foreign creditors have a linear utility function, whereas for the representative consumer $u''(c) < 0$ is possible. This means that at the margin, the valuation of defaultable debt by the representative consumer depends on the consumption level in periods 1 and 2. This is not true for the foreign creditor.

Subsection 1.5.3 assumes a linear utility function for the representative consumer and establishes a restriction on the discount factor β for debt to be sustained. The purpose of this subsection is to examine the feasible set for debt when both the representative consumer and foreign creditors have the same functional form for the period utility function $u(c)$. The analysis turns on the differences between the gross return on defaultable debt received by domestic banks and foreign lenders.

After this, we consider the case where the representative consumer has a period utility function with declining marginal utility, $u''(c) < 0$. Relative to the linear utility case, this framework adds the additional factor that a marginal unit of consumption is valued differently across different points in time and states of nature, depending on the consumption level. Numerical simulations are employed to characterize the solution of the model.

1.5.2 Haircut Decision

The following proposition reiterates the observation regarding the haircut decision in this framework.

Proposition 1.11 (Haircut Decision) *The haircut decision $h = H(\alpha, D, B_f, \tilde{R})$ in the tradable debt case is exactly the same function as in the nontradable debt case.*

1.5.3 Special Case: Linear Utility Function

We provide some analytical results for the case where the utility functions of both the domestic representative consumer and foreign creditors are linear, i.e., $u(c) = c$. The state variable λ

is always equal to unity and is redundant. Domestic banks are willing to purchase an infinite quantity of defaultable debt at any price less than

$$p_B = \beta \mathbb{E} \left\{ (1-h) \cdot \left[1 + \tilde{R}f' \left([(1-\alpha) + (1-h)\alpha] D \right) \right] \right\},$$

Foreign creditors are willing to purchase an unlimited quantity at any price below

$$q = \frac{1}{1+r} \mathbb{E}(1-h).$$

For the tradable debt case, $p_B = q$:

$$\frac{1}{1+r} \mathbb{E}(1-h) = \beta \mathbb{E} \left\{ (1-h) \cdot \left[1 + \tilde{R}f' \left([(1-\alpha) + (1-h)\alpha] D \right) \right] \right\}. \quad (1.14)$$

This is the equal marginal valuation constraint in the linear utility case. The following propositions immediately follow.

Proposition 1.12 (Feasibility of Debt) *If and only if the discount factor $\beta \in \left(0, \frac{1}{1+r}\right)$, it is feasible for the sovereign to issue debt.*

Proposition 1.13 (Maximum Level of Debt) *The maximum level of debt depends on β and is weakly lower than in the nontradable debt case.*

For $\beta = \frac{1}{1+r}$, equation (1.14) is inconsistent with a debt level $z > 0$. Consider any combination $C = (\alpha, D, B_f)$ with $B_f > 0$. Debt can be raised in period 1, i.e., $z > 0$, if there is repayment in some states of nature in period 2. For those states of nature where there is repayment, it is also true that $\tilde{R}f'(x) > 0$. The representative consumer receives a higher gross return than foreign creditors from holding debt. If they both discount future periods at the same rate, domestic debtholders value the defaultable debt more and will hold all of the debt. Foreign agents are not able to hold any defaultable debt, so no resources can be borrowed from abroad in period 1. Therefore $z = 0$. Debt cannot be supported in equilibrium. For $\beta = 0$, equation (1.14) is again inconsistent with debt. The representative consumer places a zero value on the debt. At any positive price, it will not hold any of the defaultable debt. Therefore, there is no output cost of default in period 2. The optimal policy for the government is to default on

all of its debt in the final period, which means the price of debt in period 1 must be zero by rational expectations. It is not feasible for the government to issue debt at a positive price in the first period.

The arguments in the previous paragraph illustrate the new constraint in the tradable debt case: it is important that domestic banks can be persuaded to hold some, but not all, of the defaultable debt. If they want to hold all of the debt, foreigners cannot lend to the government. If they want to hold none of the debt, the exposure mechanism examined in the previous section of the chapter immediately gives us the result that government debt has no value to the foreigners.

We turn to the optimal combination $C = (\alpha, D, B_f)$ chosen to raise a given level of debt z in the linear utility case. If the government saves abroad, the combination $C = (\alpha, D, B_f)$ is again given by the following proposition.

Proposition 1.14 (Saving) *For $z < 0$, the government chooses: (i) $\alpha = 0$; (ii) $D = \bar{D}$; (iii) $B_f = (1 + r)z$.*

Next consider $z > 0$. An amended version of Lemma 1.1 holds for the specification with tradable debt. This result allows us to focus on a restricted subset of D .

Lemma 1.2 *Define $\bar{D} = \bar{x}$. For any combination $C = (\alpha, D, B_f)$ such that $D > \bar{D}$, there exists some other combination $C' = (\alpha', D', B'_f)$ where $D' = \bar{D}$, such that C' raises the same revenues as C in period 1 and is equivalent to C in terms of repayments abroad, output and hence consumption for all values of the productivity shock \tilde{R} in period 2. C and C' satisfy the Euler condition for defaultable debt for the same value of λ .*

Corollary 1.2 *We can restrict our attention to combinations $C = (\alpha, D, B_f)$ such that $D \in [0, \bar{D}]$.*

We seek to analyze how the feasible set of debt values is different relative to the nontradable debt case. The next proposition establishes that the equal marginal valuation restriction reduces the size of the feasible set.

Proposition 1.15 (Feasible Domestic Exposure Levels) *Fix $D = D'$. The feasible set of (α, z) values is a subset of the feasible set in the nontradable debt case.*

Figure 1.9 illustrates this result. Fix $D = D'$. The bold lines are the boundaries of the feasible set in the nontradable debt case. For the tradable debt case, we show the feasible set for a typical case. The set of feasible allocations is the line $\alpha_T(z)$.

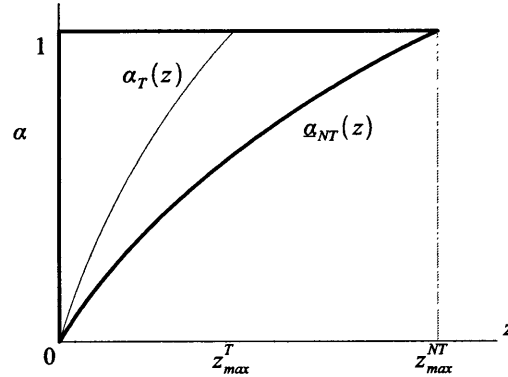


Figure 1.9: Feasible Set in the Tradable Debt Case

The intuition is as follows, for the case where the government issues debt such that in period 2, it repays debt fully in some states of nature and defaults on a portion of it in others. The exposure mechanism explained in the nontradable debt case also operates here. Therefore for fixed $D = D'$, a reduction in the exposure level α causes an increase in the haircut in those states of nature where the haircut was initially interior. This reduces the price that foreigners are willing to pay for the debt. It also reduces the price that domestic banks are willing to pay, but by a lesser amount. Therefore, in equilibrium the price q falls and the fraction of defaultable debt held by foreigners, $\gamma = \frac{B_f}{\alpha D}$, declines. Unlike in the nontradable debt case, it is not possible to vary α and γ independently, because of the equal marginal valuation constraint. Therefore, the feasible set is reduced. However, the relationship between exposure levels and the haircut is still given by the exposure mechanism described in the nontradable debt case.

Proposition 1.16 (Total Domestic Debt) *Total domestic debt may be less than \bar{D} at the optimum.*

Proposition 1.7 does not hold in the case with tradable debt. The intuition is as follows. Let us begin with total domestic debt initially set to \bar{D} , and suppose that the government wishes to increase expected consumption in period 2. There are two means by which it can implement this. Firstly, it can keep both D and the proportion of debt held by foreign relative to domestic lenders $\gamma = \frac{B_f}{\alpha D}$ fixed, and reduce the exposure level of the domestic economy. This reduces repayments in period 2, hence increasing consumption in that period. However, the exposure mechanism dictates that this policy necessarily reduces consumption in period 1 by reducing the debt that can be raised by the government in period 1. The desirability of such a policy depends on β .

Alternatively, the government can reduce both D and γ . Let us consider a reduction in D large enough to reduce output for the highest productivity state of nature. In this state of nature, it also increases the loans rate in the market for loans in period 2. This increases the return to holding defaultable debt for the domestic banks for this state of nature, which means that the proportion of defaultable debt held by domestic banks rather than foreigners must rise. Therefore, γ declines. This perturbation also increases expected consumption in period 2, but in this case it is possible (although not certain) that the debt level z rises due to a price effect on government debt. The higher proportion of debt held by domestic bondholders in period 1 reassures foreign creditors that repayments will be made by the government in period 2, and this increases the price of government debt sufficiently to increase the debt level z . Therefore, consumption in both periods may rise. In other words, in this case it is desirable (for any value of β) for the government to constrict output in the second period in order to induce domestic banks to hold debt, which reduces the interest rate on the debt.

For concave utility, additional factors work to reduce the total domestic debt level below \bar{D} . Firstly, suppose that the perturbation above reduces consumption in period 1 (via a change in z). Such a perturbation may still be desirable for a risk averse representative consumer, because consumption increases for the worst productivity shock values in period 2. The perturbation offers insurance in these states of nature. Secondly, consider a reduction in D that does not reduce output in period 2, because the haircuts are always strictly interior. Nevertheless, such a perturbation changes haircuts in different states of nature. For the linear utility case, this affects the valuation of debt by foreigners and domestic banks equiproportionately, so it does

not change the proportion of debt in the hands of foreigners. If utility is concave, the marginal utility of domestic consumption varies across different states of nature, while the same is not true for foreign creditors. This means that the proportions of government debt held by foreign and domestic debtholders may indeed change, and this may be desirable.

1.5.4 Optimal Debt Issuance Policy for Concave Utility Case

For the concave utility case, the equal marginal valuation constraint is reproduced below. The marginal unit of debt must be valued equally by foreign creditors and domestic banks:

$$\frac{1}{1+r} \mathbb{E}(1-h) = \beta \mathbb{E} \left\{ \frac{u'(c_2)}{u'(c_1)} \cdot (1-h) \cdot \left[1 + \tilde{R} f' \left([(1-\alpha) + (1-h)\alpha] D \right) \right] \right\}.$$

The difference from the linear utility case is that now, the valuation of defaultable debt by the domestic representative consumer depends on the consumption levels in periods 1 and 2. This is not true for the foreign creditor. This alters the feasible set and makes the levels of the endowment y_1 and y_2 relevant, since they affect the marginal utility of consumption in the two periods.

The analysis of the linear utility case was presented in order to provide some analytical intuition for the tradable debt specification. For the case with concave utility, we present numerical simulations rather than further analytical exercises. The numerical exercise utilizes the same functional forms and parameter values for the production function presented in subsection 1.4.4. We use a constant relative risk aversion utility function

$$u(c) = \log(c)$$

The discount factor is $\beta = 0.8$. We set $y_1 = 15$ and $y_2 = 9$. For these parameters, it is not possible to borrow more than $z_{\max} = 0.3750$ units of real resources from abroad in period 1.

Panel a of figure 1.10 shows how the optimal exposure of the domestic economy varies with the asset level. For the region $z < 0$, α is set to zero and for $z = 0$, the level of α is irrelevant. For the region $z > 0$, full domestic exposure ($\alpha = 1$) is always optimal for the functional forms and parameter configuration detailed above. The optimal level of total domestic debt is shown in panel b. When the government decides to borrow from abroad, it finds it optimal to reduce

the magnitude of total domestic debt below the output-maximizing level \bar{D} . The optimal level of foreign debt issuance is monotonically increasing in the debt level.

How much does the government borrow from abroad? Figure 1.11 allows the initial endowment y_1 to vary and displays the debt choice of the government as a function of this endowment. The government borrows less if the initial endowment is higher.

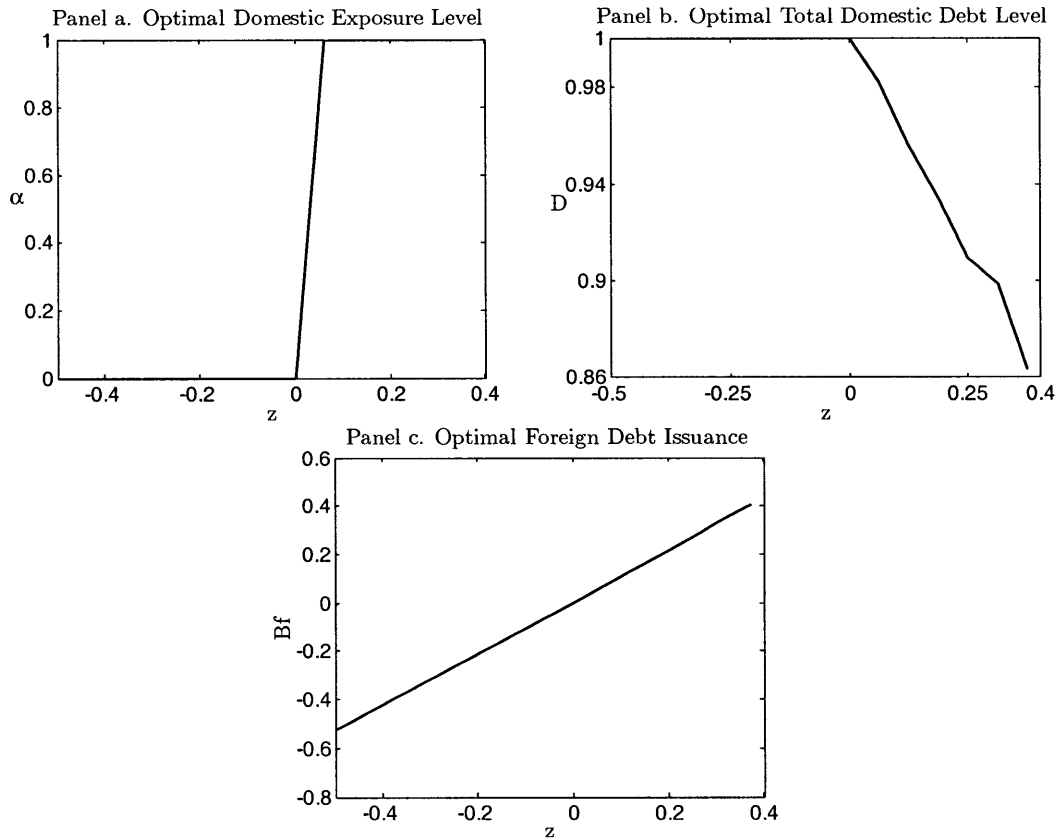


Figure 1.10: Optimal Debt Issuance Policy in the Tradable Debt Case

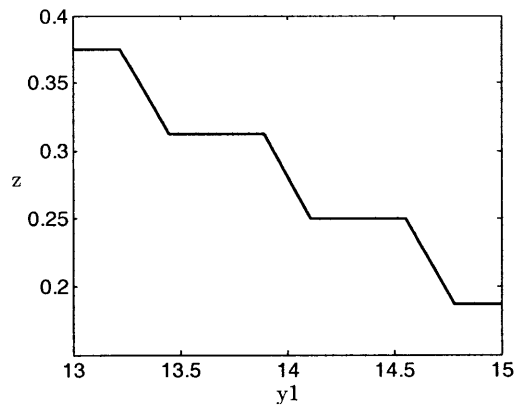


Figure 1.11: Optimal Debt Level as a Function of the Initial Endowment

1.6 Policy Implications

The model developed in this chapter provides an analytical framework within which to evaluate particular policy recommendations to improve the welfare of the economy.

1.6.1 Enhance Commitment Power

If the government can commit in period 1 to a schedule of haircuts in period 2, and if it can save abroad and issue debt at the same time, then the first best can be achieved. The key observation is that the exposure mechanism no longer applies. The government can commit to repay its debt even if domestic banks do not hold any of it. Therefore, there is no need for the economy to subject itself to output costs in period 2.

In our model, repayments to foreign creditors are increasing in the productivity shock \tilde{R} . Therefore, the debt repayments vary across states of nature in a direction that we would desire from fully contingent debt. However, the first best case has higher welfare than the lack of commitment case, for two reasons. Firstly, nothing ensures that the magnitude of variations in repayment in our model are of the same magnitude as in the first best case. In particular, for fully contingent debt with government commitment, repayments in a particular state of nature for a unit of debt would depend on the productivity realization \tilde{R} and the maximum value of

output $f(\bar{x})$. In our environment, repayments depend on \tilde{R} , and additionally on the ratio of foreign to domestic defaultable debt and the marginal product schedule $f'(x)$. The latter two are not relevant for the first best case. Secondly, the government must expose the domestic economy to output costs of default in order to be able to effectively commit to repay a portion of its debt. But ex post, such a policy results in output costs in period 2. These output costs are a deadweight welfare cost that is borne by the domestic economy.

Whether the first best can be implemented in the prevailing legal environment is questionable. In essence, courts must be able to enforce contractual adherence by the government to any promised repayment schedule. For example, the government could issue debt with court-enforceable repayments that are contingent on the productivity shock or the output level.

1.6.2 Improving Creditor Rights

Suppose that international institutions can strengthen the courts such that debtholders are able to enforce repayments of their claims on the sovereign, but that the government can only issue noncontingent debt. We consider the limiting case where the haircut is zero for all values of the productivity shock \tilde{R} . The model reduces to the framework with nondefaultable and noncontingent debt. Again, the government is able to commit to repay its debt without having to subject the domestic economy to output costs in period 2.

Such a policy has several distinct effects on welfare. Firstly, in the tradable debt case it may be desirable for the government to restrict output. It is no longer optimal to do so, and the economy benefits from this. Secondly, the economy may also benefit from an increase in the debt limit in the nondefaultable debt case as opposed to the defaultable debt case. Thirdly, the average consumption cost of borrowing falls to $1 + r$. Borrowing is no longer associated with a schedule of deadweight output losses borne by the domestic economy. The reduction in the average cost of borrowing benefits the domestic economy and tends to increase the level of borrowing in period 1. Finally, the policy may have one negative effect on welfare. In particular, the repayments on debt are no longer effectively contingent on the state of nature \tilde{R} . This hurts the domestic consumers in the case of concave utility, and tends to decrease the level of borrowing in period 1.

Numerical simulations to evaluate the effects of improving creditor rights are presented be-

low in figure 1.12. The parametrization of the model follows the specification used in subsection 1.5.4. Utility is higher in the full repayment case than in the nontradable and tradable debt specifications of the model in this chapter. The level of borrowing by the government is substantially higher in the case with improved creditor rights. This shows that the reduction in the average consumption cost of borrowing dominates the removal of the contingency of debt repayments.

Figure 1.12: Effect of Improving Creditor Rights

	Full repayment	Nontradable	Tradable
Expected Utility	4.789	4.786	4.786
α	/	1	1
D	1	1	0.927
B_f	0.980	0.248	0.230
z	0.934	0.214	0.214
Average h	0	0.094	0.022
Average ρ	1	1.025	1.025
z_{\max}	8.571	1.186	0.357

1.6.3 Reduction in Domestic Exposure

Finally, international financial institutions may instruct the government to reduce the vulnerability of the domestic economy to defaultable debt. To analyze such a policy, see figure 1.9 above. Fix the total domestic debt level D . For the nontradable debt case, a forced reduction in the level of domestic exposure α necessarily reduces the maximum level of debt z_{\max} that can be raised from abroad. For both nontradable and tradable debt cases, the domestic exposure level in our model is an optimal response of the economy to the lack of commitment of the sovereign. Therefore, any recommendation by outside institutions to reduce the level of domestic exposure below the level chosen by the government is welfare-decreasing. It does not solve the market imperfections that are the underlying reason for the high observed domestic exposure level.

1.7 Conclusion

In this chapter we have analyzed the government's optimal default and debt issuance decisions within a model framework that incorporates lack of commitment of the sovereign and domestic output costs of default. The assumed market imperfections generate an output cost from debt repudiation which depends both on the fraction of debt held by foreign as opposed to domestic lenders, and on the exposure of the domestic economy to government-issued defaultable debt. Our results fall into two broad categories. Firstly, we examine the repayment decision. We characterize the optimal default decision of the government and relate this decision to both the inherited debt variables and the productivity shock. We obtain that the government may not wish to fully default on its debt if the domestic output costs of default are sufficiently high on the margin, relative to the marginal benefits. This in turn confirms that debt can be sustained in this model by the output costs alone. Reputation effects are not required. Secondly, we examine the optimal debt issuance decision in the period prior to repayment. The government recognises that its debt issuance policy will affect the incentives to default in future periods, both by affecting the domestic output cost of default and by determining the distribution of losses from the default decision across domestic and foreign debtholders.

Two key mechanisms play an important role in the debt issuance decision. The exposure mechanism captures the fact that the government can effectively "purchase commitment" to repay its debt in future periods by exposing the domestic economy to severe output costs if the government reneges on its repayment obligations. Indeed, if the government wishes to borrow more resources from abroad, or if it wishes to raise the same amount more cheaply, it may find it necessary to increase the exposure of the domestic economy to assure foreign creditors of its intention to repay. We should expect that governments increase the fragility of the domestic economic environment, making output more vulnerable to the default decision, in order to be able to borrow more ex ante. This is an optimal response to the government's lack of commitment. We should also observe that these output costs are in fact realized when the government is confronted by adverse productivity shocks. In future states of nature where the productivity shock realization is low, the government will still find it optimal to default, because it can push some of the burden onto foreign creditors. Default imparts some contingency onto debt repayments that are otherwise contractually noncontingent.

The exposure mechanism applies whether the defaultable debt is tradable or not in the period of issue. When debt is tradable in the period of issue, a new restriction is imposed on the feasible set for debt issuance. In particular, the government can no longer force the domestic banking system to hold the quantities of cash and defaultable debt that the government chooses. Banks choose their portfolios optimally. Specifically, there is an equal marginal valuation restriction: both domestic and foreign lenders must hold debt and value the marginal unit of debt equally, if debt is to be sustained. If domestic banks value the debt more highly than foreign creditors, they purchase most of the debt and it is difficult for the government to raise resources from abroad. If domestic banks are unwilling to hold much of the debt, then foreign creditors will anticipate high defaults by the government in the subsequent period, and the price of government debt will be low. This again constrains the ability of the government to raise resources from abroad.

Whether the nontradable or tradable debt specifications are more accurate descriptions of reality is debatable. If governments are able to mandate that domestic banks hold a certain fraction of government or other assets in their portfolio, then the nontradable debt specification may be more appropriate. In general however, if domestic banks have unhindered access to secondary markets where they can purchase and sell government debt, it is more desirable to examine the predictions of the tradable debt case. Possible justifications for the government executing equal haircuts on foreign and domestic creditors include that the government cannot observe who is holding the debt at the moment of repayment, or that the existence of secondary markets imposes constraints on the government's ability to effectively execute different haircuts on different groups of lenders. For these interpretations, it is more natural to assume that the debt should also be tradable in the period of issue.

Since the exposure of the domestic economy is an optimal response to the underlying problem of the sovereign's lack of commitment, recommendations by international financial institutions to reduce the exposure of the economy to default may have the counterintuitive side-effect of a reduction in the ability of the government to borrow from abroad, and in general may reduce welfare if the advice is binding. This feature of the model derives from the benevolent government setup, since it decides to impose costs on the domestic economy optimally. Improvements in creditor rights reduce the average cost of borrowing because deadweight losses in output are

Firm profits in equilibrium are given by

$$\Pi_F = X + \tilde{R}f(X) - \left\{1 + \tilde{R}f'(X)\right\} X$$

Substitute the two government budget constraints, the expression for savings and firm profits into the consumer budget constraint:

$$c_1 \leq y_1 + qB_f$$

$$c_2 \leq y_2 - (1 - h)B_f + \tilde{R}f(x)$$

where

$$x \leq A_d + (1 - h)B_d$$

Given bank investments and firm profits, substituting the government budget constraints into the consumer budget constraint yields the resource constraints. If we include the government budget constraint and the resource constraint, we can drop the consumer budget constraint from the problem.

Equation (1.7) determines the price q , and is included as the rational expectations constraint on the problem.

Now let us apply Definition 1.2. It is an optimum for the government budget constraints to hold with equality, so the resource constraints above will hold with equality. We do require that the representative consumer's Euler conditions hold. However, the prices p_A and p_B do not appear in any other equation, and therefore the Euler equations are redundant constraints for the problem. In particular, it is possible to solve the government problem and then derive the prices p_A and p_B as residuals of the exercise. Finally, it is not necessary to keep track of T_1 and T_2 for the problem (these quantities can be calculated as residuals from the solution to the government program), so we drop the equations that define them. This yields the first specification of the government program presented in the main text.

The condition $\lim_{x \rightarrow 0} xf'(x) = 0$ establishes that the optimal values of both D and B_f can be bounded from above. The upper bound on D follows from Corollary 1.1. The upper bound on B_f follows from the existence of a maximum level of debt (shown in the proof of Proposition

1.6) and Step 1 of the proof of Proposition 1.9. See below. ■

The second specification is derived from the first as follows. We assume that y_2 is large enough so that constraint (1.9) is never binding. As mentioned in the text, a sufficient condition on the production function to ensure that this approach is valid is: $\lim_{x \rightarrow 0} x f'(x) = 0$.

Let us consider the determination of the haircut h in period 2. Notice that the haircut appears only inside the $u(c_2)$ expression in the final period. The first order condition with respect to h in this period yields:

$$u'(c_2) \left[B_f - \tilde{R}f'([(1-\alpha) + (1-h)\alpha]D) \cdot \alpha D \right] = 0.$$

If h is interior, it must satisfy this first order condition. It is also possible for the haircut h to be at the boundaries 1 or 0 if the expression inside the square brackets is always positive or negative, respectively. The equation above provides the expression for the haircut in the main text, expression (1.10). The key result is that the haircut may be written in the form

$$h = H(\alpha, D, B_f, \tilde{R}).$$

Therefore the bond price can be derived:

$$Q(\alpha, D, B_f) = \frac{1}{1+r} \mathbb{E} \left\{ 1 - H(\alpha, D, B_f, \tilde{R}) \right\}$$

Replace the haircut decision and the bond price with the above expressions. Next we reduce the number of state variables. Use the law of iterated expectations and define $\hat{U}_1 = \mathbb{E}U_1$, $\hat{U}_2(q, B_f) = \mathbb{E}U_2(q, B_f)$. The government problem in period 1 can be written:

The government problem may be written as follows. In period 1:

$$\hat{U}_1 = \max_{c_1, q, B_f} \mathbb{E} \left\{ u(c_1) + \beta \hat{U}_2(q, B_f) \right\}$$

subject to

$$c_1 = y_1 + qB_f$$

$$c_1 \geq 0$$

$$(q, B_f) \in \hat{G},$$

where the expression $\hat{U}_2(q, B_f)$ is defined by the program:

$$\hat{U}_2(q, B_f) = \max_{c_2, \alpha, D} \mathbb{E}\{u(c_2)\}$$

subject to

$$c_2 = y_2 - (1 - h)B_f + \tilde{R}f([(1 - \alpha) + (1 - h)\alpha]D)$$

$$h = H(\alpha, D, B_f, \tilde{R}).$$

$$q = Q(\alpha, D, B_f)$$

$$B_f < 0 \Rightarrow \alpha = 0$$

where \hat{G} is the set of feasible (q, B_f) pairs generated by the $q = Q(\alpha, D, B_f)$ relation. Notice that the second subproblem involves the government choosing (α, D) before the productivity shock in the second period is realized.

Finally, define

$$V_2(z) = \max_{q, B_f} \hat{U}_2(q, B_f)$$

subject to

$$qB_f = z.$$

Then we derive the second version of the government program presented in the main text. The set of feasible asset values G is unbounded below. It is bounded above by the maximum value that can be achieved by the function $Q(\alpha, D, B_f) \cdot B_f$. This value will depend upon the specification of the production function and the range of possible values for the productivity shock. ■

Proof of Formulations of the Government Problem in Subsection 1.5.1.

This first specification of the government program is derived in a similar manner as above, but with the new restriction: $p_B = q$. The representative consumer's Euler equation for defaultable debt is not redundant because the price in this equation q appears elsewhere in the government

problem. The representative consumer's Euler equation for cash is redundant. ■

The second version of the government program in the main text follows the approach for nontradable debt, but again with the new restriction: $p_B = q$. It is important to check the cases where the defaultable debt is held entirely by either domestic agents (in which case $B_f = 0$) or foreign creditors (in which case $\alpha D = 0$). We obtain the following government program.

$$V_1 = \max_{c_1, z} \mathbb{E} \{u(c_1) + \beta V_2(z, \lambda)\}$$

subject to

$$c_1 = y_1 + z$$

$$c_1 \geq 0$$

$$\lambda = u'(c_1)$$

$$(z, \lambda) \in \tilde{G}$$

for some set \tilde{G} , where the expression $V_2(z, \lambda)$ is given by

$$V_2(z, \lambda) = \max_{c_2, \alpha, D, B_f} \mathbb{E} \{u(c_2)\}$$

subject to

$$c_2 = y_2 - (1 - h)B_f + \tilde{R}f([(1 - \alpha) + (1 - h)\alpha]D)$$

$$h = H(\alpha, D, B_f, \tilde{R})$$

If $z < 0$, combinations $C = (\alpha, D, B_f)$ satisfy: $\alpha = 0, B_f < 0$.

