

# Essays on Optimal Insurance Design

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by

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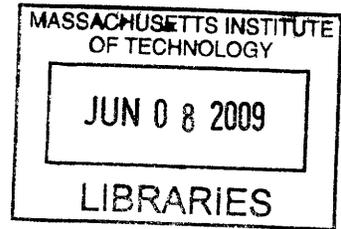
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## Abstract

This dissertation consists of three chapters analyzing the optimal design of insurance contracts. I consider three relevant contexts that change the central trade-off between the provision of insurance and the provision of incentives.

The first chapter analyzes the role of biased beliefs for the optimal design of static and dynamic insurance contracts. Biased risk perceptions change the perceived value of insurance and the perceived returns to avoiding these risks. I show empirically that unemployed workers overestimate how quickly they will find work, but underestimate the return to their search efforts. I analyze how these biases drive a wedge between social and private insurance, and between naive and optimal policy implementation.

The second chapter analyzes the role of training for the design of unemployment insurance. A worker's human capital falls upon displacement and depreciates during unemployment. Training counters the decrease in human capital, but also changes the willingness of the unemployed to search. I characterize the optimal unemployment insurance contract and analyze the optimal timing of unemployment benefits and training programs during unemployment.

The third chapter analyzes the role of heterogeneity in risk perceptions for the optimal design of screening contracts in a model with moral hazard and adverse selection. I show how optimists receive less insurance than pessimists and I contrast the distortions in insurance coverage that arise with competing and monopolistic insurers. Heterogeneity in beliefs strengthens the case for government intervention in insurance markets and can explain the negative correlation between risk occurrence and insurance coverage found in empirical studies.

Thesis Supervisor: Bengt Holmström  
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# Introduction

People face risks and dislike the variation in income due to these risks. They are willing to give up consumption in good times to increase their consumption in bad times. Whether times are good or bad often depends on their own behavior. People can mitigate the risk or reduce the probability that a loss occurs by exerting precautionary effort. Moral hazard arises when insured people do not account for the consequences of their behavior on the expected expenditures for the insurer. The insured will exert less precautionary efforts the more insured they are. Insurers thus face a fundamental trade-off between providing insurance against risks and providing incentives to avoid risks. This trade-off is central to the design of optimal insurance contracts.

The trade-off between insurance and incentives also arises for the design of social insurance contracts, like unemployment insurance. Workers may lose their job beyond their control. They are willing to pay a tax when employed in order to receive unemployment benefits when they lose their job. Unemployed workers do not fully control how rapidly they are employed again, but by exerting search efforts they can increase the probability to find a job. Unemployment insurance insures the unemployed against the loss of their labor earnings, but also reduces the incentives to search for a job.

This dissertation consists of three chapters analyzing the optimal design of insurance contracts. I consider three different contexts that change the trade-off between the provision of insurance and incentives.

Chapter 1 analyzes optimal insurance contracts when the insured have biased beliefs. Biased perceptions of risks change the perceived value of insurance and the perceived returns to avoiding these risks. Contracts equalizing the marginal smoothing benefit and

the moral hazard cost of insurance, as in the Baily formula (1978), are suboptimal when beliefs are biased. Social and private insurance diverge as well; a paternalistic social planner corrects the moral hazard cost for the distortion in the insured's effort choice, while private insurers focus on the perceived rather than the true smoothing benefits. I show empirically that unemployed workers overestimate how quickly they will find work, but underestimate the return to their search efforts. With optimistic unemployed workers, privatizing unemployment insurance may result in inefficiently low or rapidly decreasing unemployment benefits.

Chapter 2 analyzes unemployment insurance contracts that combine monetary benefits and training. Unemployment insurance trades off the provision of search incentives and insurance against both the loss of current earnings and future earnings due to the decrease in human capital. Human capital falls upon displacement and continuously depreciates during unemployment. Training counters the decrease in human capital, but also changes the willingness of the unemployed to search. I characterize the optimal insurance contract and analyze the optimal timing of consumption and training. Numerical simulations show that if the cost of training is sufficiently low, the human capital of long-term unemployed converges to a unique, positive level. Hence, the optimal contract never stops inducing search efforts. In practice, training programs are mainly targeted towards the long-term unemployed. This is optimal only if the fall in human capital upon displacement is small relative to the depreciation rate during unemployment.

Chapter 3 analyzes the optimal design of screening contracts when individuals have differing beliefs about risks they face and about their ability to mitigate these risks. Profit-maximizing firms screen identical agents with different beliefs by providing less insurance to optimists than to pessimists. Optimists perceive the risk to be less likely than pessimists given their respective choices of precautionary efforts. Depending on the nature of competition, the screening distortions in insurance coverage are determined by differences in beliefs about the likelihood or the marginal return to effort. I show that heterogeneity in beliefs strengthens the case for government intervention in insurance markets and can explain the negative correlation between risk occurrence and insurance coverage found in empirical studies.

# Chapter 1

## Unemployed but Optimistic: Optimal Insurance Design with Biased Beliefs

Insurers face the trade-off between providing insurance against risks and incentives to avoid risks. The risk perceptions of the insured are central to this trade-off. The perceived likelihood of risks determines the perceived value of insurance against these risks. The perceived return to precautionary effort determines the effectiveness of incentives to avoid risks. Both types of perceptions are often subject to systematic biases. Psychological research has shown that people often overestimate the probability of positive events and underestimate the probability of negative events (Weinstein 1980, 1982 and 1984, Slovic 2000) and can either be optimistic (Langer 1975) or discouraged about the degree to which they control outcomes (Jahoda 1971). These particular biases complement the heuristics and biases in probabilistic thinking documented by Tversky and Kahneman (1974).

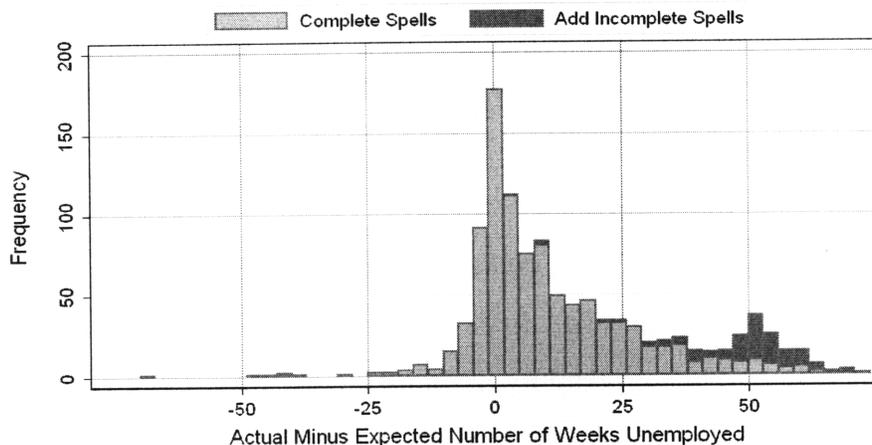
The central contribution of this paper is the theoretical and empirical analysis of unemployment insurance and the biases in beliefs held by the unemployed. On the theoretical side, I analyze how biased beliefs change the optimal design of static and dynamic insurance contracts in the presence of moral hazard. The distinction between the *baseline belief* about the probability of finding work and the *control belief* about the extent to

which search efforts increase this probability is shown to be essential. The theoretical results generalize to insurance applications with moral hazard, other than unemployment insurance. On the empirical side, I present new evidence that suggests that job seekers are highly optimistic about the probability of finding a job, but pessimistic about their control.

Using data collected by Price, Vinokur, Howe, and Caplan (1998), I link the expectations of unemployed job seekers with the actual outcome of their job search. The first empirical result is that job seekers largely underestimate the duration of their unemployment spell; on average they expect to remain unemployed for 7 weeks, but actually need 23 weeks to find new employment. Many more job seekers have underestimated rather than overestimated the length of their unemployment spell and the forecast errors are much more pronounced for the optimistic than for the pessimistic job seekers, as presented in Figure 1-1. The second empirical result is that job seekers who report searching more intensively are less optimistic about the length of their unemployment spell. Controlling for heterogeneity and endogeneity, I provide evidence that job seekers underestimate the returns to their search efforts. Job seekers who search harder expect shorter unemployment spells, but the actual reduction in the unemployment spell is larger than expected. This suggests that job seekers are at the same time baseline-optimistic and control-pessimistic; they overestimate the baseline probability of finding work, but underestimate their control over this probability.

The theoretical analysis builds on a canonical result for social insurance known as the Baily formula. Optimal insurance equalizes the benefit of smoothing consumption between states and the moral hazard cost at the margin. Baily (1978) formalized this principle for unemployment insurance in a static model with moral hazard. For unemployment insurance to be optimal, the relative difference in marginal utilities of consumption in employment and unemployment has to be equal to the elasticity of the unemployment duration to the unemployment benefit level. I show how this characterization needs to be adjusted when the insured have biased beliefs. I assume that the insurer knows the insured's beliefs and that these beliefs cannot be manipulated by the insurer, nor changed

Figure 1-1: Histogram of Differences between Actual and Expected Unemployment Duration



Source: Unemployed job seekers in Maryland and Detroit between 1996 and 1998 surveyed by Price et al. (1998).

in response to the contract being offered.<sup>1</sup>

I contrast the contracts offered by two extreme types of insurers: a social planner, who is paternalistic and maximizes the insured agent's *true* expected utility, and competing private insurers, who maximize the insured agent's *perceived* expected utility. When beliefs are unbiased, the probability weights in the respective expected utility functions are the same. The social optimum and the competitive equilibrium coincide. Moral hazard, in contrast with adverse selection, is no reason for government intervention as long as beliefs are unbiased. When beliefs are biased, the social optimum and the competitive equilibrium diverge. The implied wedge suggests a previously unexplored welfare cost of privatizing insurance.

In the social optimum the smoothing benefit and the moral hazard cost are still equalized at the margin, but with the moral hazard cost corrected for the *search internality* that arises when the insured agent misperceives the impact of her search on her own true expected utility. An increase in insurance coverage decreases the induced effort level, but when an agent is pessimistic about her control, she already exerts too little effort. Thus with control-pessimistic insurees, the moral hazard cost of insurance needs to be revised

<sup>1</sup>These assumptions correspond to a setting with different priors where the insurer and the insured 'agree to disagree'.

upward because of the search internality. The elasticity of the unemployment duration to unemployment benefits no longer provides sufficient information to implement the optimal insurance contract. A naive policy maker, who ignores the pessimistic control bias and implements the standard Baily formula, sets the unemployment benefit level suboptimally high.

Private insurers do not correct for the search internality and focus on the insured's perceived value of insurance. In the competitive equilibrium, the moral hazard cost of additional insurance is set equal to the *perceived smoothing benefit*. When an agent is optimistic about the baseline probability of finding work, she underestimates the value of unemployment insurance. Private insurers respond to this bias by offering less or even no insurance at all. This may explain the puzzle of why unemployment insurance is almost always publicly provided.<sup>2</sup> Competition disciplines insurers to charge actuarially fair prices, but not to correct people's distorted demand for insurance.

I proceed to consider a dynamic extension of the unemployment model along the lines of Hopenhayn and Nicolini (1997). The conventional wisdom in economic policy debates is that unemployment benefits should be decreasing with the length of the unemployment spell. The threat of falling benefits in the future increases the incentives for unemployed workers to search for work (Shavell and Weiss 1979). First, I show, using Baily-type conditions, that the adjustment of the optimal dynamic characterization for the presence of biases in beliefs is very similar as in the static model; the social planner corrects the moral hazard cost for the search internality, while the private insurers focus on the perceived smoothing benefits. Second, when unemployed agents underestimate the duration of unemployment, the social planner may increase welfare by providing more incentives to the short-term unemployed than to the long-term unemployed. Optimism about the duration

---

<sup>2</sup>Exceptions are unemployment insurance provided by trade unions or voluntary public unemployment insurance systems in countries like Denmark, Finland and Sweden, grown out of trade union programs (Parsons et al. 2003). The latter are heavily subsidized by the government, as expected with baseline-optimistic insurees. The existence of private information and aggregate risk and the government's advantage in coping with moral hazard have been suggested as explanations for the absence of private unemployment insurance (Chiu and Karni 1998, Barr 2001). Acemoglu and Shimer (2000) conclude: "Why unemployment insurance is almost always publicly provided, in contrast to most other insurance contracts, remains an important, unresolved question."

of unemployment makes the threat of receiving lower unemployment benefits in the future less effective in inducing search efforts. I show that in contrast with private insurers, the social planner may prefer to make unemployment benefits more rapidly decreasing at the start of the unemployment spell and more slowly later on.

I calibrate the dynamic model in order to numerically analyze the impact of biased beliefs on the optimal design of unemployment insurance. The calibration exercise also shows that the consumption subsidy required to make the agent insured by private insurers as well off as in the social optimum, increases exponentially in the baseline bias. Although the risk of an unemployment spell seems small within a lifetime, privatizing the insurance provision comes at a very high welfare cost if beliefs are strongly biased.

**Related Literature** The empirical and experimental evidence on the misperceptions of probabilities has led to two recent strands of literature. One strand proposes explanations for biases in beliefs and shows how these biases can be sustained in equilibrium. Examples are Bénabou and Tirole (2002 and 2006), Compte and Postlewaite (2004), Glaeser (2004), Van den Steen (2004), Brunnermeier and Parker (2005), Gollier (2005) and Köszegi (2006). These theoretical papers suggest that optimistic beliefs, either about the baseline probability of success or one's control, are more likely to arise and persist than pessimistic beliefs. This corresponds to the empirical evidence that I find for the unemployed's baseline beliefs, but contrasts with the empirical evidence for the unemployed's control beliefs.

The theoretical analysis in this paper is related to the second strand of literature that takes biases in risk perceptions as given and analyzes the consequences for contract design in the presence of moral hazard or adverse selection. De la Rosa (2007) and Santos-Pinto (2008) analyze how incentive contracts proposed by a profit-maximizing principal change in response to particular optimistic biases. The response depends on the extent to which the considered biases make the agent more baseline-optimistic or control-optimistic as defined here. Also, changes in control beliefs change the price of providing incentives relative to insurance. The effect of changing control beliefs on the induced effort level is unambiguous, the effect on the insurance provision is not. The main focus of this

paper is on the unambiguous comparison, for a given bias in beliefs, between social and private insurance on the one hand and optimal and naive implementation on the other hand. Jeleva and Villeneuve (2004) and Villeneuve (2005) study the effects of exogenous biased beliefs in models with adverse selection due to heterogeneity in risk. Eliaz and Spiegler (2008), Grubb (2009), Sandroni and Squintani (2007) studies adverse selection due to heterogeneity in risk perceptions. In Chapter 3, I also allow for heterogeneity in risk perceptions by relaxing the assumption made in this chapter that the agent's prior is known to the principal. I then analyze how agents are screened with contracts providing different levels of insurance coverage depending on the difference in baseline and control beliefs.

The comparison between social and private insurance relates to the policy and welfare analysis in the behavioral public economics literature, studying non-standard decision makers.<sup>3</sup> The use of the true probabilities to evaluate welfare is paternalistic, but highlights the contrast with the considerations of profit-maximizing insurers. The comparison also relates to the distinction between a paternalistic and populist government, with the latter catering to its voters' beliefs (Salanié and Treich 2009). The use of the true probabilities also assumes that these are measurable. Bernheim and Rangel (2009) argue that the presence of ancillary conditions, like framing issues, may distort people's choices. To the extent that better informing individuals alleviates ancillary conditions, the perceived probabilities after individuals are informed are more appropriate for evaluating their welfare than the perceived probabilities before they are informed. The empirical estimation of the biases in beliefs in this paper can help to identify agents' true preferences from their observed choices, as argued by Köszegi and Rabin (2007 and 2008). Finally, the comparison between the implementation of the standard and adjusted Baily formula adds to the recent literature reviewed by Chetty (2008a) that analyzes conditions under which sufficient statistic formulas for taxation and social insurance apply or need to be adjusted.

The paper is organized as follows. Section 1.1 introduces a static model of unemployment insurance and defines the baseline bias and control bias in beliefs. Section 1.2

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<sup>3</sup>For reviews, see Kanbur, Pirttila and Tuomala (2004) and Bernheim and Rangel (2007).

characterizes the optimal insurance contract given the biases in beliefs, as proposed by the social planner and private insurers. Section 1.3 extends the analysis to a dynamic framework. Section 1.4 discusses the data and shows the empirical estimates of the baseline and control bias. Section 1.5 calibrates the dynamic model given the empirical estimates in order to calculate the optimal contracts and the welfare cost of privatizing insurance numerically. Section 1.6 concludes. All proofs and tables are presented in the appendix.

## 1.1 Static Model

A risk-averse agent, whom I refer to as the *insuree*, is employed with exogenous probability  $p$  and unemployed with probability  $1 - p$ .<sup>4</sup> When unemployed, the insuree exerts unobservable search effort at utility cost  $e \in E$ . She finds work with probability  $\pi(e)$  and remains unemployed with probability  $1 - \pi(e)$ . The insuree produces  $w$  when employed and 0 when unemployed. A risk-neutral *insurer* offers a contract  $(b, \tau)$  that provides insurance against the unemployment risk. When the insuree starts the period employed, she consumes her after tax wage  $w - \tau$ . When the insuree starts the period unemployed, she consumes unemployment benefit  $b$  if she does not find work, but wage  $w$  if she does find work. This static setup follows Baily (1978) very closely.<sup>5</sup>

Central to this model is the assumption that the insuree may perceive the probability of finding work differently from the true probability. I denote by  $\hat{\pi}(e)$  the insuree's belief about the probability of finding work when she exerts effort  $e$ . Both the true probability of success  $\pi(e)$  and the perceived probability of success  $\hat{\pi}(e)$  are increasing and concave in  $e$ . I deliberately put no restrictions on how the true and perceived probability are related. The analysis, however, will show that the difference is essential in two dimensions; the difference in levels  $\hat{\pi}(e) - \pi(e)$  and the difference in margins  $\hat{\pi}'(e) - \pi'(e)$ . The difference

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<sup>4</sup>An insuree is defined in the Oxford English Dictionary as a person whose life or property is insured. I use this term in line with previous literature to clearly contrast the person providing and the person receiving insurance.

<sup>5</sup>I relax Baily's assumption that once unemployed, the agent becomes risk neutral between being unemployed and employed. However, I also assume that the unemployed agent does not pay taxes upon employment. This implies that the optimal search level does not depend on taxes and taxes can be written explicitly as a function of unemployment benefits only. I relax this assumption in the dynamic model.

in levels, the *baseline bias*, determines the difference between the true and perceived value of insurance. The difference in the margins, the *control bias*, determines the difference between the true and perceived marginal return of search and therefore the distortion in the choice of search effort.

**Definition 1** *An insuree is baseline-optimistic (baseline-pessimistic) if  $\hat{\pi}(e) \geq \pi(e)$  ( $\pi(e) \geq \hat{\pi}(e)$ ) for all  $e \in E$ .*

**Definition 2** *An insuree is control-optimistic (control-pessimistic) if  $\hat{\pi}'(e) \geq \pi'(e)$  ( $\pi'(e) \geq \hat{\pi}'(e)$ ) for all  $e \in E$ .*

For expositional purposes, I consider biases in beliefs that are the same for all effort levels, although only the local biases in beliefs matter for the optimality conditions. Baseline and control beliefs are related, as illustrated in the following two examples.

**Example I**  $\pi(e) = \theta e^\alpha$  and  $\hat{\pi}(e) = \hat{\theta} e^\alpha$

In this example the probability of finding work is complementary in the insuree's ability  $\theta$  and effort  $e$ . An insuree who overestimates her ability (i.e.  $\hat{\theta} > \theta$ ) is at the same time baseline-optimistic and control-optimistic.

**Example II**  $1 - \pi(e) = \frac{1 - e^\alpha}{\theta}$  and  $1 - \hat{\pi}(e) = \frac{1 - e^\alpha}{\hat{\theta}}$

In this example the insuree's ability  $\theta$  determines the probability of finding work when no effort is exerted. Ability and effort are now substitutes; effort increases the probability of finding work more if ability is lower. An insuree who overestimates her ability (i.e.  $\hat{\theta} > \theta$ ) is baseline-optimistic, but control-pessimistic.

I focus the analysis on baseline optimism and control pessimism. This corresponds to the second example and is in line with the empirical evidence presented in this paper. The results are opposite for baseline pessimism and control optimism.

### 1.1.1 The Insuree's Problem

The insuree's perceived expected utility from the insurance contract  $(b, \tau)$  and search effort  $e$  equals

$$\hat{U}(b, \tau, e) = pu(w - \tau) + (1 - p) [\hat{\pi}(e)u(w) + (1 - \hat{\pi}(e))u(b) - e].$$

The Bernoulli-utility  $u$  is increasing and concave in consumption. In this static model, the insuree exerts costly search effort when she starts without a job and either finds a job immediately or is unsuccessful and consumes the unemployment benefit  $b$ . The insuree weighs the uncertain outcomes of search with the perceived probabilities  $\hat{\pi}(e)$  and  $1 - \hat{\pi}(e)$ . In a dynamic setting, the periodic probability of finding a job is the inverse of the expected duration of unemployment. A baseline-optimistic insuree overestimates the probability of finding a job or, similarly, underestimates the expected duration of unemployment.

When unemployed, the insuree searches to maximize her perceived expected utility. Her effort choice  $\hat{e}(b)$  equalizes the perceived individual benefit and cost of search at the margin,

$$\hat{\pi}'(e)[u(w) - u(b)] = 1. \tag{IC}$$

Higher unemployment benefits reduce the utility gain of finding a job. The induced effort level  $\hat{e}(b)$  is thus decreasing in the unemployment benefit  $b$ . Moral hazard arises since the insuree does not internalize the impact of her effort on the insurer's budget constraint. The first-best effort level is higher than the effort choice of the insuree and the difference between the two increases with control pessimism. A control-pessimistic insuree exerts less effort than an insuree with unbiased beliefs, since she perceives the marginal return to effort to be lower than the true marginal return,  $\hat{\pi}'(e) < \pi'(e)$ . Given the concavity of the insuree's problem, the first order condition is sufficient for the unemployment benefit to be incentive compatible with search effort  $e$ .<sup>6</sup>

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<sup>6</sup>I assume that a positive level of effort is exerted in the social optimum or the competitive equilibrium. The condition that  $\hat{\pi}'(0) = \infty$  is sufficient for this to hold.

### 1.1.2 The Insurer's Problem

The expected profits for the insurer from an insurance contract  $(b, \tau)$  equal

$$P(b, \tau) \equiv p\tau - (1 - p)(1 - \pi(\hat{e}(b)))b.$$

The expected expenditures depend on the true probability that the insuree does not find employment  $1 - \pi(\hat{e}(b))$ . Since effort is not contractible, the insurer is constrained by the insuree's effort choice  $\hat{e}(b)$ . For a given contract, the insurer's profits are higher the more the unemployed insuree searches. I denote by  $\hat{\tau}(b)$  the tax required in order to keep the budget balanced,

$$\hat{\tau}(b) \equiv \frac{(1 - p)(1 - \pi(\hat{e}(b)))}{p}b.$$

I contrast two types of insurers with different objectives; a paternalistic social planner and competing private insurers.

The social planner cares about the insuree's true expected utility and thus weights the uncertain outcomes of the insuree's search effort with the true probabilities  $\pi(e)$  and  $1 - \pi(e)$ . Assuming a balanced budget, the (constrained) social optimum solves

$$\max_{b, \tau, e} U(b, \tau, e) = pu(w - \tau) + (1 - p)[\pi(e)u(w) + (1 - \pi(e))u(b) - e] \quad (1.1)$$

subject to  $(IC)$  and  $P(b, \tau) = 0$ .

Private insurers maximize their profits and compete to attract insurees. Competition drives profits to zero and insurees choose the contract that maximizes their perceived expected utility. The competitive equilibrium contract solves

$$\max_{b, \tau, e} \hat{U}(b, \tau, e) = pu(w - \tau) + (1 - p)[\hat{\pi}(e)u(w) + (1 - \hat{\pi}(e))u(b) - e] \quad (1.2)$$

subject to  $(IC)$  and  $P(b, \tau) = 0$ .

In contrast with the social planner's objective function in (1.1), the uncertain outcomes of

the insuree's search effort are weighted with the perceived probabilities  $\hat{\pi}(e)$  and  $1 - \hat{\pi}(e)$ .<sup>7</sup>

## 1.2 Optimal Insurance Contracts

An insurer faces the trade-off between smoothing consumption between employment and unemployment and providing incentives for search. The insuree's perception of the probability to remain unemployed and the returns to her search effort is central to this trade-off.

### 1.2.1 Unbiased Beliefs: the Baily Formula

If the beliefs about the returns are unbiased (i.e.  $\hat{\pi}(\cdot) = \pi(\cdot)$ ), the contracts proposed by the social planner and the private insurers in a competitive equilibrium coincide. The optimal contract equalizes the consumption smoothing benefit and the moral hazard cost of insurance at the margin.

**Consumption Smoothing** Unemployment benefits smooth the risk-averse insuree's consumption when unemployed. The smoothing benefit of further increasing the unemployment benefit  $b$  equals the relative difference in marginal utilities of consumption when unemployed and employed,

$$\frac{u'(b) - u'(w - \hat{\tau}(b))}{u'(w - \hat{\tau}(b))}.$$

Everything else equal, the smoothing benefit is decreasing in both the unemployment benefit  $b$  and the tax  $\hat{\tau}(b)$ . Less effort  $\hat{e}(b)$  increases the required tax  $\hat{\tau}(b)$  and thus decreases the marginal smoothing benefit.

**Moral Hazard** Higher benefits reduce the incentives for an unemployed insuree to search for work. A tax raise is required to balance the budget in response to an increase in the benefit  $b$ . This tax raise is higher the more search decreases. The tax  $\hat{\tau}(b)$  is thus

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<sup>7</sup>Chetty and Saez (2008) consider the optimal level of social insurance when private insurance is endogenous. I consider the insurance contract provided by either the social planner without the presence of private insurers or by competing private insurers without the presence of social insurance.

increasing in the benefit  $b$ , both because of the increased expenditures for an unemployed insuree and the increased probability that an insuree is unemployed,

$$\frac{d \log (\hat{\tau}(b))}{d \log b} = 1 + \varepsilon_{1-\pi(\hat{e}(b)),b}$$

where

$$\varepsilon_{1-\pi(\hat{e}(b)),b} \equiv \frac{d \log (1 - \pi(\hat{e}(b)))}{d \log b}.$$

The required tax increase due to moral hazard is completely determined by the elasticity  $\varepsilon_{1-\pi(\hat{e}(b)),b}$ , which describes the responsiveness of the true probability of unemployment with respect to unemployment benefits. This responsiveness determines the relative price of consumption during unemployment and employment. The lower the responsiveness, the better the rate at which consumption is being transferred from employment to unemployment.

The optimal contract equalizes the relative marginal utility and the relative price of consumption in employment and unemployment.

**Proposition 1** *With unbiased beliefs, optimal unemployment insurance is characterized by*

$$\frac{u'(b) - u'(w - \hat{\tau}(b))}{u'(w - \hat{\tau}(b))} = \varepsilon_{1-\pi(\hat{e}(b)),b}. \quad (1.3)$$

The maximization problems in (1.1) and (1.2) coincide when beliefs are unbiased. The proposition follows from the first order condition with respect to  $b$ ,

$$u'(b) - u'(w - \hat{\tau}(b)) [1 + \varepsilon_{1-\pi(\hat{e}(b)),b}] = 0.$$

The insurer sets the unemployment benefit such that the utility gain when unemployed from an increase in the benefit  $b$  equals the utility loss when employed, coming from the increase in taxes required to satisfy the budget constraint. The increase in the benefit also reduces the exerted effort. However, when insurees have unbiased beliefs, the impact of the reduced effort on the expected utility is of second order by the envelope condition. I

assume that the primitives are such that the second order condition holds globally.<sup>8</sup> This requires that  $\frac{u'(b)-u'(w-\hat{\tau}(b))}{u'(w-\hat{\tau}(b))}$  decreases more in  $b$  than  $\varepsilon_{1-\pi(\hat{e}(b)),b}$ .

If the insuree is irresponsive to incentives, the moral hazard cost disappears and full insurance is optimal. Everything else equal, a higher elasticity implies a higher moral hazard cost and therefore a lower optimal unemployment benefit. However, if a change in the fundamentals does not only increase the elasticity, but also effort, an increase in both the consumption levels during employment and unemployment becomes feasible. The effect on the optimal unemployment benefit level is ambiguous.

Using a Taylor approximation for the marginal utility in the left hand side of (1.3) leads to the standard formula derived by Baily (1978),

$$\gamma \frac{\Delta c}{c} \cong \varepsilon_{1-\pi(\hat{e}(b)),b},$$

with  $\gamma$  the relative risk aversion,  $\frac{\Delta c}{c}$  the relative change in consumption between employment and unemployment and  $\varepsilon_{1-\pi(\hat{e}(b)),b}$  the elasticity of the unemployment duration with respect to benefits. The identification of these three statistics is sufficient to test for the optimality of the current unemployment insurance system (Gruber 1997). For instance, identifying the primitives underlying the moral hazard problem is not necessary if the elasticity  $\varepsilon_{1-\pi(\hat{e}(b)),b}$  is known. Chetty (2008a) reviews the recent literature developing “sufficient statistic formulas” for social insurance and optimal taxation. In particular, Chetty (2006) shows how the Baily formula generalizes in a dynamic framework and is robust to the introduction of borrowing constraints, durable goods, search and leisure benefits during unemployment. However, in the presence of biased beliefs, the Baily formula and Chetty’s extension prescribe an insurance level that is generally suboptimally high or low. The direction of the bias depends on the nature of the bias in beliefs.

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<sup>8</sup>In the appendix, I derive the condition for global concavity of the maximization problem for the generalized model with biased beliefs.

## 1.2.2 Biased Beliefs: the Adjusted Baily Formula

If beliefs are biased (i.e.  $\hat{\pi}(\cdot) \neq \pi(\cdot)$ ), the contracts proposed by the social planner and the private insurers in a competitive equilibrium diverge. The social optimum equalizes the true smoothing benefit and the moral hazard cost at the margin, with the moral hazard cost corrected for the search internality. The competitive equilibrium equalizes the perceived smoothing benefit and the moral hazard cost, without correction for the search internality.