If $z = 0$, combinations $C = (\alpha(s), D(s), B_f(s))$ satisfy one of:

(i) $B_f = 0, \alpha D = 0$;

(ii) $B_f = 0, \alpha D > 0$ with

$$\frac{1}{1+r} \mathbb{E}(1-h) \leq \beta \mathbb{E} \left\{ \frac{u'(c_2)}{\lambda} \cdot (1-h) \cdot \left[1 + \tilde{R}f'([(1-\alpha) + (1-h)\alpha]D) \right] \right\}$$

(iii) $B_f > 0$ with

$$\frac{1}{1+r} \mathbb{E}(1-h) = 0$$

If $z > 0$, combinations $C = (\alpha, D, B_f)$ satisfy:

$$z = \frac{B_f}{1+r} \mathbb{E}(1-h)$$

$$z\lambda = \beta B_f \mathbb{E} \left\{ u'(c_2) \cdot (1-h) \cdot \left[1 + \tilde{R}f' \left([(1-\alpha) + (1-h)\alpha] D \right) \right] \right\}.$$

Our notation suppresses the dependence of h on $(\alpha, D, B_f, \tilde{R})$.

Now consider cases (ii) and (iii) for the debt level $z = 0$. Case (ii) can be replicated by issuing no defaultable debt and by issuing D units of cash instead. Case (iii) can be replicated by issuing zero defaultable debt. Therefore, without loss of generality, we may ignore these cases and assume that no defaultable debt is issued when the sovereign wishes to raise zero debt. This yields the formulation in the main text. ■

Proof of Proposition 1.1.

In the first best case, the economy has access to contingent debt. Assume that y_2 is sufficiently high so that the government wishes to make net repayments abroad in every state of nature in period 2. Then the ability to save and borrow at the same time in period 1 is redundant, and the government chooses only to issue debt. Also assume that y_2 is sufficiently high so that the government always has enough resources to repay all of its debt if it so wishes. Consider the nontradable debt case. Output in period 2 is maximized by setting $A_d \geq \bar{x}$, B_d to any value and the government problem reduces to the form written in the proposition. Results 2 and 3 of the proposition follow immediately.

For the tradable debt case, an additional constraint is added to the problem which reduces the feasible set:

$$\frac{1}{1+r} \mathbb{E}(1-h) \geq \beta \mathbb{E} \left\{ \frac{u'(c_2)}{u'(c_1)} \cdot (1-h) \cdot \left[1 + \tilde{R}f'(x) \right] \right\}.$$

This ensures that domestic banks do not purchase all of the defaultable debt, so foreign creditors hold some of the debt and the economy can borrow resources from abroad. It can be verified

that the first best optimum in the nontradable debt case remains in the feasible set of the tradable debt case, now for some specific $B_d \geq 0$. The government sets B_d at this value and issues total defaultable debt $B = B_d + B_f$. ■

Proof of Proposition 1.2.

Assume that y_2 is sufficiently high so that the government always has enough resources to repay all of its debt if it so wishes. Consider the nontradable debt case. The government can set $A_d \geq \bar{x}$ to maximize output in period 2 and B_d is set to any value. The government problem for B_f reduces to the form written in the proposition. Results 2 and 3 of the proposition follow immediately. For the tradable debt case, the additional constraint on the feasible set is now:

$$\frac{1}{1+r} \geq \beta \mathbb{E} \left\{ \frac{u'(c_2)}{u'(c_1)} \cdot [1 + \tilde{R}f'(x)] \right\}.$$

The optimum for the nontradable debt case remains feasible in the tradable debt case, now for some specific $B_d \geq 0$. The government sets B_d at this value and issues total defaultable debt $B = B_d + B_f$. ■

Proof of Lemma 1.1.

The optimal haircut by the government for any realization of the productivity shock \tilde{R} is derived by the maximization of the expression for consumption in period 2. This helps us to understand the output and consumption profiles in period 2. For any realization of the productivity shock \tilde{R} , output is given by $\tilde{R}f(x)$, where $x = [(1 - \alpha) + (1 - h)\alpha] D$. Consumption is given by the formula

$$c_2 = y_2 - (1 - h)B_f + \tilde{R}f(x).$$

Consider a combination $C = (\alpha, D, B_f)$ such that $D > \bar{D}$, or equivalently, a combination (A_d, B_d, B_f) such that $[A_d + B_d] > \bar{D}$. We consider the possible cases.

Case 1: $0 \leq A_d \leq \bar{D}$.

Keep A_d unchanged. Reduce the magnitudes of B_d and B_f equiproportionately until $[A_d + B_d] = \bar{D}$. This combination C' raises the same revenues as C in period 1 and is equivalent to C in terms

of repayments abroad, output and hence consumption for every realization of the productivity shock \tilde{R} in period 2.

Case 2: $A_d > \bar{D}$.

Set $A_d = \bar{D}$ and $B_d = B_f = 0$. This combination C' raises the same revenues as C in period 1 and is equivalent to C in terms of repayments abroad, output and hence consumption for every realization of the productivity shock \tilde{R} in period 2. ■

Proof of Corollary 1.1.

This immediately follows from Lemma 1.1. ■

Proof of Proposition 1.3.

The formula in this proposition follows from the following equation:

$$\frac{B_f}{\tilde{R}B_d} = f'([A_d + (1-h)B_d]). \quad (1.15)$$

and the restriction established by Corollary 1.1. Given the assumptions on the production function, $(f')^{-1}[\gamma]$ is strictly decreasing in the argument γ . This yields the comparative statics listed. ■

Proof of Proposition 1.4.

The first order condition with respect to the haircut h in period 2 yields equation (1.15), which is illustrated in figure 1.3. This characterizes the solution for interior h . The upper bound for the haircut is binding when

$$(f')^{-1}\left[\frac{B_f}{\tilde{R}B_d}\right] < A_d.$$

This condition may hold for states of nature with lower productivity realizations \tilde{R} . Denote these states by the set $\mathcal{S}_1 = \{\underline{R}, \dots, R^*\}$. The zero bound for the haircut is binding when

$$(f')^{-1}\left[\frac{B_f}{\tilde{R}B_d}\right] > A_d + B_d.$$

This condition may apply for states of nature with higher productivity realizations \tilde{R} . Denote these states by the set $\mathcal{S}_3 = \{R^{**}, \dots, \bar{R}\}$.

The first order condition determines the haircut when it is interior, i.e., for states between R^* and R^{**} (non-inclusive). Let us denote these states by the set \mathcal{S}_2 . The above argument establishes the formula for the total real resources raised by the government as a function of the combination (A_d, B_d, B_f) :

$$\begin{aligned} z &= \frac{1}{1+r} \sum_{\tilde{R} \in \mathcal{S}} (1-h) B_f \cdot \Pr(\tilde{R}) \\ &= \frac{1}{1+r} \sum_{\tilde{R} \in \mathcal{S}_2} \gamma \cdot \left\{ (f')^{-1} \left[\frac{\gamma}{\tilde{R}} \right] - A_d \right\} \Pr(\tilde{R}) + \frac{1}{1+r} \sum_{\tilde{R} \in \mathcal{S}_3} B_f \cdot \Pr(\tilde{R}) \end{aligned} \quad (1.16)$$

where we define $\gamma = \frac{B_f}{B_d}$.

The final step of the proof is to show that formula (1.16) can yield $z > 0$ for some choice of configuration (A_d, B_d, B_f) . Set $A_d = 0$ and $B_d = \bar{D}$. Equation (1.16) reduces to

$$z = \frac{\gamma}{1+r} \sum_{\tilde{R} \in \mathcal{S}} (f')^{-1} \left[\frac{\gamma}{\tilde{R}} \right] \Pr(\tilde{R}).$$

Choose any $\gamma > 0$. Then $(f')^{-1} \left[\frac{\gamma}{\bar{R}} \right] \in (0, \bar{x})$. The absolute value of borrowing is positive, as required. ■

Proof of Proposition 1.5.

For negative debt values $z < 0$, i.e., saving abroad, there is no default by foreigners because foreign institutions credibly commit to fully repay. The government can only issue cash to domestic residents. Therefore:

$$f([(1-\alpha) + (1-h)\alpha] D) = f(D)$$

$$z = \frac{B_f}{1+r}.$$

It is straightforward to see that the proposition follows. ■

Proof of Proposition 1.6.

Step 1: Attainment of a maximum z for each (A_d, B_d) pair.

Equation (1.16) yields

$$\begin{aligned}
 z &= \frac{1}{1+r} \sum_{\tilde{R} \in \mathcal{S}_2} \gamma \cdot \left\{ (f')^{-1} \left[\frac{\gamma}{\tilde{R}} \right] - A_d \right\} \Pr(\tilde{R}) + \frac{1}{1+r} \sum_{\tilde{R} \in \mathcal{S}_3} B_d \cdot \Pr(\tilde{R}) \\
 &= \frac{\gamma}{1+r} \sum_{\tilde{R} \in \mathcal{S}_2} (f')^{-1} \left[\frac{\gamma}{\tilde{R}} \right] \Pr(\tilde{R}) \\
 &\quad - \frac{\gamma}{1+r} \cdot A_d \left[\sum_{\tilde{R} \in \mathcal{S}_2} \Pr(\tilde{R}) \right] + \frac{\gamma}{1+r} \cdot B_d \left[\sum_{\tilde{R} \in \mathcal{S}_3} \Pr(\tilde{R}) \right]. \tag{1.17}
 \end{aligned}$$

Fix A_d and $B_d > 0$ and treat this expression as a function of $\gamma \in [0, \infty)$. It is continuous in γ . First, consider $A_d > 0$. It can be shown that there exists a value $\gamma_M(A_d, B_d)$ such that for all $\gamma > \gamma_M(A_d, B_d)$, the sets \mathcal{S}_2 and \mathcal{S}_3 are empty and therefore $z = 0$. This feature means that the supremum value of z must be achieved for γ within the compact set $[0, \gamma_M(A_d, B_d)]$. Apply the Weierstrass Theorem for the function defined on this compact set. This proves that expression (1.17) attains a maximum for some γ in this set. All values of z between 0 and the maximum value can be achieved.

Next, consider $A_d = 0$ and $B_d > 0$. A different approach is required. The set \mathcal{S}_1 is now empty, and equation (1.17) reduces to:

$$z = \frac{\gamma}{1+r} \sum_{\tilde{R} \in \mathcal{S}_2} (f')^{-1} \left[\frac{\gamma}{\tilde{R}} \right] \Pr(\tilde{R}) + \frac{\gamma}{1+r} \cdot B_d \left[\sum_{\tilde{R} \in \mathcal{S}_3} \Pr(\tilde{R}) \right]. \tag{1.18}$$

Consider each of the functions for values of the productivity shock \tilde{R} :

$$g(\gamma, \tilde{R}) = \gamma \cdot (f')^{-1} \left[\frac{\gamma}{\tilde{R}} \right].$$

An increase in γ corresponds to a reduction in x . The function $g(\gamma, \tilde{R})$ can be written as:

$$g(\gamma, \tilde{R}) = \tilde{R} f'(x) \cdot x \text{ for } x = (f')^{-1} \left[\frac{\gamma}{\tilde{R}} \right].$$

We assume that $\lim_{x \rightarrow 0} x f'(x) = 0$. This establishes that $\lim_{\gamma \rightarrow \infty} g(\gamma, \tilde{R}) = 0 \quad \forall \tilde{R}$, and hence that $\lim_{\gamma \rightarrow \infty} \mathbb{E}g(\gamma, \tilde{R}) = 0$.

Whatever the value of B_d , as γ increases there eventually comes a point when the set \mathcal{S}_3 becomes empty. Therefore, the limit of the expression (1.18) as $\gamma \rightarrow \infty$ is equal to the limit of the same expression without the second term.

$$\lim_{\gamma \rightarrow \infty} z(\gamma) = \lim_{\gamma \rightarrow \infty} \mathbb{E}g(\gamma, \tilde{R}) = 0.$$

There exists debt for this case $A_d = 0, B_d > 0$. Choose any value $\gamma > 0$, and this yields a positive value of debt, say z_1 . Let us pick a value of $\varepsilon \in (0, z_1)$. This value of ε corresponds to a number $\gamma_M(0, B_d)$ such that z is lower than ε for all $\gamma \geq \gamma_M(0, B_d)$. Then we know that the supremum level of debt that can be attained lies in the set $[0, \gamma_M(0, B_d)]$. Apply the Weierstrass Theorem for the continuous function (1.18) over this compact set. This establishes that the maximum is attained. Furthermore, all values of z between 0 and the maximum value can be achieved.

For $B_d = 0$, $z = 0$ irrespective of the value of A_d .

Step 2: Comparative statics with respect to α , given $D = D'$.

Fix $D = D'$. Consider an increase in A_d and an equal reduction in B_d . Furthermore, change B_f so as to preserve the value of the ratio $\gamma = \frac{B_f}{B_d}$. This corresponds to a reduction in α . The perturbation considered reduces the value of the function in expression (1.17) for any given value of γ , both through a fall in the second term and a possible reduction in the set of states \mathcal{S}_2 for which the upper bound on the haircut is not binding. Therefore, the maximum value of the expression given by equation (1.17) must be lower (the maximum is still attained, by repeated application of the Weierstrass Theorem). This establishes that $\underline{\alpha}(z)$ is weakly increasing in z .

For $z = 0$, we may set $\gamma = 0$. The value of A_d does not matter. Therefore $\underline{\alpha}(0) = 0$. From the argument above, the highest value of the expression (1.17) is achieved when $A_d = 0$. Therefore $\underline{\alpha}(z_{\max}(D')) = 1$. ■

Proof of Proposition 1.7.

There are two steps of the proof.

Step 1: The zero bound for the haircut is never binding at the optimal combination

$C = (\alpha, D, B_f)$.

Proof by contradiction.

Case 1: $B_f > 0$.

Suppose that the optimal combination $C = (\alpha, D, B_f)$ satisfies $D < (f')^{-1} \left[\frac{B_f}{\bar{R}B_d} \right]$. The zero bound for the haircut is binding for states in the set $\mathcal{S}_3 = \{R^{**}, \dots, \bar{R}\}$. For these states:

$$\begin{aligned} (f')^{-1} \left[\frac{B_f}{\bar{R}B_d} \right] &> D \\ \iff \frac{B_f}{\bar{R}B_d} &< f'(D). \end{aligned}$$

Consider the following perturbation: an infinitesimal equiproportionate increase in the magnitudes of B_d and B_f that preserves the value of the ratio $\gamma = \frac{B_f}{B_d}$. This perturbation is feasible. Since B_d and B_f take positive values, the perturbation involves $dB_d, dB_f > 0$.

For states of nature in the set $\mathcal{S} \setminus \mathcal{S}_3$, the perturbation does not change repayments abroad, output or consumption. For states of nature in the set \mathcal{S}_3 , the perturbation does have an effect. Note that

$$c_2(\alpha, D, B_f, \tilde{R}) = y_2 - (1-h)B_f + \tilde{R}f([(1-\alpha) + (1-h)\alpha]D)$$

where

$$h = H(\alpha, D, B_f, \tilde{R}).$$

We suppress the dependence of c_2 on $(\alpha, D, B_f, \tilde{R})$ in our notation. For values of the produc-

tivity shock $\tilde{R} \in \mathcal{S}_3$, the perturbation has the following effect on c_2 :

$$\begin{aligned} dc_2 &= -d[(1-h)B_f] + \tilde{R}df \\ &= -\frac{B_f}{B_d}dB_d + \tilde{R}f'(D)dB_d \\ &> -\frac{B_f}{B_d}dB_d + \tilde{R}\frac{B_f}{\tilde{R}B_d}dB_d = 0. \end{aligned}$$

So the perturbation increases consumption c_2 and increases repayments abroad for these states of nature. The second effect increases z in the initial period, which means higher consumption in period 1.

Therefore, the perturbation considered increases consumption in period 1 and weakly increases consumption for every value of the productivity shock \tilde{R} in period 2. The original combination cannot have been optimal. This argument means that we choose D such that:

$$D \geq (f')^{-1} \left[\frac{B_f}{\tilde{R}\alpha D} \right].$$

Case 2: $B_f = 0$.

Suppose that the optimal combination $C = (\alpha, D, B_f)$ satisfies $D < \bar{D}$. The optimal government policy in period 2 is not to default at all. Consider the following perturbation: an infinitesimal increase in the magnitude of D . This is feasible, and leaves the optimal policy unchanged. The perturbation has the following effect:

$$\begin{aligned} dc_2 &= \tilde{R}df \\ &= \tilde{R}f'(D)dD > 0, \quad \forall \tilde{R}. \end{aligned}$$

So the perturbation increases consumption c_2 for all values of the productivity shock \tilde{R} in the final period, while leaving consumption in the initial period unchanged. The original combination cannot have been optimal.

Step 2: Completion of Proof.

This immediately follows from the above. ■

Proof of Proposition 1.8.

We optimally set $D = \bar{D}$, so the set \mathcal{S}_3 is empty.

The approach in this proof is to take a given combination $C = (\alpha, \bar{D}, B_f)$ which raises debt z such that $\alpha > \underline{\alpha}(z)$, and to ask whether this combination is optimal. In particular, one perturbation we consider is to reduce the level of domestic exposure α and adjust B_f appropriately so that the same level of debt z is raised in period 1.

Step 1: Optimal size of B_f for any given exposure level.

Consider two combinations $C = (\alpha, \bar{D}, B_f)$ and $C' = (\alpha, \bar{D}, B'_f)$ which raise the same level of debt z . The exposure level is the same but for combination C' , the ratio $\gamma' = \frac{B'_f}{\alpha \bar{D}}$ is larger than $\gamma = \frac{B_f}{\alpha \bar{D}}$. It can be shown that the combination C' is not optimal. For any given level of domestic exposure, we choose the lowest value of B_f that raises any given level of debt z . For the basic idea behind this part of the proof, see the proof of Proposition 1.9. The corollary of this result is that if we plot the debt level z as a function of γ for a given level of α , a necessary condition for a combination $C = (\alpha, \bar{D}, B_f)$ to be optimal is that it lies either (i) on the upward-sloping portion of the graph, or (ii) at a local (non-global) maximum of the graph. We assume this condition holds.

Step 2: Perturbation that reduces α and increases $\gamma = \frac{B_f}{\alpha \bar{D}}$.

Let us increase A_d by an infinitesimal amount, so that the exposure level α falls. This corresponds to a downward shift in the graph of z against γ described above, with a greater downward shift for higher γ values. From the information in Step 1 of this proof, it is therefore only feasible to raise the same level of debt z by increasing γ . The increase in γ is marginal if the combination was initially on the upward-sloping portion of the graph, and it is a discrete jump in γ if the combination was initially at a local (non-global) maximum. In the latter case, there is a discrete fall in consumption and the perturbation is not optimal. The remainder of this proof concentrates on the former case. We split the perturbation into two stages.

Increase A_d , and reduce B_d and B_f equiproportionately to keep $D = \bar{D}$ and $\gamma = \frac{B_f}{B_d}$ constant. For states of nature $\tilde{R} \in \mathcal{S}_1$, repayments abroad in period 2 are unaffected, but

output (and hence consumption) is affected in the final period:

$$dc_2 = \tilde{R}f'(A_d) dA_d > 0.$$

For states of nature $\tilde{R} \in \mathcal{S}_2$, output is unaffected, but repayments fall:

$$dc_2 = \frac{B_f}{B_d} dA_d > 0.$$

This perturbation reduces the debt level z .

Increase B_f until the debt level z rises to the initial level. For states of nature $\tilde{R} \in \mathcal{S}_1$, repayments abroad, output and consumption in period 2 are unaffected.

For states of nature $\tilde{R} \in \mathcal{S}_2$, repayments rise and consumption falls. Adapting the expression in the proof of Proposition 1.9:

$$dc_2 = - \left\{ (f')^{-1} \left[\frac{\gamma}{\tilde{R}} \right] - A_d \right\} \cdot d\gamma < 0.$$

Now combine the two components of the perturbation described above, and set dA_d and $d\gamma$ so that the level of debt raised z is unchanged as a result of the perturbation. For states of nature $\tilde{R} \in \mathcal{S}_1$:

$$dc_2 = \tilde{R}f'(A_d) dA_d > 0.$$

For states of nature $\tilde{R} \in \mathcal{S}_2$:

$$dc_2 = \frac{d\gamma}{\sum_{\tilde{R} \in \mathcal{S}_2} \Pr(\tilde{R})} \cdot \left[\sum_{\tilde{R} \in \mathcal{S}_2} \left\{ (f')^{-1} \left[\frac{\gamma}{\tilde{R}} \right] + \frac{\gamma}{\tilde{R}} \left[(f')^{-1} \right]' \left[\frac{\gamma}{\tilde{R}} \right] \right\} \Pr(\tilde{R}) \right] - d\gamma \cdot \left\{ (f')^{-1} \left[\frac{\gamma}{\tilde{R}} \right] - A_d \right\}.$$

It is straightforward to prove that:

$$\mathbb{E}_{\tilde{R} \in \mathcal{S}_2} \{dc_2\} < 0, \quad \frac{d}{d\tilde{R}} \{dc_2\} < 0 \quad \forall \tilde{R} \in \mathcal{S}_2.$$

Consider the set \mathcal{S}_2 . Such a deviation depresses average consumption in the set \mathcal{S}_2 , but consumption is depressed less in states of nature which receive a worse productivity shock. For

permissible production functions $f(x)$, it is possible that the combination of deviations actually increases consumption for the worst productivity shock realizations within \mathcal{S}_2 in period 2. This may apply even if set \mathcal{S}_1 is empty. If \mathcal{S}_1 is not empty, of course, consumption necessarily increases for the worst realizations of the productivity shock \tilde{R} .

It follows that there exist some utility functions $u(c)$ with sufficiently high risk aversion such that the representative consumer finds this perturbation optimal. The sovereign finds it optimal to reduce the level of domestic exposure because this perturbation enables the sovereign to purchase some insurance across states of nature against the productivity shock realization \tilde{R} . ■

Proof of Proposition 1.9.

Step 1: Proof that it is optimal to choose the lowest level of γ that achieves a given debt level z .

We set $D = \bar{D}$, and let $u(c)$, $f(x)$ and y_2 be specified such that $\alpha = 1$ is the optimal level of exposure for all levels of debt. Then sets \mathcal{S}_1 and \mathcal{S}_3 are empty. Consider any combination $C = (1, \bar{D}, B_f)$ and define $\gamma = \frac{B_f}{\bar{D}}$. What is the effect of an infinitesimal increase in γ on consumption in period 2?

$$dc_2 = -d[(1-h)B_f] + \tilde{R}df$$

where

$$\begin{aligned} -d[(1-h)B_f] &= -d[(1-h)B_d \cdot \gamma] \\ &= -\gamma \cdot d[(1-h)B_d] - (1-h)B_d \cdot d\gamma \\ &= -\gamma \cdot \left(\left[(f')^{-1} \right]' \left[\frac{\gamma}{\bar{R}} \right] \frac{d\gamma}{\bar{R}} \right) - \left((f')^{-1} \left[\frac{\gamma}{\bar{R}} \right] \right) \cdot d\gamma \end{aligned}$$

and

$$\begin{aligned} df &= f' \left((f')^{-1} \left[\frac{\gamma}{\bar{R}} \right] \right) \left(\left[(f')^{-1} \right]' \left[\frac{\gamma}{\bar{R}} \right] \frac{d\gamma}{\bar{R}} \right) \\ &= \frac{\gamma}{\bar{R}} \cdot \left(\left[(f')^{-1} \right]' \left[\frac{\gamma}{\bar{R}} \right] \frac{d\gamma}{\bar{R}} \right). \end{aligned}$$

Therefore

$$dc_2 = - (f')^{-1} \left[\frac{\gamma}{\tilde{R}} \right] \cdot d\gamma < 0. \quad (1.19)$$

We have presented full details of the mathematical derivation, but since the haircut decision is always interior and set in an optimal manner, the final result can be reached more concisely via application of the Envelope Theorem. The expression dc_2 takes the same sign for all $\gamma \in [0, \infty)$.

Thus an increase in the ratio γ always reduces consumption c_2 for all values of the productivity shock \tilde{R} in period 2. Now suppose that the same level of z can be achieved for two levels of the ratio $\frac{B_f}{D}$ given by γ_1 and $\gamma_2 > \gamma_1$. Both values of γ achieve the same level of consumption in period 1, but γ_2 results in lower consumption than γ_1 in period 2. Therefore the lower value of γ must be optimal.

Step 2: Completion of proof.

Equation (1.17) reduces to

$$z = \frac{\gamma}{1+r} \sum_{\tilde{R} \in \mathcal{S}} (f')^{-1} \left[\frac{\gamma}{\tilde{R}} \right] \Pr(\tilde{R}).$$

The above expression is continuous in $\gamma \in [0, \infty)$. Imposing that $\lim_{x \rightarrow 0} x f'(x) = 0$, the relevant set for γ is a compact set of the form $[0, \gamma_{\max}]$ and the maximum value of z is attained on this set). Step 1 of the proof implies that the optimal B_f value will lie within the region $[0, \gamma_{\max}]$. Continuity of the $z(\gamma)$ function and application of the result in step 1 of the proof yields the result that the optimal B_f is increasing in z .

From result 1 of Proposition 1.3, an increase in B_f increases the optimal haircut for all values of the productivity shock \tilde{R} , and hence the interest rate on government debt. ■

Proof of Proposition 1.10.

We set $D = \bar{D}$. Let $u(c)$, $f(x)$ and y_2 be specified such that $\alpha = 1$ is the optimal level of exposure for all levels of debt. The condition $\lim_{x \rightarrow 0} x f'(x) = 0$ means that the set of feasible debt levels takes the form $[0, z_{\max}]$. Note also that the expression for z as a function of γ is continuous and differentiable. As the desired level of debt z varies, the optimal level of γ (and hence B_f) may exhibit discontinuities. The Euler condition needs to take this into account.

The left derivative takes the form:

$$\left(\frac{\Delta c_2}{dz}\right)_- = -(1+r) \frac{(f')^{-1} \left[\frac{\gamma}{R}\right]}{\mathbb{E} \left\{ (f')^{-1} \left[\frac{\gamma}{R}\right] + \frac{\gamma}{R} \left[(f')^{-1} \right]' \left[\frac{\gamma}{R}\right] \right\}} < 0,$$

which approaches ∞ for γ corresponding to a point of discontinuity). The right derivative takes the above form at points when the optimal B_f schedule is continuous in z , but the following form at points of discontinuity:

$$\left(\frac{\Delta c_2}{dz}\right)_+ = -(1+r) \frac{(f')^{-1} \left[\frac{\gamma+\Delta\gamma}{R(s')}\right]}{\mathbb{E} \left\{ (f')^{-1} \left[\frac{\gamma+\Delta\gamma}{R}\right] + \frac{\gamma+\Delta\gamma}{R} \left[(f')^{-1} \right]' \left[\frac{\gamma+\Delta\gamma}{R}\right] \right\}} - \int_{\gamma}^{\gamma+\Delta\gamma} (f')^{-1} \left[\frac{z}{\tilde{R}}\right] \cdot dz < 0,$$

where $\Delta\gamma$ is the jump in γ at the point of discontinuity.

The above argument proves the first claim in the proposition. The third claim follows from the formulae presented above and equation (1.19), which can be used to prove that a downward jump in γ improves consumption more for higher realizations of the productivity shock \tilde{R} .

The second claim in the proposition is proven as follows. First, consider the left derivative and define

$$\mathbb{Y}_- (z, \tilde{R}) = \frac{(f')^{-1} \left[\frac{z}{\tilde{R}}\right]}{\mathbb{E} \left\{ (f')^{-1} \left[\frac{z}{\tilde{R}}\right] + \frac{z}{\tilde{R}} \left[(f')^{-1} \right]' \left[\frac{z}{\tilde{R}}\right] \right\}}.$$

We prove that:

$$\mathbb{E} \left\{ \mathbb{Y}_- (z, \tilde{R}) \right\} = \frac{\mathbb{E} \{x\}}{\mathbb{E} \{x\} + \mathbb{E} \left\{ \frac{f'(x)}{f''(x)} \right\}} > 1,$$

which establishes the result desired. A similar argument applies for the right derivative, after taking the points of discontinuity into account. ■

Proof of Proposition 1.11.

This is identical to the proof of Proposition 1.3. ■

Proof of Proposition 1.12.

Domestic banks are willing to purchase an infinite quantity of defaultable debt at any price less than

$$p_B = \beta \mathbb{E} \left\{ (1-h) \cdot \left[1 + \tilde{R}f' \left([(1-\alpha) + (1-h)\alpha] D \right) \right] \right\}$$

Foreign creditors are willing to purchase an unlimited quantity at any price below

$$q = \frac{1}{1+r} \mathbb{E}(1-h)$$

For the tradable debt case, $p_B = q$:

$$\frac{1}{1+r} \mathbb{E}(1-h) = \beta \mathbb{E} \left\{ (1-h) \cdot \left[1 + \tilde{R}f' \left([(1-\alpha) + (1-h)\alpha] D \right) \right] \right\} \quad (1.20)$$

Only if:

Let $\beta \geq \frac{1}{1+r}$. Consider any combination $C = (\alpha, D, B_f)$ with $B_f > 0$ and $z > 0$. Debt can be raised in period 1 if there is repayment for some values of the productivity shock \tilde{R} in period 2. For those states of nature where there is repayment, it is also true that $\tilde{R}f'(x) > 0$. The right hand side of the above equation exceeds the left hand side. Equation (1.20) does not hold, so debt is not feasible.

Let $\beta = 0$. Whatever the combination $C = (\alpha, D, B_f)$, domestic banks are not willing to purchase debt at any positive price. Debt cannot be sustained.

If:

Let $\beta \in \left(0, \frac{1}{1+r}\right)$. We prove the existence of debt by construction. For this case, there exists a finite $\gamma^* = \frac{B_f}{B_d} > 0$ such that:

$$1 = \beta(1+r)[1 + \gamma^*]. \quad (1.21)$$

Set $A_d = 0$, $B_d = \bar{D}$ and $B_f = \gamma^* \cdot \bar{D} > 0$. For any value of the productivity shock in period 2, the optimal haircut decision follows:

$$\gamma^* = \tilde{R}f'([A_d + (1-h)B_d]). \quad (1.22)$$

If conditions (1.21) and (1.22) hold, it is immediate that equation (1.20) is satisfied. It is feasible for the sovereign to issue debt.

As required. ■

Proof of Proposition 1.13.

In the nontradable debt case, the maximum debt level z_{\max} is achieved for $\alpha = 1$ and $D = \bar{D}$. We set $\gamma = \frac{B_f}{D}$ so as to maximize the debt level for these values of α and D . Denote this level as $\hat{\gamma}$. For the tradable debt case, it is possible to show that the maximum level of γ is γ^* , as defined in the proof of Proposition 1.12. If $\gamma^* < \hat{\gamma}$, the maximum level of debt in the tradable debt case is clearly strictly lower than z_{\max} . If $\gamma^* > \hat{\gamma}$, then it can be proven that a lower level of γ can only be achieved for exposure level $\alpha = 1$ if the total level of domestic debt D is strictly less than \bar{D} . In fact, it is achieved for a value of D such that the zero bound for the haircut is binding. Again, the maximum debt level is strictly lower than z_{\max} . Only if $\gamma^* = \hat{\gamma}$ is the maximum level of debt equal for the nontradable and tradable debt cases.

Since γ^* depends on β , it follows the maximum level of debt in the tradable debt case depends on the the same parameter. ■

Proof of Proposition 1.14.

For $z < 0$, foreigners credibly commit not to default on the government's savings abroad. The government issues cash to domestic residents, and it chooses the value of D that maximizes domestic production. The claims in the proposition follow immediately. ■

Proof of Lemma 1.2.

For any combination $C = (\alpha, D, B_f)$ such that $D > \bar{D}$, Lemma 1.1 establishes that there exists some other combination $C' = (\alpha', D', B'_f)$ where $D = \bar{D}$, such that C' raises the same revenues as C in period 1 and is equivalent to C in terms of repayments abroad, output and hence consumption for all values of the productivity shock \tilde{R} in period 2. We construct C' from $C = (\alpha, D, B_f)$ in the same manner as described in the proof of Lemma 1.1. It remains to prove that C and C' satisfy the Euler condition for defaultable debt for the same value of λ .

Case 1: $z \leq 0$.

Construct C' from $C = (\alpha, D, B_f)$ in the same manner as in the proof of Lemma 1.1. None of the defaultable debt is issued, so no Euler condition needs to be satisfied for this debt.

Case 2: $z > 0$.

Apply Lemma 1.1. Since C' raises the same revenues as C in period 1 and is equivalent to C in terms of repayments abroad, output and hence consumption for all values of the productivity shock \tilde{R} in period 2, it also follows that the expression $f'(x)$ is also the same for C and C' for all values of the productivity shock \tilde{R} . These results are sufficient to show that C and C' satisfy the equation

$$\lambda \cdot \frac{1}{1+r} \mathbb{E}(1-h) B_f = \beta \mathbb{E} \left\{ u'(c_2) \cdot (1-h) B_f \cdot \left[1 + \tilde{R} f' \left([(1-\alpha) + (1-h)\alpha] D \right) \right] \right\}$$

for the same value of λ . ■

Proof of Corollary 1.2.

This immediately follows from Lemma 1.2. ■

Proof of Proposition 1.15.

Fix $D = D'$ such that the zero bound for the haircut is binding for some values of the productivity shock \tilde{R} in period 2. Consider an increase in A_d and an equal reduction in B_d . This corresponds to a reduction in α . Furthermore, change B_f so as to preserve the value of the ratio $\gamma = \frac{B_f}{B_d}$. This perturbation was feasible in the nontradable debt case, but it is no longer feasible in the tradable debt case. The exposure mechanism means that the haircut increases for those states of nature where the haircut was initially interior. The price that foreigners are willing to pay for the debt declines by more than the price that domestic banks are willing to pay, because domestic banks still receive a high loans rate in those states of nature when the debt is fully repaid in period 2. The right hand side of equation (1.20) exceeds the left hand side. It can only be satisfied with equality again if the ratio γ falls.

In the nontradable debt case, any value of γ is achievable for given α and D . This means that for any given α and D , the entire set of debt levels $[0, z_{\max}(D)]$ can be achieved by varying γ . In the tradable debt case, this is no longer true. The corollary of this result is that only a restricted set of z values is achievable for any given α and D . ■

Proof of Proposition 1.16.

Set D equal to the smallest value such that the zero bound on the haircut is not binding for any value of the productivity shock \tilde{R} in period 2, i.e., $D = \tilde{D}$, where

$$\tilde{D} = (f')^{-1} \left[\frac{B_f}{\tilde{R}B_d} \right].$$

The only value of $\gamma = \frac{B_f}{B_d}$ consistent with this value of D and equation (1.20) is γ^* , as given in the proof of Proposition 1.12. Let $\gamma = \gamma^*$. Now consider alternative perturbations that increase expected consumption in period 2. It can be shown that there are only two possible perturbations. We consider the effect of each on consumption in periods 1 and 2.

Perturbation 1: Increase A_d , keeping $D = \tilde{D}$ and $\gamma = \gamma^*$ fixed. Step 2 of the proof of Proposition 1.8 establishes that this perturbation increases consumption for states of nature in the sets \mathcal{S}_1 and \mathcal{S}_2 . Set \mathcal{S}_3 is empty. Therefore, expected consumption in period 2 increases. The debt level z is affected:

$$dz = -\frac{\gamma}{1+r} \left[\sum_{\tilde{R} \in \mathcal{S}_2} \Pr(\tilde{R}) \right] dA_d < 0,$$

which means that consumption in period 1 falls by this same amount.

Perturbation 2: Reduce B_d and γ , keeping A_d fixed. B_d and γ must be reduced in a manner such that equation (1.20) is still satisfied. The reduction in B_d has no effect on consumption for any value of the productivity shock. It does not affect output or the volume of debt repayments for any values of the productivity shock lower than \tilde{R} . For the highest productivity shock value, it reduces output and repayments equally at the margin, leaving consumption unchanged. Step 2 of the proof of Proposition 1.8 establishes that the reduction

in γ improves consumption for all states of nature in the sets \mathcal{S}_1 and \mathcal{S}_2 . It can be verified that this is also true for states of nature in the set \mathcal{S}_3 . Therefore, expected consumption in period 2 increases. Let it increase by the same amount as in perturbation 1. The effect on the debt level z is ambiguous. For the debt level z to rise, the condition that must be satisfied is:

$$\left[\sum_{\tilde{R} \in \mathcal{S}_2} \left\{ (f')^{-1} \left[\frac{\gamma}{\tilde{R}} \right] - A_d + \frac{\gamma}{\tilde{R}} \left[(f')^{-1} \right]' \left[\frac{\gamma}{\tilde{R}} \right] \right\} \Pr(\tilde{R}) \right] \\ + B_d \Pr(\bar{R}) + \frac{\gamma}{\bar{R} |f''(\tilde{D})|} \sum_{\tilde{R} \in \mathcal{S}_2} (1 - h) \Pr(\tilde{R}) < 0.$$

The term in the square brackets is ambiguous in sign. Let it be negative. The interpretation of this assumption is that in this range, an increase in the proportion of debt held by domestic banks increases the price of debt sufficiently to increase the total repayments for all states of nature in the set \mathcal{S}_2 . If $\Pr(\bar{R})$ is very small, and \bar{R} and $|f''(\tilde{D})|$ are very large, then the condition above may be satisfied. ■

1.8.B. Division of Debt Categories into Cash and Defaultable Debt

In the environment studied in the chapter, the government can issue two types of debt: cash (type A) and defaultable debt (type B). This section of the appendix shows that this delineation of debt types is an equilibrium outcome of a slightly more general model. In this general model, the government can issue three types of debt. Debt type A can be held by domestic residents only, debt type B can be held by both domestic and foreign lenders, and debt type M can be held by foreign creditors only. The government can choose different haircuts for the three different types of debt: h_A , h_B and h_C respectively. We prove that the government will choose to fully repay all of debt A , to repay none of debt M , and to repay the fraction $1 - h$ of debt type B . These results establish that the more general model reduces to the model presented in the main text of the chapter.

Modifications to the Model

The maximization problem of domestic agents now takes into account that all debt is defaultable. Specifically, the Euler equations of the representative consumer will be modified in the

appropriate manner to take the haircuts into consideration. The government budget constraints are altered to the following:

$$T_1 \leq p_A A_d + p_B B_d + q_B B_f + q_M M_f, \quad (1.23)$$

$$T_2 \geq (1 - h_A) A_d + (1 - h_B) [B_d + B_f] + (1 - h_M) M_f \quad (1.24)$$

Prices of foreign-held debt follow the equations:

$$q_B = \frac{1}{1+r} \mathbb{E} \{1 - h_B\} \quad (1.25)$$

$$q_M = \frac{1}{1+r} \mathbb{E} \{1 - h_M\}. \quad (1.26)$$

The definition of equilibrium follows.

Definition 1.3 *A Rational Expectations Equilibrium for this economy comprises sequences for allocation rules $\{c_1, s_1, c_2, \{\chi\}, x\}$, prices $\{p_A, p_B, q_B, q_M, \rho\}$ and policies $\{A_d, B_d, B_f, M_f, h_A, h_B, h_M, T_1, T_2\}$ that satisfy:*

(a) *Consumers choose $\{c_1, s_1, c_2\}$ to maximize utility (1.1) subject to the budget constraints (1.2), (1.3) and the nonnegativity constraints on consumption (1.4), taking prices, bank contract offers, government policies and the endowment as given.*

(b) *Banks offer contract schedules $\chi : s_1 \rightarrow S(s_1, \tilde{R})$ in period 1 to maximize expected profits, taking prices and government policies in period 2 as given.*

Banks choose lending quantity x in period 2 to maximize profits, taking the loan rate ρ as given.

(c) *Firms choose borrowing level x to maximize profits (1.8), taking the loan rate ρ as given.*

(d) *Government chooses $\{h_A, h_B, h_M, T_2\}$ in period 2 to satisfy the government budget constraint (1.24) in that period, taking $\{A_d, B_d, B_f, M_f\}$ and the shock \tilde{R} as given.*

Government chooses $\{A_d, B_d, B_f, M_f, T_1\}$ in period 1 to satisfy the government budget constraint (1.23) in that period, taking the price functions $\{p_A(A_d, B_d, B_f, M_f),$

$p_B(A_d, B_d, B_f, M_f), q_B(A_d, B_d, B_f, M_f), q_M(A_d, B_d, B_f, M_f), \rho(x)$ and government policies in period 2, $h_A(A_d, B_d, B_f, M_f, \tilde{R}), h_B(A_d, B_d, B_f, M_f, \tilde{R}), h_M(A_d, B_d, B_f, M_f, \tilde{R})$ and $T_2(A_d, B_d, B_f, M_f, \tilde{R})$, as given.

(e) All markets clear for the economy. In particular, the markets for cash, defaultable debt, goods and loans clear.

(f) Bond prices follow rational expectations: both $q_B(A_d, B_d, B_f, M_f) = \frac{1}{1+r} \mathbb{E}\{1 - h_B\}$ and $q_M(A_d, B_d, B_f, M_f) = \frac{1}{1+r} \mathbb{E}\{1 - h_M\}$ taking government policies $h_B(A_d, B_d, B_f, M_f, \tilde{R})$ and $h_M(A_d, B_d, B_f, M_f, \tilde{R})$ in period 2 as given.

The goods market clearing condition yields the resource constraints:

$$c_1 \leq y_1 + q_B B_f + q_M M_f$$

$$c_2 \leq y_2 - (1 - h_B)B_f - (1 - h_M)M_f + \tilde{R}f((1 - h_A)A_d + (1 - h_B)B_d)$$

Now we turn to the optimal policy problem for the government.

Definition 1.4 *The Government Problem is to maximize utility (1.1) over time consistent rational expectations equilibria. In particular, we must satisfy not only the equilibrium conditions above but also the additional optimization decisions:*

(g) Government chooses $\{h_A, h_B, h_M, T_2\}$ in period 2 to maximize $u(c_2)$ given $\{A_d, B_d, B_f, M_f\}$ and the shock \tilde{R} .

Government chooses $\{A_d, B_d, B_f, M_f, T_1\}$ in period 1 to maximize $u(c_1) + \beta \mathbb{E}u(c_2)$, taking the price functions $\{p_A(A_d, B_d, B_f, M_f), p_B(A_d, B_d, B_f, M_f), q_B(A_d, B_d, B_f, M_f), q_M(A_d, B_d, B_f, M_f), \rho(x)\}$ and government policies in period 2, $h_A(A_d, B_d, B_f, M_f, \tilde{R}), h_B(A_d, B_d, B_f, M_f, \tilde{R}), h_M(A_d, B_d, B_f, M_f, \tilde{R})$ and $T_2(A_d, B_d, B_f, M_f, \tilde{R})$, as given.

We consider two different scenarios. In the first specification, defaultable debt is not tradable between domestic banks and foreign creditors in the period of issue. In the second specification, defaultable debt is tradable in the period of issue. In the latter case, we impose the additional restriction:

$$p_B = q_B.$$

We adopt the same approach to the problem described in subsections 1.4.1 and 1.5.1.

Nontradable Debt

Apply the methodology for the results in the main text, to derive the following formulation. In period 1:

$$U_1 = \max_{c_1, \alpha, D, B_f, M_f} \left\{ u(c_1) + \beta \mathbb{E} U_2 \left(\alpha, D, B_f, M_f, \tilde{R} \right) \right\}$$

subject to

$$c_1 = y_1 + q_B B_f + q_M M_f$$

$$c_1 \geq 0$$

$$q_B = \frac{1}{1+r} \mathbb{E} \left\{ 1 - h_B \left(\alpha, D, B_f, M_f, \tilde{R} \right) \right\}$$

$$q_M = \frac{1}{1+r} \mathbb{E} \left\{ 1 - h_M \left(\alpha, D, B_f, M_f, \tilde{R} \right) \right\}$$

$$B_f < 0 \Rightarrow \alpha = 0$$

$t = 2$:

$$U_2 \left(\alpha, D, B_f, M_f, \tilde{R} \right) = \max_{c_2, h_A, h_B, h_M} u(c_2)$$

subject to

$$c_2 = y_2 - (1 - h_B) B_f - (1 - h_M) M_f + \tilde{R} f \left(-[(1 - h_A) A_d + (1 - h_B) B_d] \right)$$

$$c_2 \geq 0$$

$$y_2 \geq (1 - \alpha) D + (1 - h) [\alpha D + B_f] \tag{1.27}$$

$$0 \leq h_A \leq 1$$

$$0 \leq h_B \leq 1$$

$$0 \leq h_M \leq 1.$$

We again assume that y_2 is large enough so that the expression (1.27) never binds. Let us

now focus on the haircut decisions in period 2. Firstly, raising h_M to 1 improves the objective function without violating any constraints. So it is optimal to set $h_M = 1$ for all values of the productivity shock \tilde{R} . The government never makes repayments on debt type M . It immediately follows that $q_M = 0$ and that the quantity of debt issuance M_f in the initial period is payoff irrelevant for the representative consumer. We may set $M_f = 0$ without loss of generality. Secondly, lowering h_A to 0 improves the objective function without violating any constraints. It is optimal to set $h_A = 0$ for all values of the productivity shock \tilde{R} . The government never defaults on any portion of debt type A .

So the only debt type with potentially variable haircuts is debt type B , held by both domestics and foreigners. Let us define $h = h_B$. Then the above program reduces to the program in the main text of this chapter.

Tradable Debt

The argument for the nontradable debt case can be modified for this case. The same results follow.

1.8.C. Concurrent Saving and Borrowing

In the environment studied in the chapter, the government cannot save in foreign assets abroad and concurrently issue defaultable debt. $B_f < 0$ corresponds to saving abroad, and $B_f > 0$ captures the issuance of defaultable debt to foreign creditors. This section of the appendix relaxes this restriction and allows the government to both save abroad and issue defaultable debt at the same time. This is achieved by considering a slightly amended model where the sovereign has access to a richer set of assets. It can still issue defaultable debt, which is denoted by $B_f \geq 0$. In addition, it can save abroad in a third asset, $J_f \leq 0$, which yields a gross return of $1 + r$ in every state of nature of the final period. The sovereign may choose any concurrent combination of defaultable debt issuance and saving in foreign assets. The main result of this section is that the feasible set of debt values G , and in particular the maximum level of debt sustainable in a rational expectations equilibrium, remain unchanged from the case in the main text.

Modifications to the Model

The maximization problem of the consumers remains unchanged, and domestic banks have access to the same set of assets as in the main text. The government budget constraints are altered to the following:

$$T_1 \leq p_A A_d + p_B B_d + q B_f + \frac{1}{1+r} J_f, \quad (1.28)$$

$$T_2 \geq A_d + (1-h)[B_d + B_f] + J_f \quad (1.29)$$

The definition of equilibrium follows.

Definition 1.5 *A Rational Expectations Equilibrium for this economy comprises sequences for allocation rules $\{c_1, s_1, c_2, \{\chi\}, x\}$, prices $\{p_A, p_B, q, \rho\}$ and policies $\{A_d, B_d, B_f, J_f, h, T_1, T_2\}$ that satisfy:*

(a) *Consumers choose $\{c_1, s_1, c_2\}$ to maximize utility (1.1) subject to the budget constraints (1.2), (1.3) and the nonnegativity constraints on consumption (1.4), taking prices, bank contract offers, government policies and the endowment as given.*

(b) *Banks offer contract schedules $\chi : s_1 \rightarrow S(s_1, \tilde{R})$ in period 1 to maximize expected profits, taking prices and government policies in period 2 as given.*

Banks choose lending quantity x in period 2 to maximize profits, taking the loan rate ρ as given.

(c) *Firms choose borrowing level x to maximize profits (1.8), taking the loan rate ρ as given.*

(d) *Government chooses $\{h, T_2\}$ in period 2 to satisfy the government budget constraint (1.6) in that period, taking $\{A_d, B_d, B_f, J_f\}$ and the shock \tilde{R} as given.*

Government chooses $\{A_d, B_d, B_f, J_f, T_1\}$ in period 1 to satisfy the government budget constraint (1.5) in that period, taking as given the price functions $\{p_A(A_d, B_d, B_f, J_f), p_B(A_d, B_d, B_f, J_f), q(A_d, B_d, B_f, J_f), \rho(x)\}$ and also the government policies in period 2, $h(A_d, B_d, B_f, J_f, \tilde{R})$ and $T_2(A_d, B_d, B_f, J_f, \tilde{R})$.

(e) *All markets clear for the economy. In particular, the markets for cash, defaultable debt, goods and loans clear.*

(f) Bond prices for foreign debt follow rational expectations: $q(A_d, B_d, B_f, J_f) = \frac{1}{1+r} \mathbb{E}\{1 - h\}$, taking the government policy $h(A_d, B_d, B_f, J_f, \tilde{R})$ in period 2 as given.

The goods market clearing condition yields the resource constraints:

$$c_1 \leq y_1 + qB_f + \frac{1}{1+r} J_f$$

$$c_2 \leq y_2 - (1 - h)B_f - J_f + \tilde{R}f(A_d + (1 - h)B_d)$$

Now we turn to the optimal policy problem for the government.

Definition 1.6 *The **Government Problem** is to maximize utility (1.1) over time consistent rational expectations equilibria. In particular, we must satisfy not only the equilibrium conditions above but also the additional optimization decisions:*

(g) *Government chooses $\{h, T_2\}$ in period 2 to maximize $u(c_2)$ given $\{A_d, B_d, B_f, J_f\}$ and the shock \tilde{R} .*

Government chooses $\{A_d, B_d, B_f, J_f, T_1\}$ in period 1 to maximize $u(c_1) + \beta \mathbb{E}u(c_2)$, taking the price functions $\{p_A(A_d, B_d, B_f, J_f), p_B(A_d, B_d, B_f, J_f), q(A_d, B_d, B_f, J_f), \rho(x)\}$ and government policies in period 2, $h(A_d, B_d, B_f, J_f, \tilde{R})$ and $T_2(A_d, B_d, B_f, J_f, \tilde{R})$, as given.

In the first specification, defaultable debt is not tradable between domestic banks and foreign creditors in the period of issue. In the second specification, defaultable debt is tradable in the period of issue. In the latter case, we impose the additional restriction:

$$p_B = q.$$

Again, we adopt the approach to the problem described in section 1.4.1.

Nontradable Debt

Apply the methodology applied to derive the formulation in the main text. The government problem may be derived:

$$V_1 = \max_{c_1, z} \mathbb{E} \{u(c_1) + \beta V_2(z)\}$$

subject to

$$c_1 = y_1 + z$$

$$c_1 \geq 0$$

$$z \in \check{G}$$

where the expression $V_2(z)$ is defined by

$$V_2(z) = \max_{c_2, \alpha, D, B_f, J_f} \mathbb{E} \{u(c_2)\}$$

subject to

$$c_2 = y_2 - (1 - h)B_f - J_f + \tilde{R}f([(1 - \alpha) + (1 - h)\alpha]D)$$

$$h = H(\alpha, D, B_f, \tilde{R})$$

$$z_1 = Q(\alpha, D, B_f) \cdot B_f \geq 0$$

$$z_2 = \frac{1}{1 + r} J_f < 0$$

for some set \check{G} . Our notation suppresses the dependence of h on $(\alpha, D, B_f, \tilde{R})$ in the consumption equation.

Let us briefly interpret this program. The government's problem may again be decomposed into two components. The intertemporal component of the problem concerns how much to borrow in period 1, z . The intratemporal component uses the functional form for the default decision h in the final period in order to calculate the optimal combination (α, D, B_f, J_f) for the chosen z value.

The intratemporal decision is more complicated in this model than in the main text. The debt issuance decisions (α, D, B_f) correspond to a gross debt position, i.e., $z_1 \geq 0$. The saving decision J_f corresponds to a gross asset value $z_2 \leq 0$. The net asset position of the economy at

the beginning of the final period is the summation of these two gross positions: $z = z_1 + z_2$.

The feasible set of values for z_1 is the same as the positive region of the feasible set G from the model of the main text. The feasible set for z_2 is the non-positive real line. It follows that the feasible set for z is the same as in the model studied in the main text of the chapter: $\check{G} = G$. One corollary of this result is that the maximum feasible debt level is the same for this model and the environment studied in the main text. Another corollary is that the maximum level of debt z_{\max} is achieved with the same values of (α, D, B_f) as in the model of the main text, and J_f set to zero.

Tradable Debt

The arguments provided above for the nontradable debt case can be adapted for the specification with tradable debt. The feasible set of debt is the same as in the model in the main text.

1.8.D. Justification for Equal Haircuts

Throughout this chapter, we assume that the government defaults equally on domestic and foreign lenders. The appendix considers possible justifications for this setup.

Unobservability of the debtholder

If the government cannot observe the residence of debtholders at the moment of repayment, it cannot discriminate and offer different haircuts to different categories of lenders, domestic or foreign. Even if the government can observe purchases of its own bonds in period 1, the existence of secondary markets for government debt means that defaultable debt may change hands over time. In period 2, the composition of the debt holders may be quite different.

Legal Restrictions

Governments may issue several different categories of debt, with each category classified as having different risk characteristics and awarding different legal rights to the creditors. In this case, it may be possible for the government to default on different debt categories in a differential manner, but legal constraints may force the sovereign to treat all debtholders within an asset class equally.

Equal Haircuts as an Equilibrium Outcome

The government may wish to execute different haircuts on different debtholders, but the existence of secondary markets may make this impossible or ineffective.

In the environment studied in the chapter, the government cannot distinguish whether the holders of the defaultable debt are domestic or foreign. Let us now consider a model where the sovereign can indeed discern whether the debt holders are domestic banks or foreign creditors at the point of repayment, but where it is still impossible for it to direct transfers to the domestic productive sector except by repaying government debt. The government has the option to select different haircuts for debt owed to domestic banks and foreign creditors.

The timing of the model considered in this section is captured in figure 1.13. The stages of the model highlighted in bold are the steps which are not present in the model in the main text of this chapter. There is lack of commitment between periods: the government cannot credibly make a commitment in period 1 regarding the extent of repayment of debt in period 2. However, the process of default in period 2 has a particular structure. After the stochastic productivity shock and the deterministic endowment are realized, the government announces haircuts of h_d and h_f on debt held by domestic and foreign residents respectively. The government conditions the haircut not on the holder of the debt at the point of the announcement, but at the point of repayment. For example, h_f is the haircut on debt held by foreigners at the time of execution of the haircut, not on debt held by foreigners at the time of the announcement of the haircut. Following this announcement, domestic and foreign holders of defaultable debt can trade it with each other on secondary markets. After such trading is completed, the government must enforce the haircuts that it announced earlier in the same period. In other words, the government has short-term (within-period) commitment: when it comes to the execution of default, the government must follow the haircut announcements that it has made at the beginning of the period. Domestic and foreign lenders settle their secondary trading accounts before the loans market opens in period 2.

Figure 1.13: Amended Model Timeline

<p>Period 1</p> <ul style="list-style-type: none"> • Endowment y_1 realized. • Government issues debt A_d, B_d, B_f and transfers proceeds T_1 to consumers. Consumers consume c_1 goods and save s_1 in banks. Banks invest in government debt A_d, B_d. Foreigners purchase government debt B_f. <p>Period 2</p> <ul style="list-style-type: none"> • Productivity shock \tilde{R} realized. • Government announces h_d, h_f. • Secondary market trades between domestic banks and foreigners. • Government imposes lump sum taxes T_2 and applies pre-announced haircuts h_d, h_f on debt B_d, B_f. • Domestic banks and foreigners settle their secondary trading positions. • Banks lend x to firms. Firms borrow and produce $F(x, \tilde{R}) = x + \tilde{R}f(x)$. • Consumers consume c_2 goods.
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Let us specify the facilities available in secondary market trading. All holders of defaultable government debt have access to secondary market trading accounts. These accounts allow debtholders to borrow unlimited funds from abroad in order to purchase government bonds from other debtholders, but these funds must be fully repaid before the loans market opens, at a gross (within-period) interest rate of one. This feature of the secondary market trading account means that it is possible for domestic (or, indeed, foreign) debtholders to purchase all the government debt between the announcement and execution of haircuts, at a price equal to $1 - h$, where h is the haircut that corresponds to the purchaser of the debt. Foreign debtholders may also purchase all of the debt. After the execution of the haircuts, the secondary market trading markets must be settled, i.e., receipts from government debt repayments must be used to repay all borrowed funds from abroad. Then the domestic productive sector produces output.