### Social Planner: Search Internality

The insuree equalizes the perceived marginal benefit and cost of effort at the margin. If the perceived and true marginal return to search differ, the insuree does not correctly internalize the effect of her search effort on her true expected utility. When determining the optimal unemployment benefit level, the social planner does account for both the externality the insuree imposes on the social planner's budget constraint and the externality she imposes on herself by misperceiving the returns to search. I refer to the latter as the *search internality*.<sup>9</sup>

With unbiased beliefs, the effect of increasing unemployment benefits on the true expected utility through the change in effort equals

$$(1 - p) \{ \pi'(\hat{e}(b)) [u(w) - u(b)] - 1 \} \frac{d\hat{e}(b)}{db} = 0.$$

Since the insuree already chooses her effort level to maximize her true expected utility, the effect of a marginal change in effort on her true expected utility is of second order by the envelope condition. However, when the insuree is control-pessimistic,  $\pi'(\cdot) > \hat{\pi}'(\cdot)$ , she underestimates the marginal return to effort and exerts too little effort. An increase in benefits now causes a first-order decrease in the true expected utility by decreasing the

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<sup>9</sup>This is in line with the behavioral public economics literature, for instance on the taxation of cigarettes (Gruber and Köszegi 2004).

insuree's effort choice. By the *IC* constraint, this first-order loss equals

$$(1 - p) \{ \pi'(\hat{e}(b)) - \hat{\pi}'(\hat{e}(b)) \} [u(w) - u(b)] \frac{d\hat{e}(b)}{db}.$$

This loss is lower the less responsive the effort choice, but higher the more distorted the effort choice. The distortion in the effort choice is increasing in the utility gain from finding a job  $u(w) - u(b)$  and the control bias  $|\hat{\pi}'(\hat{e}(b)) - \pi'(\hat{e}(b))|$ . The loss is therefore non-monotonic in control pessimism, since it decreases the responsiveness, but increases the distortion.

The constrained social optimum still equalizes the relative utility and the relative price of consumption in unemployment and employment, but the relative price is corrected for the search internality. Since control-pessimists exert too little effort, the corrected relative price of unemployment compensation exceeds the uncorrected relative price.

**Proposition 2** *The socially optimal unemployment insurance is characterized by*

$$\frac{u'(b) - u'(w - \hat{\tau})}{u'(w - \hat{\tau})} = \varepsilon_{1-\pi(\hat{e}),b} \left[ 1 + \frac{\pi'(\hat{e}) - \hat{\pi}'(\hat{e})}{\pi'(\hat{e})} I(b) \right], \quad (1.4)$$

with  $\hat{e} = \hat{e}(b)$ ,  $\hat{\tau} = \hat{\tau}(b)$  and  $I(b) = \frac{u(w) - u(b)}{bu'(w - \hat{\tau}(b))} > 0$ .

Biased beliefs change the socially optimal unemployment benefit only if they affect the insuree's behavior. In this static model, the insuree only chooses how much effort to exert and baseline optimism does not change the insuree's choice of effort. Baseline beliefs thus do not change the social optimum.<sup>10</sup> Control pessimism, however, reduces the insuree's effort choice and affects the socially optimal unemployment benefit through three channels. The net effect is ambiguous. The first channel is through the correction for the search internality and decreases the optimal unemployment benefit. The elasticity in (1.4) is multiplied by a correction greater than 1 for  $\hat{\pi}'(\hat{e}) < \pi'(\hat{e})$ . The second channel is through the standard smoothing benefit and increases the optimal unemployment benefit.

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<sup>10</sup>Notice that this changes if the insuree chooses how much insurance coverage to buy at a given price. A baseline-optimistic insuree buys less insurance coverage than an unbiased insuree does.

The reduced effort decreases the smoothing benefit through an increase in the required tax  $\hat{\tau}(b)$ . The last channel is through the standard moral hazard cost. Control pessimism may decrease the elasticity  $\varepsilon_{1-\pi(\hat{e}(b)),b}$  and therefore decrease the optimal unemployment benefit.<sup>11</sup> The ambiguity of the net effect is not surprising. Control beliefs affect the effort of an insuree for a given level of insurance. As in a standard consumption problem with two goods (effort and insurance), an increase in the price of one good (effort) decreases the consumption of that good. The effect on the other good (insurance) is ambiguous. The increase in the price of inducing effort makes the optimal contract substitute toward providing more insurance, but at the same time, the set of feasible combinations of effort and insurance shifts inward.

**Naive Planner** Despite the ambiguous response to control beliefs, the difference between the budget balanced insurance schemes solving the standard Baily formula in (1.3) and the adjusted Baily formula in (1.4) unambiguously depends on the control bias. This comparison is relevant when a naive planner who is not aware of biases in beliefs implements the standard Baily formula.<sup>12</sup> By implementing such policy, the naive planner ignores the search internality. With control-pessimistic insurees, this implies that the planner underestimates the relative price of unemployment compensation and sets the benefit level suboptimally high.

**Corollary 1** *The standard Baily formula overestimates the socially optimal level of unemployment benefits with control-pessimistic job searchers.*

Similarly, for two societies where the consumption smoothing benefits coincide, policy makers implementing the standard Baily formula set the same level of insurance if the observed elasticities are the same. However, if job searchers in the one society are more control-pessimistic, the insurance level in that society should be lower. The corollary

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<sup>11</sup>The elasticity  $\varepsilon_{1-\pi(\hat{e}(b)),b}$  equals  $-\hat{e}'(b) \frac{\pi'(\hat{e}(b))}{1-\pi(\hat{e}(b))} b \geq 0$ . The agent's absolute response  $\hat{e}'(b)$  is larger, the higher she perceives the marginal return to her effort. However, the chosen effort level  $\hat{e}(b)$  increases with  $\hat{\pi}'(\cdot)$  as well. For the elasticity to be higher for control-optimists, it is sufficient that  $\frac{d}{de} \left( \frac{\pi'(e)}{1-\pi(e)} \right) > 0$ .

<sup>12</sup>This still assumes that the naive planner knows the insuree's utility, the elasticity of unemployment duration, as well as the tax rate  $\hat{\tau}(b)$  that keeps the budget balanced as a function of  $b$ .

emphasizes that formulas based on reduced statistics should be used cautiously when designing insurance contracts.

### Private Insurers: Perceived Consumption Smoothing

An insuree who underestimates the duration of unemployment underestimates the value of unemployment insurance. Private insurers respond by providing less insurance. In a competitive equilibrium, private insurers offer unemployment insurance that equalizes the perceived smoothing benefit and the moral hazard cost, not corrected for the search internality.

**Proposition 3** *The equilibrium contract offered by competing private insurers is characterized by*

$$\frac{1 - \hat{\pi}(\hat{e})}{1 - \pi(\hat{e})} \frac{u'(b) - u'(w - \hat{\tau})}{u'(w - \hat{\tau})} = \varepsilon_{1-\pi(\hat{e}),b}, \quad (1.5)$$

with  $\hat{e} = \hat{e}(b)$  and  $\hat{\tau} = \hat{\tau}(b)$ .

The proposition follows from the first order condition of the insurer's profit maximization (1.2), which simplifies to

$$\frac{1 - \hat{\pi}(\hat{e}(b))}{1 - \pi(\hat{e}(b))} u'(b) - u'(w - \hat{\tau}(b)) [1 + \varepsilon_{1-\pi(\hat{e}(b)),b}] = 0.$$

An increase in unemployment benefits is perceived by the insuree to be received with probability  $(1 - p)(1 - \hat{\pi}(\hat{e}))$ , but only paid by the insurer with probability  $(1 - p)(1 - \pi(\hat{e}))$ . The latter probability determines the tax increase required for the insurer to make zero profits. This explains why the marginal utility when unemployed relative to the marginal utility when employed is weighted by  $\frac{1 - \hat{\pi}(\hat{e}(b))}{1 - \pi(\hat{e}(b))}$ . Since the insuree searches to maximize her perceived expected utility, the effect through the change in search efforts is again of second order.

Baseline-optimistic beliefs lower the left-hand side in equation (1.5). The equilibrium insurance is therefore unambiguously lower when job searchers are baseline-optimistic. If

job searchers sufficiently underestimate the unemployment duration, they may receive no unemployment insurance at all in equilibrium.

**Naive Insurers** The standard Baily formula ignores the difference between the perceived and actual consumption smoothing benefits. This implies the following corollary.

**Corollary 2** *The standard Baily formula overestimates the equilibrium level of unemployment insurance with baseline-optimistic job searchers.*

While the difference between the standard and adjusted Baily formula depends on the control bias for the social optimum, it depends on the baseline bias for the competitive equilibrium. A private insurer responds to the control beliefs as well, since these beliefs affect the effort choice  $\hat{e}(b)$ . This response, however, is the same as the response by an insurer who is unaware of biased beliefs and implements the standard Baily formula.

### 1.2.3 Comparing Private and Social Insurance

Adverse selection, due to ex-ante private information about risk types, is often argued to be a reason for government intervention in insurance markets. Moral hazard, due to ex-post private information, does not raise the need for government intervention by itself; competing private insurers offer the socially optimal insurance contract, but only if beliefs are unbiased. The analysis of optimal insurance design with biased beliefs sheds a new light on the topic of privatizing unemployment insurance. First of all, the analysis suggests an alternative explanation for the puzzle of why unemployment insurance is mostly publicly provided; if people are sufficiently optimistic about the risk of unemployment, providing insurance becomes unprofitable for private insurers. Second, the analysis suggests that privatizing unemployment insurance may be undesirable because of the welfare cost due to the biases in beliefs. Competition forces private insurers to charge the actuarially fair price for insurance, but does not force them to sell the socially optimal amount of insurance. A similar caveat holds for the unemployment insurance savings accounts, as proposed by Orszag and Snower (1997) and Altman and Feldstein (2006). In the same way

that biased beliefs distort the insuree's willingness to pay for unemployment consumption, they distort the insuree's willingness to save, both before and during unemployment.

**Difference in Insurance Coverage** The nature of the regulation of private insurance markets depends in the first place on whether the insurance coverage provided in equilibrium is suboptimally high or low. Biases in baseline beliefs and control beliefs drive a wedge between the social optimum and the competitive equilibrium for different reasons. Baseline-optimistic insurees undervalue the consumption smoothing benefit of insurance. The focus of private insurers on the perceived smoothing benefit decreases the unemployment benefit in competitive equilibrium compared to the social optimum. Control-pessimistic insurees exert too little effort. The correction by the social planner for this search externality decreases the unemployment benefit in the social optimum compared to the competitive equilibrium.

If insurees were baseline-optimistic and control-optimistic, the competitive unemployment insurance would be suboptimally low. Baseline optimism and control pessimism, however, change the difference in unemployment benefits in competitive equilibrium and the social optimum in opposite directions. The actual difference depends on which bias dominates, conditional on the term  $\frac{\hat{\pi}'(e)}{-\hat{\pi}''(e)}$  which determines the curvature of the perceived probability as a function of effort.

**Corollary 3** *The equilibrium insurance provided to baseline-optimistic insurees is suboptimally low, unless the pessimistic control bias is such that*

$$\{\pi'(e) - \hat{\pi}'(e)\} \frac{\hat{\pi}'(e)}{-\hat{\pi}''(e)} > \hat{\pi}(e) - \pi(e),$$

*evaluated at the effort level chosen in the social optimum.*

The gain of correcting the control-pessimistic insuree's effort choice is increasing in the control bias  $\pi'(e) - \hat{\pi}'(e)$ . However, the social planner can only correct for the insuree's distorted effort choice if the insuree is responsive to incentives. The effort response to a change in benefits  $\frac{d\hat{e}(b)}{db}$  is increasing in  $\frac{\hat{\pi}'(e)}{-\hat{\pi}''(e)}$ . If this response is modest, as for insurees

who perceive the marginal return to search to be very low, the social planner's correction is likely to be dominated by the private insurers' focus on the perceived smoothing benefit.

**Welfare Comparison for Extreme Control Biases** The welfare cost of privatizing insurance due to biases in beliefs depends on the extent to which private insurance diverges from social insurance and the impact of this divergence on the chosen effort levels. This difference becomes very salient for extreme control beliefs.

The moral hazard cost of providing insurance arises from the fact that the insuree does not internalize the positive effect of her search effort on the insurer's profits and therefore chooses an effort level that is lower than the first-best level of effort. Control pessimism increases this wedge with the first best, since a control-pessimistic insuree underestimates even the private benefits of search. If an insuree becomes more and more pessimistic about her control, the relative price of inducing effort in terms of insurance becomes so high that the social planner substitutes away from providing incentives and provides insurance converging to full insurance. Although effort matters, in the limit it is perceived not to matter and the opportunity cost of providing insurance equals zero. Extreme control-pessimistic insurees cannot be induced to do effort, but private insurers nevertheless give less than full insurance in response to the insuree's baseline optimism.<sup>13</sup>

**Proposition 4** *When  $\hat{\pi}'(e) \rightarrow 0$  for all  $e$ , the optimal social contract converges to full insurance and the insurance  $b^p$  provided in an interior solution to the competitive equilibrium solves*

$$\frac{1 - \hat{\pi}(e^p)}{1 - \pi(e^p)} = \frac{u'(w - \hat{\tau}(b^p))}{u'(b^p)}.$$

*In both cases, the induced effort level converges to zero.*

Extreme control optimism also drives the social optimum towards full insurance, but for the opposite reason. The price of inducing effort becomes so low that the social planner prefers to increase insurance as well. If an insuree is very optimistic about her control, a small share of the risk imposed on the insuree suffices to induce the first-best effort level.

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<sup>13</sup>If the insuree were baseline-pessimistic, the private insurer would provide more than full insurance.

In the limit, the social contract approximates the first best. The competitive equilibrium diverges now from the social optimum both in terms of the level of insurance provided and the search effort induced. The next proposition considers the case where the perceived probability remains unchanged, but the true returns to effort converge to zero.<sup>14</sup> Effort is perceived to matter, but actually has no true impact in the limit.

**Proposition 5** *When  $\pi'(e) \rightarrow 0$  for all  $e$ , the interior optimal social contract converges to full insurance and efficiently induces zero effort if the probability to start employed  $p \rightarrow 1$ . The insurance provided in an interior equilibrium solves*

$$\frac{1 - \hat{\pi}(e^p)}{1 - \pi(e^p)} = \frac{u'(w - \hat{\tau}(b^p))}{u'(b^p)}.$$

*The induced effort level  $e^p = \hat{e}(b^p)$  is positive and therefore inefficient.*

The competitive equilibrium contract imposes too much risk on the baseline-optimistic insurees and induces too much effort if insurees are extremely control-optimistic as well. For baseline-optimistic insurees, the welfare cost of privatizing insurance is higher when they are extremely control-optimistic rather than control-pessimistic.

### 1.3 Dynamic Model

In this section, I extend the analysis to a dynamic framework with the unemployed continuing to search as long as they have been unsuccessful in finding employment. Static insurance contracts transfer consumption from employment to unemployment. Dynamic insurance contracts can transfer consumption between unemployment spells with different lengths as well. I consider CARA preferences and restrict the analysis to unemployment schemes for which consumption depends linearly on the unemployment duration.

First, I derive Baily-type conditions characterizing the optimal linear contract. I show how the adjustment of these conditions for the presence of biases in beliefs is similar as

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<sup>14</sup>I do not consider the extreme case with the perceived marginal return  $\hat{\pi}'(e)$  increasing without bound, since this would imply that  $\int \hat{\pi}'(e) de > 1$ , which is inconsistent with  $\hat{\pi}(e) \leq 1$  for all  $e$ .

in the static model; the private insurers focus on the perceived consumption smoothing benefits and the social planner corrects the search externality. In a dynamic setting, baseline-optimistic insurees also underestimate the utility loss of consumption decreasing during unemployment and search too little because they overvalue the continuation value of unemployment. In the competitive equilibrium, consumption decreases more rapidly during unemployment than what is socially optimal if the first effect dominates.

Second, I show that for CARA preferences the linear contract is optimal for private insurers regardless of the beliefs, while a social planner may increase welfare by decreasing consumption more rapidly for short-term unemployed than for long-term unemployed when they are baseline-optimistic. The threat of lower consumption in the future when still unemployed becomes less effective when job searchers overestimate the probability to leave unemployment. If a job searcher is very confident that she will find a job within six months, a reduction in the benefits after six months of unemployment hardly induces her to search harder today. This induces a social planner to shift incentives to the short-term unemployed. Private insurers, however, also take into account that job seekers underestimate the probability that they will experience these lower consumption levels in the future.

### 1.3.1 Setup

I follow the optimal contracting approach in Hopenhayn and Nicolini (1997), focusing on the consumption allocation throughout unemployment and upon employment for the insurees who start unemployed. This complements the static analysis of the insurance between insurees who start employed and unemployed in the previous section. The optimal dynamic contract characterized in this section can only be implemented under the assumption that savings are observable (Werning 2002, Shimer and Werning 2008). I also make the simplifying assumption that not only the true probability function of effort, but also the perceived probability function of effort does not change during the unemployment spell. Notice though that I find no empirical evidence suggesting that the optimistic bias becomes smaller during unemployment.

**Assumption 1** *Both the true probability  $\pi(e)$  and perceived probability  $\hat{\pi}(e)$  remain unchanged during the unemployment spell.*

A risk-averse insuree starts unemployed and exerts effort at cost  $e$  to find work. If the insuree does not find work in the current period, she has to search for work again in the next period. Once she finds a job, she remains employed forever. Since there is no moral hazard once the insuree is employed, it is optimal to keep consumption constant after employment. The insurer offers a consumption schedule as a function of the length of the unemployment spell  $d$ . The length of the spell is a sufficient statistic for the unemployment history. This contract can be implemented with a schedule of unemployment benefits and taxes  $\{(b_d, \tau_d)\}_d$  if no savings are possible.

### 1.3.2 Linear Unemployment Insurance

I restrict the analysis to unemployment schemes that are linear in the length of the unemployment spell. In the next section, I show that such contracts are optimal for private insurers when beliefs are unbiased or biased, and for the social planner when beliefs are unbiased.

**Assumption 2** *Contracts are linear, i.e.  $b_d = b - xd$  and  $\tau_d = \tau^u + x(d - 1)$  for  $d \geq 1$ .*

A linear contract reduces the insurer's problem to the choice of a vector of three variables  $z = (b, w - \tau^u, x)$ : the unemployment benefit  $b$  at the start of unemployment, the after-tax wage  $w - \tau^u$  if the insuree finds work after one period of unemployment and the reduction  $x$  in benefit and the after-tax wage for each additional period that the unemployment spell takes.

**Assumption 3** *The insuree has CARA preferences with monetary costs of effort  $e$ ,*

$$u(c - e) = -\exp(-\sigma(c - e)).$$

An insuree with CARA preferences makes her search decision only based on the differences in consumption levels across states. With a linear contract, the only difference

between the continuing contracts when short-term unemployed and long-term unemployed is an equal shift in all consumption levels. Hence, the insuree exerts the same search effort throughout the unemployment spell. Using the property for CARA preferences that  $u(c-x) = -u(-x)u(c)$ , it is possible to write the lifetime utility explicitly, rather than rewriting the problem recursively. The true and perceived expected utility of a contract for an insuree who starts unemployed simplify to

$$U(z, e) = \frac{u(b-e) + \beta\pi(e) \frac{u(w-\tau^u)}{1-\beta}}{1 - \beta(1 - \pi(e))(-u(-x))}$$

and

$$\hat{U}(z, e) = \frac{u(b-e) + \beta\hat{\pi}(e) \frac{u(w-\tau^u)}{1-\beta}}{1 - \beta(1 - \hat{\pi}(e))(-u(-x))}.$$

The insuree exerts effort  $e$  and consumes  $b$  during the first period of unemployment and finds employment the next period at the after-tax wage  $w - \tau^u$  with probability  $\pi(e)$ . With probability  $1 - \pi(e)$ , the insuree is still unemployed the next period and faces the exact same prospects as the period before, except that all payments are  $x$  lower. As before, the insuree's effort choice  $\hat{e}(z)$  maximizes her perceived expected utility  $\hat{U}(z, e)$ , rather than her true expected utility  $U(z, e)$ . With  $c_0 \equiv (b, w - \tau^u)$ , the initial levels of unemployment benefit and after-tax wage, the effort level  $\hat{e}(z)$  solves

$$\beta\hat{\pi}'(\hat{e}(z)) \left[ \frac{u(w - \tau^u)}{1 - \beta} - \hat{U}(c_0 - x, x, \hat{e}(z)) \right] = u'(b - \hat{e}(z)).$$

In the dynamic model, both baseline and control beliefs change the insuree's effort choice. If an unemployed insuree is baseline-optimistic, she overestimates the continuation value of remaining unemployed  $\hat{U}(c_0 - x, x, \hat{e}) > U(c_0 - x, x, \hat{e})$  and therefore exerts too little effort.

The expected cost for the insurer when facing an insuree who starts unemployed simplifies to

$$C(z, e) = \frac{b - \beta \left\{ \pi(e) \frac{\tau^u}{1-\beta} + (1 - \pi(e)) \frac{x}{1-\beta} \right\}}{1 - \beta(1 - \pi(e))}.$$

If the insuree finds work, the insurer starts receiving  $\tau^u$  from the next period on. If the insuree does not find work, the insurer has to pay unemployment benefits again in the next

period, but all future consumption levels are reduced by  $x$ . For the insurer's budget to be balanced, these expected costs when the insuree starts unemployed need to be funded with the tax paid when the insuree starts employed, as analyzed in the static model.<sup>15</sup>

I characterize the optimal contract for an insuree who starts unemployed considering two revenue-neutral changes. First, an increase in the unemployment benefit level accompanied with a decrease in the after-tax wage upon employment. Second, an increase in the starting consumption levels accompanied with a faster decrease in the consumption levels throughout unemployment. Again, for the insurance contract to be optimal, the marginal consumption smoothing benefits and the moral hazard cost of such changes have to be equal.

**Unemployment vs. Employment** I first consider an increase in the unemployment benefit  $b$ , accompanied by a decrease in the after-tax wage  $w - \tau^u$ . Keeping the reduction  $x$  constant, this implies an equal increase in all consumption levels during unemployment and an equal decrease in all consumption levels upon employment, regardless of the length of the unemployment spell. The Baily formula and the adjustments for biased beliefs generalize for this change in the dynamic contract. In order to emphasize the similarity with the static contracts, I introduce the functions  $J^\tau(z) \equiv \frac{\beta\pi(\hat{e}(z))[b + \tau^u - \frac{x}{1-\beta}]}{[1 - \beta(1 - \pi(\hat{e}(z)))]}$  and  $I^\tau\left(\frac{\pi'(\hat{e}) - \hat{\pi}'(\hat{e})}{\pi'(\hat{e})}, \frac{\hat{\pi}(\hat{e}) - \pi(\hat{e})}{\pi(\hat{e})}, z\right)$  defined in appendix and discussed below.

**Proposition 6** *The unemployment contracts in the social optimum and the competitive equilibrium are characterized by respectively*

$$\frac{u'(b - \hat{e}) - u'(w - \tau^u)}{u'(w - \tau^u)} = \varepsilon_{\frac{1}{\pi(\hat{e})}, (b, \tau^u)} J^\tau(z) \left[ 1 + I^\tau\left(\frac{\pi'(\hat{e}) - \hat{\pi}'(\hat{e})}{\pi'(\hat{e})}, \frac{\hat{\pi}(\hat{e}) - \pi(\hat{e})}{\pi(\hat{e})}, z\right) \right]$$

and

$$\frac{\frac{\pi(\hat{e}(z))}{\hat{\pi}(\hat{e}(z))} u'(b - \hat{e}) - u'(w - \tau^u)}{u'(w - \tau^u)} = \varepsilon_{\frac{1}{\pi(\hat{e})}, (b, \tau^u)} J^\tau(z),$$

with  $I_1^\tau > 0$ ,  $I_2^\tau > 0$  and  $I^\tau(0, 0, z) = 0$ .

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<sup>15</sup>Since the starting consumption level  $c_0 = (b, w - \tau^u)$  when unemployed does not change the search effort level with CARA preferences, the characterisation of the consumption allocation between the insurees who start unemployed and who start employed simplifies to  $U_{c_0}(z, \hat{e}) = \frac{u'(w - \tau)}{1 - \beta}$  and  $\hat{U}_{c_0}(z, \hat{e}) = \frac{u'(w - \tau)}{1 - \beta}$  in the social optimum and the competitive equilibrium respectively.

The consumption smoothing benefit associated with the change is again on the left-hand side of the equation, the moral hazard cost is on the right-hand side. When beliefs are unbiased,  $I^\tau(0, 0, z) = 0$  and  $\frac{\pi(\hat{e}(z))}{\hat{\pi}(\hat{e}(z))} = 1$ . The insurance contracts in the social optimum and the competitive equilibrium coincide. The moral hazard cost is determined by the elasticity  $\varepsilon_{\frac{1}{\pi(\hat{e})}, (b, \tau^u)}$ , capturing the responsiveness of the expected unemployment duration  $\frac{1}{\pi(\hat{e}(z))}$  to the considered change in benefit and tax, and by  $J^\tau(z)$ , which reflects the increase in expected costs for the insurer  $C(z, \hat{e}(z))$  from an increase in the unemployment duration.<sup>16</sup> Longer unemployment spells are more costly if the starting levels of the unemployment benefit  $b$  and the wage tax  $\tau^u$  are higher, but less costly if the consumption levels vanish quickly with the length of the unemployment spell. The consumption smoothing gain is again determined by the relative difference in the marginal utility of consumption during unemployment and upon employment.

The role of control pessimism is the same as in the static model. Baseline optimism distorts the insuree's effort choice downward as well and thus affects the search internality in the same way as control pessimism. Both biases make the social planner revise the moral hazard cost upward and therefore decrease the optimal benefit level, i.e.  $I^\tau > 0$  if  $\frac{\pi'(\hat{e}) - \hat{\pi}'(\hat{e})}{\pi'(\hat{e})} > 0$  and  $\frac{\hat{\pi}(\hat{e}) - \pi(\hat{e})}{\pi(\hat{e})} > 0$ . A naive policy maker who ignores the correction  $I^\tau$  sets the unemployment benefit level too high, like in Corollary 1. The pessimistic control bias and the optimistic baseline bias move this policy error in the same direction.<sup>17</sup> Baseline optimism decreases the perceived consumption smoothing benefits, like in the static model. Private insurers respond to this bias by decreasing the unemployment benefits and increasing the wage tax paid upon employment. This response by the private insurers needs to dominate the social planner's correction for the search internality for equilibrium insurance to be suboptimally low when insurees are baseline-optimistic and

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<sup>16</sup>In the static model, effort did not depend on the change in taxes. Here, the taxes changes the effort level as well. The response in effort captured by the elasticity  $\varepsilon_{\frac{1}{\pi(\hat{e})}, (b, \tau^u)}$  is both to the change in the unemployment benefit and the change in the tax that keeps the revenues constant.

<sup>17</sup>As with control pessimism, the effect of baseline optimism on the optimal wedge between unemployment and employment consumption is ambiguous. Both biases decrease search and increase the required tax change, but may also decrease the responsiveness to insurance coverage. The first effect decreases the smoothing benefit. The second effect decreases the moral hazard cost. The effect on the optimal starting levels  $b$  and  $w - \tau^u$  is ambiguous.

control-pessimistic.

**Short-term vs. Long-term Unemployed** I now consider an equal decrease in the starting level of the unemployment benefit  $b$  and the after-tax wage  $w - \tau^u$ , accompanied with a slower reduction  $x$  in consumption during unemployment. The slower reduction smooths the risk-averse insuree's consumption profile, but discourages her from searching for a job. In the optimum, the marginal consumption smoothing gain and moral hazard cost of this change has to be equal. The social planner corrects for the search externality, whereas the private insurers focus on the perceived smoothing cost in the presence of biased beliefs. Proposition 7 follows given the functions  $J^x(z) \equiv \frac{\beta\pi(\hat{e})[(b+\tau^u)(1-\beta)-x]}{[1-\beta(1-\pi(\hat{e}))]^2x}$  and  $I^x\left(\frac{\pi'(\hat{e})-\hat{\pi}'(\hat{e})}{\pi'(\hat{e})}, \frac{\hat{\pi}(\hat{e})-\pi(\hat{e})}{\pi(\hat{e})}, z\right)$  defined in appendix and discussed below.

**Proposition 7** *The unemployment contracts in the social optimum and the competitive equilibrium are characterized by respectively*

$$\frac{U_{c_0}(c_0 - x, x, \hat{e}) - U_{c_0}(c_0, 0, \hat{e})}{U_{c_0}(c_0 - x, x, \hat{e})} = -\varepsilon_{\frac{1}{\pi(\hat{e})}, (c_0, x)} J^x(z) \left\{ 1 + I^x\left(\frac{\pi'(\hat{e})-\hat{\pi}'(\hat{e})}{\pi'(\hat{e})}, \frac{\hat{\pi}(\hat{e})-\pi(\hat{e})}{\pi(\hat{e})}, z\right) \right\}$$

and

$$\frac{\hat{U}_{c_0}(c_0 - x, x, \hat{e}) - \frac{1-\hat{\pi}(\hat{e})}{1-\pi(\hat{e})}\hat{U}_{c_0}(c_0, 0, \hat{e})}{\hat{U}_{c_0}(c_0 - x, x, \hat{e})} = -\varepsilon_{\frac{1}{\pi(\hat{e})}, (c_0, x)} J^x(z) ,$$

with  $I_1^x > 0$ ,  $I_2^x > 0$  and  $I^x(0, 0, z) = 0$ .

The moral hazard cost is again similar in nature;  $J^x(z)$  depends on the increase in the expected costs for the insurer if the unemployment duration increases and the elasticity  $\varepsilon_{\frac{1}{\pi(\hat{e})}, (c_0, x)}$  depends on the responsiveness of the unemployment duration with respect to the considered change. Given the CARA preferences, the starting level of consumption  $c_0$  has no impact on search, whereas an increase in the reduction  $x$  increases search and thus decreases the expected unemployment duration.

The consumption smoothing gain evaluated at the contract  $(c_0, x)$  equals the difference in marginal expected utility gains from an increase in  $c_0$ , denoted by  $U_{c_0}$ , for two unemployment schemes,  $(c_0, 0)$  and  $(c_0 - x, x)$ . The first scheme  $(c_0, 0)$  equals the contract  $(c_0, x)$  offered to the insuree in the first period of unemployment, but with no decrease in

consumption in the next periods of unemployment. The second scheme  $(c_0 - x, x)$  equals the contract  $(c_0, x)$  offered to the insuree from the second period of unemployment on. That is, the consumption levels start at  $c_0 - x$  and decrease with  $x$  for every additional period of unemployment. If  $x > 0$ , the marginal utility of consumption is higher for the second contract,  $U_{c_0}(c_0 - x, x, \hat{e}) > U_{c_0}(c_0, 0, \hat{e})$ . This consumption smoothing gain is decreasing in  $x$ . If  $x = 0$ , the first and the second scheme coincide. At that point, the consumption smoothing gain of changing  $x$  is of second order. Since an increase in  $x$  increases the induced effort level and thus has a first order impact on the insurer's budget constraint,  $x$  needs to be positive to be optimal. This confirms the well-known result by Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) that with unbiased beliefs consumption should be decreasing with the length of the unemployment spell.