This timing of events preserves the liquidity constraint on the domestic production sector in the event of non-repayment of sovereign debt.

Let us now analyze the response of domestic and foreign debtholders to government haircut announcements. There are 3 cases to consider.

Case 1: $h_f > h_d$.

Foreigners value government debt at $1 - h_f$, which is lower than $1 - h_d$, the value of the debt to domestic agents. Foreign creditors are willing to sell their debt holdings at any price above $1 - h_f$. Therefore the supply of bonds S_f takes the shape in figure 1.14. Domestic debtholders are willing to purchase the debt at any price less than or equal to $1 - h_d$. Their demand for debt D_d is horizontal at this price. Therefore, the secondary market price of debt is $1 - h_d$, as shown in the figure.

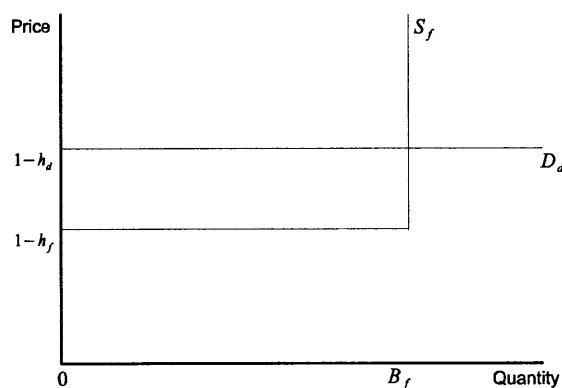


Figure 1.14: Secondary Markets for Debt

At the time of the execution of the haircut, all the debt is in the hands of domestic debtholders. The haircut of h_d is applied. Domestic debtholders make no profit on secondary market trades, since they must repay exactly this quantity to settle the secondary market trading account. Foreigners receive $1 - h_d$ for their debt. So all agents suffer a haircut of h_d on the debt. The government achieves this haircut on all of its debt.

Case 2: $h_f < h_d$.

An analogous argument to that above establishes that after the announcement and before the execution of the haircuts, foreigners purchase all of the debt from domestic debtholders at the price $1 - h_f$. The haircut applied on all of the debt, and therefore the haircut achieved by the government, is h_f .

Case 3: $h_f = h_d$.

In this case, domestic and foreign lenders value the debt at the same price. Therefore whether they purchase it from each other or not is irrelevant for the haircut imposed on the debt, and for the payoffs of domestic and foreign debtholders. The haircut applied on all of the debt is $h_f = h_d$.

Therefore, no matter the configuration of the haircuts announced by the government, both domestic and foreign debtholders effectively suffer the same haircut on the debt $h = \min\{h_d, h_f\}$, and the government achieves this haircut h on all its debt. For the rest of the model, it does not matter who ends up holding the debt after secondary market trading. Thus, the above argument proves the following result.

Lemma 1.3 *Consider any equilibrium with configuration of haircuts $\mathcal{H} = \{h_d, h_f\}$ announced by the government. Let $h = \min\{h_d, h_f\}$. There exists another equilibrium where the government announces the haircuts $\tilde{\mathcal{H}} = \{h, h\}$, which achieves the same payoffs for domestic and foreign debtholders and for the government.*

We conclude this section with a short discussion of the applicability of this result. The lag between haircut announcements and execution is designed to capture the fact that in reality, secondary markets are nearly always open for trading. Typically in the event of default, the institutional structure requires that governments announce haircuts in advance of making (partial) repayments. Secondary debt markets are always active, and in particular they will be open in the time between announcement and execution of haircuts (no matter how long this interval is in practice, especially if the secondary market is liquid). Therefore, debt can change hands in this interval. This is what we need for the mechanism in this version of the model to

be functional. Given this mechanism and the tradability of debt between domestic and foreign debtholders, haircuts are equalized across different categories of lenders.

1.9 References

1. Aiyagari, S. Rao, Albert Marcet, Thomas J. Sargent and Juha Seppälä. 2002. "Optimal Taxation without State-Contingent Debt." *Journal of Political Economy*, Vol. 110, No. 6, pp. 1220-1254.
2. Alessandro, Mauro. 2007. "Domestic Financial Development and Foreign Credit to Private Firms." Macro Lunch Presentation, MIT.
3. Amador, Manuel. 2003. "A Political Economy Model of Sovereign Debt Repayment." Working Paper, Stanford University GSB.
4. Arellano, Cristina. 2008. "Default Risk and Income Fluctuations in Emerging Economies." *The American Economic Review*, Vol. 98, No. 3, pp. 690-712.
5. Bulow, Jeremy, and Kenneth Rogoff. 1989. "Sovereign Debt: Is to Forgive to Forget?" *The American Economic Review*, Vol. 79, No. 1, pp. 43-50.
6. Broner, Fernando A., and Jaume Ventura. 2006. "Globalization and Risk Sharing." NBER Working Paper No. 12482.
7. Broner, Fernando A., Alberto Martin and Jaume Ventura. 2006. "Sovereign Risk and Secondary Markets." NBER Working Paper No. 12783.
8. Burnside, Craig, Martin Eichenbaum and Sergio Rebelo. 2001. "Hedging and Financial Fragility in Fixed Exchange Rate Regimes." *European Economic Review*, Vol. 45, pp. 1151-1193.
9. Cole, Harold L., and Patrick J. Kehoe. 1995. "The Role of Institutions in Reputation Models of Sovereign Debt." *Journal of Monetary Economics*, Vol. 35, pp. 46-64.
10. Cole, Harold L., and Patrick J. Kehoe. 1998. "Models of Sovereign Debt: Partial versus General Reputations." *International Economic Review*, Vol. 39, pp. 55-70.

11. de Bolle, Monica B., Ivetta Hakobyan and Bjoern Rother. 2006. "The Level and Composition of Public Sector Debt in Emerging Market Crises." IMF Working Paper No. 06/186.
12. De Paoli, Bianca, Glenn Hoggarth and Victoria Saporta. 2006. "Costs of Sovereign Default." Financial Stability Paper No.1, Bank of England.
13. Dooley, Michael P. 2000. "Can Output Losses Following International Financial Crises be Avoided?" NBER Working Paper No. 7531.
14. Eaton, Jonathan. 1996. "Sovereign Debt, Reputation and Credit Terms." *International Journal of Finance and Economics*, Vol. 1, pp. 25-35.
15. Eaton, Jonathan, and Mark Gersovitz. 1981. "Debt with Potential Repudiation: Theoretical and Empirical Analysis." *Review of Economic Studies*, Vol. 48, pp. 289-309.
16. Gelos, Gaston R., Ratna Sahay and Guido Sandleris. 2004. "Sovereign Borrowing by Developing Countries: What Determines Market Access?" IMF Working Paper No. 04/221.
17. Grossman, Herschel I., and John B. van Huyck. 1988. "Sovereign Debt as a Contingent Claim: Excusable Default, Repudiation and Reputation." *The American Economic Review*, Vol. 78, No. 5, pp. 1088-1097.
18. Jeanne, Olivier, and Anastasia Guscina. 2006. "Government Debt in Emerging Market Countries: A New Data Set." IMF Working Paper No. 06/98.
19. Kaminsky, Graciela L., and Carmen M. Reinhart. 1999. "The Twin Crises: The Causes of Banking and Balance-of-Payments Problems." *The American Economic Review*, Vol. 89, No. 3, pp. 473-500.
20. Kletzer, Kenneth M., and Brian D. Wright. 2000. "Sovereign Debt as Intertemporal Barter." *The American Economic Review*, Vol. 90, pp. 621-639.
21. Livshits, Igor. 2007. "Sovereign Default and Banking." Working Paper, University of Western Ontario.

22. Medeiros, Carlos, Michael Gapen and Luisa Zanforlin. 2005. "Assessing the Determinants and Prospects for the Pace of Market Access by Countries Emerging from Crisis – Further Considerations." International Capital Markets Department, IMF.
23. Mendoza, Enrique G., and Vivian Z. Yue. 2008. "A Solution to the Default Risk-Business Cycle Disconnect." NBER Working Paper No. 13861.
24. Rose, Andrew K. 2005. "One Reason Countries Pay their Debts: Renegotiation and International Trade." *Journal of Development Economics*, Vol. 77, pp. 189-206.
25. Sturzenegger, Federico, and Jeromin Zettelmeyer. 2005. "Haircuts: Estimating Investor Losses in Sovereign Debt Restructurings, 1998-2005." IMF Working Paper No. 05/137.
26. Sturzenegger, Federico, and Jeromin Zettelmeyer. 2006. *Debt Defaults and Lessons from a Decade of Crises*. Cambridge, Mass.: The MIT Press.
27. Tirole, Jean. 2003. "Inefficient Foreign Borrowing: A Dual- and Common-Agency Perspective." *The American Economic Review*, Vol. 93, No. 5, pp. 1678-1702.
28. Tomz, Michael. 2007. *Reputation and International Cooperation: Sovereign Debt across Three Centuries*. Princeton, New Jersey: Princeton University Press.
29. Werning, Iván. 2003. "Standard Dynamic Programming for Aiyagari, Marcet, Sargent and Seppälä." Mimeo, MIT.
30. Woodford, Michael. 1990. "Public Debt as Private Liquidity." *The American Economic Review*, Vol. 80, No. 2, pp. 382-388.

Chapter 2

Sovereign Debt and Fragility in an Infinite Horizon Model

2.1 Introduction

Recent sovereign default episodes have been associated with substantial output costs. The government may decide to default in order to reduce repayments abroad, but it should take into account that debt repudiation generates output costs for the domestic economy. In the previous chapter, we analyzed the effect of domestic economic costs of default on optimal government policy in a two-period model. In the model, the government issues debt in the first period and makes its repayment decision in the second. We characterized the optimal government default decision in the final period as a function of the productivity shock realized in that period, as well as inherited debt variables. The optimal haircut on debt is decreasing in the productivity shock and increasing in the volume of foreign (as opposed to domestic) debt issuance.

Then we took a step backward and analyzed the optimal government debt issuance policy in the first period. We allowed the government to manipulate three variables: the exposure of the domestic banking system to government debt, the composition of debtholders between foreign and domestic lenders, and the total level of domestic financial liquidity. The government recognizes that exposure of the domestic banking system creates a commitment device for debt repayment in the subsequent period, and it increases this exposure in order to be able to borrow more resources from abroad in the initial period. For high productivity shocks in the second

period, this commitment device works and the optimal haircut is low. For adverse productivity shocks, the benefits of sharing the low productivity realization with foreign creditors dominates the adverse domestic consequences of default. The government finds it optimal to default on most of its debt, even though domestic output is adversely affected.

It is reasonable to ask whether the results of our earlier work extend to the infinite horizon framework traditionally used in the default literature. This chapter addresses this question. We construct an infinite horizon version of the model described above, and we use it to derive three sets of additional results. We concentrate on the specification of the model with nontradable debt.

Infinite horizon models have an infinite number of equilibria depending on how we specify the coordination of foreign investors and the government to future continuation equilibria in the aftermath of the default decision. We show that if the government is not sanctioned by foreign creditors in either the period of default or future periods, the optimal haircut decision of the government is unchanged from the two-period model. In addition, the optimal government debt issuance decisions remain unchanged for the most part. The only difference is that the optimal domestic exposure level chosen by the government may differ. This specification of the consequences of default is closest to the spirit of chapter 1: we are interested in a framework where default does not lead to reduced access to international capital markets.

The infinite horizon specification allows us to examine the effects of persistent shocks. With independently and identically distributed productivity shocks, an adverse shock in a particular period does not change the feasible set of debt levels available to the domestic economy. The government wishes to borrow more in order to dampen the effect of the shock on domestic consumption. If the shock is persistent, a poor productivity realization leads to a contraction of the feasible set of debt levels, because it increases the expected haircuts in future periods. Therefore, the government finds that the minimum level of domestic exposure increases for any given level of resources borrowed from abroad. It may optimally choose to borrow less despite the negative shock. A sequence of negative shocks may lead to a simultaneous contraction of the feasible set of debt levels and an increase in the exposure of the domestic banking system to government debt.

Finally, we examine whether the government chooses to issue debt in the long run. In

an environment with noncontingent debt, the economy may find it optimal to accumulate a buffer stock of assets so as to avoid the region of debt issuance. The option of default increases the contingency of debt repayments, but the contingency is coupled with a higher expected consumption cost of debt issuance. Consider the restriction that the government must either issue debt or save abroad, but cannot do both. If the discount factor is low, then debt is included in the long run invariant distribution. However, if the discount factor is sufficiently high, debt is not observed in the long run.

What happens if the government can simultaneously issue domestic debt and save in assets abroad? We prove that irrespective of the discount factor, it is optimal for the government to both save abroad and simultaneously issue domestic debt in the long run. Savings allow the government to insure itself against adverse productivity shocks, but there is no contingency in interest repayments from this asset. Therefore, the government issues domestic debt and utilizes its ability to default in order to enjoy contingency in interest repayments across states of nature. The optimal pattern of debt issuance follows the same pattern described in the baseline infinite horizon specification.

This chapter seeks to improve our understanding of sovereign default, and it is interesting to contrast our results to other infinite horizon models in the literature. In the model of Eaton and Gersovitz (1981), the default decision by the country is followed by reduced access to international capital markets. In such a model, default improves consumption in the current period but reduces continuation utility. More recent theoretical analyses, such as the theoretical results in Arellano (2008), maintain the assumption that default is punished through coordination of foreign creditors to permanently worse continuation equilibria. Arellano relaxes this assumption in her numerical simulations. In the model considered in this chapter, default is not followed by future punishment by foreign creditors. In the current period, the optimal default decision benefits domestic consumption by reducing repayments abroad by more than the loss of domestic output. Furthermore, the default decision allows the government to reduce its debt level, which improves future continuation utility. Default benefits consumption in all periods.

Persistence of shocks introduces additional dimensions to the problem. An adverse productivity shock leads foreign investors to expect worse shocks in future periods, which raises the

optimal haircut in future periods. This leads to a contraction in the feasible set of debt levels in the current period, which changes the optimal government debt issuance decision immediately. In a different but related context, Aguiar et al. (2006) present a model of optimal investment cycles with risk of capital expropriation by the government and, crucially, persistent shocks. Following adverse productivity shocks, the government is less able to commit not to expropriate future returns, and this depresses investment even if the first best level of capital remains unchanged.

The remainder of the chapter is structured as follows. Section 2.2 summarizes the baseline model. We specify the punishment for default such that the insights of the two-period model carry over to the infinite horizon case. Section 2.3 presents the analysis of the model with persistent shocks. Section 2.4 considers the long run asset dynamics arising from the model. Section 2.5 concludes.

2.2 Infinite Horizon Model

The purpose of this section is to present an infinite horizon version of the two-period model developed in chapter 1. The precise nature of the equilibrium depends upon how we specify the coordination of foreign investors and the government to future continuation equilibria in the aftermath of the default decision. We show that if the default decision is not punished by reduced capital market access in either the period of default or future periods, the optimal haircut decision of the government is unchanged from the two-period model. In addition, most of the results regarding the optimal government debt issuance decision are still valid. These results are obtained despite the fact that it is more difficult to separate the intratemporal and intertemporal dimensions of the decisions in the infinite horizon case.

2.2.1 Model Setup

Time is discrete and the horizon is infinite: $t = 0, 1, 2, \dots$. There are five categories of actors in our framework: consumers, firms, banks, the government and foreign creditors. There is a continuum of all categories except the government. Consumers and firms both exist in unit measure.

Preferences Each identical and infinitely-lived consumer has utility function over consumption streams $\{c_t\}_{t=0}^{\infty}$ given by the expression

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right].$$

$\beta \in (0, 1)$ is the discount factor and the period utility function is continuously differentiable and strictly increasing: $u'(c) > 0$.

The benevolent government maximizes the utility of the representative consumer. Firms, banks and foreign creditors are risk neutral and maximize expected profits. They each survive for one period only.

Technology At the beginning of each period t , each consumer receives an endowment y . Then the firms in the economy have access to a production technology. An investment of x_t units of the endowment in the production sector yields $F(x_t, \tilde{R}_t)$ units of output:

$$F(x_t, \tilde{R}_t) = x_t + \tilde{R}_t f(x_t).$$

\tilde{R} is the level of domestic productivity. Its value is realized at the beginning of the period and is i.i.d. across periods. We assume $\tilde{R}_t \geq 0$, with highest and lowest values \bar{R} and \underline{R} respectively. The production function $f(x)$ is strictly increasing and strictly concave up to an input level \bar{x} , and is flat for input levels beyond this:

$$f'(x) \geq 0, f''(x) < 0 \quad \forall x \in [0, \bar{x}]$$

$$f(x) = f(\bar{x}) \quad \forall x \geq \bar{x}.$$

$f(x)$ is twice differentiable. We impose $\lim_{x \rightarrow 0} f'(x) = \infty$ and $f'(\bar{x}) = 0$. The output of the production sector cannot be reinvested in the same sector.

The economy enters period t with inherited debt level z_t . It makes repayments v_t to foreign creditors. In addition, it is possible for the economy as a whole to borrow resources z_{t+1} from foreign creditors. There is no domestic storable good between periods. The resource constraint

in period t is derived:

$$c_t \leq y + \tilde{R}_t f(x_t) - v_t + z_{t+1}$$

Foreign creditors maximize profits from their lending to the domestic economy, and they have access to an international riskless asset which yields the interest rate r between periods. The rational expectations restriction across periods is as follows:

$$z_t = \frac{1}{1+r} \mathbb{E}_{t-1} v_t.$$

Model Timeline Figure 2.1 illustrates the order of events and actions in each period t .

Figure 2.1: Model Timeline

Period t
<ul style="list-style-type: none"> • Endowment y realized.
<p style="margin-left: 2em;">Productivity shock \tilde{R}_t realized.</p>
<ul style="list-style-type: none"> • Government imposes lump sum taxes $T_{2,t}$ and applies haircut h_t on debt $B_{d,t}, B_{f,t}$.
<ul style="list-style-type: none"> • Banks lend x_t to firms. <p style="margin-left: 2em;">Firms borrow and produce $F(x_t, \tilde{R}_t) = x_t + \tilde{R}_t f(x_t)$.</p>
<ul style="list-style-type: none"> • Government issues debt $A_{d,t+1}, B_{d,t+1}, B_{f,t+1}$ and transfers proceeds $T_{1,t}$ to consumers. <p style="margin-left: 2em;">Consumers consume c_t goods and save s_t in banks.</p> <p style="margin-left: 2em;">Banks invest in government debt $A_{d,t+1}, B_{d,t+1}$.</p> <p style="margin-left: 2em;">Foreigners purchase government debt $B_{f,t+1}$.</p>

Consumers Each consumer solves the following maximization problem:

$$\max_{\{c_t, s_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \quad (2.1)$$

subject to

$$c_t \leq y - s_t + S(s_{t-1}, \tilde{R}_t) + T_{1,t} + T_{2,t} + \Pi_{B,t} + \Pi_{F,t} \quad (2.2)$$

$$c_t \geq 0 \quad (2.3)$$

In the each period t , each consumer decides on its consumption and savings decisions $\{c_t, s_t\}$. Transfers from the government $T_{1,t}$ and $T_{2,t}$ are taken as given. Savings s_t are deposited in the banks, and yield a gross investment return of $S(s_t, \tilde{R}_{t+1})$ in the subsequent period. Each consumer owns an equal share in all banks and firms. $\Pi_{B,t}$ and $\Pi_{F,t}$ denote their respective profits in period t .

Consumers, firms and banks cannot borrow from or save abroad.

Bank Deposit Contracts In period t , banks offer contracts χ to consumers in a competitive market:

$$\chi : s_t \longrightarrow S(s_t, \tilde{R}_{t+1})$$

No other transfers between consumers and banks are allowed. Consumers choose the contract that maximizes their expected utility. In competitive equilibrium, bank profits are zero ($\Pi_{B,t} = 0$) and their investment in assets in period t maximizes the expected utility of consumers. Since there is no storable good between periods, the banking system transfers resources between periods by purchasing government-issued debt. The set of government assets is described next.

Government Debt At the end of each period t , the government issues cash $A_{d,t+1}$ and defaultable debt B_{t+1} . Of the defaultable debt, $B_{d,t+1}$ is purchased by domestic banks and $B_{f,t+1}$ is purchased by foreign creditors. Government expenditure is set to zero. The government may transfer to consumers any resources raised from debt issuance:

$$T_{1,t} \leq p_{A,t}A_{d,t+1} + p_{B,t}B_{d,t+1} + q_t B_{f,t+1}. \quad (2.4)$$

Positive quantities are used to denote debt. $p_{A,t}$ is the price of cash in terms of output. $p_{B,t}$ and q_t are the prices of defaultable debt held by domestic banks and foreign creditors respectively. Notice especially that for the model considered in this chapter, defaultable debt is not tradable between domestic and foreign agents in the period of issue. Therefore, these prices may differ.

At the beginning of the next period $t + 1$, the government observes the productivity shock and then decides on its repayments to holders of the defaultable debt. The government cannot default on cash, and it must default on all holders of defaultable debt by an equal haircut h_{t+1} . The haircut is defined to be the proportion of the face value of debt that is not repaid. The

government imposes lump sum transfers on consumers in order to make repayments on its debt:

$$-T_{2,t+1} \geq A_{d,t+1} + (1 - h_{t+1}) [B_{d,t+1} + B_{f,t+1}]. \quad (2.5)$$

The government cannot commit in period t to the level of the haircut h_{t+1} in period $t + 1$.

Foreign Creditors The rational expectations restriction may be rewritten in terms of the debt variables.

$$\begin{aligned} & \max_{B_{f,t+1}} \left\{ \frac{1}{1+r} \mathbb{E}_t(1 - h_{t+1}) B_{f,t+1} - q_t B_{f,t+1} \right\} \\ \implies & q_t = \frac{1}{1+r} \mathbb{E}_t(1 - h_{t+1}) \end{aligned} \quad (2.6)$$

This equation determines q_t , the price of defaultable debt held by foreign creditors.

Loans Market in Period t Banks enter period t with holdings of government-issued cash and defaultable debt. The resources in the banking system after the default decision are given by the expression

$$X_t = A_{d,t} + (1 - h_t) B_{d,t}.$$

We assume that the government cannot transfer resources from consumers to banks except through repayment of cash and defaultable debt, and that banks have no other means of raising funds from consumers.

Banks can either hold these resources X_t until the end of the period, or to lend these resources to firms in a competitive market for loanable funds. In the latter case, firms use the loaned funds as inputs in production and repay the banks with interest before the end of the period. At the end of period t , banks transfer the promised units of output $S(s_{t-1}, \tilde{R}_t)$ to consumers.

Firms take the loan rate for funds ρ_t as given and choose to borrow x_t units of input in order to maximize profits:

$$\begin{aligned} & \max_{x_t} \left\{ x_t + \tilde{R}_t f(x_t) - \rho_t x_t \right\} \\ \implies & 1 + \tilde{R}_t f'(x_t) = \rho_t \end{aligned} \quad (2.7)$$

The equilibrium loan rate is given by

$$\rho_t = 1 + \tilde{R}_t f'(X_t).$$

The key constraint that summarizes the market imperfection on the production side of the economy is

$$x_t \leq A_{d,t} + (1 - h_t)B_{d,t}$$

In each period t , inputs into the domestic production sector are less than or equal to the total value of repaid government cash and bonds. Inputs into production in period t are constrained by the gross return on investments made in period $t - 1$.

2.2.2 Equilibrium Definition

The equilibrium definition is as follows.

Definition 2.1 *A Rational Expectations Equilibrium for this economy comprises sequences for allocation rules $\{c_t, s_t, \{\chi\}_t, x_t\}_{t=0}^\infty$, prices $\{p_{A,t}, p_{B,t}, q_t, \rho_t\}_{t=0}^\infty$ and policies $\{A_{d,t}, B_{d,t}, B_{f,t}, h_t, T_{1,t}, T_{2,t}\}_{t=0}^\infty$ that satisfy:*

(a) *Consumers choose $\{c_t, s_t\}_{t=0}^\infty$ to maximize utility (2.1) subject to the budget constraint (2.2) and the nonnegativity constraint on consumption (2.3), taking prices, bank contract offers, government policies and the endowment as given.*

(b) *Banks offer contract schedules $\chi : s_t \rightarrow S(s_t, \tilde{R}_{t+1})$ in period t to maximize expected profits, taking prices and government policies in periods t and $t + 1$ as given.*

Banks choose lending quantity x_t in period t to maximize profits, taking the loan rate ρ_t as given.

(c) *Firms choose borrowing level x_t to maximize profits (2.7), taking the loan rate ρ_t as given.*

(d) *Government chooses $\{h_t, T_{2,t}\}$ in period t to satisfy the government budget constraint (2.5) in that period, taking $\{A_{d,t}, B_{d,t}, B_{f,t}\}$ and the shock \tilde{R}_t as given.*

Government chooses $\{A_{d,t+1}, B_{d,t+1}, B_{f,t+1}, T_{1,t}\}$ in period t to satisfy the government budget constraint (2.4) in that period, taking the price functions $\{p_{A,t}(A_{d,t+1}, B_{d,t+1}, B_{f,t+1})$,

$p_{B,t}(A_{d,t+1}, B_{d,t+1}, B_{f,t+1}), q_t(A_{d,t+1}, B_{d,t+1}, B_{f,t+1}), \rho_t(x_t)$ and government policies in period $t+1$, $h_{t+1}(A_{d,t+1}, B_{d,t+1}, B_{f,t+1}, \tilde{R}_{t+1})$ and $T_{2,t+1}(A_{d,t+1}, B_{d,t+1}, B_{f,t+1}, \tilde{R}_{t+1})$, as given.

(e) All markets clear for the economy for every period t . In particular, the markets for cash, defaultable debt, goods and loans clear.

(f) Bond prices for foreign debt follow rational expectations: $q_t(A_{d,t+1}, B_{d,t+1}, B_{f,t+1}) = \frac{1}{1+r} \mathbb{E}_t \{1 - h_{t+1}\}$, taking the government policy $h_{t+1}(A_{d,t+1}, B_{d,t+1}, B_{f,t+1}, \tilde{R}_{t+1})$ in period $t+1$ as given.

The optimal policy problem for the government is described next. The government lacks commitment: it cannot credibly commit in period t to the haircut it will impose in period $t+1$.

Definition 2.2 *The Government Problem* is to maximize utility (2.1) over time consistent rational expectations equilibria. In particular, we must satisfy not only the equilibrium conditions above but also the additional optimization decisions:

(g) Government chooses $\{h_t, T_{2,t}\}$ in period t to maximize

$$\mathbb{E}_t \left[\sum_{s=t}^{\infty} \beta^{s-t} u(c_s) \right], \quad (2.8)$$

given $\{A_{d,t}, B_{d,t}, B_{f,t}\}$, the shock \tilde{R}_t and the behavior of foreign investors following any default history $\{h_s\}_{s=0}^t$.

Government chooses $\{A_{d,t+1}, B_{d,t+1}, B_{f,t+1}, T_{1,t}\}$ in period t to maximize expression (2.8), taking as given the price functions $\{p_{A,t}(A_{d,t+1}, B_{d,t+1}, B_{f,t+1}), p_{B,t}(A_{d,t+1}, B_{d,t+1}, B_{f,t+1}), q_t(A_{d,t+1}, B_{d,t+1}, B_{f,t+1}), \rho_t(x_t)\}$ and also the optimal government policies in period $t+1$, $h_{t+1}(A_{d,t+1}, B_{d,t+1}, B_{f,t+1}, \tilde{R}_{t+1})$ and $T_{2,t+1}(A_{d,t+1}, B_{d,t+1}, B_{f,t+1}, \tilde{R}_{t+1})$.

From these definitions we can immediately see that the set of equilibria is larger in the infinite horizon case than in the two-period setup. In particular, notice that the gross expected return to foreign investors from holding government debt is equal to the gross riskless rate $1+r$. Foreign investors are indifferent between lending to the domestic government and investing in

riskless assets. Therefore, Definition 2.1 part (f) is consistent with a continuum of levels of borrowing, given any choice $\{A_{d,t+1}, B_{d,t+1}, B_{f,t+1}\}$ by the government. In the aftermath of a default episode, it is possible for foreign investors to sanction the government by coordinating to a lower level of lending to the government. The precise specification of the punishment for default is important for the determination of the optimal government haircut, and therefore the optimal government debt issuance decision.

2.2.3 Optimal Policy Program

We rewrite any combination of government debt issuance $(A_{d,t}, B_{d,t}, B_{f,t})$ as a combination $(\alpha_t, D_t, B_{f,t})$ such that:

$$\begin{aligned} D_t &= A_{d,t} + B_{d,t} \\ \text{where } A_{d,t} &= (1 - \alpha_t) D_t \\ B_{d,t} &= \alpha_t D_t. \end{aligned}$$

D_t is equal to the total face value of government-issued cash and defaultable debt held by the domestic banks at the beginning of period t . α_t is the fraction of defaultable debt in total bank assets. This amended notation is used in the remainder of this chapter.

The default history of the government until period t is given by $\{h_s\}_{s=0}^t$. Consider a government that has not defaulted until the current period. We write the program for the government problem as follows. In the current period:

$$U(\alpha, D, B_f, \tilde{R}) = \max_{h, \alpha', D', B'_f} \left\{ u(c) + \beta \mathbb{E} U(\alpha', D', B'_f, \tilde{R}', h) \right\}$$

subject to

$$c = y - (1 - h)B_f + \tilde{R}f - [(1 - \alpha) + (1 - h)\alpha]D + qB'_f \quad (2.9)$$

$$c \geq 0 \text{ and } 0 \leq h \leq 1 \quad (2.10)$$

$$y \geq (1 - \alpha)D + (1 - h)[\alpha D + B_f] \quad (2.11)$$

$$q = \frac{1}{1 + r} \mathbb{E} \left\{ 1 - h'(\alpha', D', B'_f, \tilde{R}') \right\} \quad (2.12)$$

$$B'_f \in (-\infty, \overline{B}_f(h)], \text{ with } B'_f < 0 \Rightarrow \alpha' = 0. \quad (2.13)$$

The government enters the period with inherited debt variables (α, D, B_f) . It then decides the optimal haircut. As in the two-period model, it takes into account both the contemporaneous productivity shock \tilde{R} and the inherited debt variables. In the infinite horizon context, it also takes into account the impact of its default decision on the economy's capital market access. The upper bound $\overline{B}_f(h)$ on the feasible set of debt issuance in expression (2.13) captures the possibility that default in this period leads to a restricted ability to issue debt this period. Restricted access to capital markets in future periods is incorporated in the above framework by making the expected continuation utility of the government $\mathbb{E}U(\alpha', D', B'_f, \tilde{R}', h)$ depend on the current haircut h . The precise nature of foreign creditors' punishments determines the shape of the function for continuation utility $\mathbb{E}U(\alpha', D', B'_f, \tilde{R}', h)$.

At the end of the period, the government decides on its debt issuance decision. Each combination (α', D', B'_f) corresponds to a default schedule across states $h'(\alpha', D', B'_f, \tilde{R}')$ in the next period, and hence to the bond price function $q = Q(\alpha', D', B'_f)$. This function is calculated using rational expectations over the default schedule in the next period $t+1$, and is taken as given by the government in the current period t .

Expression (2.9) shows that consumption depends both on the current value of the haircut h as well as the government's debt issuance decision (α', D', B'_f) . The debt issuance decision affects both the price and volume of foreign debt issuance. Equation (2.11) states that government debt repayments must be less than or equal to the consumer endowment in that period. For the remainder of this chapter, we assume that y is large enough so that this constraint never binds. A sufficient condition on the production function to ensure that this approach is valid is: $\lim_{x \rightarrow 0} x f'(x) = 0$. We assume that this condition is satisfied.

Finally, the second part of expression (2.13) states that the government chooses either to save abroad or to issue debt. If it chooses to save abroad, it does not issue debt, and therefore the exposure level of the domestic economy is zero. This assumption is made to simplify the model; it is relaxed in subsection 2.4.2 of this chapter.

Let us define the haircut and bond price schedules:

$$h = \tilde{H}(\alpha, D, B_f, \tilde{R})$$

and

$$\tilde{Q}(\alpha', D', B_f') = \frac{1}{1+r} \mathbb{E} \left\{ 1 - \tilde{H}(\alpha', D', B_f', \tilde{R}') \right\}.$$

We now rewrite the problem in a form that is more amenable to theoretical and numerical analysis. In the amended version of the program, the government first chooses how much to raise from abroad z' , and then decides the optimal combination (α', D', B_f') that achieves this level of borrowing. The optimal combination is decided before the state of nature in the next period is realized. The government problem above is rewritten as follows.

$$V(z) = \max_{\alpha, D, B_f, z'} \mathbb{E} \{ u(c) + \beta V(z', h) \}$$

subject to

$$c = y - (1-h)B_f + \tilde{R}f([(1-\alpha) + (1-h)\alpha]D) + z'$$

$$c \geq 0$$

$$h = \tilde{H}(\alpha, D, B_f, \tilde{R})$$

$$z = \tilde{Q}(\alpha, D, B_f) \cdot B_f, \text{ with } z < 0 \Rightarrow \alpha = 0$$

$$z' \in \tilde{G}(h).$$

What are the consequences of default in the general case? Firstly, it may lead to a reduction in the government's ability to issue debt in the period of default, and this means that the set of feasible debt values $G(h)$ depends on the haircut h . Secondly, it may lead to reduced capital market access in future periods. Therefore, the value function $V(z', h)$ also depends on the haircut in this period.

It is straightforward to show that Lemma 1.1 and Corollary 1.1 from the two-period model are still valid. We can restrict attention to combinations $C = (\alpha, D, B_f)$ such that $D \in [0, \bar{D}]$.

2.2.4 First Best Case

As in the finite horizon setup, the first best case is achieved when the government can both (i) fully commit in period t to the haircut schedule in period $t+1$, and (ii) save abroad and issue debt at the same time. There is no adverse effect of default in terms of diminished current or

future capital market access by the country.

The government's optimal policy program is stationary.

Proposition 2.1 (First Best Case) *Assume that y is sufficiently high. The optimal consumption schedule $\{c_t\}_{t=0}^{\infty}$ is the same whether debt is tradable or not. It has the properties:*

1. *Production by domestic firms is equal to $\bar{x} + \tilde{R}_t f(\bar{x})$ when the productivity shock is \tilde{R}_t .*

The optimal allocation solves:

$$\max_{\{h_t, B_{f,t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u \left(y + \tilde{R}_t f(\bar{x}) - (1 - h_t) B_{f,t} + \frac{B_{f,t+1}}{1+r} \mathbb{E}_t(1 - h_{t+1}) \right) \right]$$

2. *Consumption $\{c_t\}_{t=0}^{\infty}$ is chosen to satisfy the representative consumer's Euler equation.*
3. *Consumption is equalized across states of nature \tilde{R}_t in each period t (by appropriate selection of haircuts in period t).*

The total output of domestic firms is at the maximum level in every state of nature \tilde{R} in period 2. The output of this sector does vary due to the fluctuation in the productivity shock value, but consumption is fully insured against this shock by the government. To achieve this, repayments to foreigners in period 2 vary across different states of nature.

2.2.5 No Punishment after Default

Let us return to the infinite horizon environment with lack of commitment, described in subsection 2.2.3.

Consider a specification of the model such that the government is not sanctioned by foreign creditors in response to a default decision. This scenario is closest to the spirit of chapter 1: we are interested in a framework where debt repayment is (partially) enforced by the domestic output costs of default, not by punishments available to external creditors. Mathematically, the consequences of this assumption can be written:

$$\tilde{G}(h) = \tilde{G}$$

and

$$V(z', h) = V(z').$$

In words, the default decision does not affect capital market access by the country either in the period of default or in future periods. $V(z')$ is defined in a recursive manner.

The government problem described in subsection 2.2.3 can now be rewritten as a recursive stationary problem. It can be written as follows.

$$V(z) = \max_{\alpha, D, B_f, z'} \mathbb{E} \{u(c) + \beta V(z')\}$$

subject to

$$c = y - (1 - h)B_f + \tilde{R}f([(1 - \alpha) + (1 - h)\alpha]D) + z'$$

$$c \geq 0$$

$$h = H(\alpha, D, B_f, \tilde{R})$$

$$z = Q(\alpha, D, B_f) \cdot B_f, \text{ with } z < 0 \Rightarrow \alpha = 0$$

$$z' \in G.$$

Note that we have replaced the general functions \tilde{H} , \tilde{Q} and \tilde{G} with their counterparts from the two-period model: H , Q and G .

The proposition below immediately follows.

Proposition 2.2 (Haircut Decision) *If default is not followed by reduced capital market access, then the haircut function is the same as in the two-period model. The optimal haircut decision $h = H(\alpha, D, B_f, \tilde{R})$ satisfies the following formulation:*

$$h = \max\{0, \min\{1, \theta\}\}$$

where θ satisfies

$$\frac{B_f}{\tilde{R}\alpha D} = f'([(1 - \alpha) + (1 - \theta)\alpha]D). \quad (2.14)$$

1. The haircut is (weakly) increasing in the volume of foreign debt issued B_f .

2. The haircut is (weakly) decreasing in the productivity shock \tilde{R} .

Government default does not induce any kind of sanction by foreign creditors. Therefore, the haircut is selected only to maximize consumption in the period of default, as in the two-period model. The default decision depends on the domestic productivity shock and inherited debt variables. Default is high when the domestic productivity shock \tilde{R} is low. The higher is the ratio of foreign-held to domestically-held defaultable debt $\frac{B_f}{\alpha D}$, the higher is the marginal benefit of default. Domestic output is lower at the optimum. The haircut is increasing in the volume of foreign debt issuance B_f .

How are current consumption and continuation utility affected when the government defaults? The haircut decision is selected to maximize consumption in the current period. For any given debt level chosen in this period z' , current consumption is higher owing to default. Alternatively, default allows the country to maintain the same level of consumption as in the case without default, but with a lower debt level z' . The lower debt level corresponds to higher continuation utility. In our framework, the default decision can lead to both higher current consumption and improved continuation utility. This contrasts with much of the theoretical literature, which associates default with higher current consumption and lower continuation utility. The latter effect comes from reduced capital market access in future periods.

For the remainder of the chapter, we maintain the assumption that government default is not followed by any sanctions in the form of reduced capital market access.

Proposition 2.3 (Feasibility of Debt) *It is feasible for the sovereign to issue debt in every period. The maximum debt level is the same as in the two-period model.*

What about the optimal government debt issuance decision? For the infinite horizon specification, it is more difficult to separate the intertemporal and intratemporal dimensions of the problem.

The intertemporal government problem may be written as follows.

$$V(z) = \max_{z'} \mathbb{E} \left\{ u \left(X(z, z', \tilde{R}) + z' \right) + \beta V(z') \right\}$$

subject to

$$X(z, z', \tilde{R}) + z' \geq 0$$

$$z' \in G.$$

The expression $X(z, z', \tilde{R})$ comes from the solution to the intratemporal problem. It is the optimal schedule of consumption across states of nature \tilde{R} in the current period, given the inherited debt level z and the chosen debt level this period z' :

$$X(z, z', \tilde{R}) = y - (1-h)B_f^* + \tilde{R}f([(1-\alpha^*) + (1-h)\alpha^*]D^*)$$

where (α^*, D^*, B_f^*) solves

$$\max_{\alpha, D, B_f} \mathbb{E} \left\{ u \left(y - (1-h)B_f + \tilde{R}f([(1-\alpha) + (1-h)\alpha]D) + z' \right) + \beta V(z') \right\}$$

subject to

$$h = H(\alpha, D, B_f, \tilde{R})$$

$$z = Q(\alpha, D, B_f) \cdot B_f, \text{ with } z < 0 \Rightarrow \alpha = 0$$

For the two-period model, z' equals zero by construction and the intratemporal problem can be solved independently of the intertemporal decision. For the infinite horizon specification, this separation result no longer holds. Nevertheless, Propositions 1.5 to 1.9 of the two-period model remain valid. They are renumbered and reproduced below.

Proposition 2.4 (Saving) *For $z < 0$, the government chooses: (i) $\alpha = 0$; (ii) $D = \bar{D}$; (iii) $B_f = (1+r)z$.*

Proposition 2.5 (Minimum Domestic Exposure) *Fix $D = D'$. For any level of borrowing in the set $[0, z_{\max}(D')]$ to be achieved, it is required that the level of domestic exposure is sufficiently high, i.e., $\alpha \in [\underline{\alpha}(z), 1]$. The necessary exposure level has the following properties:*

1. $\underline{\alpha}(0) = 0$.
2. $\underline{\alpha}(z)$ is weakly increasing in z .

3. $\underline{\alpha}(z_{\max}(D')) = 1$.

Proposition 2.6 (Total Domestic Debt) *It is an optimum to set $D = \bar{D}$.*

Proposition 2.7 (Optimal Domestic Exposure) *Consider a combination $C = (\alpha, \bar{D}, B_f)$ which raises debt z such that $\alpha > \underline{\alpha}(z)$. At the margin, it is feasible to raise the same level of debt z by reducing α and increasing $\gamma = \frac{B_f}{\alpha \bar{D}}$. Whether this perturbation is optimal depends on the risk aversion of the representative consumer.*

Proposition 2.8 (Foreign Debt Issuance) *Let $u(c)$ and $f(x)$ be specified such that $\alpha = 1$ is the optimal level of exposure for all levels of debt. Then for $z \in [0, z_{\max}]$:*

1. B_f is increasing in z .
2. The interest rate on government debt is increasing in z and the volume of foreign debt issuance B_f .

A caveat is in order. Proposition 2.7 establishes that the optimal domestic exposure level for the economy depends on the risk aversion of the representative consumer. The relevant risk aversion in the infinite horizon case may be different from the two-period model. This is because the choice of the debt level z' affects the average level of consumption, and the risk aversion coefficient may vary with this average consumption level. In the two-period model, z' equals zero by construction.

2.3 Persistent Shocks

Persistence of productivity shocks introduces additional dimensions to the government problem. In this section we characterize the government's optimal policy program with persistent shocks, and present both theoretical and numerical results for this case. An poor productivity realization increases the probability of adverse shocks in the future, and this increases the expected haircuts in the next period. Therefore, the feasible set of debt levels contracts today. Furthermore, remember that the government's optimal debt issuance decision is made before the productivity shock in the next period is realized. After an adverse productivity shock, the

government revises upward the probability of future adverse shocks. Correspondingly, its optimal debt issuance decision places greater weight on consumption in the worst states of nature in future periods.

We maintain the assumption that government default is not punished by reduced capital market access. The government problem described in subsection 2.2.5 was valid for a model setup with independently and identically distributed productivity shocks. It can be amended for the case for persistent shocks as follows. The value of the productivity shock realized in the previous period \tilde{R}^- is a new state variable for the problem.

$$V(z, \tilde{R}^-) = \max_{\alpha, D, B_f, z'} \mathbb{E} \left\{ u(c) + \beta V(z', \tilde{R}) \mid \tilde{R}^- \right\}$$

subject to

$$c = y - (1 - h)B_f + \tilde{R}f([(1 - \alpha) + (1 - h)\alpha]D) + z'$$

$$c \geq 0$$

$$h = H(\alpha, D, B_f, \tilde{R})$$

$$z = Q(\alpha, D, B_f, \tilde{R}^-) \cdot B_f, \text{ with } z < 0 \Rightarrow \alpha = 0$$

$$z' \in G(\tilde{R}).$$

The haircut function H is unchanged from the case with independently and identically distributed shocks. Consider any realization of the productivity shock \tilde{R} . All of the propositions proved in subsection 2.2.5 are still valid conditional on the value of the productivity shock. However, the feasible and optimal levels of domestic exposure vary across different realizations of the shock. The bond price function Q now depends on the value of the productivity shock realized in the previous period \tilde{R}^- . The feasible set of debt levels G for the current period depends on the current realization of the productivity shock \tilde{R} .

For concreteness, let us specify $\tilde{R} \in \{\underline{R}, \bar{R}\}$ such that productivity shocks follow the Markov process:

$$\Pr(\tilde{R}_{t+1} = \underline{R} \mid \tilde{R}_t = \underline{R}) = \Pr(\tilde{R}_{t+1} = \bar{R} \mid \tilde{R}_t = \bar{R}) = \pi,$$

where we set $\pi > \frac{1}{2}$.

Proposition 2.9 (Minimum Domestic Exposure) Fix $D = D'$. Define $\underline{\alpha}(z, \tilde{R}^-)$ to be the minimum domestic exposure level required to achieve debt level z after productivity shock realization \tilde{R}^- . For any level of debt z' that is feasible for $\tilde{R}^- \in \{\underline{R}, \bar{R}\}$:

$$\underline{\alpha}(z, \underline{R}) \geq \underline{\alpha}(z, \bar{R}).$$

Any given level of domestic exposure α corresponds to a schedule of haircuts across states of nature in the next period. An adverse productivity shock increases the probability of a low productivity realization in the next period. Since the haircut is higher for the low productivity shock, such a change in probabilities increases the average expected future haircut. This reduces the maximum feasible debt level for any given α . A higher level of domestic exposure is necessary in order to achieve the same debt level.

The maximum level of debt is achieved with $\alpha = 1$ and $D = \bar{D}$. The following proposition immediately follows.

Proposition 2.10 (Maximum Debt Level) Define $z_{\max}(\tilde{R})$ to be the maximum debt level after the realization of productivity shock \tilde{R} . Then $z_{\max}(\bar{R}) > z_{\max}(\underline{R})$.

An adverse productivity realization in the current period increases the probability of a high haircut in the next period. The average expected haircut increases for any given volume of debt issuance to foreigners B_f . Therefore, the maximum feasible level of debt is lower following an adverse productivity shock.

Following a low productivity shock, the government faces a reduced feasible set of debt levels. It also recognizes that the probability of an adverse productivity shock in the next period is higher. How does this affect the optimal government debt issuance decision? The government places a higher weight on consumption in the worst state of nature in future periods. In particular, the government may wish to reduce the exposure level of the domestic economy in order to insure consumption in the worst future states of nature. Its ability to do so is constrained by the fact that the feasible set G has changed, such that a higher exposure level is necessary to raise any given level of debt.

We use numerical simulations to illustrate the effect of persistent shocks on the optimal

government debt issuance decision. We utilize the constant relative risk aversion utility function

$$u(c) = \log(c).$$

The following production function is employed:

$$f(x) = \begin{cases} x^\theta - \delta x & \text{for } x \leq \bar{x} \\ \bar{x}^\theta - \delta \bar{x} & \text{for } x > \bar{x} \end{cases}$$

where \bar{x} is set to the value that maximizes $f(x)$, i.e., $\bar{x} = \left(\frac{\theta}{\delta}\right)^{\frac{1}{1-\theta}}$. This production function satisfies the assumptions in subsection 2.2.1. In addition, it satisfies the property that $\lim_{x \rightarrow 0} x f'(x) = 0$.

The parametrization of the model is as follows. The riskless rate of return is equal to $r = 0.05$ and the discount factor is set to $\beta = 0.8$. We set $y = 9$. The production function parameters are $\theta = \delta = 0.5$. The implied \bar{x} (and hence \bar{D}) is therefore equal to unity. In each period, there are two possible values of the productivity shock, $\underline{R} = 8$ and $\bar{R} = 12$. We set $\pi = 0.9$. With these model parameters, the upper bound of the set $G(\underline{R})$ is $z_{\max}(\underline{R}) = 0.9951$, and the upper bound for the set $G(\bar{R})$ is $z_{\max}(\bar{R}) = 1.3774$.

Figure 2.2 shows the minimum levels of domestic exposure associated with raising any given level of debt z . The minimum level is higher for the country experiencing the low productivity shock. Notice that the maximum level of debt $z_{\max}(\bar{R})$ is lower for the low productivity shock. Figure 2.3 plots the optimal domestic exposure level, which for this specification turns out to be $\alpha = 1$ for all levels of debt. Figure 2.4 shows the optimal volume of foreign debt issuance B_f to raise any given level of debt. The country that experiences the adverse productivity shock issues a higher face value of debt B_f in order to raise any given level of real resources z , because the adverse shock increases the expected haircut next period.

Finally, figure 2.5 plots the interest rate schedule as a function of the total volume of government debt issuance to foreigners B_f . After an adverse productivity realization, expected haircuts increase and this drives down the price of government bonds sold to foreigners. We observe a corresponding shift upward in the interest rate schedule.

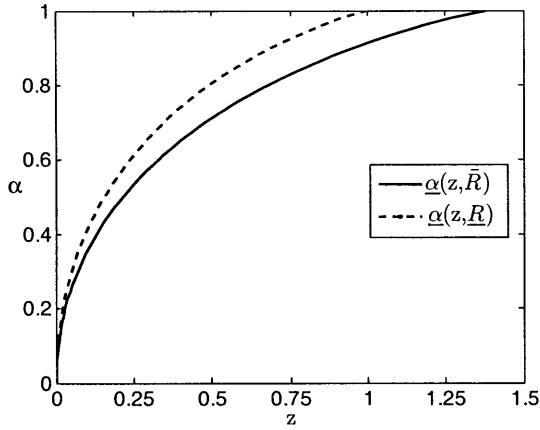


Figure 2.2: Debt Level z and Minimum Domestic Exposure Level $\underline{\alpha}(z, \tilde{R})$

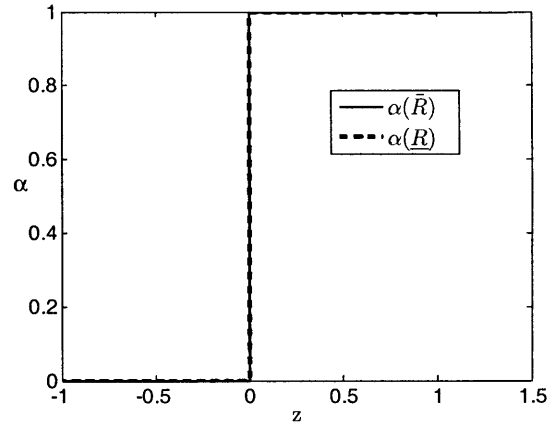


Figure 2.3: Optimal Domestic Exposure Level for $\tilde{R}^- \in \{\underline{R}, \bar{R}\}$

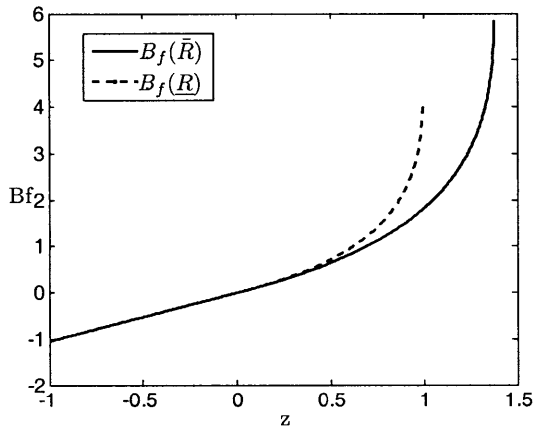


Figure 2.4: Optimal Foreign Debt Issuance for $\tilde{R}^- \in \{\underline{R}, \bar{R}\}$

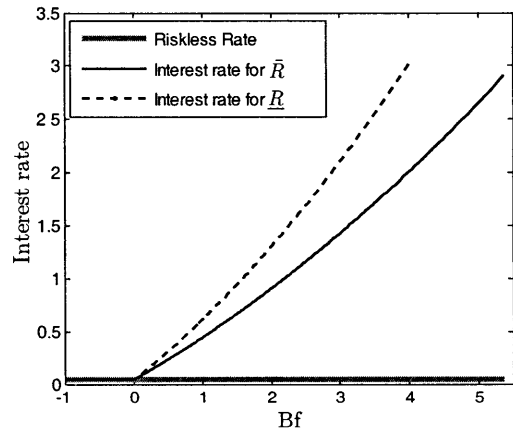


Figure 2.5: Interest Rates on Foreign Debt for $\tilde{R}^- \in \{\underline{R}, \bar{R}\}$

What is the optimal level of borrowing by the country in the aftermath of a productivity shock? The analysis in this case is more involved because of the inclusion of last period's productivity shock \tilde{R}^- as a state variable of the problem. Consider an inherited debt level z from the previous period. This is consistent with different combinations $C = (\alpha, D, B_f)$ depending on the value of the productivity shock in the last period. The optimal debt level chosen in this period z' depends on the specific combination inherited.

To remove this problem, we proceed as follows. We first set $\pi = 0.5$ (i.i.d. shocks) and simulate the model. Then we consider two scenarios. The first is that the shocks continue in an independently and identically distributed manner. The second is that there is a structural break in the productivity shock process, so that it follows the Markov process with $\pi = 0.9$ from this period onward. Figure 2.6 plots the optimal debt levels z' for any inherited level z for both scenarios. For the i.i.d. process, the productivity shock realization in this period conveys no information about the probability of future realizations. The government decides to borrow more following the low productivity realization, to partially insure consumption against the low shock.

For the persistent shock process, a low productivity realization increases the probability of low realizations in the future. For the parametrization above, the government actually decides to borrow less following a low productivity shock. Why? Firstly, the feasible set of debt levels contracts and it is more expensive to borrow. Secondly, the government anticipates that it will experience low productivity realizations in future periods and needs to build up savings for future insurance of consumption.

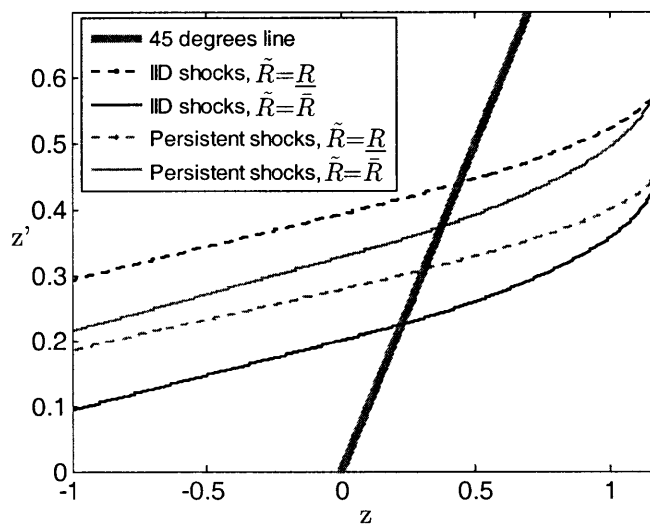


Figure 2.6: Optimal Debt Level z' as a Function of Inherited Debt Level z

2.4 Long Run Dynamics

The long run distribution of asset levels for the country depends on the productivity shock process and the country's discount factor. In an environment with noncontingent debt, the economy may find it optimal to accumulate a buffer stock of assets so as to avoid the region of debt issuance. This will be the case in our framework when the discount factor is sufficient high, because the savings technology is noncontingent. Subsection 2.4.1 describes this result.

Therefore, it may appear that the optimal government debt issuance and default decisions described in this and the previous chapters are not observed in the long run. However, this is an artifact of our simplifying assumption that the government must either issue debt or save abroad, but cannot do both. If we allow the government to simultaneously issue domestic debt and save in assets abroad, then it is optimal for the government to use both debt and savings in the long run, irrespective of the discount factor. Savings allow the government to self-insure itself against adverse productivity shocks, but there is no contingency in interest repayments from this asset. Therefore, the government issues domestic debt and utilizes its ability to default in order to enjoy contingency in interest repayments across states of nature. The analyses of optimal government debt issuance and default decisions described earlier in this chapter are still valid and relevant, since the government chooses to issue debt.

2.4.1 Evolution of the Debt Level

By construction, the savings technology available to the government is a noncontingent asset: its return does not vary with the value of the productivity shock. Debt contracts issued by the government are formally noncontingent, but the ability of the government to default on its debt imparts contingency to actual debt repayments. For low values of the discount factor β , the government decides to borrow in response to poor productivity shocks, and positive debt is part of the support of the long run distribution of assets.

Figure 2.7 presents the policy functions for the model parametrization described in section 2.3. The specification with independently and identically distributed shocks is used. The discount factor is set to $\beta = 0.8$ and the riskless rate of return is equal to $r = 0.05$. Therefore, $\beta(1+r) < 1$. For these parameters, the numerical simulation shows that the long run distri-

bution of debt levels has positive support only in the region of debt issuance, i.e. positive z . Debt is observed in the long run.

Figure 2.8 shows the corresponding policy functions when we set $\beta = 0.99$, while keeping the riskless rate unchanged. In this case, $\beta(1+r) > 1$. For this value of the discount factor, the government wishes to accumulate savings. The interest payments on savings are noncontingent. Therefore, the government faces the problem of self-insurance against uninsurable shocks using noncontingent assets. The optimal policy is for savings to grow, and the probability of the government remaining in the debt region in the long run is zero.