Biased beliefs change the induced effort level and the responsiveness to  $x$ . The impact on the optimal level of  $x$  is again ambiguous. However, insurees who overestimate the probability to leave unemployment clearly underestimate the utility cost of a fast decrease in benefits or a fast increase in taxes for longer unemployment spells. This effect on the perceived consumption smoothing tends to increase the equilibrium level of the  $x$ . The social planner, however, wants to correct for the search externality. This tends to increase the socially optimal level of  $x$  if insurees are more baseline-optimistic or control-pessimistic. If the perceived consumption smoothing effect dominates the search externality effect, equilibrium consumption decreases suboptimally fast during unemployment.

### 1.3.3 Optimal Unemployment Insurance

As in the static model, the social optimum and competitive equilibrium coincide for unbiased beliefs. Werning (2002) shows that with CARA preferences the optimal unemployment schedule is linear of the form  $z = (b, w - \tau^u, x)$  using recursive techniques. The dual problem that minimizes the expected cost of an insurance scheme providing a given level of expected lifetime utility  $V$  can be written recursively. The lifetime utilities promised last period to the insuree conditional on unemployment or employment summarizes all relevant aspects of the insuree's unemployment history. The promised utility  $V$

is therefore the unique state variable during unemployment. Moreover, starting from an optimal contract which assigns expected utility  $V$ , the optimal response to an increase in  $V$  is to increase all consumption levels by the same amount today and in the future, while employed and unemployed. The reason is that with CARA preferences search effort only depends on differences in consumption across different states. The provision of search incentives and utility becomes separable. Once the differences in consumption levels are chosen to induce the optimal levels of search effort, the consumption levels can be set to assign the required utility level. Hence, two consecutive periods of unemployment only differ by an equal shift in all expected consumption levels. This implies that the ratio of promised utilities in two consecutive periods of unemployment remains unchanged. The optimal contract is linear with the shift in consumption for an additional period of unemployment being constant throughout the unemployment spell.

It is important that this argument continues to hold for private insurers, but not for the social planner in the presence of biased beliefs. With biased beliefs, the unemployed's search behavior is determined by the perceived expected lifetime utility  $\hat{V}$ . Since a private insurer does not care about the true expected utility, the recursive problem has still a unique state variable, i.e. the perceived expected lifetime utility  $\hat{V}$ . The same argument holds as with unbiased beliefs, but now in terms of  $\hat{V}$ . Linear contracts are still optimal.

**Proposition 8** *The profit-maximizing contract offered by competing insurers is linear, whether or not beliefs are biased.*

In contrast to private insurers, the social planner cares about the insuree's true expected utility. However, the perceived expected utility still determines the search behavior. The optimal contract is not linear anymore. If the unemployed worker overestimates the probability of leaving unemployment, decreasing future benefits become an ineffective instrument for inducing effort. Starting from the optimal linear contract, the social planner improves the trade-off between consumption smoothing and inducing effort by making the consumption steeper at the beginning of the unemployment spell and flatter afterwards. Such a variation induces more effort at the start of the unemployment spell, but less effort in any later period. This may increase or decrease the search internality.

Hence, the considered deviation from the optimal linear contract increases the insuree's welfare if the effect on the search internality is positive or negative and small.

**Proposition 9** *If beliefs are unbiased, the social optimum is a linear contract. With baseline-optimistic beliefs and  $I^x \approx 0$ , making consumption steeper at the start of unemployment (i.e.  $x_1 = x + \varepsilon$ ) and flatter afterwards (i.e.  $x_d = x - \delta$  for all  $d > 1$ ) to balance the budget increases welfare for small  $\varepsilon, \delta > 0$ .*

The proposition only shows one local variation that increases welfare. If similar variations for longer unemployment spells lead to the optimal contract, this suggests that the long-term unemployed should be incentivized less than the short-term unemployed. For private insurers this effect of improved incentives is offset by the fact that insurees need to be compensated less in terms of current consumption for decreases in future consumption the further these decreases are in the future. The timing of incentives during unemployment is therefore another dimension along which privatizing insurance may decrease welfare in the presence of biased beliefs.

## 1.4 Empirical Analysis

In this section, I analyze empirically the baseline and control bias in the beliefs held by unemployed job seekers. Linking the expected duration of unemployment with the actual duration of unemployment, I find strong evidence that job seekers are baseline-optimistic. The identification of the control bias is more difficult. I consider the differential impact of search efforts on the unemployed's expectations and their actual employment outcomes. Controlling for heterogeneity and endogeneity, the estimates suggest that job seekers are control-pessimistic, but that the control bias is less pronounced than the baseline bias. Finally, I analyze how baseline beliefs change during unemployment and find no evidence that unemployed workers are less biased the longer or the more often they have been unemployed.

### **1.4.1 Data**

I use data collected by Price et al. (1998) in a study about preventing depression in couples facing job loss. The study was conducted in and around two major urban areas in Michigan and Maryland from 1996 to 1998. All participants were recruited through state unemployment offices. Initial screening retained 1487 job seekers, who were part of a couple. All retained subjects were unemployed for less than 15 weeks and looking for work, but did not expect to be recalled to their former job. About one month after the initial screening, the retained subjects and their partners were interviewed for the first time. Two follow-up interviews were organized about six months and twelve months later. A third follow-up interview was organized one month after the first interview, but only for a subsample of the initial group. In Table 1 in appendix, I show sample averages of the demographics of the retained job seekers. The average subject has 13.6 years of education, earned 2595 dollars in the month before unemployment and has been unemployed for 6.9 weeks at the time of the first interview.

### **1.4.2 Baseline Beliefs: Actual and Expected Duration**

The subjects are asked about their expectations to find a job. One question asks: “How many weeks do you estimate it will actually be before you will be working more than 20 hours a week?” I interpret the subjects’ answers as the number of weeks they expect to remain unemployed. The average expected remaining duration of unemployment at the time of the first interview equals 6.8 weeks. The cumulative distribution is shown in Figure 1-2. The median expected duration is 4 weeks. More than 90 percent of the subjects expect that they will have found employment within the next 3 months.

In follow-up interviews, subjects are asked when they actually started working. 86 percent of the subjects found work for more than 20 hours a week before the last interview, about one year after the first interview. The average time they needed to find such a job was 17.0 weeks. I compute the minimum duration of an unemployment spell, assuming that the other 14 percent of the subjects found work on the date of the last interview. The average minimum duration for the entire sample equals 23.0 weeks, again starting

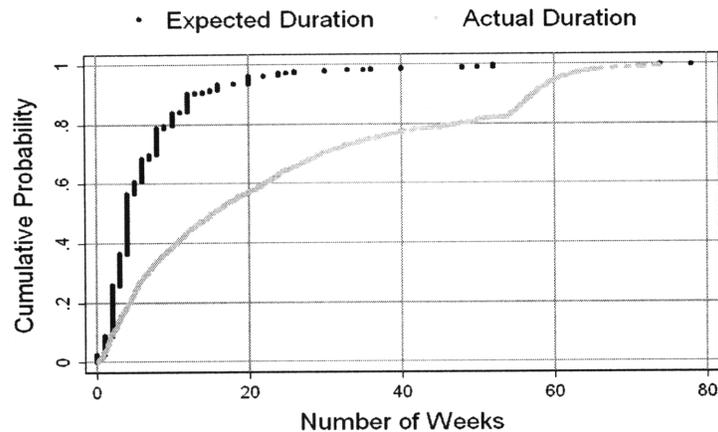


Figure 1-2: Empirical Cumulative Distributions: Actual Duration vs. Expected Duration

from the first interview.<sup>18</sup> This suggests that on average the unemployment spell lasts more than three times longer than expected.

Matching the expectations and the actual realizations shows that 80 percent of the job seekers underestimate the duration of their unemployment spell and that the number of weeks by which the durations are underestimated exceed by far the number of weeks by which the durations are overestimated. The distribution of the differences between the actual and expected number of weeks of unemployment is shown in Figure 1-1 in the introduction. The difference between the minimum actual duration and the expected duration is shown in dark grey for the job seekers who have not found work before the last interview. The optimistic bias in baseline beliefs also appears clearly in Figure 1-2, comparing the empirical distributions of the expected and actual unemployment durations. The cumulative distribution of the expected duration stochastically dominates the cumulative distribution of the minimum duration.<sup>19</sup> For any number of weeks, the num-

<sup>18</sup>In the follow-up interviews, subjects are asked about the start date of their current job only. I do not include subjects who report they have found a different job before the one they are currently working on, but for which no start date is known ( $n = 97$ ). Including these subjects with the date they started their current job when interviewed increases the average optimism with 1.3 weeks. I also do not consider a subject to be reemployed if he or she works less than 20 hours at the start of the new job. This is reported in the data set for 116 subjects. Including these subjects would decrease the average optimism by less than a week.

<sup>19</sup>The kink in the cumulative distribution of the actual duration is due to the fact that I include the

ber of job seekers who expect their unemployment spell will end within that time span exceeds the number of job seekers for whom the unemployment spell actually ends within that time span.

**Selection Effects** In this sample, job seekers largely underestimate the duration of unemployment. Selection effects seem to play a minor role in explaining this optimistic bias. First, the average unemployment duration decreases in the US between 1996 and 1998, as did the average unemployment rates in four out of the five counties considered in the sample. It seems unlikely that job seekers were surprised by an unexpected deterioration of economic conditions. Second, by screening through state unemployment offices, only job seekers who are filing for unemployment benefits are selected. These job seekers are the most policy relevant group of unemployed workers. Moreover, this selection effect does not necessarily increase the estimate of baseline optimism either. Anderson and Meyer (1997) document that the main reason why displaced workers do not take up unemployment benefits is that they expect that the unemployment spell will be short.<sup>20</sup> Third, the sample characteristics are similar to the characteristics of the unemployed in Maryland and Michigan between 1996 and 1998 in the Current Population Survey.<sup>21</sup> Fourth, the job seekers in this sample have been unemployed for 7 weeks on average at the time of the first interview. This implies that both job seekers with ex post short unemployment spells and baseline-pessimistic job seekers, who search more intensively, are likely to be under-represented in the sample. However, the average baseline-optimistic bias is hardly smaller for the newly unemployed. For the 249 job seekers who have been unemployed for 3 weeks

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minimum duration for the job seekers with incomplete spells. These minimum durations are bunched around 52 weeks, which is the average time between the first and the last interview.

<sup>20</sup>Anderson and Meyer (1997) find that 37 percent of the job losers and leavers eligible for UI give 'Expected to get another job soon/be recalled' as the reason for not applying for UI, whereas no other single reason is given by more than 7 percent of them.

<sup>21</sup>Out of the 425 unemployed in Maryland and Michigan in the March CPS between 1996 and 1998, 54 percent are male and 69 percent are white, compared to 53 percent and 73 percent respectively in the sample considered in this paper. The unemployed in the CPS sample have less education and are younger. This may be explained by the fact that this sample is restricted to couples. Compared with the married unemployed in the CPS, the distributions of education and age are more similar. Notice that baseline optimism is significantly higher for the less educated and not significantly lower for the young job seekers.

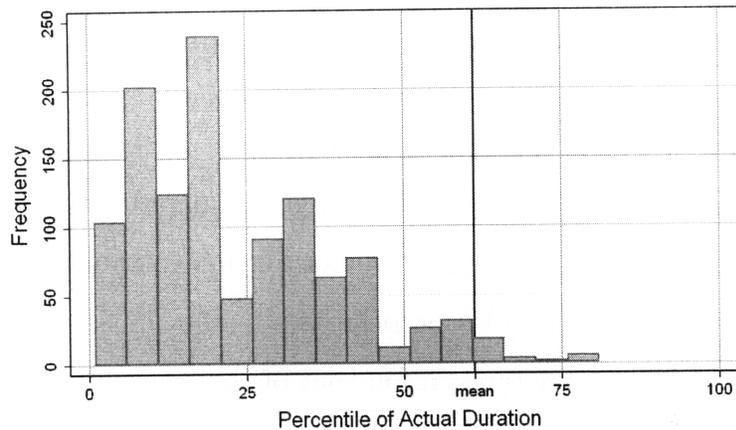


Figure 1-3: Histogram of the reported expectations with bins labeled by the percentiles of the actual duration

or less, the average optimistic bias equals 14.5 weeks. The Wilcoxon rank-sum test does not reject that the baseline bias has the same distribution for the recently displaced job seekers and the other job seekers ( $p$ -value = .79). Finally, exit rates tend to decrease with the duration of unemployment, which may explain why the average remaining duration in the sample considered here is high. The average duration of unemployment for newly unemployed is about 14 weeks in the US in 1996 (Valletta 1998). This is still twice as long as the average expectation in the sample.

**Reported Expectations** One may be concerned about the extent to which the duration predictions capture the job seekers' expectations on which they act. First, the job seekers are not explicitly incentivized to report their expectations truthfully. I do not observe actual behavior either, like their savings for instance, to verify to what extent their behavior is explained by the reported expectations. The expectations do however explain almost as much variation in the actual duration of the unemployment spells as all other demographic and employment variables together.<sup>22</sup> Also, the growing literature on the measurement of expectations confirms the predictive value of surveyed expecta-

<sup>22</sup>The  $R^2$  for regression (1) in Table 2, which regresses actual duration on all considered covariates, increases from .07 to .13 when including the expected duration as an explanatory variable.

tions for both actual outcomes and future behavior (Manski 2004, Gan, Gong, Hurd and McFadden 2004). Second, I interpret the job seekers' reported point predictions as their subjective means. However, some job seekers may report different distributional features as their point predictions, like the median or any other percentile.<sup>23</sup> Figure 1-3 suggests that it is unlikely that these alternative interpretations of the question play an important role in explaining the optimistic bias. The figure shows the distribution of the reported expectations by the percentiles of the actual duration distribution. That is, for each job seeker it shows the percentile he or she should have had in mind if his or her reported point prediction were to be accurate ex post. This assumes that the population distribution is the true distribution that all job seekers are facing. The point predictions are centered around the 20th percentile of the actual duration distribution, and more than 90 percent of the predictions are below the median (and thus below the mean).

**Probability of Finding Work** The subjects are asked a second question about their expectations: "How likely is it that you will be employed more than 20 hours a week in the next two months?" The subjects have the choice among five options; 'very unlikely', 'unlikely', 'neither likely, nor unlikely', 'likely', 'very likely'. I interpret the first two and last two options as the beliefs that the probability to be employed is smaller than a half ( $\hat{\pi} < 1/2$ ) and greater than a half ( $\hat{\pi} > 1/2$ ) respectively. I find the following distribution of subjects who do and don't find employment within two months;<sup>24</sup>

	$\hat{\pi} < 1/2$	$\hat{\pi} > 1/2$
not-employed	0.14	0.47
employed	0.05	0.34

Although the answers do not allow to quantify the bias, they suggest similar optimism about the baseline probability of finding work. Among those who believe that the probability of becoming employed within two months is strictly greater than one half, less than

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<sup>23</sup>Engelberg, Manski and Williams (2009) argue that the elicitation of probabilistic forecasts is therefore more instructive. Notice however that the use of these point predictions about the duration of unemployment does avoid bunching issues that arise when eliciting probabilities.

<sup>24</sup>I exclude the subjects whose answer is 'neither likely, nor unlikely'.

half actually do. Moreover, while 47 percent believe that the probability is greater than a half and do not find work, only 5 percent believe that the probability is smaller than a half, but do find work.

### 1.4.3 Control Beliefs: Actual and Perceived Returns to Effort

Subjects are asked how frequently they have searched for work during the month before the interview. The questions ask about reading the newspaper for job opportunities, checking with employment agencies, checking with friends, sending out resumes, etc.<sup>25</sup> I aggregate the answers to these questions, giving each answer the same weight. A unit increase in the resulting search index corresponds to doubling the frequency in every search dimension. The correlation between this search index and any of its nine components varies between 0.48 and 0.70. The partner of each subject is asked the exact same questions about the subject's efforts. The correlation between the search index as reported by the job seekers and their partners is 0.57.

I estimate the impact of the search index on the actual and expected duration of unemployment. The main regression of interest is

$$y_i = \beta \text{search}_i + X_i\gamma + \varepsilon_i, \quad (1.6)$$

with the dependent variable  $y_i$  equal to respectively the (minimum) actual duration of unemployment since the first interview, the expected duration of unemployment since the first interview, and the difference between the two. Table 2 reports the ordinary least squares estimates for these three regressions. Unemployment spells are both shorter and expected to be shorter for unemployed workers who report to search more intensively. The first effect is stronger than the second effect. If the search index increases by one unit, the

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<sup>25</sup>The questions are: "During the past month, how often have you; read the newspaper and other publications for job opportunities? checked with employment agencies? talked to friends, family, or other people you know to get information about jobs? used, or sent out a resume? filled out application forms for a job? telephoned, written or visited potential employers? done things to improve the impression you would make in a job interview? contacted a public employment service? went out on information interviews?" The answer options are; 1. Not at all, 2. Once every 3 to 4 weeks, 3. Once every couple of weeks, 4. Every week, 5. Two or three times a week, 6. Every day.

actual unemployment spell is 3.0 weeks shorter, but the expected unemployment spell is only 2.2 weeks shorter. Both effects are significant at the 1 percent level. This suggests that job seekers underestimate the returns to search and thus are control-pessimistic. Higher search levels correspond to significantly lower optimism about the duration of unemployment. The control bias is however less pronounced than the baseline bias.<sup>26</sup>

The baseline-optimistic bias does not only change with search efforts. I control for many covariates, as reported in Table 2. Optimism about the duration is more than 5 weeks lower for white and married job seekers. White job seekers have significantly shorter unemployment spells, but do not have different expectations. The same is true for married job seekers. Unemployed workers who earned more at their last steady job believe their unemployment spell will last longer, but it does not actually last longer, making them significantly less optimistic. Notice that heterogeneity in beliefs is ignored in the theoretical analysis here. This is analyzed in Chapter 3.

**Timing of Search** The search index measures the search efforts exerted in the month before the first interview. For the actual duration of unemployment starting from the first interview, the search effort actually exerted matters. For the expected duration of unemployment, the anticipated search effort matters. Unless job seekers perfectly anticipate their efforts, it is not clear whether past effort or actually exerted effort approximate the anticipated effort better. I have an imperfect measure of effort exerted after the first interview. These efforts span the month before the second interview if subjects are still unemployed and the month before they did find work if they are already employed. The

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<sup>26</sup>When restricting the sample to the completed spells, the relation between search and the actual duration of the unemployment spell weakens and becomes insignificant (Panel C of Table 3). The data set is subject to left-truncation as well. I try to control for this by including in the benchmark regression how many weeks the subject has been unemployed so far. I also estimate the differential impact of search in a hazard model with Weibull distribution, conditioning on the fact that job seekers have been unemployed for a while and that the duration for the unsuccessful job seekers goes past the last interview date. Here, the actual and expected increase in the hazard rate for job seekers who search more intensively is almost the same (Panel D of Table 3). Finally, all incomplete spells can also be used without censoring when considering the binary outcome whether or not a job seeker expects to find and actually finds work within  $m$  months. The estimates in a linear probability model suggest that the actual return to search exceeds the expected return to search for  $m \geq 3$ , confirming that job seekers are control-pessimistic. However, the results reverse for  $m \leq 2$ , suggesting that job seekers are control-optimistic in the short run.

measure is only available for a subsample. If the search intensity according to this later measure doubles, actual unemployment spells are on average 4.5 weeks shorter, but only expected to be 1.0 week shorter, as reported in Panel A of Table 3. The estimated effect on the actual duration is larger than for the earlier search variable, but the estimated effect on the expected duration is smaller. The suggested pessimistic bias in control beliefs is larger. Another issue is that low search efforts may be correlated with a lower willingness to work or accept job offers. Job seekers are asked when exactly they would like to start working, if they could choose deterministically. When controlling for this preference, the estimates of the coefficient on search reduce to respectively  $-2.0$  and  $-1.3$  in the actual and expected duration regressions respectively (Panel B of Table 3).

**Heterogeneity and Endogeneity** The theoretical analysis considers the difference between the actual and perceived impact of search efforts on the duration of unemployment. The causal nature of this relation is essential, but may be inconsistently estimated due to unobserved heterogeneity and endogeneity. Some job seekers may be more employable and have shorter actual unemployment spells, although they search less. This channel suggests that ordinary least squares underestimate the actual returns to search. However, if job seekers accurately perceive the impact of their employable nature on the actual duration of unemployment, the estimate of the effect of search on optimism is still consistent. Another problem is that search efforts depend on the perceived value of remaining unemployed. The theory suggests that someone who believes that it is very likely to leave unemployment in the near future is less inclined to search hard today. This channel suggests that ordinary least squares underestimate the perceived returns to search. This may not be solved by considering the difference between actual and expected duration. I try to correct for endogeneity using instrumental variables.

Both the utility difference between employment and unemployment and the marginal cost of search determine how intensively someone searches for a job. Candidates for instruments are variables that change either the utility differential or the cost of searching, but do not change the difference between the actual and expected duration of unemployment in other ways than through search. I consider two instruments that affect the utility

differential: the potential unemployment benefit level and the importance of working to the job seeker. I do not observe the unemployment benefits received, but I calculate what a job seeker would have received if eligible, conditional on his or her monthly earnings before unemployment.<sup>27</sup> The schedule is approximately linear in earnings up to a maximum amount. The identification of the impact of search comes from the non-linearity in the benefit schedule. The identifying assumption is that by including monthly earnings linearly I control for the underlying relation between earnings and the duration of unemployment, actual or expected. The impact of search is identified only by the difference between the non-linear benefit schedule and the smooth relation between earnings and the duration of unemployment. For the importance of work, I use the job seeker's answer to the question: "How important is work to you as part of your daily life?" The identifying assumption is that the error term for the regression of interest is not correlated with the importance people attach to work. This assumption seems plausible for the job seeker's optimism, the main regression of interest. However, if people who attach more importance to work are more likely to be wishful thinkers and thus more optimistic about the duration of the unemployment spell, the estimate of the pessimistic control bias would be biased downward.

Table 4 reports the two stage least squares estimates and the first stage. The estimated impact of search on the job seeker's optimism increases. The optimistic bias decreases with  $-2.3$  weeks when the job seekers double their search intensity. This confirms the control-pessimistic bias suggested by the least squares estimates. The estimate becomes insignificant though, since the standard error increases even more. The first stage regression shows that the potential unemployment benefit level is a weak instrument. The decrease in search when potential unemployment benefits increase is insignificant. However, job seekers who attach more importance to work search significantly more. Considering the actual and expected duration separately, we see that the estimated impact of search increases in both regressions compared to the least squares estimates. A job seeker who doubles search efforts reduces the unemployment spell by 6.3 weeks, but expects this

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<sup>27</sup>The replacement rates are different in Maryland and Michigan. I use the UI calculator used in Chetty (2008b) to calculate the replacement rates based on the reported hourly wage before unemployment.

reduction to be only 4.6 weeks.<sup>28</sup>

#### 1.4.4 Change in Beliefs during Unemployment

In the dynamic model, I have made the simplifying assumptions that both the true and perceived probability of finding work are not affected by the duration of unemployment. The first assumption contradicts negative dependence of the exit rates on the duration of the unemployment spell. The second assumption contradicts learning by unemployed workers. However, the beliefs reported by the job seekers suggest that not much learning is going on. The optimistic bias does not disappear with unemployment experience.

First, job seekers who have been unemployed before are not less optimistic about the duration of the current unemployment spell. The number of times a job seeker has been unemployed in the last three years does not significantly lower his or her optimism about the current unemployment spell, as shown in Table 2. Second, job seekers who have been unemployed for longer in the current unemployment spell, are not less optimistic either. One extra week of unemployment before the first interview increases the expected unemployment spell from the first interview on with .17 weeks ( $p$ -value = .01) and the actual unemployment spell with .32 weeks ( $p$ -value = .04). The long-term unemployed are thus more optimistic about the remaining unemployment spell than the short-term unemployed. Both results are cross-sectional and do not necessarily rule out that the optimistic baseline bias decreases when job seekers become more experienced. Job seekers who are less optimistic about finding a job may search more and leave unemployment earlier. Perfect learning however would overcome this selection effect. A final argument is that unsuccessful job seekers hardly increase their expectations throughout the unem-

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<sup>28</sup>The job seeker's control pessimism is confirmed when I add alternative instruments that make the first stage stronger. A first additional instrument is the job seeker's partner's opinion about how much the job seeker ought to work as an instrument. This instrument changes the cost of searching for the job seeker. The IV estimate of search on optimism is now  $-7.53$ , with robust standard error 2.62. A second additional instrument is to use the job seeker's search as reported by the partner. This instrument does solve the endogeneity problem that arises because of measurement error in the search index variable, but does not solve the potential endogeneity problem indicated before. The IV estimate of search on optimism is now  $-3.73$  with robust standard error 1.61. The results are also similar when considering the binary outcome whether or not a job seeker expects to find and actually finds work within  $m$  months.

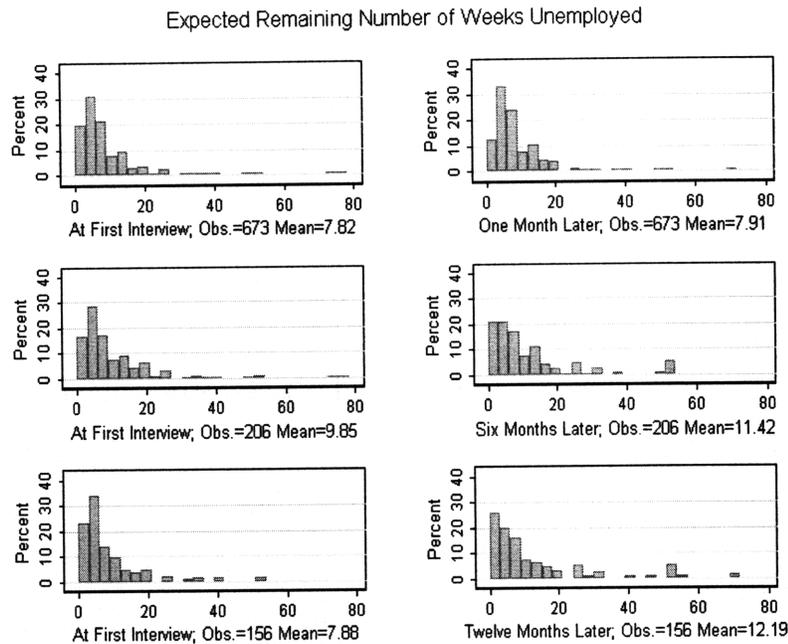


Figure 1-4: Expectations about Remaining Duration of Unemployment

ployment spell. I compare the expectations of the same job seekers at different lengths of the unemployment spell in Figure 1-4. The figure suggest that the distribution of the expected remaining number of weeks of unemployment is very stable throughout the unemployment spell. The average of the expectations at the first interview is not significantly different from the average of the expectations one month or six months later. Only the expectations twelve months later are significantly higher. Together these results suggest that if some learning about the bias is going on, it is very modest.<sup>29</sup>

<sup>29</sup>Notice that if job seekers are uncertain about their ability to find a job at the start, a longer unemployment spell should make them revise their beliefs about the remaining duration upward. The data suggests that they are revising their beliefs upward, at least after twelve months, however they may not revise sufficiently and become more optimistic compared to an unbiased job seeker who is Bayesian updating, the longer they are unemployed. This is what Falk, Huffman and Sunde (2006a) find in a laboratory experiment.

## 1.5 Numerical Analysis

In this section, I use my empirical estimates to gauge the importance of the biases in beliefs for insurance design. I calibrate the full dynamic model in order to numerically analyze the impact on the social optimum and the competitive equilibrium. I discuss the implied welfare consequences of both the privatization of insurance and the naive implementation ignoring the presence of biases in beliefs.

**Calibration** The true probability function and perceived probability function in this numerical exercise are of the form

$$\pi(e) = \pi_0 + \pi_1 e^\rho \text{ and } \hat{\pi}(e) = \hat{\pi}_0 + \hat{\pi}_1 e^\rho.$$

I choose values for the parameters of these functions to match the beliefs and exit rates as a function of effort for the job seekers in the sample considered in the previous section. For the default specification, I consider a pessimistic control bias  $\frac{\hat{\pi}'(e) - \pi'(e)}{\pi'(e)} = \frac{\hat{\pi}_1 - \pi_1}{\pi_1}$  of  $-35$  percent, which corresponds to the least squares estimates in Table 2, and an optimistic baseline bias  $\frac{\hat{\pi}(e) - \pi(e)}{\pi(e)} = \frac{\hat{\pi}_0 - \pi_0 + (\hat{\pi}_1 - \pi_1)e^\rho}{\pi_0 + \pi_1 e^\rho}$  of 100 percent evaluated at the average effort level, which is less extreme than the bias in the sample. For the parameters of the cost of effort function I choose values such that the monthly exit rate in the calibrated model given the current UI system equals 0.188 and the implied elasticity of the unemployment duration to unemployment benefits equals  $-0.5$ . These values correspond to respectively the average exit rate in the sample and the empirical estimates of duration elasticities reviewed in Krueger and Meyer (2002). The details of the calibration are presented in Appendix B.

**Optimal Static Contracts** I first consider contracts  $(b, \tau)$  transferring consumption from those who start employed to those who start unemployed. This corresponds to the static contracts in Section 1.2 with  $x = 0$  and  $\tau^u = 0$ . With a baseline bias of 100 percent and a control bias of  $-35$  percent, the unemployment benefit  $b$  is significantly lower in the competitive equilibrium than in the social optimum. The respective unemployment

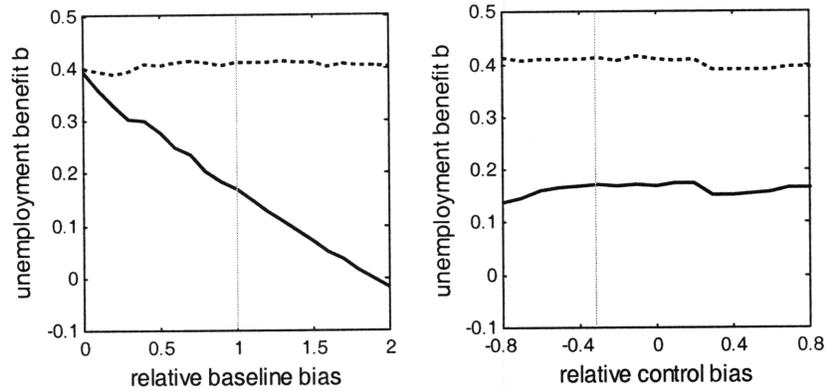


Figure 1-5: Unemployment Benefit in Static Competitive Equilibrium (full) and Static Social Optimum (dash) for Different Baseline and Control Biases

benefit levels are .17 and .41. I scaled the individual output level to 1 such that the unemployment benefit level  $b$  can be interpreted as a replacement rate. On the one hand, private insurers respond to the perception of the value of insurance held by the baseline-optimistic insurees, which makes them offer lower unemployment benefits than what is socially optimal. On the other hand, private insurers do not correct for the low effort level exerted by the baseline-optimistic and control-pessimistic insurees, which makes them offer higher unemployment benefits than what is socially optimal. The former effect strongly dominates the latter effect for this numerical example. Private insurers hardly offer any insurance against unemployment despite its value. Given the lower replacement rate, the insurees exert more search effort in the competitive equilibrium than in the social optimum. The monthly exit rate is .187 in the competitive equilibrium and .175 in the social optimum.

The full line and dashed line in Figure 1-5 shows the optimal static contract in respectively the competitive equilibrium and the social optimum for different biases in beliefs underlying the data. For every alternative beliefs specification, I recalibrate the cost function to match the exit rate and duration elasticity. In the left panel, I present the respective replacement rates for a baseline bias  $\frac{\hat{\pi}(e) - \pi(e)}{\pi(e)}$  ranging from 0 to 200 percent,

evaluated at the average effort level. I change the baseline bias by changing  $\hat{\pi}_0$ , which leaves the control bias unaffected. Private insurance is much more responsive to changes in the baseline beliefs than social insurance, accommodating the baseline optimists' changing perception of the value of insurance. The private insurers decrease the rate from .39 to even negative values for sufficiently high baseline optimism. The social planner only responds to the changed price of inducing effort and corrects for the search externality due to the baseline-optimistic beliefs. The socially optimal replacement rate varies between .39 and .42 for the considered range of baseline optimistic biases. In the right panel of Figure 1-5, I present the respective replacement rates for a control bias  $\frac{\hat{\pi}'(e) - \pi'(e)}{\pi'(e)}$  ranging from  $-80$  to  $80$  percent. I change the control bias by changing  $\hat{\pi}_1$ . I also change  $\hat{\pi}_0$  such that the baseline bias, evaluated at the average effort level, remains at  $100$  percent. The two panels together clearly show that the wedge between social insurance and private insurance is predominantly driven by the baseline bias rather than by the control bias. Both the responses by private insurers and the social planner to a change in the control bias are relatively modest. This would be different if effort is modeled along the extensive rather than the intensive margin.

**Optimal Dynamic Contracts** I now allow the insurers to impose a wage tax  $\tau^u$  on the unemployed from the moment they find employment and to decrease the unemployment benefit  $b$  and the after-tax wage  $w - \tau^u$  by  $x$  for any additional month of unemployment. This corresponds to the linear contracts considered in Section 1.3. For the benchmark specification, search effort is induced both by rewarding a successful job seeker with a net wage  $w - \tau^u$  that exceeds the unemployment benefit  $b$  and by punishing an unsuccessful job seeker by decreasing all future consumption levels by  $x$ . Both the reward  $w - \tau^u - b$  and the punishment  $x$  are much larger in the competitive equilibrium. The unemployment benefit level starts at .58 in the social optimum and at  $-.02$  in the competitive equilibrium. The monthly decrease in consumption during unemployment, expressed as a percentage of production, equals only .4 percentage points in the social optimum, but 10 percentage points in the competitive equilibrium. Consumption jumps by .42 upon employment in the social optimum and by 1.30 in the competitive equilib-

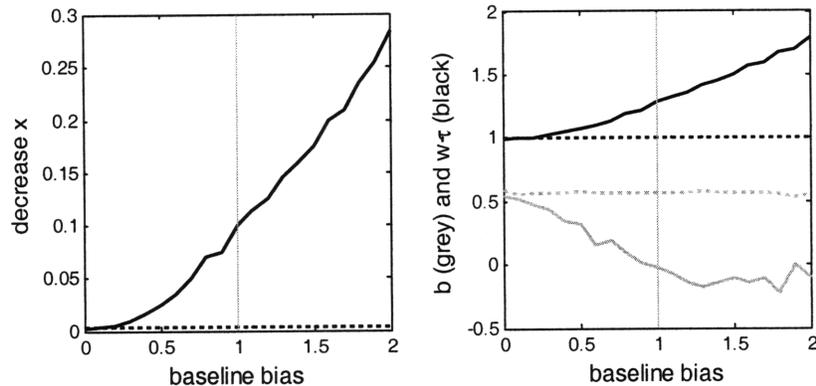


Figure 1-6: Dynamic Contract in Competitive Equilibrium (full) and Social Optimum (dash) as a Function of the Baseline Bias

rium. Figure 1-6 shows the dynamic contracts in the social optimum and the competitive equilibrium for different baseline biases underlying the data (with the cost function recalibrated). In the social optimum,  $x$  is slightly higher when insurees are baseline-optimistic. Still the change is very small compared to the exponential increase in  $x$  in the competitive equilibrium. For an optimistic baseline bias of 200 percent, consumption falls at a rate of 0.28 for each extra month of unemployment. Also the reward upon employment  $w - \tau^u - b$  increases much more with baseline optimism in the competitive equilibrium than in the social optimum.

**Naive Policies** A policy maker uses data to test for the optimality of current policies and to implement new policies. When unaware of biases in beliefs, the policy maker would miscalibrate his model by matching the empirical moments under the assumption that the job seekers' beliefs are unbiased. In the spirit of this calibration exercise, the policy maker who naively assumes that job seekers have correct beliefs, would use the cost function that matches the exit rate and unemployment duration elasticity if beliefs were to be unbiased. This miscalibrated cost function leads the policy maker to reward and punish the job seekers too little. He sets the reward for finding employment  $w - \tau^u - b$  at .39 and the monthly decrease in consumption  $x$  at only .25 percentage points. The socially

optimal contract has  $w - \tau^u - b = .42$  and  $x = .4$ , if the baseline bias is 100 percent and the control bias is  $-35$  percent. This is in line with Corollary 1. The naive policy maker ignores that the additional incentives increase welfare by correcting the lowered incentives due to the bias in beliefs. Figure 1-5 and 1-6 show how the changes in the social optimum when assuming different biases in beliefs underlying the data are small relative to the difference between the social optimum and the competitive equilibrium. This suggests that the impact of miscalibration is small relative to the impact of privatizing insurance.

**Welfare Effects** An insuree does not internalize the impact of her effort on the insurer's budget constraint. This moral hazard problem lowers the insuree's welfare in the social optimum below the first best. Baseline optimism and control pessimism decrease the effort choice further and aggravate the moral hazard problem. The true expected utility in the social optimum is therefore decreasing in both biases. If in contrast insurees are sufficiently baseline-pessimistic or control-optimistic, the social planner could approximate the first best.

In the competitive equilibrium, the true expected utility is not only lower than in the social optimum when agents have biased beliefs, but also tends to decrease more than in the social optimum when insurees become more baseline-optimistic or control-pessimistic. I calculate the consumption subsidy  $\Delta c$  required in every period of the insuree's life such that she achieves the same true expected utility in the competitive equilibrium as in the social optimum. For an optimistic baseline bias of 100 percent and a pessimistic control bias of 35 percent, this consumption subsidy is 14 percent of the output when employed. That means that 14 percent of the economy's production is needed to make people with competitive unemployment insurance as well off as they would be with an insurance system that is optimally designed. The consumption subsidy increases exponentially in the baseline bias, as shown in Figure 1-7. When the baseline bias is small, the consumption subsidy is approximately zero. However, the subsidy exceeds 100 percent of output when the optimistic baseline bias is large. The exponential increase in welfare cost reflects the exponential increase in the monthly consumption reduction  $x$  and the wedge  $w - \tau^u - b$  in the competitive equilibrium.

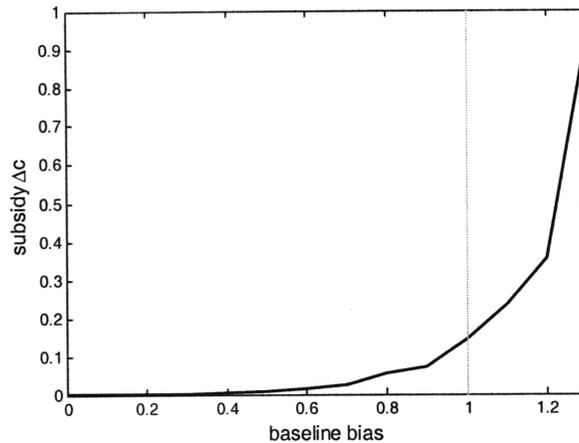


Figure 1-7: Welfare Cost of Privatizing Insurance as a Function of the Baseline Bias

The welfare cost is mostly driven by the dynamic component of the contract. When the insurers are restricted to static contracts, the required consumption subsidy never exceeds 1 percent of output for the beliefs considered. In contrast with the static contracts, the dynamic contracts allow the private insurer to exploit both the fact that the insuree overestimates the probability of finding work, by giving higher consumption levels upon employment, and the fact that the insuree underestimates the probability of being unemployed for a long term, by offering steeper consumption profiles. The equilibrium contract implies a strong disparity in lifetime utility between the long-term unemployed on the one hand and the employed and the short-term unemployed on the other hand. People accept this disparity, but only because they underestimate the probability of being among the long-term unemployed.

## 1.6 Conclusion

The perception of risk is at the heart of optimal insurance design. This paper focuses on the optimal design of unemployment insurance, presenting new evidence that suggests that job seekers are optimistic about the probability of finding a job, but pessimistic about the returns to their search effort. The theoretical analysis applies to insurance

and incentive contracts in other contexts in which biases in beliefs may be important, like for instance car insurance, health insurance and labor contracts. Young drivers may overestimate the probability of avoiding car accidents, but underestimate the returns to driving safely. Women may overestimate the probability of being spared from breast cancer, but underestimate the returns to preventive care. Employees may overestimate the probability of good outcomes and at the same time their control over this probability.

The analysis assumes that the bias in beliefs is representative and stable. The assumption that the bias is representative is restrictive if insurers can offer a menu of contracts. People have heterogeneous perceptions of risks (Slovic 2000) and this heterogeneity is typically not observable to insurers. In Chapter 3, I explore how insurers screen insurees with different private perceptions. The assumption that the bias in beliefs is stable excludes a natural way to correct for behavioral distortions due to biased beliefs, that is by informing the insuree about her biased perception. Changing biased perceptions seems important, but has proven to be difficult though. Moreover, insurers may prefer not to inform insurees or even mislead them such that the information provided by insurers loses credibility. A paternalistic government always prefers an insuree to be more control-optimistic, because control optimism mitigates the moral hazard problem. A profit-maximizing insurer always prefers an insuree to be more baseline-pessimistic, because baseline pessimism increases the willingness to pay for insurance.

The biases in baseline and control beliefs result in an unambiguous difference between optimal and naive insurance design, on the one hand, and social and private insurance, on the other hand. First, policy makers should be aware of people's perceptions when evaluating or implementing policies. Given the lack of information, the design of policies is often based on the responsiveness of observable outcomes, like the response in employment to unemployment benefits, in health outcomes to the copay and deductible, in production to taxes or in retirement decisions to pension benefits. These statistics play a different role when people's perceptions are biased. Second, policy makers have a reason to intervene when insurance is provided to insurees with biased beliefs by private insurers. Although competition disciplines private insurers to charge actuarially fair prices, it does not induce them to correct the insurees' distorted choices. The welfare gains from intervening are

exponentially increasing in the biases in beliefs.

The analysis is restricted to the design of the monetary structure of unemployment insurance. The perceptions of the unemployed are central to the evaluation of other unemployment policies as well. The empirical analysis suggests that job seekers search too little, since they underestimate the returns to search and overestimate the probability of leaving unemployment. This sheds a new light on the role of active labor market policies. The biased perceptions unambiguously increase the value of policies that monitor job search efforts or induce the unemployed to search harder. An alternative policy that reduces moral hazard is the replacement of unemployment insurance by individual unemployment savings accounts, as proposed by Altman and Feldstein (2006). If the choice to save is given to individuals, similar issues arise as with the privatization of insurance. Workers who are optimistic about the probability of finding work would choose to save too little in employment and dissave too fast in unemployment. Saving mandates are therefore an indispensable part of such a policy.

## 1.7 Appendix A: Proofs

### Proof of Proposition 1

With unbiased beliefs, the two maximization problems (1.1) and (1.2) coincide. The first order condition of this problem equals

$$(1-p)(1-\pi(\hat{e}(b)))u'(b) - pu'(w-\hat{\tau}(b))\frac{d\hat{\tau}(b)}{db} + (1-p)[\pi'(\hat{e}(b))[u(w)-u(b)]-1]\frac{d\hat{e}(b)}{db} = 0. \quad (1.7)$$

The last term equals zero by (IC). Under the assumption that  $\pi(\hat{e}) < 1$ , dividing by  $(1-p)(1-\pi(\hat{e}))$  gives

$$u'(b) - u'(w-\hat{\tau}(b))\frac{b}{\hat{\tau}(b)}\frac{d\hat{\tau}(b)}{db} = 0.$$

Using  $\frac{d\hat{\tau}(b)}{db}\frac{b}{\hat{\tau}(b)} = 1 + \varepsilon_{1-\pi(\hat{e}(b)),b}$ , the Baily formula (1.3) follows by dividing both terms by  $u'(w-\hat{\tau}(b))$ .  $\square$

### Proof of Proposition 2

The first order condition of the social planner's problem (1.1) equals

$$(1-p)(1-\pi(\hat{e}(b)))u'(b) - pu'(w-\hat{\tau}(b))\frac{d\hat{\tau}(b)}{db} + (1-p)\{\pi'(\hat{e}(b))[u(w)-u(b)]-1\}\frac{d\hat{e}(b)}{db} = 0.$$

Using the incentive compatibility constraint to substitute for the marginal cost of search 1 by the perceived and dividing by  $(1-p)(1-\pi(\hat{e}(b)))$ , this simplifies to

$$u'(b) - u'(w-\hat{\tau}(b))(1 + \varepsilon_{1-\pi(\hat{e}(b)),b}) + \{\pi'(\hat{e}(b)) - \hat{\pi}'(\hat{e}(b))\}[u(w) - u(b)]\frac{\pi'(\hat{e}(b))b}{\pi'(\hat{e}(b))b(1-\pi(\hat{e}(b)))} \frac{1}{b} \frac{d\hat{e}(b)}{db} = 0.$$

Rewritten in elasticities, this becomes

$$u'(b) - u'(w-\hat{\tau}(b)) = u'(w-\hat{\tau}(b))\varepsilon_{1-\pi(\hat{e}(b)),b} + \frac{\hat{\pi}'(\hat{e}(b)) - \pi'(\hat{e}(b))}{\pi'(\hat{e}(b))} \frac{u(w) - u(b)}{b} \varepsilon_{1-\pi(\hat{e}(b)),b}.$$

The adjusted Baily formula (1.4) for the social optimum follows by dividing both sides by  $u'(w - \hat{\tau}(b))$ .  $\square$

### Proof of Proposition 3

The first order condition of the social planner's problem (1.2) equals

$$(1-p)(1-\hat{\pi}(\hat{e}(b)))u'(b) - pu'(w - \hat{\tau}(b))\frac{d\hat{\tau}(b)}{db} + \\ (1-p)\{\hat{\pi}'(\hat{e}(b))[u(w) - u(b)] - 1\}\frac{d\hat{e}(b)}{db} = 0.$$

The last term equals zero by the incentive compatibility constraint. Dividing by  $(1-p)(1-\pi(\hat{e}(b)))$  gives

$$\frac{1-\hat{\pi}(\hat{e}(b))}{1-\pi(\hat{e}(b))}u'(b) - u'(w - \hat{\tau}(b))\frac{b}{\hat{\tau}(b)}\frac{d\hat{\tau}(b)}{db} = 0.$$

Using  $\frac{d\hat{\tau}(b)}{db}\frac{b}{\hat{\tau}(b)} = 1 + \varepsilon_{1-\pi(\hat{e}(b)),b}$ , the adjusted Baily formula (1.5) for the competitive equilibrium follows by dividing both sides by  $u'(w - \hat{\tau}(b))$ .  $\square$

### Proof of Proposition 4

The interior social optimum  $(b^s, e^s)$  satisfies

$$[(1-\pi(e^s))u'(b^s) - (1-\pi(e^s))u'(w - \hat{\tau}(b^s))] + \\ \{\pi'(e^s)[u(w) - u(b^s)] - 1 + \pi'(e^s)u'(w - \hat{\tau}(b^s))b^s\}\frac{d\hat{e}(b^s)}{db} = 0 \quad (1.8)$$

, whereas an interior competitive equilibrium  $(b^p, e^p)$  satisfies

$$(1-\hat{\pi}(e^p))u'(b^p) - (1-\pi(e^p))u'(w - \hat{\tau}(b^p)) + \\ \{\hat{\pi}'(e^p)[u(w) - u(b^p)] - 1 + \pi'(e^p)u'(w - \hat{\tau}(b^p))b^p\}\frac{d\hat{e}(b^p)}{db} = 0. \quad (1.9)$$

Since  $\hat{e}(b)$  solves  $\hat{\pi}'(e)[u(w) - u(b)] = 1$ , both  $\frac{d\hat{e}(\cdot)}{db}$  and the induced effort level  $\hat{e}(\cdot)$  converge to 0 when  $\hat{\pi}'(e) \rightarrow 0$ . The conditions (1.8) and (1.9) converge to respectively

$$(1-\pi(e^s))[u'(b^s) - u'(w - \hat{\tau}(b^p))] \rightarrow 0 \text{ and} \\ [(1-\hat{\pi}(e^p))u'(b^p) - (1-\pi(e^p))u'(w - \hat{\tau}(b^p))] \rightarrow 0.$$

The proposition immediately follows.  $\square$

### Proof of Proposition 5

With  $\pi'(e) \rightarrow 0$ , the optimality condition (1.8) for an interior solution  $(b^s, e^s)$  converges to

$$(1 - \pi(e^s)) [u'(b^s) - u'(w - \tau(e^s, b^s))] - \frac{d\hat{e}(b^s)}{db} \rightarrow 0.$$

For any benefit level  $b \leq w - \hat{\tau}(b)$ , the left hand side is strictly positive. This cannot be socially optimal in the limit, except for an upper bound on  $b$ . If insurers are restricted not to overinsure, full insurance maximizes the true expected utility in the limit. Evaluated at  $b^s = w - \tau(e^s, b^s)$ , the utility gain of a further increase in  $b^s$  is indeed  $-\frac{d\hat{e}(b^s)}{db} > 0$ . Moreover, if  $p \rightarrow 1$ ,  $b^s \rightarrow w$ . Hence, the induced effort level converges to 0, which is efficient for  $\pi'(e) \rightarrow 0$ .

Using the *IC* constraint, the optimality condition (1.9) for an interior solution  $(b^p, e^p)$  converges to

$$(1 - \hat{\pi}(e^p)) u'(b^p) - (1 - \pi(e^p)) u'(w - \tau(e^p, b^p)) \rightarrow 0.$$

With  $\hat{\pi}(e^p) > \pi(e^p)$ ,  $b^p \in (0, w - \tau(e^p, b^p))$  and the induced effort level is positive if  $\hat{\pi}'(0) [u(w) - u(b^p)] > 0$ .  $\square$

### Proof of Proposition 6

I consider an increase in  $b$  together with an increase in  $\tau^u$  such that the budget constraint, accounting for the changes in  $\hat{e}(z)$ , is still satisfied. That is,

$$\frac{-db + \beta\pi(\hat{e}(z)) \frac{d\tau^u}{1-\beta}}{1 - \beta(1 - \pi(e))} - C_e(z, e) \left\{ \frac{\partial \hat{e}(z)}{\partial b} db + \frac{\partial \hat{e}(z)}{\partial \tau^u} d\tau^u \right\} = 0,$$

with

$$C_e(z, e) = -\frac{\beta\pi'(e)}{[1 - \beta(1 - \pi(e))]^2} \left\{ b + \tau - \frac{x}{1 - \beta} \right\},$$

the decrease in the expected cost of the contract for the insurer when  $e$  increases. Notice  $C_e(z, e) < 0$  if  $b + \tau > \frac{x}{1-\beta}$ . This always holds in the numerical simulations considered. Denote the elasticity of unemployment duration  $\frac{1}{\pi(\hat{e})}$  with respect to an increase in  $b$ ,

balanced by an increase in  $\tau^u$ , by

$$\varepsilon_{\frac{1}{\pi(\hat{e})},(b,\tau^u)} = -\frac{\pi'(\hat{e}(z))}{\pi(\hat{e}(z))} \left\{ \frac{\partial \hat{e}(z)}{\partial b} + \frac{\partial \hat{e}(z)}{\partial \tau^u} \frac{d\tau^u}{db} \right\} b > 0,$$

then the revenue-neutral change implies

$$\frac{d\tau^u}{db} = \frac{1-\beta}{\beta\pi(\hat{e}(z))} \left\{ 1 + \frac{\beta\pi(\hat{e}(z))}{1-\beta(1-\pi(\hat{e}(z)))} \frac{b + \tau^u - \frac{x}{1-\beta}}{b} \varepsilon_{\frac{1}{\pi(\hat{e})},(b,\tau^u)} \right\}.$$

The gain in true expected utility from an increase in  $b$ , balanced by an increase in  $\tau^u$  equals zero if

$$\frac{\partial U(z, \hat{e}(z))}{\partial b} + \frac{\partial U(z, \hat{e}(z))}{\partial \tau^u} \frac{d\tau^u}{db} + \frac{\partial U(z, \hat{e}(z))}{\partial e} \left\{ \frac{\partial \hat{e}(z)}{\partial b} db + \frac{\partial \hat{e}(z)}{\partial \tau^u} d\tau^u \right\} = 0,$$

with

$$\begin{aligned} \frac{\partial U(z, \hat{e})}{\partial b} &= \frac{u'(b-\hat{e})}{1-\beta(1-\pi(\hat{e})) \exp(\sigma x)} \\ \frac{\partial U(z, \hat{e})}{\partial \tau^u} &= \frac{-\beta\pi(\hat{e}) \frac{u'(w-\tau^u)}{1-\beta}}{1-\beta(1-\pi(\hat{e})) \exp(\sigma x)} \\ \frac{\partial U(z, \hat{e})}{\partial e} &= \left\langle [\hat{\pi}(\hat{e}) - \pi(\hat{e})] + [\pi'(\hat{e}) - \hat{\pi}'(\hat{e})] / \sigma \left\{ 1 - \frac{1-\beta \exp(\sigma x)}{(1-\beta) \exp(\sigma x)} \frac{u(w-\tau^u)}{u(b-\hat{e})} \right\} \right\rangle \\ &\quad \times \frac{u'(b-\hat{e})[\beta \exp(\sigma x)]}{\{1-\beta(1-\pi(\hat{e})) \exp(\sigma x)\}^2}. \end{aligned}$$

For the expression for  $\frac{\partial U(z, \hat{e})}{\partial e}$ , I make use of the fact that  $\hat{e}(z)$  maximizes  $\hat{U}(z, e)$ . Notice that  $\frac{\partial U(z, \hat{e})}{\partial e}$  is increasing in  $\hat{\pi}(\hat{e}) - \pi(\hat{e})$  and in  $\pi'(\hat{e}) - \hat{\pi}'(\hat{e})$ , since both  $\frac{1-\beta \exp(\sigma x)}{(1-\beta) \exp(\sigma x)}$  and  $\frac{u(w-\tau^u)}{u(b-\hat{e})}$  are smaller than 1. Using the same algebraic manipulations as in the proof of Proposition 2, I find the first result in the proposition with the correction for the search internality

$$I^\tau \left( \frac{\pi'(\hat{e}) - \hat{\pi}'(\hat{e})}{\pi'(\hat{e})}, \frac{\hat{\pi}(\hat{e}) - \pi(\hat{e})}{\pi(\hat{e})}, z \right) \equiv \frac{\frac{\partial U(z, \hat{e})}{\partial e}}{-\frac{(1-\beta)(b+\tau^u) - x}{1-\beta(1-\pi(\hat{e}))} \frac{\partial U(z, \hat{e})}{\partial \tau^u}}.$$

The function depends on the baseline bias and control bias through  $\frac{\partial U(z, \hat{e})}{\partial e}$ . We find  $I^\tau(0, 0, z) = 0$  and  $I_1^\tau > 0$  and  $I_2^\tau > 0$  if  $C_e(z, \hat{e}) < 0$ .

The gain in perceived expected utility from an increase in  $b$ , balanced by an increase

in  $\tau^u$  equals zero if

$$\frac{\partial \hat{U}(z, \hat{e}(z))}{\partial b} + \frac{\partial \hat{U}(z, \hat{e}(z))}{\partial \tau^u} \frac{d\tau^u}{db} = 0,$$

with

$$\frac{\partial \hat{U}(z, \hat{e})}{\partial b} = \frac{u'(b-\hat{e})}{1-\beta(1-\hat{\pi}(\hat{e}))\exp(\sigma x)} \quad \text{and} \quad \frac{\partial \hat{U}(z, \hat{e})}{\partial \tau^u} = \frac{-\beta\hat{\pi}(\hat{e})\frac{u'(w-\tau^u)}{1-\beta}}{1-\beta(1-\hat{\pi}(\hat{e}))\exp(\sigma x)}.$$

The effect through effort on the perceived utility is of second order by the envelope condition. Using the same algebraic manipulations as in the proof of Proposition 3, I find the second result in the Proposition.  $\square$

### Proof of Proposition 7

I consider an increase in  $x$  together with an increase in  $c_0$  (i.e. both  $b$  and  $w - \tau^u$ ) such that the budget constraint, accounting for the changes in  $\hat{e}(z)$ , is still satisfied. That is,

$$-dc_0 + \frac{\beta(1-\pi(\hat{e}(z)))}{1-\beta(1-\pi(\hat{e}(z)))} dx - C_e(z, \hat{e}(z))(1-\beta) \frac{\partial \hat{e}(z)}{\partial x} dx = 0.$$

Only the change in  $x$  affects the effort choice, since the insuree has CARA preferences. Denote the elasticity of unemployment duration  $\frac{1}{\pi(\hat{e})}$  with respect to an increase in  $c_0$  together with an increase in  $x$  by  $\varepsilon_{\frac{1}{\pi(\hat{e})}, (c_0, x)} \equiv -\frac{\pi'(\hat{e}(z))}{\pi(\hat{e}(z))} \frac{\partial \hat{e}(z)}{\partial x} \frac{dx}{dc_0} x > 0$ , then revenue-neutrality implies

$$\frac{dx}{dc_0} = \frac{1-\beta(1-\pi(\hat{e}(z)))}{\beta(1-\pi(\hat{e}(z)))} \left\{ 1 + \frac{\beta\pi(\hat{e}(z))}{[1-\beta(1-\pi(\hat{e}(z)))]^2} \frac{(b+\tau)(1-\beta)-x}{x} \varepsilon_{\frac{1}{\pi(\hat{e})}, (c_0, x)} \right\}.$$

The gain in true expected utility from an increase in  $c_0$ , balanced by an increase in  $x$  equals zero if

$$\frac{\partial U(z, \hat{e}(z))}{\partial c_0} + \frac{\partial U(z, \hat{e}(z))}{\partial x} \frac{dx}{dc_0} + \frac{\partial U(z, \hat{e}(z))}{\partial e} \frac{\partial \hat{e}(z)}{\partial x} \frac{dx}{dc_0} = 0$$

$$\text{with } \frac{\partial U(z, \hat{e})}{\partial c_0} = -\sigma U(z, \hat{e}(z)) \quad \text{and} \quad \frac{\partial U(z, \hat{e})}{\partial x} = \sigma U(z, \hat{e}(z)) \frac{\beta(1-\pi(\hat{e}))\exp(\sigma x)}{1-\beta(1-\pi(\hat{e}))\exp(\sigma x)} < 0.$$

Using similar algebraic manipulations as in the proof of Proposition 6, I find

$$\frac{\frac{u'(b-\hat{e})+\beta\pi(\hat{e})\frac{u'(w-\tau^u)}{1-\beta}}{1-\beta(1-\pi(\hat{e}))} - \frac{u'(b-\hat{e})+\beta\pi(\hat{e})\frac{u'(w-\tau^u)}{1-\beta}}{1-\beta(1-\pi(\hat{e}))\exp(\sigma x)} \exp(\sigma x)}{\frac{u'(b-\hat{e})+\beta\pi(\hat{e})\frac{u'(w-\tau^u)}{1-\beta}}{1-\beta(1-\pi(\hat{e}))\exp(\sigma x)} \exp(\sigma x)} = J^x(z) \varepsilon_{\frac{1}{\pi(\hat{e}(z))}, (c_0, x)} \left\{ 1 + I^x \left( \frac{\pi'(\hat{e})-\hat{\pi}'(\hat{e})}{\pi'(\hat{e})}, \frac{\hat{\pi}(\hat{e})-\pi(\hat{e})}{\pi'(\hat{e})}, z \right) \right\}$$

with

$$J^x(z) = \frac{\beta\pi(e)}{[1-\beta(1-\pi(e))]^2} \frac{(b+\tau^u)(1-\beta)-x}{x}$$

$$I^x \left( \frac{\pi'(\hat{e})-\hat{\pi}'(\hat{e})}{\pi'(\hat{e})}, \frac{\hat{\pi}(\hat{e})-\pi(\hat{e})}{\pi'(\hat{e})}, z \right) = \frac{1}{\frac{(b+\tau^u)(1-\beta)-x}{[1-\beta(1-\pi(e))]} \frac{\pi'(e)}{(1-\pi(\hat{e}))} - \frac{\partial U(z, \hat{e}(z))}{\partial x}} \frac{\frac{\partial U(z, \hat{e}(z))}{\partial e}}{\frac{\partial U(z, \hat{e}(z))}{\partial x}}.$$

The first result in Proposition 7 immediately follows.  $I^x(0, 0, z) = 0$ ,  $I_1^x > 0$ ,  $I_2^x > 0$  again follows from the derivative from  $\frac{\partial U(z, \hat{e}(z))}{\partial e}$  with respect to the biases in beliefs, as in Proposition 6.