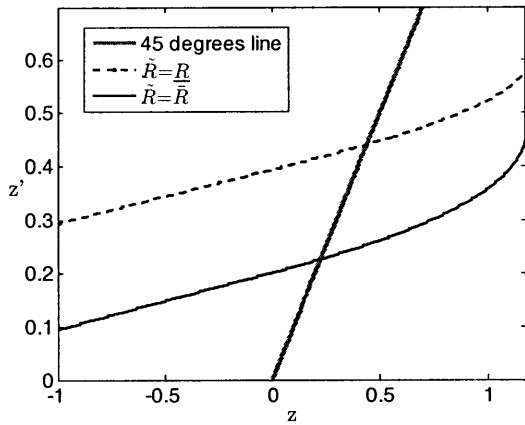


Figure 2.7: Policy Functions for $\beta = 0.8$

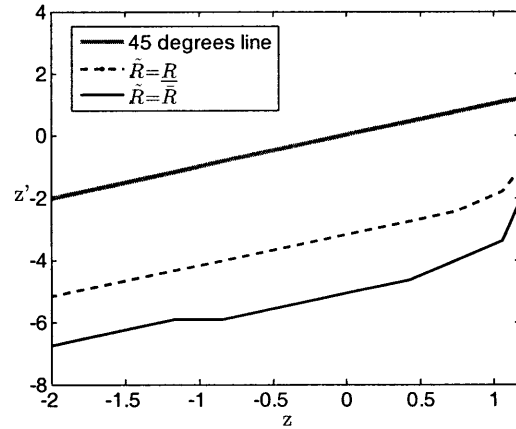


Figure 2.8: Policy Functions for $\beta = 0.99$

2.4.2 Concurrent Saving and Borrowing

The analysis in the previous subsection might give the impression that our analysis of optimal debt issuance and default decisions is relevant only in the short run, or only for countries with a low discount factor. However, this conclusion is actually an artifact of our simplifying assumption that the government must either issue debt or save abroad, but cannot do both. In this subsection we allow the government to simultaneously issue domestic debt and save in assets abroad. This case has been analyzed for the two-period model in the appendix of chapter 1. For the case where government default is not punished by reduced capital market access,

the infinite horizon formulation of the government's optimal policy program is as follows.

$$V(z) = \max_{\alpha, D, B_f, J_f, z'} \mathbb{E} \{u(c) + \beta V(z')\}$$

subject to

$$c = y - (1 - h)B_f - J_f + \tilde{R}f([(1 - \alpha) + (1 - h)\alpha]D) + z'$$

$$c \geq 0$$

$$h = H(\alpha, D, B_f, \tilde{R})$$

$$z = z_1 + z_2$$

$$z_1 = Q(\alpha, D, B_f) \cdot B_f \geq 0$$

$$z_2 = \frac{1}{1 + r} J_f < 0$$

$$z' \in G.$$

The net indebtedness of the economy at the beginning of the period is z . This value is the summation of the gross debt position z_1 and the gross asset position z_2 . Of course, the division of z into z_1 and z_2 is determined optimally. The analysis of the gross debt position is exactly in line with the results of the previous sections of this chapter, and with chapter 1. The haircut function H and debt price function Q are the same functions as in the two-period model. The ability of the government to default imparts some contingency to debt repayments. The return on saving is noncontingent and equal to the riskless rate. The feasible set for debt levels in this period is equal to the feasible set in the two-period model, G .

Proposition 2.11 (Positive Gross Debt) *Assume $u''(c) < 0$. For any net position $z < 0$, it is optimal for the government to choose $z_1 > 0$.*

Even if the government has a net position amounting to negative indebtedness (a positive net asset position), it is still optimal for the government to issue some gross debt. Consider the case where the government starts with z_1 equal to zero. The government can issue marginally more debt and save the additional resources raised abroad, so that the net debt position is

unchanged. In expected terms, this perturbation has no effect on consumption. This derives from the fact that starting from a zero gross debt position, it is possible to raise debt at the margin in this period at an expected consumption cost of $1 + r$ units in the next period. This, of course, is equal to the riskless rate. Although the perturbation has no effect on expected consumption in the next period, it does increase expected utility. This is because the ability to default induces some contingency in the pattern of debt repayments in the next period, which is valuable to a risk averse representative consumer.

In the long run the discount factor of the country determines the evolution of net indebtedness. However, irrespective of the level of this net position, it is always optimal for the government to have a positive gross debt position. The optimal issuance of this debt, and the optimal repayments on it, follow the patterns explored earlier in this chapter and in chapter 1.

2.5 Conclusion

In the previous chapter, we analyzed the effect of domestic economic costs of default on optimal government policy in a two-period model. We examined the implications of such costs for the optimal government debt issuance decision, and on the optimal haircut decision in the event of default. In this chapter, we extend the insights of the two-period model to an infinite horizon context. In the infinite horizon context, there are an infinite number of equilibria depending on how we specify the sanctions exercised by foreign creditors in the event of government default. The sanctions available in our framework entail coordination by foreign investors to continuation equilibria with reduced capital market access for the country. In our earlier work, we were most interested in an environment where default leads to domestic costs, but not reduced access to international capital markets. Therefore, this is the specification we adopt for the infinite horizon model. For this specification, all of the results for optimal government debt issuance and repayment derived in the two-period model are still valid for the infinite horizon case.

The remainder of the chapter focuses on dimensions of the problem that are easier to analyze in the infinite horizon context as opposed to a two-period model. First, we examine the effects of persistent shocks. We show that if productivity shocks are persistent, then an adverse productivity realization in this period increases the probability of poor shocks in future periods.

This has a number of effects. Firstly, this increases the expected haircut in the next period, and therefore leads to a contraction in the feasible set of debt levels. Secondly, the government places more weight on consumption in the worst states of nature in the next period, when it makes the optimal government debt issuance decision. It may even save more in response to a poor productivity shock, because it expects a sequence of poor productivity realizations in the future and wants to be able to insure future consumption against these shocks.

Finally, we address the long run dynamics of debt levels as predicted by our model. By construction, the savings technology in our model is noncontingent, and a country with a high discount factor may find it optimal to accumulate large savings accounts to partially self-insure its future consumption against productivity shocks. In the long run, the probability that such a country issues debt goes down to zero. We show that this outcome is an artifact of our simplifying assumption that the government must either issue debt or save abroad, but cannot do both. We analyze a more general model in which the government is able to both save abroad and simultaneously issue domestic debt. For this model, the discount factor determines the net asset position of the country, which may indeed rise to infinity. However, it is always optimal for a country with risk averse consumers to maintain a positive gross debt position. The results for optimal government debt issuance and optimal debt repayments that we have derived both in this chapter and in earlier work are still valid for the analysis of the gross debt position. We observe a positive gross debt position in the long run.

2.6 Appendix: Proofs of Results in the Main Text

Proof of Proposition 2.1.

This follows directly from the specification that there is no punishment after default, and from the proof of Proposition 1.1 in chapter 1. ■

Proof of Proposition 2.2.

If default is not followed by reduced capital market access, then the value of the haircut h has no impact on future continuation utility except through its indirect impact on the optimal debt level. The proof of Proposition 1.3 in chapter 1 can be amended for this case. ■

Proofs of Propositions 2.3-2.8.

These are amended versions of the proofs of Propositions 1.4 to 1.9 respectively in chapter 1.

■

Proof of Proposition 2.9.

The proof of Proposition 1.6 in chapter 1 establishes the attainment of a maximum for each (A_d, B_d) pair, and the comparative statics with respect to α , given $D = D'$. The level of resources raised from abroad z is given by the expression:

$$\begin{aligned} z &= \frac{1}{1+r} \sum_{\tilde{R} \in \mathcal{S}} (1-h) B_f \cdot \Pr(\tilde{R} | \tilde{R}^-) \\ &= \frac{\gamma}{1+r} \sum_{\tilde{R} \in \mathcal{S}_2} (f')^{-1} \left[\frac{\gamma}{\tilde{R}} \right] \Pr(\tilde{R} | \tilde{R}^-) \\ &\quad - \frac{\gamma}{1+r} \cdot A_d \left[\sum_{\tilde{R} \in \mathcal{S}_2} \Pr(\tilde{R} | \tilde{R}^-) \right] + \frac{\gamma}{1+r} \cdot B_d \left[\sum_{\tilde{R} \in \mathcal{S}_3} \Pr(\tilde{R} | \tilde{R}^-) \right]. \end{aligned}$$

For an adverse productivity realization in the previous period $\tilde{R}^- = \underline{R}$, the probability of an adverse productivity realization in the current period $\tilde{R} = \underline{R}$ increases. The optimal haircut is higher for poor productivity shocks. Therefore, z is lower in the current period for any given combination $C = (\alpha, D, B_f)$. This establishes the desired result. ■

Proof of Proposition 2.10.

In order to achieve the maximum level of debt, set $\alpha = 1$ and $D = \bar{D}$. The claim in the proposition follows from the argument in the proof of Proposition 2.9. ■

Proof of Proposition 2.11.

Suppose that the country begins the current period with net indebtedness level $z < 0$, and $z_1 = 0$. The representative consumer receives gross return $-(1+r)z > 0$ in consumption units from its asset position. Is this optimal? We prove not.

Consider a perturbation which maintains the net indebtedness level z , but increases z_1 and reduces z_2 by equal amounts. A marginal increase in z_1 necessitates a marginal increase in

$\gamma = \frac{B_f}{\alpha D}$ from zero. Let us restrict ourselves to a perturbation such that $\alpha = 1$, which is feasible. From the proof of Proposition 1.10 in chapter 1, such a perturbation induces an expected gross consumption cost of $1 + r$ units in this period. However, the equal marginal reduction in z_2 has an expected gross consumption benefit of $1 + r$ units. Therefore, expected consumption in this period remains unchanged. However, the gross debt position is repaid across states of nature according to the haircut function H . Although the expected repayment is the gross rate of $1 + r$, the country repays less in the worst states of nature and more in the best. This is valuable to the representative consumer, if it is risk averse. Therefore, the perturbation increases expected utility in this period, while keeping future consumption levels unchanged. ■

2.7 References

1. Aguiar, Mark, Manuel Amador and Gita Gopinath. 2006. "Efficient Expropriation: Sustainable Fiscal Policy in a Small Open Economy." Federal Reserve Bank of Boston Working Paper No. 06-9.
2. Aiyagari, S. Rao, Albert Marcet, Thomas J. Sargent and Juha Seppälä. 2002. "Optimal Taxation without State-Contingent Debt." *Journal of Political Economy*, Vol. 110, No. 6, pp. 1220-1254.
3. Arellano, Cristina. 2008. "Default Risk and Income Fluctuations in Emerging Economies." *The American Economic Review*, Vol. 98, No. 3, pp. 690-712.
4. Bulow, Jeremy, and Kenneth Rogoff. 1989. "Sovereign Debt: Is to Forgive to Forget?" *The American Economic Review*, Vol. 79, No. 1, pp. 43-50.
5. Broner, Fernando A., and Jaume Ventura. 2006. "Globalization and Risk Sharing." NBER Working Paper No. 12482.
6. Broner, Fernando A., Alberto Martin and Jaume Ventura. 2006. "Sovereign Risk and Secondary Markets." NBER Working Paper No. 12783.
7. de Bolle, Monica B., Ivetta Hakobyan and Bjoern Rother. 2006. "The Level and Composition of Public Sector Debt in Emerging Market Crises." IMF Working Paper No.

06/186.

8. De Paoli, Bianca, Glenn Hoggarth and Victoria Saporta. 2006. "Costs of Sovereign Default." Financial Stability Paper No.1, Bank of England.
9. Dooley, Michael P. 2000. "Can Output Losses Following International Financial Crises be Avoided?" NBER Working Paper No. 7531.
10. Eaton, Jonathan. 1996. "Sovereign Debt, Reputation and Credit Terms." *International Journal of Finance and Economics*, Vol. 1, pp. 25-35.
11. Eaton, Jonathan, and Mark Gersovitz. 1981. "Debt with Potential Repudiation: Theoretical and Empirical Analysis." *Review of Economic Studies*, Vol. 48, pp. 289-309.
12. Gelos, Gaston R., Ratna Sahay and Guido Sandleris. 2004. "Sovereign Borrowing by Developing Countries: What Determines Market Access?" IMF Working Paper No. 04/221.
13. Grossman, Herschel I., and John B. van Huyck. 1988. "Sovereign Debt as a Contingent Claim: Excusable Default, Repudiation and Reputation." *The American Economic Review*, Vol. 78, No. 5, pp. 1088-1097.
14. Jeanne, Olivier, and Anastasia Guscina. 2006. "Government Debt in Emerging Market Countries: A New Data Set." IMF Working Paper No. 06/98.
15. Kaminsky, Graciela L., and Carmen M. Reinhart. 1999. "The Twin Crises: The Causes of Banking and Balance-of-Payments Problems." *The American Economic Review*, Vol. 89, No. 3, pp. 473-500.
16. Kocherlakota, Narayana R. 1996. "Reconsideration-Proofness: A Refinement for Infinite Horizon Time Inconsistency." *Games and Economic Behavior*, Vol. 15, No. 1, pp. 33-54.
17. Kletzer, Kenneth M., and Brian D. Wright. 2000. "Sovereign Debt as Intertemporal Barter." *The American Economic Review*, Vol. 90, pp. 621-639.
18. Medeiros, Carlos, Michael Gapen and Luisa Zanfornin. 2005. "Assessing the Determinants and Prospects for the Pace of Market Access by Countries Emerging from Crisis – Further Considerations." International Capital Markets Department, IMF.

19. Mendoza, Enrique G., and Vivian Z. Yue. 2008. "A Solution to the Default Risk-Business Cycle Disconnect." NBER Working Paper No. 13861.
20. Sturzenegger, Federico, and Jeromin Zettelmeyer. 2005. "Haircuts: Estimating Investor Losses in Sovereign Debt Restructurings, 1998-2005." IMF Working Paper No. 05/137.
21. Sturzenegger, Federico, and Jeromin Zettelmeyer. 2006. *Debt Defaults and Lessons from a Decade of Crises*. Cambridge, Mass.: The MIT Press.
22. Tirole, Jean. 2003. "Inefficient Foreign Borrowing: A Dual- and Common-Agency Perspective." *The American Economic Review*, Vol. 93, No. 5, pp. 1678-1702.
23. Werning, Iván. 2003. "Standard Dynamic Programming for Aiyagari, Marcet, Sargent and Seppälä." Mimeo, MIT.
24. Woodford, Michael. 1990. "Public Debt as Private Liquidity." *The American Economic Review*, Vol. 80, No. 2, pp. 382-388.

Chapter 3

Optimal IMF Policy with Private Capital Flows

3.1 Introduction

What is the relationship between IMF intervention and country moral hazard? Governments can take actions that reduce the probability of adverse macroeconomic outcomes, but these actions are costly and difficult to observe. If governments know that IMF support is available in the event of macroeconomic crises, they may exert suboptimal effort *ex ante* to avoid such outcomes. Therefore, IMF policies may induce moral hazard, and this has been a reason for much criticism of the institution.

This chapter analyzes optimal IMF policy in an environment with moral hazard followed by adverse selection. Country welfare is determined by both IMF transfers and private capital inflows. In particular, we present a framework where IMF transfers to the worst performing countries *ex post* actually ameliorates the moral hazard problem *ex ante*. In the baseline model, government actions to improve the productivity of domestic firms are not always effective, and the government learns of the success of its actions before foreign investors. Without the IMF, it is not possible for foreign investors to discern the quality of the domestic production sector. Therefore, there only exists a pooling equilibrium *ex post*, which leads to low effort *ex ante* because the returns to good macroeconomic performance are low.

Now we introduce the IMF. The IMF can structure its crisis intervention policy so as to

reveal the government's private information to foreign investors in a separating equilibrium. The IMF provides limited transfers to countries with poor domestic productivity ex post. These countries face high interest rates on international capital markets. Countries which do not accept transfers are identified as having strong fundamentals and are rewarded with low interest rates on international capital markets. The key result is that IMF transfers to low productivity countries ex post improve the consumption of high productivity countries. The difference between ex post consumption in the high and low productivity states increases, which increases government effort ex ante.

Optimal IMF policy is the solution to a mechanism design problem in the presence of imperfectly informed competitive markets. We allow the IMF to implement redistributive transfers from high to low productivity countries (or vice versa) ex post, subject to both its budget constraint and to the rational expectations condition that foreign investors set prices according to the information revealed in equilibrium. The optimal scheme for the mechanism designer in this context must take into account that the zero profits condition for foreign investors changes as a result of the scheme. Foreign interest rates respond to the separation decision by country governments. The incentives for the government to self-select in a separating equilibrium ex post depend on a combination of the IMF's policies and the contracts offered by foreign investors. This is central to the result obtained. Countries with weak fundamentals ex post choose to receive high IMF transfers and low private capital inflows, while countries with strong fundamentals choose to refuse IMF transfers because this refusal is associated with high private capital inflows.

So the IMF designs the optimal scheme taking the market structure as given. The specific tool available to the mechanism designer is a system of redistributive transfers ex post. Can this be implemented by competitive markets instead? Yes. If the government can purchase insurance at actuarially fair rates before its effort decision, then it will choose a level of (partial) insurance that corresponds exactly to the IMF scheme. However, if such contracts are only available after the government's type is revealed, the government will not purchase the ex ante optimal level of insurance. In this case, the IMF should commit ex ante to a schedule of redistributive transfers.

Finally, we generalize the result of our baseline model that ex ante insurance can be welfare-improving because it expands the set of feasible separating equilibria. We consider an amended

version of the model where foreign investors have sufficient tools to separate countries ex post even in the absence of IMF intervention, but such separation is associated with output distortions for the country with strong fundamentals. Ex ante insurance can still result in an increase in welfare in this framework. In particular, with ex ante insurance it is no longer necessary for output to be distorted ex post for any country in a separating equilibrium. However, ex ante insurance results in lower effort by the government ex ante than the separating equilibrium without IMF intervention. IMF transfers are a feature of the optimal mechanism if the benefits of reduced output distortions outweigh the moral hazard costs.

This chapter contributes to the literature on IMF intervention and moral hazard. Some empirical evidence on IMF-induced creditor and debtor moral hazard is summarized in Dreher (2004). On the theoretical side, Jeanne and Zettelmeyer (2004) argue that if the IMF provides loans at an actuarially fair interest rate, then it cannot induce moral hazard because any changes in government effort are efficient. The catalytic finance literature proposes a separate channel by which IMF loans may affect country effort (Morris and Shin 2006, Corsetti, Guimarães and Roubini 2006). If IMF intervention reduces the risk of inefficient liquidation of projects ex post, then this sometimes induces governments to exert higher effort ex ante. IMF lending would be associated with private capital inflows. However, the empirical evidence for such a catalytic effect of IMF lending is far from conclusive (Bird and Rowlands 2002, Edwards 2006).

Our characterization of the IMF assumes that all private information is in the possession of the government, and that both the IMF and foreign investors are equally uninformed. The IMF uses its ability to make redistributive transfers, in order to reveal the information of the government to foreign investors. This is a stark characterization of the role of the IMF, and is one of many possible modeling approaches. Jeanne and Zettelmeyer (2004) model the IMF as an institution that can extract higher payments from the country (as a fraction of output) than private creditors. Marchesi and Thomas (1999) examine the role of conditionality in IMF lending. Arregui (2009) models the IMF as having an imperfect monitoring technology that it can use to certify the quality of a country.

The results in this chapter have implications for empirical work. We identify an environment such that it is optimal for the IMF to follow a scheme where official and private financing are negatively correlated, and this correlation is crucial in terms of providing incentives to

governments to reveal their type. Therefore, empirical findings affirming this correlation are not evidence for poor IMF policy. Instead, the model predicts that such a scheme should have a strictly limited size of transfers ex post, in order to minimize moral hazard concerns. Furthermore, the model predicts that the full benefits of IMF intervention cannot be discerned by looking solely at IMF program countries. IMF transfers to low productivity countries improves outcomes for high productivity countries, because it enables the latter to reveal their type in equilibrium. Countries that refuse IMF transfers benefit from the existence of the IMF, because the refusal reveals their high productivity and induces high private capital inflows.

This chapter proceeds as follows. Section 3.2 summarizes the baseline model in the absence of the IMF. Section 3.3 introduces the IMF and analyzes the mechanism design problem described above. Section 3.4 generalizes our result that ex ante insurance is welfare-improving, in an environment where foreign investors have sufficient tools to separate countries ex post even in the absence of IMF intervention. Section 3.5 concludes.

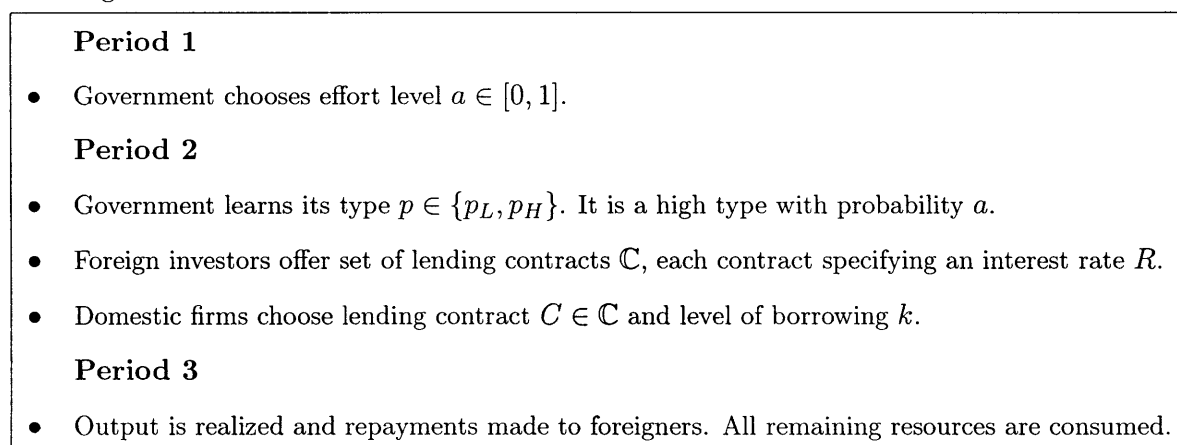
3.2 Model Without IMF

3.2.1 The Environment

The model has 3 periods. There are four categories of agents: consumers, firms, foreign investors and the government. Each category of agents exists in unit measure, except the government.

Model Timeline The order of events and actions is detailed in the figure below.

Figure 3.1: Timeline without IMF



The effort level a chosen by the government in period 1 is not observable to other agents. In period 2, the type p is revealed only to the government and not to other agents. Foreign investors compete in the provision of loans to domestic firms. Each lending contract specifies the relationship between a foreign investor and a domestic firm. It specifies only the interest rate R , and the firm can freely choose its borrowing level. Firms select their most preferred choice out of the set of offered lending contracts, and choose their level of borrowing k . They invest borrowed funds in their production technology. In period 3, the output of the firms' production technology is realized. Repayments are made to foreign investors and all remaining resources are consumed.

Payoffs for Agents The representative consumer has expected utility given by the expression

$$\mathbb{E}c - \psi(a),$$

where c denotes consumption and $\psi(a)$ is the cost function for government effort level $a \in [0, 1]$. $\psi(a)$ is twice differentiable and satisfies: $\psi'(a) \geq 0$, $\psi''(a) > 0$, with $\psi'(0) = 0$ and $\lim_{a \rightarrow 1} \psi'(a) = \infty$.

The government is benevolent and maximizes the utility of the representative consumer. It chooses a in period 1 so as to maximize the above expression for expected utility.

Each domestic firm has access to a project in period 2, in which it invests k units of capital borrowed from abroad. With probability p , the project is successful and yields $f(k)$ units of output in period 3. The firm repays R to foreign investors. With probability $1 - p$, the project fails and output in period 3 is zero. No payments to foreign creditors can be enforced in this case. $f(k)$ is twice differentiable and satisfies: $f'(k) > 0$, $f''(k) < 0$ with $\lim_{k \rightarrow 0} f'(k) = \infty$ and $\lim_{k \rightarrow \infty} f'(k) = 0$. Given R , each firm chooses its capital level to maximize expected profits:

$$\max_k p[f(k) - Rk] \implies f'(k) = R.$$

By inspection, the firm selects the lending contract $C \in \mathbb{C}$ that offers the lowest interest rate R .

The probability of project success is independent and identical across firms, so there is no

aggregate uncertainty. Total consumption for an economy of type p is equal to $p[f(k) - Rk]$. We specify $p_H > p_L$. The type p of the country determines the proportion of projects that are successful in period 3.

Foreign investors offer the set of lending contracts \mathbb{C} , each contract specifying the interest rate R . They can lend capital elsewhere at riskless rate r . The zero profits condition for foreign investors can be written:

$$p^e R - r = 0 \iff R = \frac{r}{p^e},$$

where p^e denotes foreign investors' beliefs about p .

3.2.2 First Best Benchmark

a and p are observable to all agents. Solve the model by backward induction.

Consider the actions of agents in period 2. For the high type, foreign investors offer contracts with interest rate $R_H = \frac{r}{p_H}$, which is low because p_H is high. Domestic firms accept these contracts and choose k_H such that $f'(k_H) = R_H$. Total realized consumption in the economy in period 3 is equal to $p_H[f(k_H) - R_H k_H]$. For the low type country, foreign investors offer contracts with interest rate $R_L = \frac{r}{p_L}$, which is high because p_L is low. In response, domestic firms choose k_L to satisfy $f'(k_L) = R_L$. Total realized consumption in period 3 is $p_L[f(k_L) - R_L k_L]$. Denote $F_H = [f(k_H) - R_H k_H]$ and $F_L = [f(k_L) - R_L k_L]$.

Government effort level a in period 1 solves:

$$\begin{aligned} & \max_a \{a \cdot p_H F_H + (1 - a) \cdot p_L F_L - \psi(a)\} \\ & = \max_a \{p_L F_L + a \cdot z^{FB} - \psi(a)\}, \end{aligned}$$

where $z^{FB} = [p_H F_H - p_L F_L]$. The solution to this maximization problem is effort level a^{FB} .

3.2.3 Imperfect Information Case

Again, solve the model by backward induction. Domestic firms in both high and low type countries choose the lending contract that offers the lowest interest rate R . It follows that foreign investors cannot offer contracts that induce the government to reveal its type.

Proposition 3.1 (Separating without IMF) *There exists no separating equilibrium.*

Proposition 3.2 (Pooling without IMF) *There exists at least one pooling equilibrium. In any pooling equilibrium, effort level $a^* < a^{FB}$.*

In a pooling equilibrium, foreign investors offer lending contracts with interest rate R_P :

$$R_P = \frac{r}{a^e p_H + (1 - a^e) p_L}, \quad (3.1)$$

given their beliefs of the government effort level a^e . Domestic firms choose their capital level k_P such that $f'(k_P) = R_P$. Total consumption for an economy of type p is equal to pF_P , where we denote $F_P = [f(k_P) - R_P k_P]$. In period 1, the government chooses effort level a to solve:

$$\begin{aligned} & \max_a \{a \cdot p_H F_P + (1 - a) \cdot p_L F_P - \psi(a)\} \\ & = \max_a \{p_L F_P + a \cdot z^* - \psi(a)\}, \end{aligned} \quad (3.2)$$

where $z^* = [p_H - p_L] F_P$ and the beliefs of foreign investors a^e (and hence F_P) are taken as given. The solution to this maximization problem is effort level a^* .

The beliefs of foreign investors are formed via rational expectations. Therefore, the pooling equilibrium is a fixed point for the equations (3.1) and (3.2), such that $a^e = a^*$. It is straightforward to show that such a fixed point exists, and that $a^* \in (0, 1)$. It follows that $R_L > R_P > R_H$ and $F_L < F_P < F_H$. Furthermore,

$$\begin{aligned} [p_H - p_L] F_P &< p_H F_H - p_L F_L \\ &\Leftrightarrow z^* < z^{FB}. \end{aligned}$$

Therefore, $a^* < a^{FB}$. Government effort in the pooling equilibrium is below the first best level.