The gain in perceived expected utility from an increase in  $c_0$ , balanced by an increase in  $x$  equals zero if

$$\frac{\partial \hat{U}(z, \hat{e}(z))}{\partial c_0} + \frac{\partial \hat{U}(z, \hat{e}(z))}{\partial x} \frac{dx}{dc_0} = 0,$$

with

$$\frac{\partial \hat{U}(z, \hat{e})}{\partial c_0} = -\sigma \hat{U}(z, \hat{e}(z)) \text{ and } \frac{\partial \hat{U}(z, \hat{e})}{\partial x} = \sigma \hat{U}(z, \hat{e}(z)) \frac{\beta(1-\hat{\pi}(\hat{e}))\exp(\sigma x)}{1-\beta(1-\hat{\pi}(\hat{e}))\exp(\sigma x)} < 0.$$

Using the same manipulations again, the second result immediately follows as well.  $\square$

### Proof of Proposition 8

Private insurers only care about the perceived expected utility of the contract they offer.

The equilibrium contract solves

$$C(\hat{V}) = \min_{c^u, V', V^e, e} c^u + \beta \left[ \pi(e)C^e(V^e) + (1-\pi(e))C(\hat{V}^u) \right]$$

such that

$$u(c^u - e) + \beta[\hat{\pi}(e)V^e + (1 - \hat{\pi}(e))\hat{V}^u] = \hat{V}$$

$$e \in \arg \max u(c^u - e) + \beta \left[ \hat{\pi}(e)V^e + (1 - \hat{\pi}(e))\hat{V}^u \right],$$

and  $C(\hat{V}) = 0$ . The true expected utility of the contract plays no role. Starting from an optimal contract assigning expected utility  $\hat{V}$ , the optimal response to an increase in  $\hat{V}$  is to increase all consumption levels, today and in the future, while employed and unemployed, by the same amount. This leaves the margins for search effort unchanged. Since an increase in  $\hat{V}$  is accommodated by an equal increase in all consumption levels and  $\hat{V}$  is the only state variable in the recursive problem, the optimal policy functions satisfy

$$\frac{\hat{V}}{\hat{V}^u(\hat{V})} = \frac{\hat{V}}{V^e(\hat{V})} = \frac{\hat{V}^u(\hat{V})}{V^e(\hat{V}^u(\hat{V}))} \text{ for any } \hat{V}.$$

This implies that the optimal contract is linear.  $\square$

### Proof of Proposition 9

I consider an increase in  $x$  for the first period of unemployment  $dx_0$  and a decrease in  $x$  for all later periods  $dx_+$ , such that the budget constraint is still satisfied,

$$\frac{dx_0}{dx_+} = -\frac{\beta(1 - \pi(\hat{e}))}{1 - \beta(1 - \pi(\hat{e}))}.$$

With the effect on the search internality small, this increases welfare if and only if

$$\frac{\partial U}{\partial x_0} dx_0 - \frac{\partial U}{\partial x_+} dx_+ + \lambda \left[ \frac{\partial \Pi}{\partial e_0} \left( \frac{\partial \hat{e}_0}{\partial x_0} dx_0 - \frac{\partial \hat{e}_0}{\partial x_+} dx_+ \right) + \frac{d\Pi}{de_+} \left( \frac{\partial \hat{e}_+}{\partial x_0} dx_0 - \frac{\partial \hat{e}_+}{\partial x_+} dx_+ \right) \right] > 0, \quad (1.10)$$

with  $\hat{e}_0$  the effort exerted in the first period and  $\hat{e}_+$  the effort exerted in all later periods.

Using the fact that the linear contract is optimal, i.e.

$$\begin{aligned} \frac{\partial U}{\partial x} + \lambda \left[ \frac{\beta (1 - \pi(\hat{e}))}{(1 - \beta) [1 - \beta (1 - \pi(\hat{e}))]} + \frac{d\Pi}{de} \frac{d\hat{e}}{dx} \right] &= 0 \\ \frac{\partial U}{\partial c_0} &= \frac{\lambda}{1 - \beta}, \end{aligned}$$

condition (1.10) simplifies to

$$(\hat{\pi}(\hat{e}) - \pi(\hat{e})) (-\sigma U) \beta \exp(\sigma x) \left[ \frac{\beta (1 - \pi(\hat{e})) [\exp(\sigma x) - 1]}{(1 - \beta (1 - \pi(\hat{e}))) (1 - \beta (1 - \pi(\hat{e})) \exp(\sigma x))} \right] > 0,$$

which holds if and only if  $\hat{\pi}(\hat{e}) > \pi(\hat{e})$ .  $\square$

### Proof of Corollary 1

When  $\hat{\pi}'(\hat{e}) \leq \pi'(\hat{e})$ ,  $1 + \frac{\pi'(\hat{e}) - \hat{\pi}'(\hat{e})}{\pi'(\hat{e})} I(b) \geq 1$ . By assumption,  $\frac{[u'(b) - u'(w - \hat{\tau})]/u'(w - \hat{\tau})}{\varepsilon_{1 - \pi(\hat{e}), b}}$  is decreasing in  $b$ . In the standard Baily formula (1.3) this term needs to be equal to 1. In the adjusted Baily formula (1.4) this term needs to be greater than 1. Hence, the benefit implemented by the standard Baily formula exceeds the benefit implemented by the adjusted Baily formula.  $\square$

### Proof of Corollary 2

When  $\hat{\pi}(\hat{e}) \geq \pi(\hat{e})$ ,  $\frac{u'(b) - u'(w - \hat{\tau})}{u'(w - \hat{\tau})} \geq \frac{1 - \hat{\pi}(\hat{e})}{1 - \pi(\hat{e})} \frac{u'(b) - u'(w - \hat{\tau})}{u'(w - \hat{\tau})}$ . In the standard Baily formula (1.3) and the adjusted Baily formula for the competitive equilibrium (1.5), respectively the left hand side and the right hand side need to be equal to 1. Since  $\frac{[u'(b) - u'(w - \hat{\tau})]/u'(w - \hat{\tau})}{\varepsilon_{1 - \pi(\hat{e}), b}}$  is decreasing in  $b$ , the benefit implemented by the standard Baily formula exceeds the benefit implemented by the adjusted Baily formula.  $\square$

### Proof of Corollary 3

I consider the marginal rate of transformation (MRT) and the marginal rate of substitution (MRS) between  $e$  and  $b$ , with  $\tau$  being determined by the budget constraint. The MRT is the same for social planner and private insurer, determined by the  $IC$  constraint. The MRS is different.

The interior social optimum  $(b^s, e^s)$  with  $e^s = \hat{e}(b^s)$  satisfies

$$\begin{aligned} & [(1 - \pi(e^s)) u'(b^s) - (1 - \pi(e^s)) u'(w - \hat{\tau}(b^s))] + \\ & \quad \{ \pi'(e^s) [u(w) - u(b^s)] - 1 + \pi'(e^s) u'(w - \hat{\tau}(b^s)) b^s \} \frac{d\hat{e}(b^s)}{db} = 0 \end{aligned}$$

Given global concavity, a private insurer in the competitive equilibrium offers more insurance if and only if an increase in insurance, evaluated at the social optimum, increases the perceived expected utility. That is, if and only if

$$\begin{aligned} & [(1 - \hat{\pi}(e^s)) u'(b^s) - (1 - \pi(e^s)) u'(w - \hat{\tau}(b^s))] + \\ & \quad \{ \hat{\pi}'(e^s) [u(w) - u(b^s)] - 1 + \pi'(e^s) u'(w - \tau(e^s, b^s)) b^s \} \frac{d\hat{e}(b^s)}{db} > 0. \end{aligned}$$

Using the condition for the social optimum, this inequality simplifies to

$$\{ \pi(e^s) - \hat{\pi}(e^s) \} u'(b^s) + \{ \hat{\pi}'(e^s) - \pi'(e^s) \} [u(w) - u(b^s)] \frac{d\hat{e}(b^s)}{db} > 0.$$

With  $\frac{d\hat{e}(b^s)}{db} = \frac{\hat{\pi}'(e^s)}{\hat{\pi}''(e^s)} \frac{u'(b^s)}{u(w) - u(b^s)}$ , the result immediately follows.  $\square$

### Condition for Concavity of the Maximization Problem

The program is strictly concave for the social planner if

$$\begin{aligned} & \eta + \pi''(e) [u(w) - u(b)] + \pi'(e) \left( 2 \frac{u'(w - \tau)}{u'(b)} - 1 \right) \xi + \\ & \quad (1 - \pi(e)) \left( 1 - \frac{u'(w - \tau)}{u'(b)} \right) \left( \zeta + 2\xi \frac{\hat{\pi}''(e)}{\hat{\pi}'(e)} \right) < 0, \end{aligned}$$

and for the private insurer if

$$\begin{aligned} & \eta + \pi'(e) \left[ 2 \frac{u'(w - \tau)}{u'(b)} - \frac{\hat{\pi}'(e)}{\pi'(e)} \right] \xi + \\ & \quad (1 - \hat{\pi}(e)) \left[ 1 - \frac{1 - \pi(e)}{1 - \hat{\pi}(e)} \frac{u'(w - \tau)}{u'(b)} \right] \left( \zeta + 2\xi \frac{\hat{\pi}''(e)}{\hat{\pi}'(e)} \right) < 0, \end{aligned}$$

for all  $(e, b, \tau)$  satisfying IC and BC/ZPC with

$$\begin{aligned} \eta &= \frac{(1-p)}{p} u''(w-\tau) \left\langle \pi'(e)b - \frac{1-\pi(e)}{u'(b)} \xi \right\rangle^2 \\ &\quad + \pi''(e) u'(w-\tau)b + (1-\pi(e)) \frac{u'(w-\tau)}{u'(b)} \frac{u''(b)}{u'(b)^2} \xi^2 \\ \xi &= \frac{\hat{\pi}''(e)}{\hat{\pi}'(e)^2} \\ \zeta &= \frac{\hat{\pi}'''(e)}{\hat{\pi}'(e)^2}. \end{aligned}$$

Notice that every single term in  $\eta$  is negative as is  $\xi$ . The last term in the conditions may be positive, but is small if  $\frac{u'(w-\tau)}{u'(b)}$  is close to 1. One can find  $\bar{p} < 1$  such that for all  $p > \bar{p}$  the optimization problem is globally concave.  $\square$

## 1.8 Appendix B: Calibration of the Dynamic Model

The unit of time is one month. The monthly discount factor equals  $\beta = 0.9956$ , which corresponds to a yearly discount factor equal to 0.95. I assume that the monthly output equals 1 when employed and 0 when unemployed. I consider the probability of starting employed  $p = 1/2$ .<sup>30</sup> The agents have CARA preferences with monetary costs of efforts and absolute risk aversion  $\sigma = 2$ . Both the true and perceived monthly probability of finding work are assumed not to change throughout the unemployment spell, other than through changes in effort.

**True Probability of Finding Work** I assume that effort  $e$  is linear in the number of times a job seeker reports to have engaged in any of the search activities discussed in section 1.4.3. I rescale this effort variable such that  $e = 0$  corresponds to not having searched in any dimension during the entire month and  $e = 1$  corresponds to having searched every day in every dimension, averaged over the entire month. In this interpretation,  $e = 0.15$  corresponds to the sample average of search effort (i.e. search in all dimensions between ‘once every couple of weeks’ and ‘every week’). For these three values of search effort, the probability function

$$\pi(e) = \pi_0 + \pi_1 \times e^{0.62} \text{ with } \pi_0 = 0.1448 \text{ and } \pi_1 = 0.1417$$

approximates the average duration of unemployment, estimated using ordinary least squares (Table 2).

**Perceived Probability of Finding Work** The empirical section suggests strong baseline optimism and some control pessimism. I assume that the true monthly probability

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<sup>30</sup>The simplification that employment is an absorbent state and the horizon is infinite makes a calibrated choice of  $p$  difficult. The NLSY 1979 shows that only 10.1 percent of the individuals does not experience any unemployment spell between age 18 and 40. However, many of the individuals who are unemployed at some point, are unemployed when they are young. Between age 36 and 40, 74.7 percent of the individuals do not experience any unemployment spell.

of finding work as a function of effort equals

$$\hat{\pi}(e) = \hat{\pi}_0 + \hat{\pi}_1 \times e^{0.62} \text{ with } \hat{\pi}_0 = 0.3488 \text{ and } \hat{\pi}_1 = 0.0921.$$

This implies an optimistic relative baseline bias  $\frac{\hat{\pi}(e) - \pi(e)}{\pi(e)}$  equal to 100 percent (at the average effort level  $e = 0.15$ ) and a pessimistic relative control bias  $\frac{\hat{\pi}'(e) - \pi'(e)}{\pi'(e)}$  equal to  $-35$  percent. Notice that the baseline bias is more modest than the average baseline bias in the sample of about 200 percent. The control bias corresponds to the relative ratio of the least squares estimates of the actual and perceived impact of search (Table 2).

**Monetary Cost of Effort** I finally calibrate the monetary cost of search function

$$\psi(e) = \psi_0 e^{\psi_1},$$

in order to match the empirical exit rate and unemployment duration elasticity. I assume that the monetary cost of effort when employed equals the monetary cost of searching daily in every dimension  $\psi(1) = \psi_0$ . For the standard specification of beliefs, I find  $\psi_0 = 0.475$  and  $\psi_1 = 2.34$ . I recalibrate these parameters for the alternative beliefs specifications such that the implied exit rate and unemployment duration elasticity given the current UI system remain constant. If beliefs were to be unbiased, the calibrated parameter values are  $\psi_0 = .477$  and  $\psi_1 = 1.16$ .

**Implied Exit Rate and Search Elasticity** In the US, unemployed workers are eligible for unemployment benefits for six months. The mean and median replacement rate for which the unemployed workers are eligible equal respectively 0.43 and 0.48. When implementing a contract that pays  $b = 0.45$  in the first six months and  $b = 0$  afterwards, the standard specification predicts an average monthly probability of finding work equal to 0.19. This equals the average monthly exit rate in the sample. Moreover, the implied elasticity of unemployment duration with respect to a constant benefit level  $b = 0.45$  equals  $-.5$ . This corresponds to the empirical estimates reviewed in Krueger and Meyer (2002).

## 1.9 Appendix C: Tables

**Table 1:** Summary Statistics

	Obs.	Mean	Std. Dev.
<b>Demographics</b>			
Male	1339	.55	.50
Age	1339	38.48	9.96
White	1339	.67	.47
Married	1339	.81	.39
Children	1339	1.30	1.25
Education	1334	13.63	2.14
Maryland	1339	.45	.50
<b>Partner</b>			
Employed	1139	.79	.41
Education	1137	13.50	2.20
<b>Employment</b>			
Monthly wage before unemp.	1320	2595	1720
Times unemployed	1339	.34	.47
Weeks since displacement	1339	6.91	4.16
Search (at First Interview)	1249	3.34	.87
Search (at Third Interview)	1249	3.35	.95

**Table 2:** OLS Estimates of the Effect of Search and Covariates on the Actual Duration of Unemployment (1), the Expected Duration of Unemployment (2) and the Difference Between the Actual and the Expected Duration of Unemployment (3)

	Actual duration (1)	Expected duration (2)	Optimism (3)
Search	-2.961** (.747)	-2.193** (.371)	-1.359 (.780)
<b>Demographics</b>			
Male	-4.045** (1.351)	-1.816** (.446)	-1.579 (1.389)
Age	.197** (.071)	.049* (.020)	.153* (.072)
White	-5.122** (1.412)	-.856 (.620)	-5.278** (1.544)
Married	-3.946* (1.806)	.397 (.528)	-5.373** (1.888)
Children	.809 (.513)	.409 (.257)	.373 (.544)
Education	-.457 (.357)	.275* (.127)	-.659 (.362)
Maryland	-2.456 (1.297)	-.185 (.478)	-3.091* (1.343)
<b>Partner</b>			
Employed	-2.098 (1.565)	.464 (.477)	-3.022 (1.641)
Education	-.059 (.319)	.240* (.119)	-.192 (.333)

*Continued*

**Table 2 (continued)**

	Actual duration	Expected duration	Optimism
	(1)	(2)	(3)
<b>Employment</b>			
Monthly wage before unemp.	-.0006 (.0004)	.0006** (.0002)	-.001** (.0004)
Times unemployed	-.723 (1.307)	-.558 (.435)	-.925 (1.317)
Weeks since displacement	.319* (.157)	.170** (.065)	.294 (.163)
Obs.	1120	1095	1007
R <sup>2</sup>	.073	.125	.078

Robust standard errors are in parentheses. \* denotes statistical significance at the 5 percent level, \*\* at the 1 percent level.

**Table 3:** OLS Estimates of the Effect of Search for Alternative Specifications. Dependent Variables: the Actual Duration of Unemployment (1), the Expected Duration of Unemployment (2) and the Difference Between the Actual and the Expected Duration of Unemployment (3)

	Actual Duration (1)	Expected Duration (2)	Optimism (3)
Panel A: Using Later Measure of Search			
Search	-4.535** (.716)	-.956** (.308)	-3.587** (.747)
Obs.	942	868	852
Panel B: Include Ideal Duration under Certainty			
Search	-1.990** (.764)	-1.281** (.363)	-.991 (.817)
Ideal duration	.328** (.118)	.376** (.109)	.094 (.118)
Obs.	1098	1081	994
Panel C: Complete Spells Only			
Search	-1.096 (.613)	-1.436** (.301)	-.103 (.684)
Obs.	976	886	886
Panel D: Hazard Model with Weibull Distribution			
Search	1.19** (.046)	1.32** (.049)	
Obs.	1120	1068	

Robust standard errors are in parentheses. \* denotes statistical significance at the 5 percent level, \*\* at the 1 percent level. Other covariates are as in Table 2.

**Table 4:** 2SLS Estimates of the Effect of Search. Dependent Variables: Actual Duration of Unemployment (1), Expected Duration of Unemployment (2) Difference Between the Actual and the Expected Duration of Unemployment (3), and the First Stage Regression (4)

	Actual Duration (1)	Expected duration (2)	Optimism (3)	First Stage (4)
Search	-6.315* (3.159)	-4.634** (1.442)	-2.339 (3.021)	
Potential Benefit				-.0004 (.0006)
Job Importance				.270** (.032)
Covariates	✓	✓	✓	✓
Observations	1116	1092	1004	1004
R <sup>2</sup>	.059	.069	.077	.167
Overidentification	.015	.097	0.149	

Robust standard errors are in parentheses. \* denotes statistical significance at the 5 percent level, \*\* at the 1 percent level. Other covariates are as in Table 2. Overidentification reports the p-value for the overidentification test using Hansen J statistic.

## Chapter 2

# Training and Search during Unemployment

Optimal unemployment insurance trades off the provision of incentives to search for work and the insurance against the consequences of unemployment. The obvious consequence of unemployment is the foregone wage while unemployed. However, after returning to work, many still have substantially lower wages than before displacement. In the US, one fourth of the re-employed have wages that were at least 25% lower than in their previous job (Kling 2006). It has been argued that these future income losses for the unemployed are due to the loss of human capital. Displaced workers lose human capital the moment they lose their job and their human capital continues to depreciate during unemployment. Unemployment insurance should therefore insure the unemployed against both the loss of current earnings and the expected loss of future earnings. At the same time, incentives for search are more important for a given level of human capital if finding a job avoids further depreciation of human capital.

Effective training programs counter the loss of human capital. Many countries are increasing the emphasis on training to re-integrate the unemployed in the workforce. Spending on labor market programs, active and passive, averages 3 percent of GDP in the OECD countries. The proportion of spending on active labor market programs rather than on unemployment benefits has increased to 40-50 percent in most European countries, of

which on average 40 percent is spent on training. The impact of training programs has been estimated in the empirical literature. An important conclusion of this literature is the heterogeneity in impact of the different programs (Heckman et al. 1999). More recent work supports the positive long-run effect of training programs with a substantial human capital component (Jacobson et al. 2005, Jespersen et al. 2004, Lechner et al. 2005, Winter-Ebmer 2006).

This paper analyzes the role of training for the design of unemployment insurance. I characterize the optimal unemployment insurance contract, specifying both consumption and training contingent on the duration of the unemployment spell. I consider a model in which a worker's human capital decreases during unemployment, but training efforts counter this decrease.<sup>1</sup> The training efforts are imposed by the social planner, while the search efforts to find a job are chosen by the unemployed worker. The unemployed worker bears the cost of both the search and training efforts, which are allowed to interact as in Holmström and Milgrom (1991). More training may increase the marginal cost of search. I assume that the same training technology is not available on the job. One justification is that employers are not willing to provide training that is not specific to their firm.

If the training technology is sufficiently effective, the unemployed worker is in one of three states depending on the level of human capital:

- In the *training state*, the level of human capital is so low that no search is induced. Training efforts are imposed to increase the level of human capital. Since no incentives are needed, the social planner can fully smooth the unemployed's consumption.
- In the *training-and-search state*, human capital is sufficiently high so that search efforts are induced. The social planner faces the trade-off between providing insurance and incentives. The depreciation of human capital increases both the value of insurance and the need for incentives. By mitigating the depreciation, training efforts relax the trade-off. The design of the optimal contract for an unemployed agent in the training-and-search state will be dependent on the complementarity

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<sup>1</sup>I ignore the use of training programs to screen and target unemployment benefits (Akerlof 1978, Besley and Coate 1992).

between search and training in the expected value of finding a job and the rivalry in the cost structure.

- In a *stationary state*, the social planner makes the unemployed maintain the same level of human capital by following training programs. At the same time they are given incentives to search for a job.

I characterize analytically how consumption during unemployment and upon re-employment depends on the length of the unemployment spell. As long as search is induced, the introduction of training does not change the result by Shavell and Weiss (1979) that unemployment consumption should be decreasing over time when preferences are additive in consumption and search efforts. However, in the training state, no search is induced and unemployment consumption remains constant. The intuition of Hopenhayn and Nicolini (1997) that taxes upon re-employment should increase with the duration of the unemployment spell does not generalize with the introduction of human capital depreciation and training. The social planner wants to protect the unemployed against human capital losses and may prefer to subsidize employment, even after long unemployment spells.

I perform numerical simulations for CARA preferences with monetary costs of efforts. I show that for such preferences the state space of the recursive problem becomes one-dimensional. The numerical simulations suggest that the human capital of the long-term unemployed converges globally to a unique stationary level. This has two important policy implications. First, if training costs are not too high, it is never optimal to discourage the unemployed worker from search activity, whatever the length of the unemployment spell. This contrasts with Pavoni (2009) and Pavoni and Violante (2007). Without training technology, they show that after a finite number of unsuccessful searches, the social planner switches to social assistance, an absorbent policy characterized by constant unemployment benefits and no active participation. Second, the difference between this unique, stationary level and the level of human capital at the start of the unemployment spell determines the optimal timing of training. If the initial level of human capital is lower, training is more intensive towards the start of unemployment. If the initial level of human capital is higher, training becomes more intensive throughout unemployment.

The human capital level at the start and the stationary level are determined by respectively the fall in human capital upon displacement and the depreciation in human capital during unemployment. Although in practice training is more focused towards the long-term unemployed, this is only optimal if the depreciation in human capital is relatively more important than the fall upon displacement. Upon displacement, the unemployed may lose firm-specific human capital. They also lose human capital specific to the industry if they are re-employed in a different industry (Neal 1995, Ljungqvist and Sargent 1998). These losses may become very important in an economy with declining industries or industries shifting production abroad. The depreciation during unemployment can be interpreted as the explicit loss of skills during unemployment or as a process of “unlearning by not doing” (Coles and Masters 2000). If unemployment spells persist for a long time, unemployed workers can get detached from the labor market, lose work habits and confidence in the own skills (Falk et al. 2006b). Although the empirical evidence for the fall and depreciation of human capital is mixed<sup>2</sup>, both have been central in explaining the persistence of unemployment and the European unemployment dilemma (Pissarides 1992, Ljungqvist and Sargent 1998, Machin and Manning 1999), as well as the negative duration dependence of exit rates (Blanchard and Diamond 1994, Acemoglu 1995).

This paper builds on a recent literature that departs from stationary search models (Shavell and Weiss 1979, Hopenhayn and Nicolini 1997) with the introduction of depreciating human capital. Shimer and Werning (2006) analyze the optimal timing of benefits in a McCall search model, assuming that savings are not observable. Human capital depreciation reduces the arrival rate of job offers or deteriorates the distribution of the wages being paid on the job. Pavoni (2009) analyzes the optimal unemployment insurance contract when the unemployed agent has the binary choice to exert costly search effort or not. The depreciation of human capital reduces the output upon re-employment and the probability to become employed if searching. In this paper, I assume that human capital only determines the output. Since search is a continuous choice in my model, the

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<sup>2</sup>Whereas Frijters and van der Klaauw (2006) show that the wage distribution deteriorates significantly, in particular during the first six months, Card et al. (2007) and Van Ours and Vodopivec (2006) find no significant effect of an increase in unemployment duration on either the wage or the duration of employment in the new job.

decrease in output due to the depreciation reduces the returns to search. The probability to become employed endogenously decreases during the unemployment spell if no training technology is available. Pavoni and Violante (2007) introduce costly job monitoring as an alternative to the provision of incentives and analyze the optimal sequencing of different unemployment policies. Pavoni and Violante (2005) and Wunsch (2008) also introduce a training technology in the numerical simulations of the model in Pavoni and Violante (2007). In contrast with my approach, training efforts cannot be imposed, but are induced by rewarding the unemployed for high values of human capital with higher unemployment benefits. Training and search efforts are also assumed to be extreme rivals and cannot be both exerted in the same period.

The paper is organized as follows. In Section 2.1, the model is presented. In Section 2.2 and 2.3, I set up the social planner's problem and I characterize the optimal unemployment insurance contract. In Section 2.4, I show how the recursive problem simplifies for CARA preferences with monetary costs of efforts. Using these simplifications, I present some numerical exercises in Section 2.5. I focus on the determinants of the value and optimal timing of training. I also consider the optimal policy for the long-term unemployed. The last section concludes.

## 2.1 Model

I consider an agent with human capital  $\theta$  at the start of an unemployment spell. During each period of unemployment, the agent exerts efforts in two dimensions, search and training. Search effort  $s$  increases the probability  $\pi(s)$  to find a job with  $\pi' > 0 \geq \pi''$ . Training effort  $t$  increases the unemployed's human capital  $\theta$ . Efforts are costly and the marginal cost of search may increase with the level of training. I assume a convex cost function  $\psi(s, t)$  with  $\psi_{s,t} \geq 0$ .<sup>3</sup> When re-employed, the agent produces  $y(\theta)$  with

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<sup>3</sup>Organizing training sessions may be costly for the social planner. Training can also capture temporary employment in public jobs and therefore be productive. We assume that all costs are borne by the unemployed and captured by  $\psi(s, t)$ . However, the social planner compensates the unemployed agent for these costs.

$y' > 0 \geq y''$  and  $y(0) = 0$ .<sup>4</sup> I assume that employment is an absorbent state. Once the unemployed agent has found a job, he remains employed forever.<sup>5</sup> I denote the consumption level during unemployment and upon re-employment by  $c$  and  $c^e$  respectively.

**Preferences** I denote by  $u(c, \psi(s, t))$  and  $u^e(c^e)$  the per-period utility during unemployment and employment respectively. The expected life-time utility for an agent starting in unemployment equals

$$u(c_0, \psi(s_0, t_0)) + \sum_{\tau=1}^{\infty} \beta^{\tau} [\pi_{\tau}^e u^e(c_{\tau}^e) + (1 - \pi_{\tau}^e) u(c_{\tau}, \psi(s_{\tau}, t_{\tau}))],$$

where the probability to be employed in period  $\tau$  equals  $\pi_{\tau}^e = \pi_{\tau-1}^e + \pi(s_{\tau-1})(1 - \pi_{\tau-1}^e)$ . I focus on two preference specifications: preferences additive in consumption and cost of efforts,

$$u(c, \psi(s, t)) = u(c) - \psi(s, t) \text{ and } u^e(c^e) = u(c^e),$$

and CARA preferences with monetary cost of efforts,

$$u(c, \psi(s, t)) = -\exp(-\sigma(c - \psi(s, t))) \text{ and } u^e(c^e) = -\exp(-\sigma c).$$

**Human Capital** Human capital decreases during unemployment. First, human capital falls immediately when the agent loses his job. Second, human capital continuously depreciates during unemployment. I only model the depreciation in human capital explicitly, but characterize the optimal contract as a function of the level of human capital at the start of the unemployment spell. Training increases human capital. I assume that no training choice is available during employment such that the level of human capital remains constant once re-employed. The depreciation may capture the loss of job-skills, self-confidence or work habits, changing preferences of employers during unemployment

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<sup>4</sup>I ignore efforts during employment. With monetary costs of efforts, this is only a rescaling of the net-output produced during employment.

<sup>5</sup>I only model incentive problems during unemployment. Wang and Williamson (1996), Zhao (2000) and Hopenhayn and Nicolini (2009) also analyze incentive problems on the job and assume multiple unemployment spells.

or even the foregone learning-by-doing.

An unemployed agent with human capital  $\theta_\tau$ , exerting training effort  $t_\tau$  in period  $\tau$ , will have human capital at  $\tau + 1$  equal to

$$\theta_{\tau+1} = m(\theta_\tau, t_\tau) \text{ with } m_\theta(\theta_\tau, t_\tau) > 0, m_t(\theta_\tau, t_\tau) > 0 \text{ and } m(\theta_\tau, 0) \leq \theta_\tau.$$

I focus on exponential depreciation with linear training technology,  $m(\theta, t) = \theta(1 - \delta) + t$ , such that both the foregone income and the decrease in expected future income due to unemployment are increasing in the level of human capital. Without training, the human capital of long-term unemployed converges to 0 for which there is no added-value of being employed.

## 2.2 Social Planner's Problem

I assume that the social planner has three instruments at his disposal: unemployment consumption  $\{c_\tau\}$ , employment consumption  $\{c_\tau^e\}$  (or wage taxes) and training  $\{t_\tau\}$ . This consumption profile can be implemented by setting unemployment benefits equal to  $c_\tau$  and taxes upon re-employment equal to  $y(\theta_\tau) - c_\tau^e$ , but only if savings can be restricted (Werning 2002, Shimer and Werning 2008). I also assume that human capital is observable, so the social planner can make the consumption and training levels dependent on the level of human capital.<sup>6</sup> Search efforts are not observable, but have to be induced through the contract scheme.