Proposition 3.3 (Welfare under Pooling) *The country is worse off ex ante with imperfect information than with perfect observability.*

In the pooling equilibrium, capital is misallocated in period 2. The capital level in the high type economy is lower than the efficient level, and the capital level in the low type economy

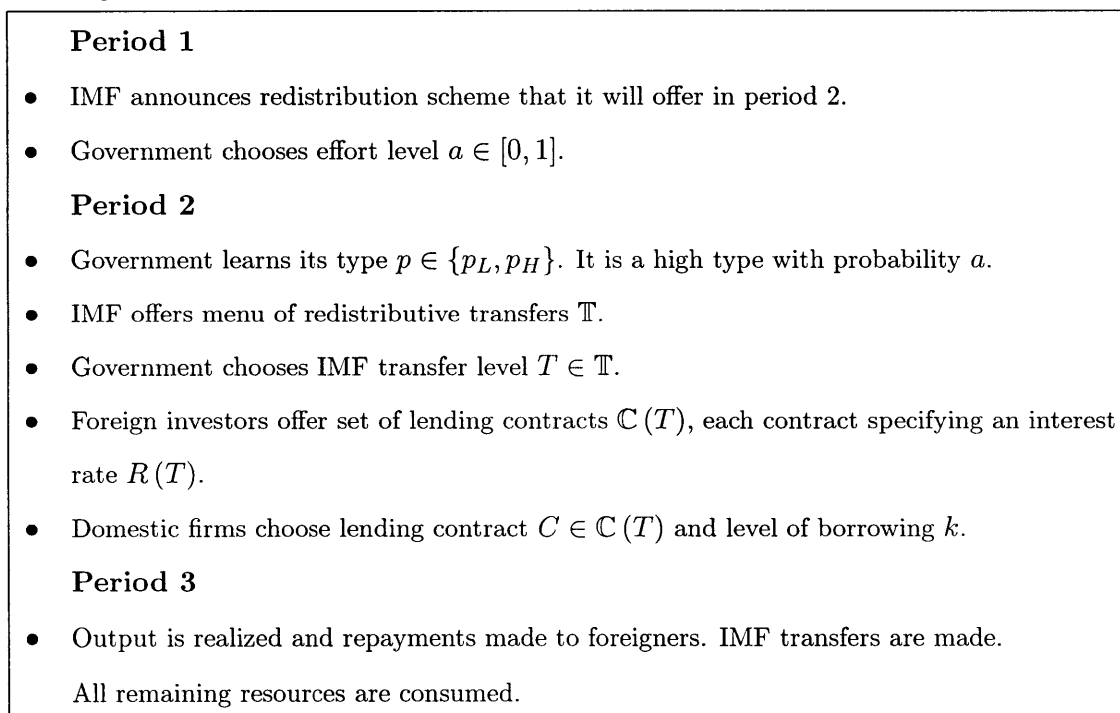
is higher than the efficient level. Since capital misallocation reduces the difference in ex post consumption for high and low type countries, the government finds it optimal to exert less effort than the first best level.

3.3 Model With IMF

3.3.1 The Amended Environment

We model the IMF as an institution that commits to a system of redistributive transfers between countries. In period 2, it offers a menu of redistributive transfers \mathbb{T} to the country. The government then selects its most preferred level of transfers $T \in \mathbb{T}$. The menu of transfers is pre-announced in period 1 (before the government exerts effort a), and promised transfers are delivered to the country after output is realized in period 3. The amended timeline is presented in the figure below.

Figure 3.2: Timeline with IMF



The IMF offers the menu of redistributive transfers \mathbb{T} to a continuum of ex ante identical countries with independent realizations of p . Any transfers made to a country by the IMF must

be financed by contributions from other countries. In other words, the IMF's budget constraint states that the aggregate level of net transfers is zero. This is equivalent to the condition that in period 1, the expected level of transfers to any particular country is zero:

$$\mathbb{E}T = 0.$$

Foreign investors first observe the government's choice of the transfer level from the IMF, and then compete in the provision of loans to domestic firms. The set of contracts offered by foreign investors can be conditioned on the transfer level chosen: $\mathbb{C}(T) = \{R(T)\}$.

As before, the government is benevolent and maximizes the utility of the representative consumer. It chooses a in period 1 so as to maximize ex ante expected utility. In period 2, the effort cost $\psi(a)$ is a sunk cost and the government knows its true type p . It chooses the transfer level $T \in \mathbb{T}$ so as to maximize consumption in period 3. It recognizes that its choice can affect the set of contracts $\mathbb{C}(T)$ offered by foreign investors to domestic firms.

Definition 3.1 *A Perfect Bayesian Equilibrium for this economy is a set of strategies $\{a, T(p), \mathbb{C}(T), k(\mathbb{C})\}$, beliefs $p^e(T)$ and a menu of transfers \mathbb{T} such that:*

1. *Government sets a in period 1 to maximize expected utility.
Government chooses $T \in \mathbb{T}$ in period 2 to maximize consumption c in period 3, given private information p , the expected set of contracts $\mathbb{C}(T)$ and the expected contract choice by domestic firms $k(\mathbb{C})$.*
2. *Foreign investors observe T and offer set of contracts $\mathbb{C}(T) = \{R(T)\}$ that maximize expected profits given their beliefs $p^e(T) = \mathbb{E}[p|T]$, updated via Bayes' Rule on the equilibrium path (and unrestricted otherwise).*
3. *Domestic firms choose the contract $C \in \mathbb{C}(T)$ and capital level k that maximize expected profits.*
4. *IMF satisfies its budget constraint. In period 1: $\mathbb{E}T = 0$.*

The last condition is needed to ensure that the perfect Bayesian equilibrium is feasible.

3.3.2 Mechanism Design Problem

Given any system of redistributive transfers, there may exist equilibria as defined above. The different equilibria correspond to different patterns of private capital inflows in period 2, and different levels of government effort in period 1. The IMF takes into account that its policies affect the revelation of information in equilibrium, and therefore (via rational expectations) the zero profit conditions of foreign investors. It also takes into account the effect on government effort. The IMF is benevolent and designs the redistribution scheme to maximize ex ante expected utility of the country.

Therefore, optimal IMF policy is the solution to a mechanism design problem in the presence of imperfectly informed competitive markets. Apply the Revelation Principle to derive the following result. We define $F(p)$ to be the net output of each successful domestic firm in a country of type p in period 2. $T(p)$ is the transfer received by the country from the IMF.

For simplicity, throughout this chapter we restrict attention to mechanisms that are implementable as pure strategy perfect Bayesian equilibria.

Proposition 3.4 (Mechanism Design Problem) *Optimal IMF policy is the solution to the following mechanism design problem:*

$$\max_{z, T(p_H), T(p_L)} W = \{[p_L F(p_L) + T(p_L)] + a \cdot z - \psi(a)\}$$

subject to

$$z = [p_H F(p_H) + T(p_H)] - [p_L F(p_L) + T(p_L)]$$

$$a = (\psi')^{-1}[z] \tag{3.3}$$

$$a \cdot T(p_H) + (1 - a) \cdot T(p_L) = 0 \tag{3.4}$$

$$p_H F(p_H) + T(p_H) \geq p_H F(p_L) + T(p_L) \tag{3.5}$$

$$p_L F(p_L) + T(p_L) \geq p_L F(p_H) + T(p_H). \tag{3.6}$$

Pooling equilibria:

$$F(p_H) = F(p_L) = F_P. \tag{3.7}$$

Separating equilibria:

$$F(p_H) = F_H, F(p_L) = F_L. \quad (3.8)$$

We have rewritten the objective function in terms of z , the difference in ex post consumption between the high type and low type countries. Constraint (3.3) relates how the government's optimal effort level is related to z . The IMF's budget constraint is given by equation (3.4). Equations (3.5) and (3.6) are the incentive compatibility constraints for the high and low type governments in period 2. No participation constraint for the country is given. This reflects the assumption that the government must choose out of the menu of transfers offered by the IMF in period 2. Equations (3.7) and (3.8) capture the additional constraints imposed on the mechanism designer owing to the presence of competitive markets that are imperfectly informed ex ante. We discuss these next.

Proposition 3.5 (Pooling with IMF) *The set of pooling equilibria the IMF can achieve is the same as in the model without the IMF. The government chooses effort level a^* .*

Pooling equilibria are achieved when foreign investors remained uninformed about the type p of the country when they offer lending contracts. Therefore, they offer the same interest rate R to domestic firms from all countries. The net output of each successful firm in both types of country is F_P . The incentive compatibility constraints for the government in period 2 immediately imply that in a pooling equilibrium, $T(p_H)$ and $T(p_L)$ are equal. Substituting into the IMF's budget constraint, we obtain:

$$T(p_H) = T(p_L) = 0.$$

The proposition above follows by inspection.

Figure 3.3 illustrates the pooling equilibrium in (T, F) space. The indifference curve of a country of type p is a line with slope equal to $-\frac{1}{p}$. IC_H and IC_L denote the indifference curves of high and low type countries respectively. The point A is located at $(0, F_P)$.

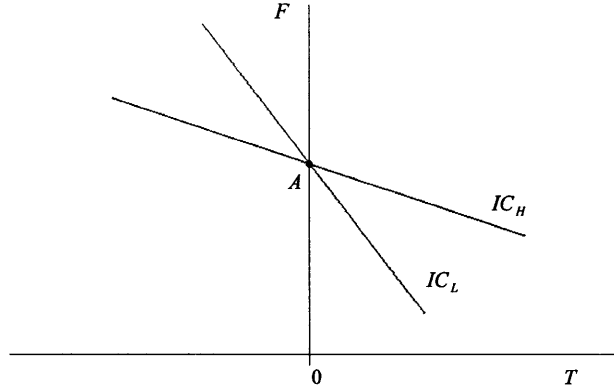


Figure 3.3: Pooling Equilibrium

Next, we turn to the set of separating equilibria that the IMF can induce.

Proposition 3.6 (Separating with IMF) *The IMF can induce a continuum of separating equilibria, each corresponding to a particular level of government effort in period 1. The feasible set of effort levels can be described:*

$$a \in [\underline{a}, \bar{a}],$$

where we define $\underline{a} = (\psi')^{-1} \{[p_H - p_L] F_L\}$ and $\bar{a} = (\psi')^{-1} \{[p_H - p_L] F_H\}$.

In a separating equilibrium, the type p of the country is revealed in equilibrium. Foreign investors offer lending contracts at interest rate R_H to firms in the high type country and interest rate R_L to firms in low type country. This explains the expressions for each successful firm's net output given in constraint (3.8). Substituting these expressions into the incentive compatibility constraints and rearranging, we obtain:

$$[p_H - p_L] F_L \leq z \leq [p_H - p_L] F_H. \quad (3.9)$$

The lower bound for z is achieved when the incentive compatibility constraint for the high type is binding, and the upper bound is achieved when the incentive compatibility constraint for the low type is binding. These correspond, via equation (3.3), to a restriction on the set of feasible

government effort levels as defined above:

$$\underline{a} \leq a \leq \bar{a}.$$

There is a continuum of separating equilibria, each corresponding to a different menu of transfers $\mathbb{T} = \{T(p_H), T(p_L)\}$ offered by the IMF. In equilibrium, $T(p_L) > 0$ and $T(p_H) = -\left(\frac{1-a}{a}\right)T(p_L) < 0$. Consider the contracts offered by foreign investors in a separating equilibrium. Foreign investors condition the set of contracts offered to firms on the transfer level chosen by the government. If the government chooses transfer level $T(p_H)$, it is identified as a high type country. Foreign investors offer contracts at interest rate R_H , and net output of each successful domestic firm is F_H . If the government chooses transfer level $T(p_L)$, it is identified as a low type country. The interest rate offered is R_L , which means that net output of each successful domestic firm is F_L .

Figures 3.4 and 3.5 illustrate the separating equilibria corresponding to effort levels \underline{a} and \bar{a} respectively. Effort level \underline{a} is achieved when the incentive compatibility constraint for the high type is binding. At the other extreme, effort level \bar{a} is achieved when the incentive compatibility constraint for the low type is binding. For each of the figures, $A = (T(p_H), F_H)$ and $B = (T(p_L), F_L)$.

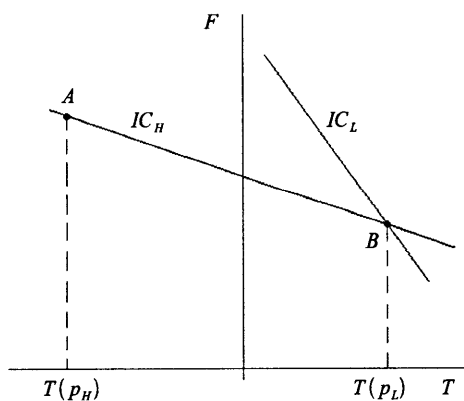


Figure 3.4: Effort level \bar{a}

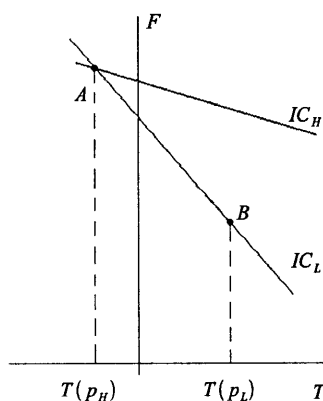


Figure 3.5: Effort level \underline{a}

This completes the description of the feasible set of perfect Bayesian equilibria. Which of

these equilibria does the IMF choose? The most preferred outcome within the set of pooling equilibria (if there exists more than one) is the equilibrium with the lowest offered interest rate R_P , and hence the highest level of net output F_P per successful firm. Now consider the set of separating equilibria. Notice that the objective function of the IMF can be written:

$$a \cdot [p_H F_H + T(p_H)] + (1 - a) \cdot [p_L F_L + T(p_L)] - \psi(a)$$

which, given the IMF's budget constraint, reduces to

$$a \cdot p_H F_H + (1 - a) \cdot p_L F_L - \psi(a).$$

This is the same objective function that the government faces in the first best case, and the global maximum of this expression is attained for effort level a^{FB} . It can be shown that the maximum level of z that the IMF can achieve is below z^{FB} . It immediately follows that $\bar{a} < a^{FB}$. The separating equilibrium that maximizes expected utility in period 1 corresponds to the highest effort level possible, which is \bar{a} .

Proposition 3.7 (Optimal IMF Policy) *The optimal allocation is a separating equilibrium with effort level \bar{a} , where $a^* < \bar{a} < a^{FB}$. The menu of redistributive transfers $\mathbb{T} = \{T(p_H), T(p_L)\}$ offered by the IMF satisfies:*

$$\begin{aligned} T(p_L) &= \bar{a} \cdot p_L [F_H - F_L] \\ T(p_H) &= - \left(\frac{1 - \bar{a}}{\bar{a}} \right) \cdot T(p_L). \end{aligned}$$

The incentive compatibility constraint for the low type country is binding.

Optimal IMF policy takes into account that both foreign investors and the government respond to the redistributive scheme $\mathbb{T} = \{T(p_H), T(p_L)\}$ offered. In a separating equilibrium, the interest rates offered by foreign investors responds to the government's choice of $T \in \mathbb{T}$, because this choice reveals the government's type p . Now consider the government's optimal decision. The incentives for the government to self-select in a separating equilibrium in period 2 depends on a combination of the IMF's policies and the government's expectations of contracts offered by foreign investors $\mathbb{C}(T)$ (in equilibrium, the expectations of the government regarding

$\mathbb{C}(T)$ are fulfilled). This is central to the result. Countries with weak fundamentals in period 2 choose to receive high levels of IMF transfers. As a consequence, they face high interest rates on international capital markets, which leads to low private capital inflows. Countries with strong fundamentals in period 2 value access to international capital markets more than low type countries. They choose to make contributions to the IMF and thereby receive low interest rates on international capital markets. They enjoy high levels of private capital inflows.

It seems counterintuitive that at the optimum, the IMF should redistribute resources towards countries with adverse realizations of economic fundamentals. However, notice that such redistribution is implemented not to decrease the difference in ex post consumption between high and low type countries, but to increase it. The IMF induces a separating equilibrium and thereby reveals information about the government's type to foreign investors. This generates capital reallocation on international capital markets from low type to high type countries in period 2. In turn, this increases the optimal level of effort by the government in period 1.

By inducing a separating equilibrium, the IMF solves the capital misallocation problem in period 2 that is associated with a pooling equilibrium. Within the set of separating equilibria, the IMF chooses the lowest level of redistributive transfers possible, so as to maximize government effort in period 1. The highest level of effort that can be sustained is \bar{a} . If redistributive transfers are reduced further with the intention of generating a higher effort level, the difference between ex post consumption of high type and low type countries would be too low to be consistent with a separating equilibrium. In period 2, the low type government would mimic the high type.

It can be proved that $a^* < \bar{a} < a^{FB}$, which establishes the above result. The upper limit \bar{a} on the set of feasible effort levels yields the following corollary.

Proposition 3.8 (Welfare with IMF) *The country is still worse off ex ante than with perfect observability.*

3.3.3 Implementation using Ex Ante Insurance Contracts

Can the optimal IMF allocation be implemented using competitive markets? Yes. Suppose that before the government chooses effort level a in period 1, it is able to purchase insurance contracts at actuarially fair rates. An insurance contract specifies payoffs $X(p_H)$ and $X(p_L)$

for high and low type countries respectively. Define $\mathbb{X} = \{X(p_H), X(p_L)\}$. After learning its type in period 2, the government reports its type to insurance providers and claims its payoff $X \in \mathbb{X}$. Foreign investors observe the government's report and then offer lending contracts $\mathbb{C}(X)$ to domestic firms. The set of contracts offered can be conditioned on the government's report. The payoffs from the insurance contract are delivered to the government in period 3, after output is realized.

The zero profits condition of competitive insurance providers is given by the expression:

$$a \cdot X(p_H) + (1 - a) \cdot X(p_L) = 0. \quad (3.10)$$

Equation (3.10) and the amended versions of the incentive compatibility constraints (3.5) and (3.6) together indicate that any allocation achievable by the IMF is also achievable using feasible insurance contracts. Given these constraints, it can easily be verified that the insurance contract that maximizes expected utility in period 1 satisfies:

$$X(p_H) = T(p_H), \quad X(p_L) = T(p_L),$$

where $T(p_H)$ and $T(p_L)$ are the redistributive transfers offered by the IMF at its optimal allocation. The government selects this contract.

Proposition 3.9 (Ex Ante Insurance) *The optimal IMF allocation can be implemented via ex ante insurance contracts.*

It is worth drawing attention here to two features of the result above. Firstly, the insurance contracts are feasible despite the fact that the type of the country p is not observable to the insurance providers in period 2. The government is induced to truthfully reveal its type in period 2, given the insurance contracts $\mathbb{X} = \{X(p_H), X(p_L)\}$ that it has signed in period 1 and the set of contracts offered by foreign investors as a function of the government's report, $\mathbb{C}(X)$. Secondly, ex ante insurance is desirable for the country even though the representative consumer is risk neutral. The optimal scheme involves partial insurance, in order to address the adverse selection problem in period 2.

The role of ex ante insurance contracts is explored further in Section 3.4.

3.3.4 Implementation using Government Debt

An alternative implementation of the IMF optimum can be achieved via issuance of government debt with type-contingent interest rates. Before the government chooses effort level a in period 1, it can issue debt level D . The debt contract promises repayments $B(p_H)$ and $B(p_L)$ for high and low type countries respectively. Define $\mathbb{B} = \{B(p_H), B(p_L)\}$. The government announces its repayment choice after it learns its type in period 2, and delivers the repayments in period 3 after output is realized. Foreign investors observe the government's announcement and then offer domestic firms the set of contracts $\mathbb{C}(B)$.

Let the riskless interest rate between periods 1 and 2 be zero. The government's debt issuance problem in period 1 is isomorphic to the IMF's mechanism design problem, with $[D - B(p_H)]$ replacing $T(p_H)$ and $[D - B(p_L)]$ replacing $T(p_L)$.

Proposition 3.10 (Government Debt) *The optimal IMF allocation can be implemented via government debt contracts with type-contingent interest rates.*

At the optimum, the government decides to pay a higher interest rate on its debt after a good realization of economic fundamentals, and a lower interest rate after the realization of adverse economic conditions. It is optimal for the government to reveal its type because the decision to make high debt repayments is associated with higher private capital inflows. Countries that decide to make low debt repayments ex post face high interest rates on international capital markets.

3.3.5 Timing of Contract Offers and Feasible Effort Levels

Now consider a version of the model with the second period modified as in figure 3.6.

Figure 3.6: Modified Period 2

<p>Period 2</p> <ul style="list-style-type: none"> • Government learns its type $p \in \{p_L, p_H\}$. It is a high type with probability a. • IMF offers menu of redistributive transfers \mathbb{T}. • Foreign investors offer set of lending contracts $\{\mathbb{C}(T)\}$. • Government chooses IMF transfer level $T \in \mathbb{T}$. • Domestic firms choose lending contract $C \in \mathbb{C}(T)$ and level of borrowing k.

Immediately after the IMF offers the menu of transfers \mathbb{T} , foreign investors offer a set of lending contracts $\{\mathbb{C}(T)\}$. Each element of this set specifies the set of lending contracts $\mathbb{C}(T)$ that are available to domestic firms if (later in period 2) the government chooses the transfer level $T \in \mathbb{T}$. The government observes \mathbb{T} and $\{\mathbb{C}(T)\}$, and then makes its choice of the transfer level. The equilibrium definition is amended appropriately to take account of the change in timing.

Definition 3.2 A *Perfect Bayesian Equilibrium* for this economy is a set of strategies $\{a, T(p), \{\mathbb{C}(T)\}, k(\mathbb{C})\}$, beliefs $p^e(T)$ and a menu of transfers \mathbb{T} such that:

1. *Government sets a in period 1 to maximize expected utility.*
Government chooses $T \in \mathbb{T}$ in period 2 to maximize consumption c in period 3, given private information p , the set of contracts $\{\mathbb{C}(T)\}$ and the expected contract choice by domestic firms $k(\mathbb{C})$.
2. *Foreign investors offer the set of contracts $\{\mathbb{C}(T)\}$ that maximize expected profits, given any transfer level T and their beliefs as a function of T , $p^e(T) = \mathbb{E}[p|T]$. Beliefs are updated using Bayes' rule on the equilibrium path (and are unrestricted otherwise).*
3. *Domestic firms choose the contract $C \in \mathbb{C}(T)$ and capital level k that maximize expected profits.*
4. *IMF satisfies its budget constraint. In period 1: $\mathbb{E}T = 0$.*

What is the effect of this change of timing on the set of feasible equilibria? The set of pooling equilibria achievable by the IMF is unaffected. However, the set of feasible separating equilibria

is reduced. Figure 3.7 presents an example of an allocation which was feasible under the previous timing but not under this one. Consider the timeline in figure 3.2. If the government expects the set of contracts $\{\mathbb{C}[T(p_H)] = \{R_H\}, \mathbb{C}[T(p_L)] = \{R_L\}\}$, then it decides to reveal its type as shown before. Given this separation decision, foreign investors find it optimal to offer precisely these contracts. In equilibrium, $A = (T(p_H), F_H)$ and $B = (T(p_L), F_L)$. Now consider the timeline in figure 3.6. Suppose that the same contracts are offered in period 2. It is then profitable for foreign investors to offer the contract corresponding to point D . This is a pooling contract conditional upon acceptance of IMF transfers $T(p_L)$. The amended set of contracts is $\{\mathbb{C}[T(p_H)] = \{R_H\}, \mathbb{C}[T(p_L)] = \{R_L, R_L - \varepsilon\}\}$, for some $\varepsilon > 0$. Given this set of contracts, both high and low type governments choose IMF transfers of $T(p_L)$ in period 2, and firms choose the contract offering interest rate $R_L - \varepsilon$. High and low type countries prefer point D . The initial allocation is not an equilibrium.

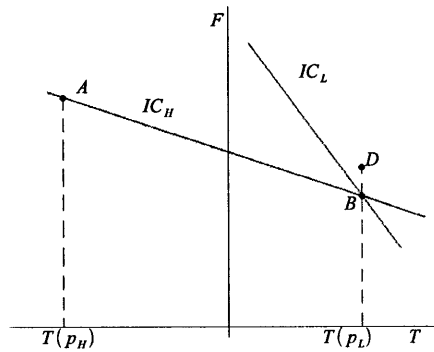


Figure 3.7: Effect of Change in Timing

For the model timeline provided in figure 3.2, points A and B above correspond to the separating equilibrium with the highest level of redistributive transfers. This configuration induces the lowest level of government effort \underline{a} in period 1. A reduction in the level of redistribution promised by the IMF increases the relative ex post consumption of the high type country, which makes it more difficult to tempt it to select a pooling contract. However, the reduction in promised redistributive transfers also increases the effort level a in period 1, which reduces R_P (and hence increases F_P) in the best feasible pooling equilibria. The latter effect makes it

possible to offer a better pooling contract to tempt the high type country. These competing effects yield the following proposition.

Proposition 3.11 (Feasible Effort Levels) *The modification to the sequence of events in period 2 reduces the set of effort levels consistent with a separating equilibrium. If the feasible set of effort levels is non-empty, it can be described:*

$$a \in [a_1, a_2] \cup [a_2, a_3] \cup \dots \cup [a_{n-1}, a_n],$$

where $a_1 > \underline{a}$ and $a_n \leq \bar{a}$.

The most preferred separating equilibrium is the one that corresponds to the highest level of effort a_n .

The modified timing amounts to an additional restriction on the set of contract offers. Given the set of offered contracts, there exists no other contract that, if offered, would both make a positive profit and increase the utility of at least one type of country. This notion of equilibrium is similar to the definition used in Rothschild and Stiglitz (1976). In their framework, the existence of separating equilibria depends upon the relative fractions of high and low types. In our model, the possibility of a deviation using pooling contracts reduces the feasible set of effort levels. If the feasible set of effort levels is non-empty, the separating equilibrium which induces the highest level of government effort in period 1 is the most preferred. The IMF compares this equilibrium to the pooling equilibrium, and chooses the equilibrium that maximizes expected utility in period 1.

3.4 Ex Ante Insurance and Separating Equilibria

IMF redistributive transfers can be interpreted as (partial) ex ante insurance, as described in subsection 3.3.3. In this section we generalize the result that ex ante insurance expands the set of feasible separating equilibria, and thereby may improve country welfare. We examine an amended version of the model where foreign investors have sufficient tools to separate countries in period 2 even in the absence of IMF intervention. However, these separating equilibria are necessarily associated with a distortion of output for the high type country. Then we consider

optimal policy by the IMF. The possibility of IMF redistributive transfers increases the set of feasible separating equilibria. The allocation in the absence of IMF intervention remains feasible, because the IMF can always commit to offer no transfers at all. In addition, the IMF can also achieve separating equilibria without any distortion of output ex post. However, these equilibria involve nonzero redistributive transfers by the IMF, and induce lower government effort in period 1. Which separating equilibrium is optimal? Nonzero IMF transfers are a feature of the optimal mechanism if the benefits of reduced output distortions outweigh the moral hazard costs.

3.4.1 Model without IMF

Figure 3.8 presents the timeline for the model considered in this subsection.

Figure 3.8: Timeline without IMF

<p>Period 1</p> <ul style="list-style-type: none"> • Government chooses effort level $a \in [0, 1]$. <p>Period 2</p> <ul style="list-style-type: none"> • Government learns its type $p \in \{p_L, p_H\}$. It is a high type with probability a. • Foreign investors offer set of lending contracts $\{\mathbb{C}(\tau)\}$. • Government chooses the tax per lending contract τ. • Domestic firms choose lending contract $C \in \mathbb{C}(\tau)$ and level of borrowing k. <p>Period 3</p> <ul style="list-style-type: none"> • Output is realized and repayments made to foreigners. All remaining resources are consumed.
--

In this framework, the government has an additional instrument. It can require that every lending contract from foreign investors to domestic firms in period 2 is associated with a tax payment of τ units of output from foreign investors to the government in period 3. Of course, a requirement of $\tau < 0$ corresponds to a subsidy. Foreign investors again compete in the provision of loans to domestic firms, but they can condition the set of contract offers on the government's taxation decision τ . Each lending contract specifies an interest rate $R(\tau)$.

Notice that the government chooses the tax per lending contract τ after the foreign investors offer the set of lending contracts. The equilibrium definition is amended appropriately.

What is the feasible set of contract offers? The zero profits condition for foreign investors can be written:

$$\begin{aligned}
 p^e Rk - rk - \tau &= 0 \\
 \iff \tau &= (p^e R - r)k
 \end{aligned}
 \tag{3.11}$$

where p^e denotes foreign investors' beliefs about p given τ . Equation (3.11) determines the level of $R(\tau)$ consistent with zero profits. Domestic firms select the lending contract $C \in \mathcal{C}(\tau)$ with the lowest interest rate R . The net output of each successful domestic firm is given by the expression:

$$F = f(k) - Rk, \tag{3.12}$$

where k satisfies:

$$f'(k) = R. \tag{3.13}$$

Equations (3.11) and (3.12), together with the restriction (3.13), describe the locus of the set of lending contracts consistent with zero profits in (τ, F) space.

For illustrative purposes, let us first characterize the equilibrium in period 2 when all countries are of the same type p . The equilibrium is shown in figure 3.9. The line ZP plots the zero profits condition. All contracts to the left of this line are associated with positive profits, and those to the right yield negative profits. The indifference curve of the country is tangent to the zero profits line at point A . This denotes the lending contract with $\tau = 0$ and $R = \frac{r}{p}$. The zero profits line is steeper than the indifference curve to the right of this point, and it has lower slope than the indifference curve to the left of it.