Rather than maximizing the expected life-time utility subject to the social planner's budget constraint, I follow the dual approach and minimize the expected costs of the insurance scheme providing a given level of expected life-time utility  $V$ . As in Spear and Srivastava (1987), the optimal contract can be written recursively. Two state variables, the level of human capital and the expected discounted utility promised last period to the

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<sup>6</sup>I assume that the depreciation rate only depends on the level of human capital. If the duration of the unemployment spell is contractible as well, it suffices that the level of human capital at the beginning of the unemployment spell is contractible and the depreciation function  $m(\theta, t)$  is known. The wage before becoming unemployed may be a good proxy for the level of human capital at the start of unemployment.

unemployed agent, summarize all relevant aspects of the agent's unemployment history.

The optimal contract  $\{c(V, \theta), V^e(V, \theta), V^u(V, \theta), s(V, \theta), t(V, \theta)\}$ , assigning a given expected life-time utility level  $V$  to the unemployed individual with human capital  $\theta$ , solves

$$C(V, \theta) = \min_{c, V^e, V^u, s, t} c + \beta [\pi(s)C^e(V^e, m(\theta, t)) + (1 - \pi(s))C(V^u, m(\theta, t))]$$

such that

$$\begin{aligned} u(c, \psi(s, t)) + \beta[\pi(s)V^e + (1 - \pi(s))V^u] &\geq V \\ s &\in \arg \max u(c, \psi(s, t)) + \beta[\pi(s)V^e + (1 - \pi(s))V^u]. \end{aligned}$$

The expected cost for the social planner consists of the cost this period and the expected cost from the next period on. The cost this period equals the unemployment consumption  $c$ . The expected cost from tomorrow on depends on whether the agent finds work today, the respective promised utilities  $V^e$  and  $V^u$  and the level of human capital. The social planner is restricted to contracts for which the agent's expected utility is higher than  $V$ . This is captured by the promise-keeping constraint. Also, the search efforts of the unemployed agent cannot be observed. The agent chooses the search level that maximizes his expected utility given the contract, which is captured by the incentive compatibility constraint. When choosing search efforts, the unemployed agent does not take into account the change in the expected cost for the social planner. Therefore, the search efforts are smaller than what the social planner would impose if search was contractible. In the second-best contract, the social planner refrains from providing full insurance in order to give incentives for search. To guarantee the incentive compatibility of the contract, the first order condition of the agent's optimization problem is sufficient if  $u(c, \psi(s, t))$  is concave in  $s$ .

The expected cost for the social planner to assign  $V^e$  to the agent when he found a job, equals

$$C^e(V^e, m(\theta, t)) = \min_{c^e, \hat{V}^e} c^e - y(m(\theta, t)) + \beta C^e(\hat{V}^e, m(\theta, t))$$

such that

$$\frac{u^e(c^e)}{1-\beta} \geq V^e.$$

Since there is no asymmetric information once the agent is re-employed, it is optimal to keep promised utility constant and give the same level of consumption in every future period. Hence,

$$C^e(V^e, m(\theta, t)) = \frac{(u^e)^{-1}((1-\beta)V^e) - y(m(\theta, t))}{1-\beta}.$$

The social planner's problem during unemployment simplifies to

$$C(V, \theta) = \min_{c, V^u, V^e, s, t} c + \beta \left[ \pi(s) \frac{(u^e)^{-1}((1-\beta)V^e) - y(m(\theta, t))}{1-\beta} + (1-\pi(s))C(V^u, m(\theta, t)) \right]$$

such that

$$V - u(c, \psi(s, t)) - \beta[\pi(s)V^e + (1-\pi(s))V^u] \leq 0 \quad (\lambda)$$

$$u_\psi(c, \psi(s, t))\psi_s(s, t) + \beta\pi'(s)[V^e - V^u] \leq 0. \quad (\mu)$$

I proceed under the assumption that  $C(V, \theta)$  is convex for the relevant pairs  $(V, \theta)$ .<sup>7</sup> The first order conditions and the two envelope conditions are in appendix.

## 2.3 Optimal Insurance Contract

In this section, I characterize how training and consumption change during unemployment. Three different states are relevant if the cost of training is sufficiently low. In the *training state*, the level of human capital is so low that no search is induced. The unemployed agent only trains to increase his human capital above a threshold  $\underline{\theta}$  for which it is optimal to start looking for work. In this *search and training state*, the unemployed agent exerts efforts in the two dimensions. The human capital of the unemployed agent who has never been successful in finding a job converges to a stationary level  $\theta^*$ . In this *stationary*

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<sup>7</sup>In the numerical simulations, I find that for  $y(\theta)$  sufficiently concave the value function is indeed convex.

*state*, the unemployed agent maintains his human capital with a constant level of training  $t^* = \delta\theta^*$ , while at the same time he spends efforts on search. This contrasts with the *social assistance state* which is characterized by constant lifetime payments and no active participation by the unemployed agent (Pavoni and Violante 2007). The social planner switches to social assistance in finite time if two conditions hold. First, no training technology is available or, if available, the cost of the technology is too high. Second, the marginal cost of search exceeds the marginal benefits at the level to which human capital depreciates without training.

Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) show for additive preferences that unemployment consumption and consumption upon re-employment should be strictly decreasing with the length of the unemployment spell. With the introduction of training, unemployment consumption  $c$  is still strictly decreasing in the search and training state, but constant in the training state. For CARA preferences with monetary costs of efforts, promised utility  $V$  is strictly decreasing in the search and training state, while net-consumption  $c - \psi$  is constant in the training state. For both preferences, it cannot be optimal that consumption upon re-employment  $c^e$  is always increasing with the length of the unemployment spell. All proofs are in the appendix.

### 2.3.1 Training State

If the level of human capital at the start of the unemployment spell is too low, the social planner starts training the unemployed before inducing any search. Since the desired level of search is zero,  $\mu$ , the Lagrange multiplier on the incentive compatibility constraint, is equal to zero as well. This implies that the first order conditions for the consumption levels coincide with those in the first best. The shadow price of the promised utility, i.e. the Lagrange multiplier on the promise-keeping constraint  $\lambda$ , remains constant during the unemployment spell, as long as no search is induced.

**Proposition 10** *If no search is induced ( $s_\tau = 0$ ),  $\Delta_{\lambda,\tau} = \lambda_\tau - \lambda_{\tau+1} = 0$ , as in the first best.*

Since  $\lambda$  equals the inverse of the marginal utility of consumption if  $\mu = 0$ , the marginal utility of consumption remains constant during unemployment in the training state. This leads to the following results for additive preferences and preferences with monetary costs.

**Corollary 4** *If no search is induced, unemployment consumption  $c$  is constant for additive preferences.*

**Corollary 5** *If no search is induced, unemployment net-consumption  $c - \psi$  is constant for preferences with monetary costs of efforts.*

If human capital is too low and training is available, no incentives for search are needed, as in the first best. Consequently, the social planner can completely insure the unemployed agent. For additive preferences, this implies constant unemployment consumption, in contrast with the result of Shavel and Weiss (1979). However, the result that only training will be positive depends on the assumption that a similar training technology is not available on the job.

**Optimal Training** If the agent remains unemployed with certainty, the Euler equation characterizing training simplifies for both additive and CARA preferences with monetary costs to

$$\psi_t(0, t_{\tau-1}) = \beta(1 - \delta) \psi_t(0, t_{\tau}).$$

When increasing training at  $\tau - 1$  by one unit, training at  $\tau$  can be decreased by  $1 - \delta$  units without changing the level of human capital at  $\tau + 1$ . With no prospects for employment, deferring training is desirable, because the effect of training depreciates over time and the cost of future training is discounted.

**Proposition 11** *As long as no search is induced ( $s_{\tau-1} = s_{\tau} = 0$ ), the optimal level of training is increasing over time ( $t_{\tau-1} < t_{\tau}$ ) for additive and CARA preferences and exponential decay.*

### 2.3.2 Search and Training State

When the level of human capital is high enough, the social planner is willing to give up full insurance in order to induce the unemployed to search for a job. This implies that the shadow price of promised utility is decreasing during unemployment in the search and training state. The intuition is that increasing promised utility tomorrow relaxes the promise-keeping constraint today, but also decreases the incentives to search for a job today. Hence, the shadow price of promised utility tomorrow equals the shadow price of promised utility today minus its impact on incentives for search today.

**Proposition 12** *If search is positive ( $s_\tau > 0$ ),  $\Delta_{\lambda,\tau} = \lambda_\tau - \lambda_{\tau+1} > 0$ .*

If search is positive, the result by Shavell and Weiss (1979) that unemployment consumption is decreasing still holds when human capital is depreciating or training is introduced, as long as the utility of consumption and the utility cost of efforts remain additive. If unemployment consumption is constant, the social planner could increase consumption this period and decrease consumption next period such that the social planner's expected costs for the initial level of search remain the same. Since the consumption levels and therefore the marginal utilities are initially the same, the change in the consumption pattern has only a second order effect on the expected utility of the unemployed agent. The reduction in tomorrow's unemployment consumption will induce a higher search level though. The change in search will have no first-order effect on the expected utility of the agent, but will decrease the expected payments to be made by the social planner.

**Corollary 6** *If search is positive, unemployment consumption  $c$  decreases for additive preferences.*

However, if preferences are not additive, the Shavell and Weiss argument may not hold for two reasons. First, if efforts are changing over time, equality of unemployment consumption may not imply equality of marginal utility with respect to consumption. Second, the decrease in consumption next period may increase the marginal cost of search next period. The implied decrease in search next period may outweigh the increase in

search this period. With monetary costs of efforts, equality of net-consumption implies equality of marginal utilities. However, a decrease in net-consumption still increases the marginal cost of search.

**Corollary 7** *If search is positive, the promised utility during unemployment  $V$  decreases for CARA preferences with monetary costs of efforts.*

The optimal contract induces search by spreading incentives over all future periods of unemployment, giving more insurance to the unemployed. In general, this does not result in a decreasing unemployment consumption path.

The same improvement in trade-off between incentives and insurance is obtained by changing taxes upon re-employment depending on the duration of the unemployment spell. From the first order conditions with respect to  $V^e$  and  $V^u$ , I find that

$$C_V(V, \theta) = \pi(s)C_V^e(V^e, m(\theta, t)) + (1 - \pi(s))C_V(V, m(\theta, t)). \quad (2.1)$$

The shadow price of promised utility when unemployed equals the weighted average of the shadow prices of promised utility when respectively employed and unemployed tomorrow. With preferences that are additive in consumption and efforts, the shadow price of promised utility equals the inverse of the marginal utility of consumption, i.e.  $C'_V(V, \theta) = \frac{1}{u'(c)}$ . Hence, for additive preferences condition (2.1) is exactly the Rogerson condition that the expectation of the inverse of the marginal utilities has to remain constant over time. Hopenhayn and Nicolini (1997) find that without human capital decay and training, this implies that consumption (taxes) upon re-employment is decreasing (increasing) for long-term unemployed, spreading the punishment over all future states. I generalize this result in Corollary 8.

**Corollary 8** *If search is positive, consumption upon re-employment in the next period must exceed unemployment consumption and net-consumption this period, for additive and CARA preferences respectively. Moreover, consumption upon re-employment cannot remain constant or always be increasing with the length of the unemployment spell.*

In the framework of Hopenhayn and Nicolini (1997) without decay of human capital, decreasing consumption upon re-employment and during unemployment implies increasing taxes and decreasing replacement ratio's with respect to the potential wage. With decay of human capital, the social planner insures the unemployed against the loss of human capital (or the costs of regaining human capital through training) as well. The level of human capital and its expected depreciation determine the need for incentives and the need for insurance through training and transfers, but the sequence of consumption levels will also be determined by the consumption smoothing preferences of the unemployed. This disconnects the consumption levels during unemployment and upon re-employment from the (potential) productivity of the agent. Hence, depending on the human capital and the promised utility at the start of unemployment, taxes can be increasing or decreasing and positive or negative. Similarly, the replacement ratio with respect to potential wage is not necessarily monotone and decreasing over the unemployment spell. Unemployment consumption may be even higher than the unemployed's productivity.

**Optimal Training** The depreciation of human capital increases the value of search efforts as well as the value of insurance. However, the consumption schedule cannot provide more incentives for search without reducing insurance. By countering the depreciation, training is a valuable alternative. Training efforts interact with the unobservable search efforts. Higher training implies higher output upon re-employment and therefore increases the returns to search. However, training may also increase the marginal cost of search. Hence, search and training are complements with respect to the expected return to finding work, but may be substitutes with respect to costs. Since the marginal return to training is higher for low levels of human capital if  $y''(\theta) \leq 0$ , training and search are more complementary for low levels of human capital. Compared to the training state, the impact of training on the incentives for search and on production, if in the next period a job is

found, becomes relevant. For additive preferences, the Euler equation equals

$$\frac{\psi_t(s_{\tau-1}, t_{\tau-1})}{u'(c_{\tau-1})} + \mu_{\tau-1} \psi_{s,t}(s_{\tau-1}, t_{\tau-1}) = \beta \pi(s_{\tau-1}) \frac{y'(\theta_\tau)}{1-\beta} + \beta(1-\pi(s_{\tau-1})) \left[ \frac{\psi_t(s_\tau, t_\tau)}{u'(c_\tau)} + \mu_\tau \psi_{s,t}(s_\tau, t_\tau) \right] (1-\delta).$$

For CARA preferences with monetary costs, the Euler equation equals

$$\psi_t(s_{\tau-1}, t_{\tau-1}) + \mu_{\tau-1} u'(c_{\tau-1} - \psi_{\tau-1}) \psi_{s,t}(s_{\tau-1}, t_{\tau-1}) = \beta \pi(s_{\tau-1}) \frac{y'(\theta_\tau)}{1-\beta} + \beta(1-\pi(s_{\tau-1})) \left[ \psi_t(s_\tau, t_\tau) + \mu_\tau u'(c_\tau - \psi_\tau) \psi_{s,t}(s_\tau, t_\tau) \right] (1-\delta).$$

For both types of preferences, increasing  $t_{\tau-1}$  increases the cost of training efforts, but changes also the incentives by increasing the marginal cost of search if  $\psi_{s,t} > 0$ . On the other hand, the increase in training at  $\tau - 1$  increases human capital at  $\tau$ . This increases the output when a job is found. When no job is found, this increase allows to decrease training at  $\tau$  to bring human capital back to the same level at  $\tau + 1$  if  $t_{\tau-1}$  had not been increased. In section 2.5, I use numerical simulations to get more insights in the value of training and its timing during the unemployment spell.

### 2.3.3 Stationary State

In a stationary state, a positive level of human capital is maintained with training effort equal to the depreciation if the cost of training is low enough. At the same time, the unemployed agent spends constant effort on search and leaves unemployment with positive probability. In the next section, I show how the recursive problem simplifies for CARA preferences with monetary costs of efforts. For these preferences, both net-consumption  $c - \psi$  and consumption upon re-employment  $c^e$  are decreasing with the length of the unemployment spell in the stationary state. I also characterize the stationary levels of search and training without solving for the value function  $C(V, \theta)$ .

The numerical simulations in section 2.5 suggest that the human capital of an agent who has never been successful in finding a job converges to a positive stationary level

of human capital  $\theta^*$  if the cost of training is sufficiently low. The convergence is global. Independent of the level of human capital at the start  $\theta_0$ , human capital converges to this unique level. Finally, the numerical simulations suggest that this convergence is monotone.

## 2.4 CARA Preferences with Monetary Costs

In this section, I focus on CARA preferences as in Werning (2002) and Shimer and Werning (2006) and assume monetary costs of efforts as in Holmström and Milgrom (1991). For this specification, I show that the value function is additive in  $V$  and  $\theta$ , i.e.  $C(V, \theta) = h(V) - g(\theta)$ . I guess and verify  $h(V)$ , which only leaves  $g(\theta)$  to be approximated numerically.

The optimal response to an increase in promised utility  $V$  is to increase all consumption levels by the same amount, regardless of the level of human capital. Increasing the consumption levels equally, today and in the future, while employed and unemployed, leaves the margins for search and training unchanged. For search, this is clear from the incentive compatibility constraint and the properties of CARA preferences. Since  $u(x + y) = -u(x)u(y)$  and  $u(x) = -\frac{u'(x)}{\sigma}$ , the promised utilities  $V^e$  and  $V^u$  and marginal utility  $u'(c - \psi)$  are all rescaled by  $-u(\varepsilon)$  after an  $\varepsilon$ -increase in all consumption levels. Hence, the incentive compatibility constraint,

$$\beta\pi'(s)[V^e - V^u] = u'(c - \psi)\psi_s(s, t),$$

remains binding after an equal increase in all consumption levels. The fact that an equal increase in all consumption levels is an optimal response to an increase in  $V$  implies that the promised utilities  $V^e$  and  $V^u$  and current-period utility  $u(c - \psi)$  are proportional to life-time utility  $V$ , for a given level of human capital  $\theta$ . I can rewrite the optimal contract

as  $\{\alpha_u(\theta), \alpha_{V^e}(\theta), \alpha_{V^u}(\theta), s(V, \theta), t(V, \theta)\}$  with

$$V^e(V, \theta) = \alpha_{V^e}(\theta)V$$

$$V^u(V, \theta) = \alpha_{V^u}(\theta)V$$

$$u(c - \psi) = \alpha_u(\theta)V.$$

Since in a stationary state  $\theta$  is constant and  $V$  does never increase, I can strengthen Corollaries 7 and 8 for CARA preferences with monetary costs.

**Corollary 9** *In a stationary state with CARA preferences and monetary costs, both net-consumption  $c - \psi$  and consumption upon re-employment  $c^e$  are constant or decreasing with the length of the unemployment spell.*

Since the optimal increase in the consumption levels in response to an increase in  $V$  is independent of the level of human capital and does not interact with search or training, the value function is additive in  $\theta$  and  $V$ .

**Proposition 13** *For CARA preferences with monetary cost of efforts, we have*

$$C^e(V^e, \theta') = -\frac{\ln(-V^e(1 - \beta))}{\sigma(1 - \beta)} - \frac{y(\theta')}{1 - \beta}$$

and

$$C(V, \theta) = -\frac{\ln(-V(1 - \beta))}{\sigma(1 - \beta)} - \frac{g(\theta)}{1 - \beta}$$

for some unknown function  $g(\theta)$ .

I rewrite the Bellman equation for  $C(V, \theta)$  in terms of  $g(\theta)$  using the expressions in Proposition 13. In the appendix I show how this simplifies to

$$g(\theta) = \max_{\alpha, s, t} \beta [\pi(s) y(\theta(1 - \delta) + t) + (1 - \pi(s))g(\theta(1 - \delta) + t)] - (1 - \beta)\psi(s, t) \\ + \frac{1}{\sigma} \left[ (1 - \beta) \ln\left(\frac{\alpha_u}{1 - \beta}\right) + \beta\pi(s) \ln(\alpha_{V^e}) + \beta(1 - \pi(s)) \ln(\alpha_{V^u}) \right]$$

such that

$$\alpha_u + \beta[\pi(s)\alpha_{V^e} + (1 - \pi(s))\alpha_{V^u}] = 1 \quad (\lambda)$$

$$\alpha_{V^e} - \alpha_{V^u} \leq -\frac{\sigma\alpha_u\psi_s(s, t)}{\beta\pi'(s)}. \quad (\mu)$$

I solve numerically for  $g(\theta)$  in the following section. However, I do not need a closed form solution for  $g(\theta)$  to characterize the level of human capital  $\theta^*$  in the stationary state, which the unemployed agent maintains with a level of training  $t^* = \delta\theta^*$ . I can use the unconstrained Bellman equation and the envelope condition to substitute for  $g(\theta)$  and  $g'(\theta)$  in the first order conditions with respect to training and search. Hence, I have a system of equations which fully characterizes the stationary state. In appendix, I show that in the second-best stationary state,  $(\theta^*, s^*, (\frac{\alpha_{V^u}}{\alpha_u})^*)$  must satisfy

$$\begin{aligned} \beta\pi'(s^*)\left[\frac{y(\theta^*) - \psi(s^*, \delta\theta^*) + \kappa/(1 - \beta)}{1 - \beta(1 - \pi(s^*))}\right] + \frac{\partial\kappa}{\partial s}/(1 - \beta) &= \psi_s(s^*, \delta\theta^*) \\ \beta\pi(s^*)\frac{y'(\theta)/(1 - \beta)}{1 - \beta(1 - \pi(s^*))(1 - \delta)} + \frac{\partial\kappa}{\partial t}/(1 - \beta) &= \psi_t(s^*, \delta\theta^*) \\ \frac{\partial\kappa}{\partial \frac{\alpha_{V^u}}{\alpha_u}} &= 0. \end{aligned}$$

The influence of the incentive compatibility is completely reflected in the function  $\kappa(s, \delta\theta, \frac{\alpha_{V^u}}{\alpha_u})$ , which equals 0 in the first best and is described in appendix.

Notice for comparison that in the first best  $\alpha_{V^e}(\theta) = \alpha_{V^u}(\theta) = \frac{\alpha_u(\theta)}{1 - \beta}$  such that the value function simplifies to

$$g^{\text{FB}}(\theta) = \max \beta \{ \pi(s)y(\theta(1 - \delta) + t) + (1 - \pi(s))g^{\text{FB}}(\theta(1 - \delta) + t) \} - (1 - \beta)\psi(s, t)$$

The first-best stationary state  $(\theta^{\text{FB}}, s^{\text{FB}})$  satisfies

$$\begin{aligned} \beta\pi'(s^{\text{FB}})\frac{y(\theta^{\text{FB}}) - \psi(s^{\text{FB}}, \delta\theta^{\text{FB}})}{1 - \beta(1 - \pi(s^{\text{FB}}))} &= \psi_s(s^{\text{FB}}, \delta\theta^{\text{FB}}) \\ \beta\pi(s^{\text{FB}})\frac{y'(\theta^{\text{FB}})/(1 - \beta)}{1 - \beta(1 - \pi(s^{\text{FB}}))(1 - \delta)} &= \psi_t(s^{\text{FB}}, \delta\theta^{\text{FB}}). \end{aligned}$$

Ignoring the incentives, the expected gain of search is the increased probability to produce  $y(\theta)$  rather than search and train at cost  $\psi(s, \delta\theta)$ . The expected gain from training is to increase the production  $y'(\theta)$  upon re-employment. The difference in discounting between search and training comes from the fact that training efforts add to a depreciating stock of human capital, whereas search efforts vanish every period.

## 2.5 Numerical Simulations

In this section I perform some numerical exercises. I solve numerically for the value function and optimal policy functions. I characterize the optimal policy for long-term unemployed, analyze the optimal timing of training and search during the unemployment spell, and study the value of training. The numerical methodology is based on value function iteration with discretization of the state space. I calibrate the model for CARA preferences with monetary effort costs since this essentially makes the state space one-dimensional as shown in Proposition 13. Regarding the needed inputs for the returns and costs of search and training, the empirical evidence is very incomplete and, to the extent it exists, very mixed. I use some simple guidelines for this calibration. Further empirical research could improve the match between simulated and empirical moments.

### 2.5.1 Calibration

**CARA Preferences** The unit of time is set to be one month and the monthly discount factor  $\beta = 0.996$  to match an annual discount factor of 0.95. The unemployed individual has CARA preferences  $u(c, \psi(s, t)) = -\exp(-\sigma(c - \psi(s, t)))$  with CARA coefficient  $\sigma = 1.5$ .

**Exponential Depreciation** I assume exponential depreciation of human capital. I assume a monthly depreciation rate  $\delta = 0.005$  in the standard specification. This is twice the effect of joblessness in a Spanish study by Alba-Ramirez and Freeman (1990). Production output  $y$  when employed with human capital  $\theta$  is given by  $y(\theta) = \theta^\omega$  with  $\omega = 0.85$  in the standard specification.

**Search and Training** I assume convex monetary cost functions  $\psi(s, t)$  with  $\psi_{s,t}(s, t) = 0$  in the standard specification. The probability to find a job given search  $s$  equals  $\pi(s) = 1 - \exp(-\rho s)$ . Finally, I assume a linear training technology such that human capital in the next month equals  $m(\theta, t) = (1 - \delta)\theta + t$ . I choose parameters following two guidelines. First, the probability to find a job for the first best effort levels does not exceed 0.4 for the relevant levels of human capital. The search cost of finding a job with probability  $1/3$  equals the training cost of increasing human capital with 1 percent at the stationary level. In a Danish experiment (Westergaard-Nielsen 1993), two to four weeks of vocational training increase wages upon re-employment by 1 percent. The estimates are the same for low-skilled and high-skilled workers, but only significant for the former.

## 2.5.2 Policy Functions

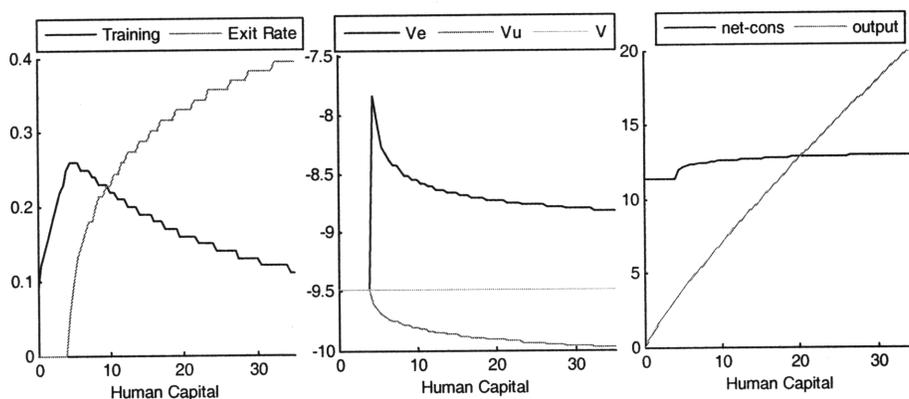


Figure 2-1: Policy Functions: Training and Exit Rate; Promised Utilities; Net-Consumption and Potential Output

Figure 2-1 shows how the different policy functions depend on human capital  $\theta$ , for a given level of expected utility  $V$ . For levels of human capital below a cut-off  $\underline{\theta}$ , it is optimal for the social planner to induce no search at all, as shown in the left panel. The unemployed agent is in the training state and only exerts efforts to increase the level of human capital. In this state, training is indeed increasing in the level of human capital, as shown in Proposition 12. Since no search is induced, the promised utilities when

re-employed or unemployed next month are equal, as shown in the center panel.

For human capital higher than this cut-off  $\underline{\theta}$ , the level of human capital is high enough for the social planner to give incentives for search. The induced search efforts and thus the exit rate are increasing in the level of human capital. The exponential depreciation is essential for this result. With exponential depreciation, both the foregone output and the expected loss in future output are increasing with the level of human capital. Hence, the social cost of remaining unemployed is higher, the higher the level of human capital. When search is positive, the slope of the policy function for training depends on the cost substitutability between training and search  $\psi_{s,t}$  and the concavity of the productivity function  $y(\theta) = \theta^\omega$ . For the standard specification ( $\psi_{s,t} = 0$  and  $\omega = 0.85$ ), training is lower for higher levels of human capital. With increased substitutability in the costs ( $\psi_{s,t} > 0$ ), this difference is even more pronounced. However, when the returns to human capital do not decrease ( $\omega = 1$ ), training is higher for higher levels of human capital.

In order to induce search, the wedge between the promised utilities  $V^e$  and  $V^u$  is positive. Search increases with human capital. Both promised utility levels are nevertheless decreasing with human capital, as we see in Figure 2-1. Since the expected utility  $V$  does not change with human capital, the probability to become employed and the net-consumption  $c - \psi$  have to compensate for the decrease of the promised utilities with human capital. For the standard specification, both net-consumption and consumption are increasing with human capital. I ignore in this analysis the incentives for individuals to build up human capital when they are young or working, but to preserve such incentives the expected lifetime utility for unemployed agents with higher levels of human capital should be higher. This would strengthen the effect that unemployment net-consumption is increasing in human capital as happens with unemployment benefit ratio's in practice. An increase in the expected utility  $V$  shifts the consumption levels during unemployment and upon re-employment, but does not change search and training incentives. The right panel of Figure 2-1 shows that depending on the level of human capital, unemployment net-consumption may be above or below the potential output. Similarly, employment may be taxed or subsidized.

### 2.5.3 Stationary State vs. Social Assistance



Figure 2-2: Human Capital  $\theta$ , Unemployment Consumption  $c$  and Net-Consumption  $c - \psi$  during unemployment, starting with  $\theta_0 > \theta^*$  (solid),  $\theta_0 < \theta^*$  (dashed) and  $\theta_0 = 0$  (dotted).

The human capital of the long-term unemployed converges to a positive level. This stationary level is unique, independent of the level of human capital at the start, as we see in the left panel of Figure 2-2. The social planner imposes training efforts that maintain the stationary level of human capital. At the same time, the social planner provides incentives for the unemployed to search for work. This result contrasts with the optimality of social assistance when no training technology is available (Pavoni and Violante 2007). Social assistance is characterized by constant unemployment payments and no active participation by the agent in job search. Pavoni and Violante assume that human capital depreciates to zero and that the marginal cost of the first unit of search is positive. In their framework without training, the depreciation of human capital causes the potential production to be too small compared to the cost of inducing search after a finite time of unemployment. Hence, the unemployed enter social assistance within finite unemployment spells. In addition, once they enter social assistance, they never leave again.

With the introduction of training during unemployment, the social planner never gives up on the unemployed. Only for the unemployed with the lowest levels of human capital in the training state, the social planner gives no incentives for search and keeps net-

consumption constant, as shown in Corollary 5. However, this is with the goal to increase human capital before inducing search. The training state is never absorbent. So even the unemployed with the lowest levels of human capital at the start converge to the same stationary level of human capital, as long as the cost of training is sufficiently low.

In Figure 2-2, we also see that, in contrast with Shavell and Weiss (1979), consumption and even net-consumption may be increasing during unemployment. In the stationary state, the level of human capital is constant, but net-consumption  $c - \psi$  and consumption upon re-employment  $c^e$  continue to decrease, as in Corollary 9. This does not necessarily imply that taxes should increase with the length of the unemployment spell (Hopenhayn and Nicolini 1997). When  $\theta_0 > \theta^*$ , the decrease in human capital may be faster than the decrease in  $c - \psi$  or  $c^e$ , in which case the taxes are first decreasing before they start increasing with the length of the unemployment spell. So it may be optimal to give higher subsidies for workers who have found a job after a longer unemployment spell, as happens in practice when firms are subsidized for hiring long-term unemployed. The opposite holds for the ratio of net-consumption with respect to the potential output.