Point A represents the equilibrium allocation. The government chooses the tax payment τ , which corresponds to the horizontal position of the economy in (τ, F) space. The choice of lending contracts by domestic firms determines F , and hence the vertical position of the allocation.

Now we return to the model described in the timeline above, with types p_H and p_L in period 2.

Proposition 3.12 (Pooling without IMF) *There exists no pooling equilibrium.*

Proposition 3.13 (Separating without IMF) *There may exist a separating equilibrium.*

Propositions 3.12 and 3.13 are the analogs of the results in Rothschild and Stiglitz (1976). A pooling equilibrium does not exist because the single-crossing property is satisfied by the indifference curves of the high and low type countries. If a separating equilibrium exists, it takes the form shown in figure 3.10. The lines ZP_H and ZP_L represent the zero profits conditions for foreign investors lending to firms in the high and low type countries respectively. ZP_L lies everywhere to the left of ZP_H . Lending contracts corresponding to points $A = (\tilde{\tau}(p_H), \tilde{F}(p_H))$ and $B = (0, F_L)$ are selected by domestic firms in equilibrium. The incentive compatibility constraint of the low type country is binding, and the net output of successful domestic firms is distorted for the high type country. The high type country provides a subsidy to foreign investors, who lend to domestic firms at an interest rate lower than the first best interest rate R_H . As a result, the net output of successful domestic firms exceeds F_H . Since the subsidy from the country is large enough to ensure that foreign investors satisfy their zero profits condition, the subsidy is welfare-reducing relative to the first best allocation. The ex post consumption of the high type country is lower in the imperfect information equilibrium than in the first best case (represented by point $C = (0, F_H)$).

This separating equilibrium generates a difference in ex post consumption between the high and low type countries, which induces the government to exert effort level \tilde{a} in period 1. This effort level determines the position of the zero profits line for foreign investors who offer pooling contracts, ZP_P . For the separating equilibrium described in figure 3.10 to exist, the line ZP_P must lie everywhere below IC_H (following the argument from subsection 3.3.5).

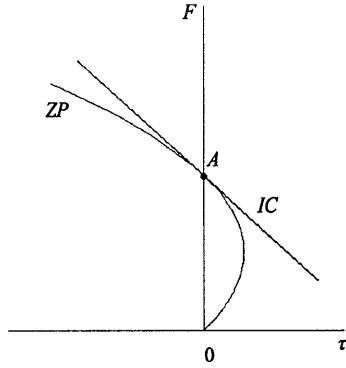


Figure 3.9: Zero Profit Line

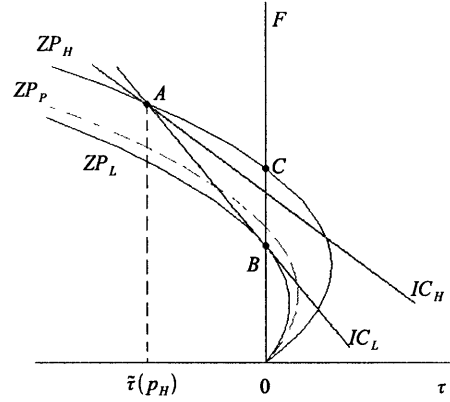


Figure 3.10: Separating Equilibrium

Assume a separating equilibrium exists. Figure 3.10 illustrates the properties of the equilibrium. The high type country imposes a tax per lending contract of $\tilde{\tau}(p_H) < 0$. Each firm faces the interest rate $\tilde{R}(p_H) < R_H$ and in the case of project success, generates net output $\tilde{F}(p_H) > F_H$. As explained above,

$$p_H \tilde{F}(p_H) + \tilde{\tau}(p_H) < p_H F_H.$$

The low type country's allocation is unchanged from the first best level: $\tilde{\tau}(p_L) = 0$. Domestic firms face interest rate R_L and produce net output F_L in the case of project success. The incentive compatibility constraint for the low type country is binding:

$$p_L F_L = p_L \tilde{F}(p_H) + \tilde{\tau}(p_H).$$

Finally, the effort level \tilde{a} of the government in period 1 solves the equation:

$$\begin{aligned} & \max_a \left\{ a \cdot \left[p_H \tilde{F}(p_H) + \tilde{\tau}(p_H) \right] + (1-a) \cdot p_L F_L - \psi(a) \right\} \\ & = \max_a \{ p_L F_L + a \cdot \tilde{z} - \psi(a) \}, \end{aligned}$$

where $\tilde{z} = [p_H - p_L] \tilde{F}(p_H)$. Notice that since $\tilde{F}(p_H) > F_H$, this expression also establishes that $\tilde{a} > \bar{a}$, where \bar{a} denotes the effort level with optimal IMF intervention in subsection 3.3.2. However, ex ante expected utility is not necessarily higher under the current setup, since the ex post consumption of the high and low type countries are different from those in the earlier subsection.

3.4.2 Model with IMF

The IMF is introduced into the above framework in the expected manner.

Figure 3.11: Timeline with IMF

Period 1

- IMF announces redistribution scheme that it will offer in period 2.
- Government chooses effort level $a \in [0, 1]$.

Period 2

- Government learns its type $p \in \{p_L, p_H\}$. It is a high type with probability a .
- IMF offers menu of redistributive transfers \mathbb{T} .
- Government chooses the IMF transfer level $T \in \mathbb{T}$.
- Foreign investors offer set of lending contracts $\{\mathbb{C}(\tau; T)\}$.
- Government chooses the tax per lending contract τ .
- Domestic firms choose lending contract $C \in \mathbb{C}(\tau; T)$ and level of borrowing k .

Period 3

- Output is realized and repayments made to foreigners. IMF transfers are made. All remaining resources are consumed.

Foreign investors offer a set of lending contracts $\{\mathbb{C}(\tau; T)\}$ after observing the government's choice of the transfer level $T \in \mathbb{T}$. Each element of this set takes T as given and specifies the set of lending contracts $\mathbb{C}(\tau; T)$, that are available to domestic firms if (later in period 2) the government chooses the tax per lending contract τ . The government makes its choice of the transfer level given its type p and its expectations regarding the set of contracts offered by foreign investors $\mathbb{C}(\tau; T)$. It then observes the set of contracts offered before making its taxation

decision τ . Finally, domestic firms make their contract choice. The equilibrium definition is amended appropriately. The IMF must satisfy its budget constraint.

Again, optimal IMF policy is the solution to a mechanism design problem in the presence of imperfectly informed competitive markets. Apply the Revelation Principle, and restrict attention to pure strategy perfect Bayesian equilibria.

Proposition 3.14 (Mechanism Design Problem) *Optimal IMF policy is the solution to the following mechanism design problem:*

$$\max_{z, T(p_H), T(p_L)} W = \{[p_L F(p_L) + \tau(p_L) + T(p_L)] + a \cdot z - \psi(a)\}$$

subject to

$$z = [p_H F(p_H) + \tau(p_H) + T(p_H)] - [p_L F(p_L) + \tau(p_L) + T(p_L)]$$

$$a = (\psi')^{-1}[z]$$

$$a \cdot T(p_H) + (1 - a) \cdot T(p_L) = 0$$

$$p_H F(p_H) + \tau(p_H) + T(p_H) \geq p_H F(p_L) + \tau(p_L) + T(p_L)$$

$$p_L F(p_L) + \tau(p_L) + T(p_L) \geq p_L F(p_H) + \tau(p_H) + T(p_H).$$

Pooling equilibria:

$$F(p_H) = F(p_L)$$

$$T(p_H) = T(p_L), \tau(p_H) = \tau(p_L).$$

Separating equilibria:

(i) *Separation induced by foreign investors' contract offers (if such an equilibrium exists):*

$$T(p_H) = T(p_L)$$

$$F(p_L) = F_L, \tau(p_L) = 0$$

and $(F(p_H), \tau(p_H))$ solve

$$\max_{F(p_H), \tau(p_H)} \{p_H F(p_H) + \tau(p_H) + T(p_H)\}$$

subject to

$$p_L F_L + T(p_L) \geq p_L F(p_H) + \tau(p_H) + T(p_H).$$

(ii) Separation induced by IMF transfer choice:

$$F(p_L) = F_L, F(p_H) = F_H, \tau(p_L) = \tau(p_H) = 0.$$

We now describe the set of feasible perfect Bayesian equilibria.

For a pooling equilibrium to exist, foreign investors must be unable to distinguish between the high and low type countries. This necessarily implies that both the transfer levels and taxes chosen by the two types of countries are identical. Suppose that the IMF transfer level chosen is indeed the same, such that $T(p_H)$ and $T(p_L)$ are equal. From the IMF's budget constraint:

$$T(p_H) = T(p_L) = 0.$$

This takes us back to subsection 3.4.1, the environment without IMF intervention. Proposition 3.12 applies. Consistent with the result in the previous subsection, there exists no feasible pooling equilibrium in period 2. If there exist any equilibria, they must be separating equilibria.

There may exist two categories of separating equilibria. In particular, the government may reveal its type through its choice of the IMF transfer level $T \in \mathbb{T}$, or through its tax decision τ . Condition (i) considers the case where information revelation occurs via the taxation decision. If both high and low type governments select identical transfer levels $T(p_H)$ and $T(p_L)$, the budget constraint of the IMF again establishes that these transfer levels should be equal to zero. We return to the model of international capital markets in the absence of IMF intervention. There may exist one separating equilibrium in this case, as shown in subsection 3.4.1. The diagram illustrating this equilibrium is reproduced as figure 3.12. The lending contracts are

plotted in (S, F) space, where we define:

$$S = \tau + T.$$

S is the sum of taxes and transfers received by the country. The government chooses the position of the economy on the horizontal dimension. The choice of lending contracts by domestic firms determines the vertical position. Lending contracts corresponding to points $A = (\tilde{\tau}(p_H), \tilde{F}(p_H))$ and $B = (0, F_L)$ are selected by domestic firms in equilibrium. Notice that for the separating equilibrium to exist, the line ZP_P must lie everywhere below IC_H .

Now let us turn to condition (ii), where the type of the country is revealed via its choice of the IMF transfer level. In this scenario, foreign investors know the type of the country when they make contract offers. Therefore, foreign investors offer all contracts that make non-negative profits conditional upon the country type. The government then chooses the contract that maximizes consumption. The optimal taxation decisions for both types of government are:

$$\tau(p_L) = \tau(p_H) = 0.$$

This establishes that the net output levels of successful domestic firms in the low and high type countries satisfy:

$$F(p_L) = F_L, F(p_H) = F_H.$$

Therefore, the set of feasible separating equilibria satisfying condition (ii) is identical to the set of separating equilibria that were feasible in section 3.3. Our analysis in that section proved that the most preferred separating equilibrium in this set was the one which involved the lowest level of redistributive transfers by the IMF. This equilibrium is illustrated in figure 3.13. Lending contracts corresponding to points $A = (T(p_H), F_H)$ and $B = (T(p_L), F_L)$ are selected by domestic firms in equilibrium. The line ZP_P is plotted in the diagram, but it is not relevant for this timing of actions and events.

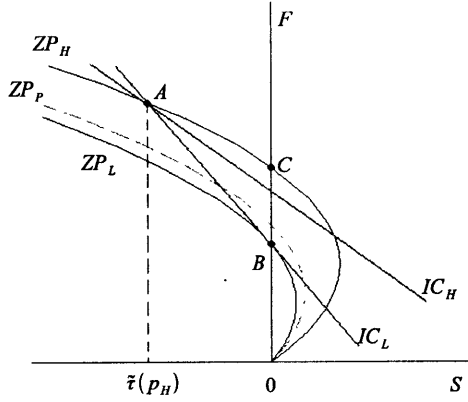


Figure 3.12: Separating Equilibrium with
Output Distortion

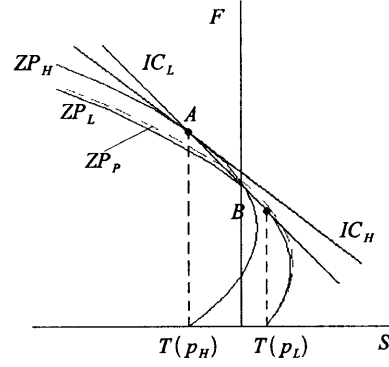


Figure 3.13: Best Separating Equilibrium
without Output Distortion

Proposition 3.15 (Optimal IMF Policy) *The optimal allocation is one of two candidate equilibria:*

(a) *The separating equilibrium induced by foreign investors' contract offers and zero IMF transfers (if it exists):*

$$T(p_L) = T(p_H) = 0$$

$$F(p_L) = F_L, \tau(p_L) = 0$$

$$F(p_H) > F_H, \tau(p_H) < 0.$$

The government effort level in period 1 is \bar{a} .

(b) *The separating equilibrium which is the optimal allocation in subsection 3.3.2, the environment with IMF transfers but without taxes on lending contracts:*

$$T(p_L) = \bar{a} \cdot p_L [F_H - F_L]$$

$$T(p_H) = -\left(\frac{1 - \bar{a}}{\bar{a}}\right) \cdot T(p_L)$$

$$F(p_L) = F_L, \tau(p_L) = 0$$

$$F(p_H) = F_H, \tau(p_H) = 0.$$

The government effort level in period 1 is $\bar{a} < \tilde{a}$.

Which separating equilibrium is preferred depends on the relative importance of moral hazard and adverse selection distortions.

IMF redistributive transfers increase the set of feasible separating equilibria. By setting all of the transfer levels in its menu to zero, the IMF can still induce the separating equilibrium that was feasible in the absence of the IMF. This separating equilibrium has high government effort in period 1, and a distortion in the net output of successful firms in the high type country in period 2: $F(p_H) > F_H$. In addition, the possibility of ex ante insurance via IMF transfers makes other separating equilibria feasible. In particular, it is no longer necessary for output to be distorted ex post for any country in a separating equilibrium. However, the elimination of output distortions is necessarily associated with a lower level of government effort in period 1.

Which of the candidate separating equilibria are optimal? It depends on the relative welfare costs of moral hazard and adverse selection. If the moral hazard problem is sufficiently severe, then the government effort is substantially lower in the allocation with nonzero IMF redistributive transfers. This adverse effect on welfare outweighs the benefit of reducing the output distortions in the absence of IMF intervention. The optimal allocation involves no role for the IMF.

Alternatively, the moral hazard problem may be small relative to the adverse selection problem. For this case, the negative welfare effects of lower government effort may be outweighed by the benefits of reducing output distortions ex post. The IMF offers a separating menu of transfers as part of the optimal allocation. For the extreme case of a model with only adverse selection, output is not distorted at the optimum.

3.5 Conclusion

Optimal IMF policy is the solution to a mechanism design problem in the presence of imperfectly informed competitive markets. The IMF implements redistributive transfers between high and low productivity countries ex post, subject to both its budget constraint and to the rational expectations condition that foreign investors set prices according to the information revealed

in equilibrium. The zero profits condition for foreign investors changes as a result of the IMF redistribution scheme. The incentives for the government to self-select in a separating equilibrium depends on a combination of the IMF's policies and the contracts offered by foreign investors. This is central to the description of the optimal allocation chosen by the IMF. Countries with weak fundamentals in the second period choose to receive high levels of IMF transfers. As a consequence, they face high interest rates on international capital markets, which leads to low private capital inflows. Countries with strong fundamentals in the second period value access to international capital markets more than low type countries. They make contributions to the IMF and thereby receive low interest rates from foreign investors. They enjoy high levels of private capital inflows.

For the baseline model considered, the IMF's redistribution scheme is implemented not to decrease the difference in ex post consumption between high and low type countries, but to increase it. This is achieved by generating capital reallocation on international capital markets from low type to high type countries in the second period. Therefore, government effort is higher ex ante.

IMF redistributive transfers expand the set of feasible separating equilibria, and thereby may be welfare-improving. This result holds in an amended version of the model where foreign investors have sufficient tools to separate countries ex post even in the absence of IMF intervention. IMF transfers reduce the output distortions that are necessary for a separating equilibrium to exist ex post. However, the government effort level ex ante is lower in the presence of IMF intervention. Whether it is optimal for the IMF to intervene and offer a menu of nonzero redistributive transfers depends on the trade-off between moral hazard and adverse selection concerns.

Markets can be used to decentralize the IMF's optimal allocation. In particular, the same allocation is obtained if the government can purchase insurance at actuarially fair rates before its effort decision. However, if such contracts are only available after the government's type is revealed, the government will not purchase the ex ante optimal level of insurance. In this case, the IMF should commit ex ante to a schedule of redistributive transfers.

Risk neutrality of the representative consumer simplifies some dimensions of the optimal mechanism design problem. By inducing a separating equilibrium, the IMF solves the capital

misallocation problem in the second period that is associated with a pooling equilibrium. This is desirable ex ante under risk neutrality. However, the ex post consumption of the low type country may lie below the level in a pooling equilibrium. In this case, a risk averse representative consumer may prefer the imperfect information allocation to the first best configuration, even from an ex ante perspective. Furthermore, the IMF encounters commitment problems with a concave objective function. Specifically, it is tempted to renege on its schedule of promised transfers ex post, and instead implement more redistribution. Commitment problems under risk aversion are explored further in Netzer and Scheuer (2009).

Our model predicts that the full benefits of IMF intervention cannot be discerned by looking solely at IMF program countries. IMF transfers to low productivity countries improves outcomes for high productivity countries, because they enable the latter to reveal their type in equilibrium. Countries that refuse IMF transfers benefit from the existence of the IMF, because the refusal reveals their high productivity and induces high private capital inflows. This mechanism has implications for empirical work into the effects of IMF intervention, and for the design of IMF programs (for detailed existing work, see Bird 2001).

3.6 Appendix: Proofs of Results in the Main Text

Proof of Proposition 3.1.

Domestic firms' choice of capital level k does not depend on the type of the country p . Firms from both types of countries prefer the contract with the lowest offered interest rate R . The interest rate is the only variable of the contract that foreign investors can propose, and there is no communication between the government and foreign investors. Therefore, it is not possible for foreign investors to offer a separating contract. ■

Proof of Proposition 3.2.

In a pooling equilibrium, foreign investors remain uninformed about the type of the country. Therefore, they offer lending contracts with interest rate R_P given by expression (3.1) in the main text, given their beliefs of the government effort level a^e . The government chooses the effort level a^* in period 1 to maximize expression (3.2), taking a^e (and hence F_P) as given.

Define

$$a^* = \Gamma(a^e)$$

to be the government's effort choice in period 1, given foreigners' beliefs a^e . A pooling equilibrium is defined as the fixed point:

$$a^* = a^e = \Gamma(a^e).$$

Now we establish the properties of $\Gamma(a)$. The first order condition for the government's maximization problem (3.2) is:

$$\begin{aligned} \psi'(a^*) &= z^* \\ \implies a^* &= (\psi')^{-1}[z^*]. \end{aligned} \tag{3.14}$$

Given the convexity of the cost function, this first order condition identifies the global maximum for expected utility in period 1. Since the cost function is twice differentiable and convex, the government's effort choice a^* is continuous and increasing in z^* . Twice differentiability and concavity of the production function establishes that z^* is continuous and increasing in a^e . Therefore, $\Gamma(a)$ is continuous and increasing in a .

Even if foreign investors expect the government to exert zero effort, the value of z^* is positive. From the condition $\psi'(0) = 0$ and the property that a^* is increasing in z^* , we obtain that $\Gamma(0) > 0$. If foreign investors expect the government to exert the maximum feasible effort level of unity, the value of z^* is positive and finite. Since $\lim_{a \rightarrow 1} \psi'(a) = \infty$, we derive $\Gamma(1) < 1$.

Therefore, there exists at least one pooling equilibrium. In any pooling equilibrium, $a^* \in (0, 1)$. $a^* < a^{FB}$ from the argument in the main text. ■

Proof of Proposition 3.3.

Consider welfare in the pooling equilibrium, which we denote as W^* .

$$W^* = a^* \cdot p_H F_P + (1 - a^*) \cdot p_L F_P - \psi(a^*).$$

Add to this expression the expected profits of foreign investors from the pooling contract, which is zero:

$$\begin{aligned}
W^* &= a^* \cdot p_H F_P + (1 - a^*) \cdot p_L F_P - \psi(a^*) \\
&\quad + a^* \cdot [p_H R_P k_P - r k_P] + (1 - a^*) \cdot [p_L R_P k_P - r k_P] \\
&= a^* \cdot [p_H F_P + p_H R_P k_P - r k_P] \\
&\quad + (1 - a^*) \cdot [p_L F_P + p_L R_P k_P - r k_P] - \psi(a^*)
\end{aligned}$$

By definition:

$$\begin{aligned}
p_H F_P + p_H R_P k_P - r k_P &= p_H f(k_P) - r k_P \\
&= p_H [f(k_P) - R_H k_P] < p_H F_H,
\end{aligned}$$

and

$$\begin{aligned}
p_L F_P + p_L R_P k_P - r k_P &= p_L f(k_P) - r k_P \\
&= p_L [f(k_P) - R_L k_P] < p_L F_L.
\end{aligned}$$

This establishes that

$$\begin{aligned}
W^* &< a^* \cdot p_H F_H + (1 - a^*) \cdot p_L F_L - \psi(a^*) \\
&< a^{FB} \cdot p_H F_H + (1 - a^{FB}) \cdot p_L F_L - \psi(a^{FB}).
\end{aligned}$$

As required. ■

Proof of Proposition 3.4.

This follows immediately from application of the Revelation Principle. For pooling equilibria, foreign investors remain uninformed about the type of the country irrespective of the information revealed by the country to the mechanism designer. Therefore, they must offer a pooling contract:

$$F(p_H) = F(p_L) = F_P.$$

For separating equilibria, the type of the country is revealed to foreign investors. Therefore, they offer lending contracts that satisfy their full information zero profit conditions:

$$F(p_H) = F_H, F(p_L) = F_L.$$

This establishes the result in the text. ■

Proof of Proposition 3.5.

For pooling equilibria, foreign investors remain uninformed about the type of the country. Note that in the model timeline, foreign investors observe the transfer level $T \in \mathbb{T}$ chosen by the government. We require that foreign investors do not learn the country's type from their choice of IMF transfers. This requires that $T(p_H)$ and $T(p_L)$ are equal. From the IMF's budget constraint, we obtain:

$$T(p_H) = T(p_L) = 0.$$

Substituting into the constrained mechanism design problem, we obtain the result required. ■

Proof of Proposition 3.6.

For separating equilibria, foreign investors learn the type of the country. The incentive compatibility constraints may be rewritten:

$$p_H F_H + T(p_H) \geq p_H F_L + T(p_L)$$

$$p_L F_L + T(p_L) \geq p_L F_H + T(p_H).$$

These expressions yield the equations in the text. ■

Proof of Proposition 3.7.

As shown in the main text, the objective function of the IMF is

$$a \cdot p_H F_H + (1 - a) \cdot p_L F_L - \psi(a).$$

We know that the unique stationary point and global maximizer of this expression is the effort level a^{FB} .

From the argument in the text, the IMF-induced separating equilibrium that maximizes expected utility in period 1 corresponds to the highest effort level possible, which is \bar{a} . It remains to prove that this equilibrium dominates any pooling equilibrium that the IMF can induce. Notice that

$$[p_H - p_L] F_P < [p_H - p_L] F_H < p_H F_H - p_L F_L,$$

which establishes that

$$a^* < \bar{a} < a^{FB}.$$

In the proof of Proposition 3.3, we showed that the welfare in the pooling equilibrium W^* satisfies the following expression:

$$W^* < a^* \cdot p_H F_H + (1 - a^*) \cdot p_L F_L - \psi(a^*).$$

The desired result immediately follows. ■

Proof of Proposition 3.8.

This follows directly from the argument in the proof of Proposition 3.7. ■

Proof of Propositions 3.9 and 3.10.

By inspection. ■

Proof of Proposition 3.11.

The separating equilibrium does not exist if the point $(T(p_L), F_P)$ lies above the indifference curve of the high type country through the allocation $A = (T(p_H), F_H)$. The shape of the feasible set of effort levels follows from the argument in the text. The objective function of the IMF is unchanged from Proposition 3.7, so the most preferred separating equilibrium is still the one corresponding to the highest level of effort. In this case it is a_n . ■

Proof of Proposition 3.12.

This follows immediately from the single-crossing property in the enriched contract space. ■

Proof of Proposition 3.13.

The structure of separating equilibria follows from the single-crossing property. The separating equilibrium does not exist if the point $(T(p_L), F_P)$ lies above the indifference curve of the high type country through the allocation $A = (T(p_H), F_H)$. ■

Proof of Proposition 3.14.

Apply the Revelation Principle. The formulation can then be derived by inspection. ■

Proof of Proposition 3.15.

Condition (i) of Proposition 3.14 is satisfied by only one separating equilibrium (if it exists), and it is summarized as candidate equilibrium (a). The set of feasible separating equilibria satisfying condition (ii) is identical to the set of separating equilibria that were feasible in section 3.3. The proof of Proposition 3.7 establishes that the best separating equilibrium in this set can be written as candidate equilibrium (b).

For equilibrium (a):

$$\tilde{W} = \tilde{a} \cdot [p_H \tilde{F}(p_H) + \tilde{\tau}(p_H)] + (1 - \tilde{a}) \cdot p_L F_L - \psi(\tilde{a}), \quad (3.15)$$

where the difference in ex post consumption between the high and low types is given by

$$\tilde{z} = [p_H - p_L] \tilde{F}(p_H).$$

For equilibrium (b):

$$\bar{W} = \bar{a} \cdot p_H F_H + (1 - \bar{a}) \cdot p_L F_L - \psi(\bar{a}), \quad (3.16)$$

with difference in ex post consumption given by

$$\bar{z} = [p_H - p_L] F_H.$$

The expression in square brackets of equation (3.15) is lower in the first term of equation (3.16). However, the government effort level \tilde{a} is higher than \bar{a} .

The difference between $\left[p_H \tilde{F}(p_H) + \tilde{\tau}(p_H) \right]$ and F_H depends on the production function $f(k)$ and the exogenous parameters of the model. The difference in government effort between the two separating equilibria depends additionally on the cost function $\psi(a)$. Therefore, the comparison is ambiguous and the statement in the proposition holds. ■

3.7 References

1. Arregui, Nicolas. 2009. "IMF Contingent Credit Lines and the Entry Problem." International Breakfast Presentation, MIT.
2. Bird, Graham. 2001. "IMF Programs: Do They Work? Can They be Made to Work Better?" *World Development*, Vol. 29, No. 11, pp. 1849-1865.
3. Bird, Graham, and Dane Rowlands. 2002. "Do IMF Programmes Have a Catalytic Effect on Other International Capital Flows?" *Oxford Development Studies*, Vol. 30, No. 3, pp. 229-249.
4. Corsetti, Giancarlo, Bernardo Guimarães and Nouriel Roubini. "International Lending of Last Resort and Moral Hazard: A Model of IMF's Catalytic Finance." *Journal of Monetary Economics*, Vol. 53, No. 3, pp. 441-471.
5. Dreher, Axel. 2004. "Does the IMF cause Moral Hazard? A Critical Review of the Evidence." International Finance Working Paper 0402003, EconWPA.
6. Edwards, Martin S. 2006. "Signalling Credibility? The IMF and Catalytic Finance." *Journal of International Relations and Development*, Vol. 9, No. 1, pp. 27-52.
7. Fischer, Stanley. 1999. "On the Need for an International Lender of Last Resort." *Journal of Economic Perspectives*, Vol. 13, No. 4, pp. 85-104.
8. Jeanne, Olivier, and Jeromin Zettelmeyer. 2001. "International Bailouts, Moral Hazard and Conditionality." CESifo Working Paper No. 563.

9. Jeanne, Olivier, and Jeromin Zettelmeyer. 2004. "The Mussa Theorem (and Other Results on IMF-Induced Moral Hazard)." IMF Working Paper No. 04/192.
10. Lane, Timothy, and Steven Phillips. 2000. "Does IMF Financing Result in Moral Hazard?" IMF Working Paper No. 00/168.
11. Marchesi, Silvia, and Jonathan P. Thomas. 1999. "IMF Conditionality as a Screening Device." *The Economic Journal*, Vol. 109, No. 454, pp. C111-C125.
12. Morris, Stephen, and Hyun Song Shin. 2006. "Catalytic Finance: When Does it Work?" *Journal of International Economics*, Vol. 70, No. 1, pp. 161-177.
13. Rothschild, Michael, and Joseph Stiglitz. 1976. "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information." *The Quarterly Journal of Economics*, Vol. 90, No. 4, pp. 629-649.
14. Netzer, Nick, and Florian Scheuer. 2009. "Competitive Markets Without Commitment." University of Zurich Socioeconomic Institute, Working Paper No. 0814.