## 2.5.4 Optimal Timing of Training

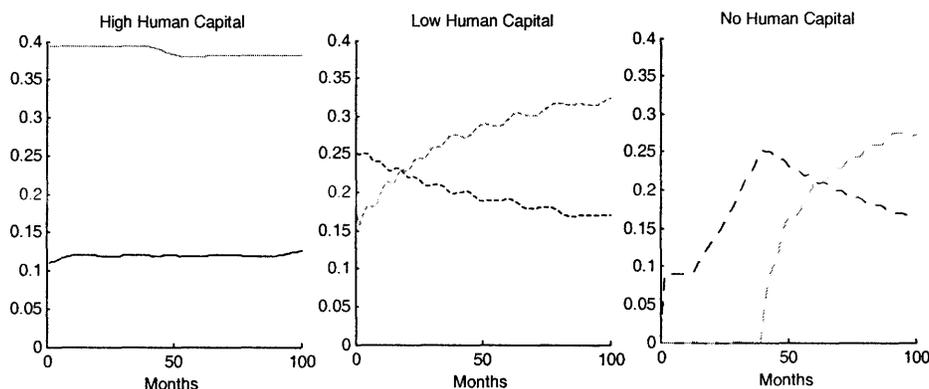


Figure 2-3: Training (black) and Exit Rate (grey) during unemployment, starting with  $\theta_0 > \theta^*$ ,  $\theta_0 < \theta^*$  and  $\theta_0 < \underline{\theta}$ .

The level of human capital to which long-term unemployed converge is unique and

the convergence to this level is monotone. Hence, two unemployed individuals who are identical except for the level of human capital at the start of the unemployment spell converge to the same level of human capital. For the long-term unemployed, the optimal training levels converge to the stationary level  $t^* = \delta\theta^*$ , regardless of the level of human capital at the start.

The implications of this result for training policies are straightforward. First, training efforts are generally lower the higher the human capital level at the start of the unemployment, except for in the training state. Second, the difference between the human capital level at the start of the unemployment spell  $\theta_0$  and in the stationary state  $\theta^*$  determines the optimal timing of training. If  $\theta_0 > \theta^*$ , training is less intensive at the beginning of the unemployment spell and becomes more intensive during the unemployment spell. Vice versa, for  $\underline{\theta} < \theta_0 < \theta^*$ , training is more intensive at the beginning of the unemployment spell and becomes less intensive during the unemployment spell. For  $\theta_0 < \underline{\theta}$ , the unemployed agent starts in the training state. Training will be increasing at the beginning of the unemployment spell, but starts decreasing once  $\theta$  passes  $\underline{\theta}$  and the unemployed agent starts searching. For the standard specification, training and search are substitutes such that more training increases the marginal cost of search. In addition, search is more valuable when human capital is high, whereas training is more valuable when human capital is low. Figure 2-3 clearly shows that these effects dominate the complementarity between search and training with respect to the expected future output. The optimal levels of training and search follow opposite trends during the unemployment spell.

The difference between  $\theta_0$  and  $\theta^*$  can be linked to the two sources for the loss of human capital for the unemployed. Human capital falls upon job loss and depreciates during unemployment. The fall in human capital when losing a job is reflected in the  $\theta_0$ . Two identical agents will have different levels of human capital at the start of the unemployment spell, if the firm-specific or industry-specific capital they lose when losing their job is different. The depreciation in human capital is reflected in  $\theta^*$ . Increasing the rate of depreciation decreases the stationary level of human capital as we see in Figure 2-4. Therefore, the more important the fall in human capital upon job loss, relative to the depreciation of human capital during unemployment, the more important training

becomes towards the beginning of the unemployment spell.

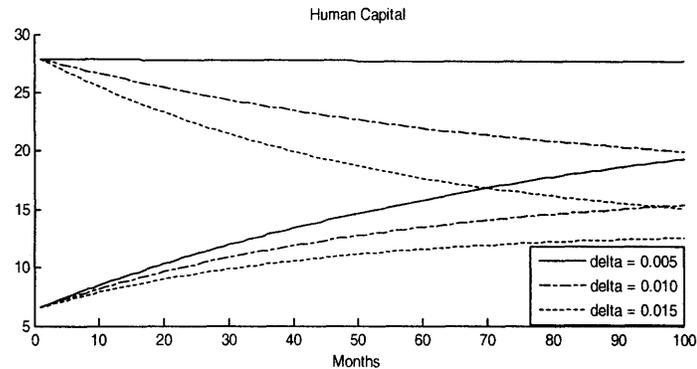


Figure 2-4: Convergence of Human Capital with different rates of depreciation  $\delta$ .

In practice, training requirements are only imposed on the long-term unemployed to remain eligible for unemployment benefits. Recent examples are the ‘New Deal’ in the United Kingdom and the ‘Activation of Search Behavior’ in Belgium. Such programs are more desirable if the loss of firm-specific and industry-specific human capital when becoming unemployed is modest relative to the loss of job skills or alienation of the job market during unemployment. However, training is often subsidized or allows the unemployed to refuse jobs offered by the Public Employment Service from the first day of unemployment.<sup>8</sup> This shift of focus towards short-term unemployed becomes more important if the loss of work goes hand in hand with the loss of human capital. Examples are the large public training programs organized in transition countries, like in East-Germany, for the unemployed who had been active in declining industries.

## 2.5.5 Value of Training

### Insurance without Training Technology

Without training technology, the depreciation of human capital cannot be countered. Search becomes more important to avoid the depreciation of human capital. In Figure

<sup>8</sup>In Belgium, for example, the unemployed may be eligible for subsidized training programs from the first day. However, if the demand exceeds the supply, priority is given to the long-term unemployed.

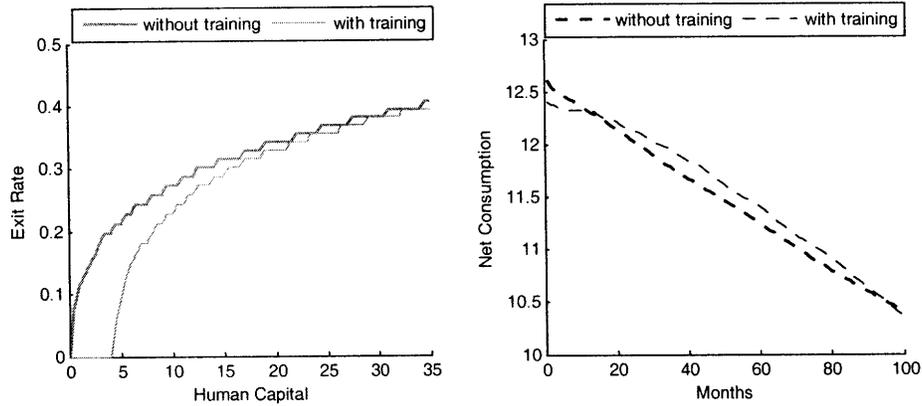


Figure 2-5: Comparison of the Exit Rate and Net-Consumption with and without Training Technology.

2-5, we see how the optimal level of search is higher for every level of human capital. However, since human capital decreases faster, search will be smaller for the long-term unemployed.

If the social planner cannot provide training programs, the consumption scheme is its only instrument to insure against unemployment and provide incentives for search. Since training mitigates the effect of human capital depreciation, it reduces the need for search. To the extent that training does not increase the cost of providing incentives ( $\psi_{s,t} \approx 0$ ), it allows the social planner to focus more on insurance. If the depreciation of human capital is equally important in Europe and in the US, this result suggests that training as an active labor market policy is more complementary to the typical European unemployment insurance with low incentives for search (high and slowly decreasing benefits) than to the US unemployment insurance with high incentives for search (low and quickly decreasing).

The numerical simulations show that the optimal trade-off between insurance and incentives provision results in decreasing net-consumption during unemployment and consumption upon re-employment over the unemployment spell. If the social planner can provide training, the decrease in net-consumption will be less steep in the beginning of the unemployment spell, as shown in the second panel of Figure 2-5. However, since without training technology, the optimal policy converges to social assistance, the decrease in

net-consumption for the long-term unemployed is stronger with training technology.

### First Best vs. Second Best

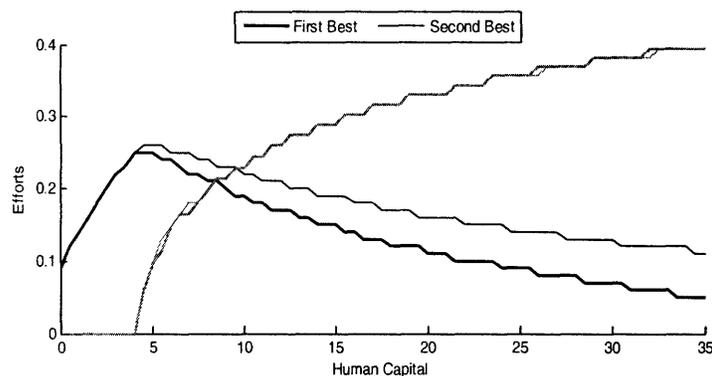


Figure 2-6: Training (black) and Exit Rate (grey) in the first best and the second best

Training allows the social planner to focus more on insurance and smooth the marginal utility of consumption. This value of training disappears in the first best, since the unemployed are perfectly insured, with or without training technology. However, the value of training also depends positively on the probability that the searching agent finds a job and search is induced without incentive costs in the first best. The numerical simulations suggest that the first effect dominates. The social planner wants to impose more intensive training in the second best than in the first best, as shown in Figure 2-6. The cut-off level of human capital  $\underline{\theta}$  for the unemployed agents to be in the training state is also slightly lower in the first best compared to the second best.

### Impact of training

The value of training depends on the returns and costs of training. Some empirical studies suggests that the impact of training may be very low. I modify the human capital accumulation process in the following way,

$$m(\theta, t) = \theta(1 - \delta) + zt.$$

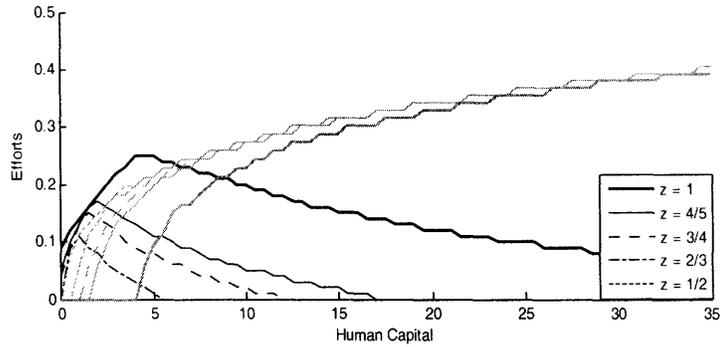


Figure 2-7: Training (black) and Exit Rate (grey) for different levels of cost-effectiveness  $z$ .

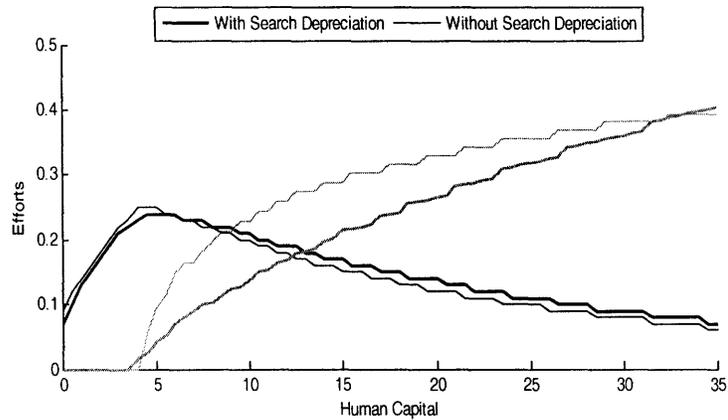


Figure 2-8: Training (black) and Exit Rate (grey) with and without search depreciation.

If the impact of training  $z$  decreases, the desired level of training is lower, whereas the desired level of search is higher. Moreover, the social planner induces search for lower levels of human capital. In Figure 2-7, we see how the cut-off level of human capital  $\underline{\theta}$  moves to the left. The search policy function shifts up and to the left. The training policy function shifts down and to the left.

I assumed that the depreciation of human capital only affects the output upon re-employment. Since with exponential depreciation a decrease in human capital leads to a decrease in search, the depreciation of human capital implies that the exit rate depends

negatively on the duration of the unemployment spell. I now introduce this negative dependence directly as well through *search depreciation* (Shimer and Werning 2006). I assume

$$\pi(s, \theta) = 1 - \exp(-\rho(\alpha + \beta\theta)s)$$

such that  $\pi_{s,\theta} > 0$ . The marginal return to search now depends directly on the level of human capital, changing the slope of the search policy function, as we see in Figure 2-8. This also implies that training does not only increase the output upon re-employment, but also the transition rates. The optimal level of training decreases in the training state, but increases in the training and search-state.

## 2.6 Conclusion

The secular trend of increasing production mobility, technological innovations and shifts in consumer demand forces workers to switch jobs (and industries) more frequently. Displaced workers are often reemployed at lower wages. Kling (2006) proposes to restructure unemployment insurance towards a program that provides wage-loss insurance for the reemployed workers. I have approached training during unemployment as an effective alternative to deal with this wage loss. However, the effectiveness of training programs has often been questioned. Some have argued that training programs are only effective in deterring the unemployed from not searching or refusing jobs. The deterrence argument (Besley and Coate 1992) may help explain the focus of training programs on long-term unemployed in practice. However, if switching jobs implies a change in the required skills, training programs should be targeted towards the newly unemployed. This also changes the optimal structure of the monetary benefits. The skill set of a displaced worker may have become redundant such that it may be undesirable to give strong incentives to search for work at the start of unemployment. If less incentives are needed, the benefits can be used to provide more insurance.

I also argued that the provision of incentives and insurance should be spread over the employment and unemployment spell. This strongly depends on the assumption that

training efforts can be imposed. The introduction of taxes and subsidies upon reemployment would distort the incentives of the unemployed agent to follow effective training programs. Finally, I have not discussed the role of firms. The loss of skills increases the importance of inducing firms to internalize the costs of displacing workers. Firms could also be subsidized to hire and train low-skilled workers who are not employable otherwise. The expected cost of the optimal insurance contract that I characterized in this paper places a lower bound on this subsidy that a social planner should be willing to pay to firms to hire and train the unemployed.

## 2.7 Appendix A: Proofs

### Proof of Proposition 10

By the  $EC_V$  at  $\tau + 1$ , we have

$$C_V(V_{\tau+1}, m(\theta_\tau, t_\tau)) = \lambda_{\tau+1}.$$

Since  $\mu_\tau = 0$  if  $s_\tau = 0$ , I find from  $FOC_{V^u}$  at  $\tau$

$$C_V(V_{\tau+1}, m(\theta_\tau, t_\tau)) = \lambda_\tau.$$

Hence,  $\lambda_\tau = \lambda_{\tau+1}$ .  $\square$

### Proof of Proposition 11

With  $s_\tau = 0$ ,  $EV_\theta$  at  $\tau$  simplifies to

$$C_\theta(V_\tau, \theta_\tau) = \beta C_\theta(V_{\tau+1}, \theta_{\tau+1}) (1 - \delta).$$

Substituting for  $C_\theta(V_\tau, \theta_\tau)$  and  $C_\theta(V_{\tau+1}, \theta_{\tau+1})$  from the  $FOC_t$  at  $\tau - 1$  and  $\tau$  respectively,

I find

$$\lambda_{\tau-1} u_\psi(c_{\tau-1}, \psi_{\tau-1}) \psi_t(s_{\tau-1}, t_{\tau-1}) = \beta (1 - \delta) \lambda_\tau u_\psi(c_\tau, \psi_\tau) \psi_t(s_\tau, t_\tau).$$

Since  $\lambda_{\tau-1} = \lambda_\tau$  by Proposition 10 and  $u_\psi(c_{\tau-1}, \psi_{\tau-1}) = u_\psi(c_\tau, \psi_\tau)$  for the respective preferences, I find

$$\psi_t(s_{\tau-1}, t_{\tau-1}) = \beta (1 - \delta) \psi_t(s_\tau, t_\tau).$$

Since  $\psi$  is convex,  $s_{\tau-1} = s_\tau = 0$  and  $\beta (1 - \delta) < 1$  imply  $t_\tau > t_{\tau-1}$ .  $\square$

### Proof of Proposition 12

From the  $FOC_{V^u}$  and  $EC_V$ , I find that

$$\Delta_{\lambda, \tau} \equiv \lambda_\tau - \lambda_{\tau+1} = \mu_\tau \frac{\pi'(s_\tau)}{(1 - \pi(s_\tau))}.$$

From  $FOC_s$ , I find

$$\mu = \frac{\beta\pi'(s) [C^e(V^e, m(\theta, t)) - C(V^u, m(\theta, t))]}{(u_{\psi, \psi} (\psi_s)^2 + u_{\psi} \psi_{s,s} + \beta\pi''(s) [V^e - V^u])}.$$

First,  $s_\tau > 0$  only if  $C^e - C^u < 0$ . Hence, the numerator is negative. Second, the denominator equals the second derivative of the agent's expected utility with respect to search. This derivative is negative. Therefore, we have that  $\mu > 0$  and with  $\pi'(s_\tau) > 0$  this implies  $\Delta_{\lambda, \tau} > 0$ .  $\square$

### Proof of Proposition 13

Since an equal increase in all consumption levels only rescales the utility levels, I expect only the utility ratio's to be dependent on human capital. Here, I rescale all promised utilities with this period's utility  $u(c - \psi)$  which allows me to explicitly solve the two side-constraints for  $\bar{V}^e \equiv V^e/u$  and  $\bar{V}^u \equiv V^u/u$ . The expected cost of insuring an unemployed agent becomes

$$C(V, \theta) = \min c + \beta[\pi(s)C^e(\bar{V}^e u, \theta(1 - \delta) + t) + (1 - \pi(s))C(\bar{V}^u u(c - \psi(s, t)), \theta(1 - \delta) + t)]$$

such that

$$1 + \beta[\pi(s)\bar{V}^e + (1 - \pi(s))\bar{V}^u] \leq \frac{V}{u} \quad (\lambda)$$

$$\pi'(s)\beta[\bar{V}^e - \bar{V}^u] = \psi'_s(s, t)\frac{u'}{u}. \quad (\mu)$$

Using the CARA properties, the IC constraint simplifies to

$$\pi'(s)\beta[\bar{V}^e - \bar{V}^u] = -\sigma\psi'_s(s, t).$$

The IC constraint does not depend on the level of consumption and the IR can remain satisfied after an  $\varepsilon$ -increase in  $V$  by increasing  $u(c)$  by  $\varepsilon$  (and therefore  $V^e$  and  $V^u$  with

$\varepsilon$ ). The first order condition with respect to  $c$  equals

$$1 + \beta [\pi(s)C_V^e u' \bar{V}^e + (1 - \pi(s))C_V u' \bar{V}^u] - \lambda \frac{V}{u^2} u' = 0$$

$\Leftrightarrow$

$$1 + \beta [\pi(s)C_V^e u' \bar{V}^e + (1 - \pi(s))C_V u' \bar{V}^u] = -\lambda \frac{\sigma V}{u}.$$

Notice also that the envelope condition with respect to  $V$  states that

$$C_V(V, \theta) = \frac{\lambda}{u}$$

or, using the first order condition,

$$C_V(V, \theta) = -\frac{1 + \beta [\pi(s)C_V^e u' \bar{V}^e + (1 - \pi(s))C_V u' \bar{V}^u]}{\sigma V}.$$

With CARA preferences,

$$\begin{aligned} C^e(V^e, \theta') &= \frac{-\ln(-V^e(1 - \beta))}{\sigma(1 - \beta)} - \frac{\theta'}{1 - \beta} \\ C_V^e(V^e, \theta') &= \frac{-1}{\sigma(1 - \beta)V^e}. \end{aligned}$$

Substituting, I find

$$C_V(V, \theta) = -\frac{1 + \beta \left[ \frac{\pi(s)}{1 - \beta} + (1 - \pi(s))C_V(-\sigma u) \bar{V}^u \right]}{\sigma V}. \quad (2.2)$$

I guess and verify whether  $C(V, \theta) = -\frac{\ln(-V(1 - \beta))}{\sigma(1 - \beta)} - \frac{g(\theta)}{\sigma(1 - \beta)}$  for some function  $g(\theta)$ . For our guess, we have  $C_V(V, \theta) = -\frac{1}{\sigma(1 - \beta)V}$ . When I plug this into (2.2), I get

$$\begin{aligned} C_V(V, \theta) &= -\frac{1 + \beta \left[ \frac{\pi(s)}{1 - \beta} + \frac{1 - \pi(s)}{1 - \beta} \right]}{\sigma V} \\ &= -\frac{1}{\sigma(1 - \beta)V}. \end{aligned}$$

Since this holds for any pair  $(V, \theta)$ , our guess and verify method has pinned down the first

term of the cost function. That is,

$$C(V, \theta) = \frac{\left(-\frac{1}{\sigma}\right) \ln(-V(1-\beta))}{1-\beta} - \frac{g(\theta)}{1-\beta}.$$

The effect on the cost for social planner of an increase in  $V$  does not depend on the level of human capital.  $\square$

#### Proof of Corollary 4

From  $FOC_c$ ,  $\frac{1}{u'(c)} = \lambda$ . The result follows by Proposition 10.  $\square$

#### Proof of Corollary 5

With  $\mu = 0$ , I find from  $FOC_c$  that  $\frac{1}{u'(c-\psi)} = \lambda$ . The result follows by Proposition 10.  $\square$

#### Proof of Corollary 6

From  $FOC_c$ ,  $\frac{1}{u'(c)} = \lambda$ . The result follows by Proposition 12.  $\square$

#### Proof of Corollary 7

From  $EC_V$ ,  $C_V(V, \theta) = \lambda$ . In Proposition 13, I will show that  $C_V(V, \theta) = \frac{-1}{\sigma(1-\beta)V}$  for CARA preferences. The result follows by Proposition 12.  $\square$

#### Proof of Corollary 8

Using  $\Delta_{\lambda, \tau} = \mu_{\tau} \frac{\pi'(s_{\tau})}{(1-\pi(s_{\tau}))}$  from Proposition 12, I find from  $FOC_{V^e}$ ,

$$\frac{1}{u^e(c_{\tau+1}^e)} - \lambda_{\tau} = \Delta_{\lambda, \tau} \frac{1 - \pi(s_{\tau})}{\pi(s_{\tau})} \quad (2.3)$$

and from  $FOC_c$ ,

$$\lambda_{\tau} - \frac{1}{u_c(c_{\tau}, \psi_{\tau})} = \frac{\Delta_{\lambda, \tau} \beta (1 - \pi(s_{\tau})) - u_{\psi, c}(c_{\tau}, \psi_{\tau}) \psi_s(s_{\tau}, t_{\tau})}{\lambda_{\tau} \beta \pi'(s_{\tau})},$$

where the last factor denotes the change in  $V^e - V^u$  to keep the IC-constraint binding when  $c$  changes. For additive preferences, this factor equals 0. For CARA preferences with monetary costs, this factor equals  $\frac{\sigma u_c \psi_s}{\beta \pi'} > 0$ . Since  $\Delta_{\lambda, \tau} > 0$  by Proposition 12, for

both types of preferences

$$\lambda_\tau - \frac{1}{u_c(c_\tau, \psi_\tau)} \geq 0 > \lambda_\tau - \frac{1}{u^{el}(c_{\tau+1}^e)}.$$

Hence,  $c_{\tau+1}^e > c_\tau$  for additive preferences and  $c_{\tau+1}^e > c_\tau - \psi_\tau$  for CARA preferences, which proves the first part of the corollary.

From (2.3) at  $\tau$  and  $\tau - 1$ , I find

$$\frac{\Delta_{\lambda, \tau-1}}{\pi(s_{\tau-1})} = \left[ \frac{1}{u^{el}(c_\tau^e)} - \frac{1}{u^{el}(c_{\tau+1}^e)} \right] + (1 - \pi(s_\tau)) \frac{\Delta_{\lambda, \tau}}{\pi(s_\tau)}.$$

Moreover,  $\Delta_{\lambda, \tau} \rightarrow 0$  for  $\tau \rightarrow \infty$ . Either  $\Delta_{\lambda, \tau} = 0$  or  $\Delta_{\lambda, \tau} > 0$  by Proposition 10 and 12. If for CARA preferences  $\Delta_{\lambda, \tau} > 0$ , then  $V_\tau > V_{\tau+1}$ , since  $\Delta_{\lambda, \tau} = \frac{-1}{\sigma(1-\beta)V_\tau} - \frac{-1}{\sigma(1-\beta)V_{\tau+1}}$ . Either  $V$  converges to  $\bar{V}$  or  $V$  converges to  $-\infty$ . In both cases,  $\Delta_{\lambda, \tau} \rightarrow 0$ . If for additive preferences  $\Delta_{\lambda, \tau} > 0$ , then  $c_\tau > c_{\tau+1}$  since  $\Delta_{\lambda, \tau} = \frac{1}{u'(c_\tau)} - \frac{1}{u'(c_{\tau+1})}$ . Either  $c$  converges to  $\bar{c}$  or  $c$  converges to  $-\infty$ . In both cases,  $\Delta_{\lambda, \tau} \rightarrow 0$  as well. By integration,

$$\frac{\Delta_{\lambda, \tau-1}}{\pi(s_{\tau-1})} = \left[ \frac{1}{u^{el}(c_\tau^e)} - \frac{1}{u^{el}(c_{\tau+1}^e)} \right] + \sum_{k=1}^{\infty} \left( \prod_{l=1}^{k-1} (1 - \pi(s_{\tau+l})) \right) \left[ \frac{1}{u^{el}(c_{\tau+k}^e)} - \frac{1}{u^{el}(c_{\tau+k+1}^e)} \right].$$

Since  $\Delta_{\lambda, \tau-1} > 0$  for  $s_{\tau-1} > 0$ , we cannot have that  $c_{\tau+k}^e \leq c_{\tau+k+1}^e$  for all  $k > 0$ . Hence, whenever search is positive during unemployment, it must be that at some later time consumption upon re-employment is strictly decreasing with the length of the unemployment spell.  $\square$

### Proof of Corollary 9

From Proposition 13, we have

$$\begin{aligned} c^e &= (u^e)^{-1} \left( (1 - \beta) \alpha_{V^e}(\theta) V \right) \\ c - \psi &= u^{-1} \left( \alpha_u(\theta) V \right). \end{aligned}$$

with the inverse functions  $(u^e)^{-1}$  and  $u^{-1}$  strictly increasing and  $\alpha_{V^e}(\theta)$  and  $\alpha_u(\theta)$  positive. Hence, since  $\theta$  is constant in a stationary state and  $V$  does never increase by Corollary 5

and 7, the result follows.  $\square$

## 2.8 Appendix B: First Order and Envelope Conditions

The social planner minimizes the expected costs of the insurance scheme given an individual rationality constraint and incentive compatibility constraint with Lagrange multipliers  $\lambda$  and  $\mu$  respectively

$$C(V, \theta) = \min_{c, V^u, V^e, s, t} c + \beta \left[ \pi(s) \frac{(u^e)^{-1}((1-\beta)V^e - y(m(\theta, t)))}{1-\beta} + (1 - \pi(s))C(V^u, m(\theta, t)) \right]$$

such that

$$V - u(c, \psi(s, t)) - \beta[\pi(s)V^e + (1 - \pi(s))V^u] \leq 0 \quad (\lambda)$$

$$u_\psi(c, \psi(s, t))\psi_s(s, t) + \beta\pi'(s) [V^e - V^u] \leq 0. \quad (\mu)$$

For an interior solution, the first order conditions are

$$0 = 1 - \lambda u_c - \mu u_{c,\psi} \psi_s \quad (FOC_c)$$

$$0 = C_V^e(V^e, m(\theta, t)) - \lambda - \mu \frac{\pi'(s)}{\pi(s)} \quad (FOC_{V^e})$$

$$0 = C_V^u(V^u, m(\theta, t)) - \lambda + \mu \frac{\pi'(s)}{(1 - \pi(s))} \quad (FOC_{V^u})$$

$$0 = \beta\pi'(s) [C^e - C^u] - \mu(u_{\psi,\psi}(\psi_s)^2 + u_\psi\psi_{s,s} + \beta\pi''(s) [V^e - V^u]) \quad (FOC_s)$$

$$0 = \beta [\pi(s)C_\theta^e(V^e, m) + (1 - \pi(s))C_\theta(V^u, m)] m_t \quad (FOC_t)$$

$$-\lambda u_\psi \psi_t - \mu [u_{\psi,\psi} \psi_s \psi_t + u_\psi \psi_{s,t}]$$

The envelope conditions are

$$C_V(V, \theta) = \lambda \quad (EC_V)$$

$$C_\theta(V, \theta) = \beta [\pi(s)C_\theta^e(V^e, m) + (1 - \pi(s))C_\theta(V^u, m)] m_\theta \quad (EC_\theta)$$

## 2.9 Appendix C: CARA Preferences with Monetary Cost of Efforts

**Bellman Equation for  $g(\theta)$**  I rewrite the Bellman equation for  $C(V, \theta)$  in terms of  $g(\theta)$ . I use the expressions in Proposition 13 to substitute for  $C^e(\alpha_{V^e}V, \theta(1 - \delta) + t)$  and  $C(\alpha_{V^u}V, \theta(1 - \delta) + t)$  with

$$C(V, \theta) = \min c + \beta \left[ \pi \frac{-\ln(-\alpha_{V^e}V(1 - \beta))}{\sigma(1 - \beta)} + (1 - \pi) \frac{-\ln(-\alpha_{V^u}V(1 - \beta))}{\sigma(1 - \beta)} \right] - \beta \left[ \pi(s) \frac{y(\theta)}{1 - \beta} + (1 - \pi(s)) \frac{g(\theta)}{1 - \beta} \right]$$

such that

$$\alpha_u + \beta[\pi(s)\alpha_{V^e} + (1 - \pi(s))\alpha_{V^u}] = 1 \quad (\lambda)$$

$$\alpha_{V^e} - \alpha_{V^u} \leq -\frac{\sigma\alpha_u\psi_s(s, t)}{\beta\pi'(s)}. \quad (\mu)$$

The first two terms in the objective function can be rewritten to

$$\begin{aligned} & -\frac{\ln(-u)}{\sigma} + \psi(s, t) - \frac{\beta\ln(-V(1 - \beta))}{\sigma(1 - \beta)} + \beta \left[ \pi \frac{-\ln(\alpha_{V^e})}{\sigma(1 - \beta)} + (1 - \pi) \frac{-\ln(\alpha_{V^u})}{\sigma(1 - \beta)} \right] \\ = & -\frac{\ln u/V}{\sigma} - \frac{\ln(-V)}{\sigma} + \psi(s, t) + \frac{\ln(-V(1 - \beta))}{\sigma} - \frac{\ln(-V(1 - \beta))}{\sigma(1 - \beta)} + \beta \left[ \pi \frac{-\ln(\alpha_{V^e})}{\sigma(1 - \beta)} + (1 - \pi) \frac{-\ln(\alpha_{V^u})}{\sigma(1 - \beta)} \right] \\ = & -\frac{\ln(-V(1 - \beta))}{\sigma(1 - \beta)} + \psi(s, t) - \frac{1}{\sigma(1 - \beta)} \left[ (1 - \beta) \ln\left(\frac{\alpha_u}{1 - \beta}\right) + \beta\pi \ln(\alpha_{V^e}) + \beta(1 - \pi) \ln(\alpha_{V^u}) \right]. \end{aligned}$$

Since

$$C(V, \theta) = -\frac{\ln(-V(1 - \beta))}{\sigma(1 - \beta)} - \frac{g(\theta)}{1 - \beta},$$

I find that the Bellman equation for  $g(\theta)$  equals

$$g(\theta) = \max_{\alpha, s, t} \beta [\pi(s) y(\theta(1 - \delta) + t) + (1 - \pi(s))g(\theta(1 - \delta) + t)] - (1 - \beta)\psi(s, t) \\ + \frac{1}{\sigma} \left[ (1 - \beta) \ln\left(\frac{\alpha_u}{1 - \beta}\right) + \beta\pi(s) \ln(\alpha_{V^e}) + \beta(1 - \pi(s)) \ln(\alpha_{V^u}) \right]$$

such that the IR and IC constraint hold.

**Stationary State for CARA Preferences** Assuming an interior solution, the value function  $g(\theta)$  in the second best can be characterized by an unconstrained maximization after substituting in for the IR and IC constraint. That is,

$$g(\theta) = \max \beta \{ \pi(s)y(\theta(1 - \delta) + t) + (1 - \pi(s))g(\theta(1 - \delta) + t) \} \\ - (1 - \beta)\psi(s, t) + \kappa(s, t, \frac{\alpha_{V^u}}{\alpha_u})$$

with

$$\kappa(s, t, \frac{\alpha_{V^u}}{\alpha_u}) = \frac{1}{\sigma} \{ \beta\pi(s) \ln(\tilde{\kappa}(s, t) + \frac{\alpha_{V^u}}{\alpha_u}) + \beta(1 - \pi(s)) \ln(\frac{\alpha_{V^u}}{\alpha_u}) \\ - \ln(1 + \beta[\pi(s) \tilde{\kappa}(s, t) + \frac{\alpha_{V^u}}{\alpha_u}]) - (1 - \beta) \ln(1 - \beta) \}$$

and

$$\tilde{\kappa}(s, t) = -\frac{\sigma\psi_s(s, t)}{\beta\pi'(s)}.$$

Notice that only three control variables remain. With  $g(\theta)$  concave, the first order conditions with respect to  $s$ ,  $t$  and  $\frac{\alpha_{V^u}}{\alpha_u} (= \frac{V^u}{u})$  are

$$\beta\pi'(s)[y(\theta(1 - \delta) + t) - g(\theta(1 - \delta) + t)] - (1 - \beta)\psi_s(s, t) + \frac{\partial\kappa}{\partial s} = 0 \\ \beta \{ \pi(s)y'(\theta(1 - \delta) + t) + (1 - \pi(s))g'(\theta(1 - \delta) + t) \} - (1 - \beta)\psi_t(s, t) + \frac{\partial\kappa}{\partial t} = 0 \\ \frac{\partial\kappa}{\partial \frac{\alpha_{V^u}}{\alpha_u}} = 0.$$

The envelope condition equals

$$g'(\theta) = \beta \{ \pi(s)y'(\theta(1-\delta) + t) + (1 - \pi(s))g'(\theta(1-\delta) + t) \} (1 - \delta).$$

In a steady state  $\theta^*$ , training equals the depreciation in human capital to maintain the same level, that is  $t^* = \delta\theta^*$ . Hence, from the objective function and envelope condition, I get

$$\begin{aligned} g(\theta) &= \frac{\beta\pi(s(\theta))y(\theta) - (1 - \beta)\psi(s(\theta), \delta\theta) + \kappa(s, t, \frac{\alpha v u}{\alpha_u})}{1 - \beta(1 - \pi(s(\theta)))} \\ g'(\theta) &= \frac{\beta\pi(s(\theta))y'(\theta)(1 - \delta)}{1 - \beta(1 - \pi(s(\theta)))(1 - \delta)}. \end{aligned}$$

Substituting this in the first order conditions, we have

$$\begin{aligned} \beta\pi'(s) \left[ \frac{y(\theta) - \psi(s, \delta\theta) + \kappa/(1 - \beta)}{1 - \beta(1 - \pi(s))} \right] + \frac{\partial \kappa}{\partial s} / (1 - \beta) &= \psi_s(s, \delta\theta) \\ \beta\pi(s) \frac{y'(\theta) / (1 - \beta)}{1 - \beta(1 - \pi(s))(1 - \delta)} + \frac{\partial \kappa}{\partial t} / (1 - \beta) &= \psi_t(s, \delta\theta) \\ \frac{\partial \kappa}{\partial \frac{\alpha v u}{\alpha_u}} &= 0. \end{aligned}$$

## Chapter 3

# Insurance and Perceptions: How to Screen Optimists and Pessimists

The perception of risk is inherently subjective.<sup>1</sup> Financial traders disagree about the risk of investments, mortgage bankers about the risk of defaulting homeowners, homeowners and renters about the risk of flooding, old and young drivers about the risk of a car accident. One person may perceive a risk as very likely, while another may perceive the same risk as unlikely. At the same time, the perception of the extent to which precautionary efforts mitigate the risk may differ as well. Both the perception of the likelihood of the risk and the perception of control are central to the design of insurance contracts. *Baseline-pessimistic* insurees, who underestimate the baseline likelihood of the risk, are willing to pay more for insurance. *Control-optimistic* insurees, who overestimate the marginal return to effort, exert more precautionary efforts and are therefore cheaper to insure.

This paper analyzes the role of heterogeneity in risk perceptions for the optimal design of screening contracts. In a model with moral hazard and adverse selection, I show how incentive compatibility imposes a very simple structure on the equilibrium contracts and I contrast the distortions in insurance coverage that arise with competing and monopolistic

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<sup>1</sup>Slovic (2000) surveys the research documenting the heterogeneity in the perception of risk and its determinants.

insurers. On the positive side, heterogeneity in risk perceptions offers an alternative explanation for the negative correlation between risk occurrence and insurance coverage found in empirical studies. On the normative side, the presence of agents with biased beliefs improves or worsens the welfare of agents with unbiased beliefs depending on the market structure and the differences in beliefs.

I consider a simple model with two states. Effort exerted by the insuree decreases the probability that a risk occurs, but insurees can have different perceptions about the probability of the risk as a function of effort. The insurer cannot observe the belief held by the insuree, but perceives her risk as independent of her belief. The insuree does not change her belief in response to the menu of insurance contracts being offered. That is, the insurer and the insurees ‘agree to disagree’ about the true underlying risk. The preferences satisfy a single-crossing property if the one insuree perceives the likelihood of the risk as lower than the other insuree for any given insurance contract. This is conditional on the effort levels chosen by the respective insurees. Optimism can therefore arise for two reasons; first of all, if an insuree is more optimistic about the baseline likelihood of the risk for the same level of effort and, second, if an insuree is more optimistic about the marginal return of effort and therefore exerts higher effort for the same insurance contract. If the single-crossing property is satisfied, the insurer can only separate the (more) optimistic insuree by offering her less insurance coverage than the (more) pessimistic insuree. This monotonicity property is independent of the nature of competition between insurers.

Optimistic agents receive less insurance, but still may be more risky ex-post if they are pessimistic about their control and exert less precautionary effort. This contrasts with the property of positive correlation between insurance coverage and risk occurrence that arises in the standard adverse selection framework (Rothschild and Stiglitz 1976). However, many empirical papers find a correlation that is not significantly positive (Chiappori and Salanié 1997 and 2000, Cardon and Hendel 2001) or even negative (Cawley and Philipson 1999, De Meza and Webb 2001, Finkelstein and McGarry 2006). With two types of insurees who only differ in their beliefs, I show that it is sufficient that the one type is more baseline-optimistic and control-optimistic for the equilibrium to satisfy the positive correlation property. For the correlation to be negative, it is necessary that the control-

pessimistic type is also more optimistic about the likelihood of the risk.

A prime issue for characterizing optimal contracts with private information is determining which incentive compatibility constraints are binding and thus which types' contracts are distorted compared to the case without private information. I show how this depends on the interaction between the nature of competition and the dimension in which beliefs are biased. Competing insurers distort the contract offered to the insuree who can be insured at lower cost, which depends on the exerted precautionary effort and thus the insuree's control beliefs. A monopolistic insurer distorts the contract offered to the insuree whose willingness to pay is lower, which depends on the insuree's baseline beliefs. Compared to someone who is unbiased, an optimist's willingness to pay is lower for an *insurance contract* providing more insurance than her outside option, but higher for an *incentive contract* providing less insurance than her outside option.

The distortions due to the screening of types imply that agents with heterogeneous perceptions impose information externalities on each other. An agent with biased beliefs imposes a negative externality on an agent with unbiased beliefs, when private insurers distort the unbiased agent's contract to discourage the biased agent from taking this contract. The externality is only positive when a monopolistic insurer pays a rent to the unbiased agent not to take the contract offered to the biased type. For agents with biased beliefs, the screening distortions may aggravate the distortion due to the biases in their beliefs, as analyzed in Chapter 1. Hence, heterogeneity in optimistic beliefs may strengthen the case for (paternalistic) government intervention through mandating insurance. This contrasts with the result in Sandroni and Squintani (2007) that heterogeneity in beliefs reduces the scope for government intervention. The heterogeneity in optimistic beliefs they consider implies that some agents with different risks perceive their risk to be the same and are pooled in equilibrium. The heterogeneity I consider implies that agents with the same underlying risk are separated.

**Related Literature** The paper studies the role of biased beliefs in the presence of both moral hazard and adverse selection. In Chapter 1, I consider only moral hazard, assuming that the bias in beliefs is known to the insurer. Jeleva and Villeneuve (2004),

Chassagnon and Villeneuve (2005) and Villeneuve (2005) consider only adverse selection. They introduce heterogeneity in risk types, but risk types may misperceive their risk. Sandroni and Squintani (2007) also introduce heterogeneity in risk types, but some agents of the high-risk type may be optimistic about being a low-risk type.

A small theoretical literature has suggested explanations for the advantageous selection with heterogeneous types that leads to negative correlation between risk occurrence and insurance coverage. Koufopoulos (2008) and Huang, Liu and Tzeng (2007) assume the presence of one type who exerts no precautionary effort, but is still more optimistic about the likelihood of the risk than the other type who exerts precautionary effort. This paper generalizes this intuition driven by heterogeneity in perceptions and characterizes how the correlation between risk occurrence and insurance coverage depends on the correlation between baseline and control beliefs. De Meza and Webb (2001) and Jullien, Salanié and Salanié (2006) explain the presence of advantageous selection by heterogeneity in risk preferences. Chiappori, Jullien, Salanié and Salanié (2006) show that such heterogeneity is not sufficient to explain the negative correlation if the competition in the insurance market is perfect. The correlation results in this paper are independent of the nature of competition.

This paper also relates to the literature that explores how firms exploit the bounded rationality of consumers, surveyed in Ellison (2006). In particular, Grubb (2009) and Eliaz and Spiegel (2008) analyze how firms exploit differences in overconfidence and optimism about future demand respectively with a menu of screening contracts. I also consider the externalities that biased agents and unbiased agents impose on each other. In a similar spirit, DellaVigna and Malmendier (2004) and Gabaix and Laibson (2006) analyze how sophisticated and non-sophisticated types affect each others' welfare.

The remainder of the paper is organized as follows. Section 3.1 introduces the model and defines the agent's beliefs. Section 3.2 analyzes properties of incentive compatible contracts with heterogeneity in beliefs. Section 3.3 characterizes the optimal screening contracts, contrasting the competitive equilibrium and the monopolistic optimum. Section 3.4 discusses welfare and policy implications. Section 3.5 presents a simple application with continuous output and linear contracts, as in Holmström and Milgrom (1987). Sec-

tion 3.6 concludes the paper. All proofs are in the appendix.

## 3.1 Model

I consider a principal-agent model with two states. In the good state, the total endowment equals  $W$ . In the bad state, a loss  $L$  is incurred and the total endowment equals  $W - L$ . The agent's unobservable choice of effort determines the probability that the good or bad state occurs. When she exerts effort at additive cost  $e \in E$ , the good state occurs with probability  $\pi(e)$  with  $\pi' \geq 0, \pi'' < 0$ . The bad state occurs with probability  $1 - \pi(e)$ . A risk-neutral principal offers a contract  $(w, \Delta)$  to the risk-averse agent. With this contract, the agent can consume  $w$  in the good state and  $w - \Delta$  in the bad state. Hence, the second argument  $\Delta$  is the deductible, which determines the consumption risk left to the agent. The higher the deductible, the less insured the agent is. When the agent's outside opportunity is  $(w_0, \Delta_0)$ , the difference  $w_0 - w$  denotes the insurance premium that the agent pays to reduce her consumption risk from  $\Delta_0$  to  $\Delta$ .

I will allow the agent's outside option  $(w_0, \Delta_0)$  to be different from the no insurance outcome  $(W, L)$ . If the contract's deductible  $\Delta < \Delta_0$ , I call the contract an *insurance contract*. If the contract's deductible  $\Delta > \Delta_0$ , I call the contract an *incentive contract*. The principal's outside option equals  $(W - w_0, L - \Delta_0)$  and the set of contracts that he can offer is restricted to

$$C \equiv \{(w, \Delta) \mid \Delta \in [0, L], w \in [\Delta, W]\}.$$

The agent cannot be overinsured, i.e.  $\Delta \geq 0$ , which follows immediately if the agent could make the bad state occur with certainty at zero cost.

### 3.1.1 The Agent's Beliefs

The agent's perception of the probability of success as a function of effort may differ from the true probability. I denote the agent's belief as  $\hat{\pi}(e)$  with  $\hat{\pi}' \geq 0, \hat{\pi}'' < 0$ . I introduce these beliefs in the most general way, but the analysis shows that the differences in the

levels and margins of the perceived probability functions are essential.

**Definition 3** *Agent  $i$  is baseline-optimistic if  $\hat{\pi}_i(e) \geq \pi(e)$  for all  $e \in E$ . Agent  $i$  is more baseline-optimistic than agent  $j$  if  $\hat{\pi}_i(e) \geq \hat{\pi}_j(e)$  for all  $e \in E$ .*

**Definition 4** *Agent  $i$  is control-optimistic if  $\hat{\pi}'_i(e) \geq \pi'(e)$  for all  $e \in E$ . Agent  $i$  is more control-optimistic than agent  $j$  if  $\hat{\pi}'_i(e) \geq \hat{\pi}'_j(e)$  for all  $e \in E$ .*

For expositional purposes, I consider the sign of the differences to be the same for all effort levels. Baseline and control beliefs are related, but optimism in the one dimension does not exclude pessimism in the other dimension. Whether agents who are more optimistic about the baseline probability are also more optimistic about their control depends on the context, as in the following two examples.

**Example I**  $\pi(e) = \theta e$  and  $\hat{\pi}(e) = \hat{\theta}e$  with  $e \in [0, \min\{1/\theta, 1/\hat{\theta}\}]$

When for a project the probability of success is complementary in the entrepreneur's ability  $\theta$  and effort  $e$ , an entrepreneur who overestimates his ability (i.e.  $\hat{\theta} > \theta$ ) is at the same time baseline-optimistic and control-optimistic.

**Example II**  $1 - \pi(e) = \phi(1 - e)$  and  $1 - \hat{\pi}(e) = \hat{\phi}(1 - e)$  with  $e \in [0, 1]$

A driver who underestimates the probability to have an accident when exerting no effort (i.e.  $\hat{\phi} < \phi$ ) is baseline-optimistic, but control-pessimistic.

The first two definitions are about the primitives of the probability functions. I introduce a third definition which involves the endogenous choice of effort by the respective agents and allows to describe a single-crossing property for the preferences of agents with different beliefs.

**Definition 5** *Agent  $i$  is more optimistic than agent  $j$  if  $\hat{\pi}_i(\hat{e}_i(c)) \geq \hat{\pi}_j(\hat{e}_j(c))$  for all  $c \in C$ .*

An agent can be more optimistic either because she perceives the likelihood of the good state to be higher for the same level of effort or because she perceives the marginal return to effort to be higher and thus exerts more efforts.

**Lemma 1** *An agent who is more baseline- and control-optimistic is more optimistic as well.*

### 3.1.2 The Agent's Preferences

The agent chooses the effort level that maximizes her perceived expected utility. Given the contract  $(w, \Delta)$ , the agent solves

$$U(w, \Delta) = \max_e \hat{\pi}(e) u(w) + (1 - \hat{\pi}(e)) u(w - \Delta) - e.$$

The agent's choice of effort  $\hat{e}(w, \Delta)$  solves

$$\hat{\pi}'(\hat{e}(w, \Delta)) [u(w) - u(w - \Delta)] = 1.$$

The second order condition is satisfied since  $\hat{\pi}'' < 0$ . The effort choice is increasing in the agent's perceived return to search  $\hat{\pi}'(\cdot)$  and the deductible  $\Delta$ . When the outside option is chosen, the agent's perceived expected utility equals

$$U(w_0, \Delta_0) \equiv \hat{\pi}(e(w_0, \Delta_0)) u(w_0) + (1 - \hat{\pi}(e(w_0, \Delta_0))) u(w_0 - \Delta_0) - e(w_0, \Delta_0).$$

The utility in the outside option is increasing in the baseline belief about the probability  $\hat{\pi}(\cdot)$  that the good state occurs. This increase is higher, the less insurance the outside option provides.

## 3.2 Incentive Compatibility with Heterogeneity in Beliefs

Pessimistic agents are willing to pay more for insurance coverage than optimistic agents because they perceive the risk as more likely. This single-crossing property of the preferences implies that only contracts providing more insurance to pessimistic agents than to optimistic agents can be incentive compatible. Whether pessimistic agents are also more risky ex-post depends on the agents' efforts and thus the agents' control beliefs. The monotonicity in insurance coverage implies simple conditions for the correlation between risk occurrence and insurance coverage to be positive or negative.

I consider two types of agents who only differ in their beliefs. Type 1 and type 2 hold the beliefs  $\hat{\pi}_1(\cdot)$  and  $\hat{\pi}_2(\cdot)$  respectively, with  $\hat{\pi}_1(\cdot) \neq \hat{\pi}_2(\cdot)$ . These beliefs are unobservable to the insurers. The true probability of success  $\pi(\cdot)$  is the same function of effort for both types. For the characterization of the equilibrium contracts, it does not matter whether these probability functions are actually the same or only perceived to be the same by the insurer. The outside option is the same for both types, but the perceived expected utility of the outside option may be different.

### 3.2.1 Single-Crossing Property

If the one type always perceives the probability of the good state to be greater than the other type for any possible contract, the two types' preferences satisfy a single-crossing property.

**Assumption 4** *Type 1 is more optimistic than type 2.*

The higher the perceived probability that the bad state occurs, the higher the willingness to give up wealth  $w$  to decrease the deductible  $\Delta$ . The perceived marginal rate of substitution between  $w$  and  $\Delta$  for type  $i$  equals

$$\left. \frac{d\Delta}{dw} \right|_{\hat{\pi}_i} = \frac{\hat{\pi}_i(\hat{e}_i(w, \Delta))}{1 - \hat{\pi}_i(\hat{e}_i(w, \Delta))} \frac{u'(w)}{u'(w - \Delta)} + 1.$$

The effect through changes in effort on the perceived expected utility in response to  $dw$  and  $d\Delta$  is of second order because of the envelope condition and does not impact the marginal rate of substitution. For different types, the marginal rates of substitution for a given contract  $(w, \Delta)$  is ranked based on the respective perceived probability of success  $\hat{\pi}_i(\hat{e}_i(w, \Delta))$ . If type 1 is more optimistic than type 2, the marginal rates of substitution are ranked the same for any contract.

**Lemma 2** *For any  $c \in C$ ,*

$$\left. \frac{d\Delta}{dw} \right|_{\hat{\pi}_1} \geq \left. \frac{d\Delta}{dw} \right|_{\hat{\pi}_2}.$$

The profit-maximizing insurer cannot observe the type of insured he is facing. By the revelation principle, we can restrict the analysis to contracts that are incentive compatible such that the different types will self-select into the contracts designed for them. A pair of contracts  $\{(w_1, \Delta_1), (w_2, \Delta_2)\}$  is incentive compatible if and only if

$$U^i(w_i, \Delta_i) \geq U^i(w_j, \Delta_j) \text{ for } i, j = 1, 2,$$

with

$$U^i(w, \Delta) \equiv \max_e \hat{\pi}_i(e) u(w) + (1 - \hat{\pi}_i(e)) u(w - \Delta) - e.$$

Clearly, for any pair of incentive compatible contracts, one contract cannot offer more consumption in both states than the other contract. That is, if  $w_1 > w_2$ , then  $w_1 - \Delta_1 < w_2 - \Delta_2$  and vice versa. I introduce the relation  $x \triangleright y$  to describe that the contract  $x$  provides less insurance than contract  $y$  in the sense that  $x$  provides lower coverage at a lower insurance premium than contract  $y$ .

**Notation 1**  $(w_i, \Delta_i) \triangleright (w_j, \Delta_j) \Leftrightarrow w_i > w_j \text{ and } w_i - \Delta_i < w_j - \Delta_j$

**Notation 2**  $(w_i, \Delta_i) \trianglerighteq (w_j, \Delta_j) \Leftrightarrow w_i \geq w_j \text{ and } w_i - \Delta_i \leq w_j - \Delta_j$

I use the particular notation because  $(w_i, \Delta_i) \triangleright (w_j, \Delta_j)$  implies  $(w_i, \Delta_i) > (w_j, \Delta_j)$ . Notice that the opposite does not hold.

### 3.2.2 Monotonicity

In standard adverse selection problems the incentive compatibility constraints imply a monotonicity constraint on the separating contracts offered to different types, if the preferences satisfy a single-crossing property. The same is true here despite the presence of moral hazard.

The utility from one insurance contract can be expressed as the utility from any other insurance contract, plus the sum of the utility gains, positive or negative, from the

incremental changes that lead from the latter to the former insurance contract. That is,

$$U^i(w_i, \Delta_i) = U^i(w_j, \Delta_j) + \int_{\Delta_j}^{\Delta_i} \{U_w^i(\tilde{w}(\Delta), \Delta) \tilde{w}'(\Delta) + U_\Delta^i(\tilde{w}(\Delta), \Delta)\} d\Delta,$$

for any continuous, differentiable function  $\tilde{w}(\Delta)$  with  $\tilde{w}(\Delta_j) = w_j$  and  $\tilde{w}(\Delta_i) = w_i$ . I denote the gain in perceived expected utility for type  $i$  from switching from contract  $(w_2, \Delta_2)$  to  $(w_1, \Delta_1)$  by

$$\phi^i[(w_1, \Delta_1), (w_2, \Delta_2)] \equiv U^i(w_1, \Delta_1) - U^i(w_2, \Delta_2).$$

For contracts to be incentive compatible, the gain from switching to the other type's contract has to be negative for both types,

$$\phi^1[(w_1, \Delta_1), (w_2, \Delta_2)] \geq 0 \quad (IC_1)$$

$$\phi^2[(w_2, \Delta_2), (w_1, \Delta_1)] \geq 0. \quad (IC_2)$$

When choosing between two contracts, the more optimistic type puts relatively more weight on the change in consumption when successful and relatively less weight on the change in consumption when unsuccessful. This difference in weights is not sufficient to sign the difference for two types in utility gains from switching contracts, because the exerted effort levels differ as well. However, the single-crossing property can be used to evaluate the utility gains from all marginal changes in  $\Delta$  and  $\tilde{w}(\Delta)$  for which changes in effort are of second order. When changing the contract from  $(w_j, \Delta_j)$  to  $(w_i, \Delta_i)$ , the sign of the difference in utility gains for type  $i$  and type  $j$  from the marginal changes along the linear function  $\tilde{w}(\Delta) = w_j + (\Delta - \Delta_j) \frac{w_i - w_j}{\Delta_i - \Delta_j}$  exactly equals the sign of the difference in perceived likelihoods,  $\hat{\pi}_i(\hat{e}_i(\tilde{w}(\Delta), \Delta)) - \hat{\pi}_j(\hat{e}_j(\tilde{w}(\Delta), \Delta))$ . The more optimistic type suffers less from the marginal increase in  $\Delta$  and gains more from the marginal increase in  $\tilde{w}(\Delta)$ . This observation implies the following lemma.

**Lemma 3** *If  $(w_1, \Delta_1) \triangleright (w_2, \Delta_2)$ , then*

$$\phi^1 [(w_1, \Delta_1), (w_2, \Delta_2)] > \phi^2 [(w_1, \Delta_1), (w_2, \Delta_2)].$$

The utility gain from switching to an insurance contract for which the insurance coverage and the insurance premium is lower, is greater for someone who is more optimistic about the risk not occurring. This implies that for two contracts to be incentive compatible, the insurance contract designed for the more optimistic type must provide less insurance, but at a lower insurance premium.

**Proposition 14** *Type 1 receives less insurance than type 2 in any incentive compatible equilibrium, i.e.*

$$(w_1, \Delta_1) \supseteq (w_2, \Delta_2).$$

This monotonicity property follows immediately from the incentive compatibility constraints and Lemma 3. Assume, by contradiction, that  $(w_2, \Delta_2)$  provides less insurance than  $(w_1, \Delta_1)$ . Since type 1 is more optimistic than type 2, the utility gain from switching to the contract providing less insurance is higher for type 1 than for type 2. However, for  $(w_2, \Delta_2)$  to be incentive compatible for type 2, her gain from switching from  $(w_1, \Delta_1)$  to  $(w_2, \Delta_2)$  must be positive, which implies that the gain from switching from  $(w_1, \Delta_1)$  to  $(w_2, \Delta_2)$  is positive for type 1 as well. By consequence,  $(w_1, \Delta_1)$  is not incentive compatible for type 1.

### 3.2.3 Positive vs. Negative Correlation

With heterogeneity in perceptions, either positive or negative correlation can arise between the ex-post probability that the risk occurs for a type and the insurance coverage provided to that type. An optimistic type necessarily receives more insurance than a pessimist, but whether the optimistic type is more risky depends on both her control beliefs and the insurance coverage.

**Corollary 10** *If type 1 is more optimistic and control-optimistic than type 2, any separating equilibrium satisfies the ‘positive correlation’-property, i.e.*

$$(w_1, \Delta_1) \succeq (w_2, \Delta_2) \text{ and } \pi(\hat{e}_1(w_1, \Delta_1)) \geq \pi(\hat{e}_2(w_2, \Delta_2)).$$

If type 1 is more control-optimistic, she exerts more effort than type 2 for the same level of insurance. Since in addition type 1 receives less insurance, she exerts more effort in equilibrium and is less likely to suffer a loss. The observed correlation between risk occurrence and insurance coverage is positive.

**Corollary 11** *Only if the optimistic type 1 is more control-pessimistic than type 2, a separating equilibrium may satisfy the ‘negative correlation’-property, i.e.*

$$(w_1, \Delta_1) \triangleright (w_2, \Delta_2) \text{ and } \pi(\hat{e}_1(w_1, \Delta_1)) < \pi(\hat{e}_2(w_2, \Delta_2)).$$

If type 1 is more control-pessimistic, she exerts less effort than type 2 for the same level of insurance. If she is sufficiently more control-pessimistic, she will still exert less effort despite receiving less insurance as well. The negative correlation between optimism and control-optimism across types is necessary for the negative correlation between risk occurrence and insurance coverage to occur.

Negative correlation arises naturally when one type believes his effort has no impact at all, but still perceives the probability that the good state occurs to be more likely than the other type. This extreme example is considered by Koufopoulos (2008) and Huang et al. (2007). Several recent papers show empirical evidence for negative correlation in insurance. Heterogeneity in risks and preferences cannot explain this negative correlation if insurance markets are competitive. Chiappori et al. (2006) show that the positive correlation between ex post risk and insurance is a robust property of competitive markets. However, with heterogeneity in risk aversion and imperfect competition, Jullien et al. (2007) show that the correlation can be negative as well. In contrast, Corollary 10 and 11 are independent of the nature of competition in the insurance market. The empirical question that arises is whether baseline beliefs and control beliefs are positively or nega-

tively correlated. This will depend on the particular risk being considered. For instance, young drivers tend to overestimate the probability to have an accident, but underestimate the returns to driving safely (Finn and Bragg 1986, Tränkle et al. 1990). Similarly, women who overestimate the probability not to have breast cancer are less likely to take mammograms (Katapodi et al. 2004), plausibly because they underestimate the returns to preventive efforts, as argued by Polednak et al. (1991).

### 3.3 Optimal Insurance Contracts

I contrast the insurance contracts offered by competing insurers and a monopolistic insurer who cannot observe the beliefs of the insuree they are facing. Heterogeneity in beliefs drives a wedge between the insurer's cost of providing insurance and the insuree's willingness to pay for being insured. On the one hand, the insurer's cost of providing insurance depends on the insuree's effort choice, which is increasing in her control beliefs. When an insuree of type  $i$  accepts the contract  $(w, \Delta)$ , the insurer's expected profit equals

$$\Pi^i(w, \Delta) = W - w - (1 - \pi(\hat{e}_i(w, \Delta)))(L - \Delta).$$

On the other hand, an insuree's willingness to pay for insurance is decreasing in her baseline beliefs. Similarly, her willingness to accept risk is increasing in her baseline beliefs.

The wedge between cost and valuation implies that whether a type's contract is distorted compared to the full-information contract crucially depends on the nature of competition between insurers. Competing insurers distort the contract offered to the 'low-cost' type to discourage the 'high-cost' type from pretending she has low cost. Control beliefs are thus central under competition. A monopolistic insurer distorts the contract to the 'low-valuation' type to discourage the 'high-valuation' type from pretending she has low valuation. Baseline beliefs are thus central under monopoly. Notice that when insurees only differ in risk, a riskier type values insurance more, but is also more costly to insure.

For the competitive equilibrium, I assume that insurers are competing as in Rothschild

and Stiglitz (1976) with any contract offered in equilibrium making zero profit in expectation. For the monopolistic optimum, the insurees' participation constraints are central to the analysis. For a contract to be accepted, the insuree needs to expect higher utility from that contract than from her outside option. For the competitive case, I assume that the outside option provides no insurance and that the participation constraints are never binding in the competitive equilibrium. I relax both assumptions for the monopolistic case.

### 3.3.1 Full-Information Benchmark

I first characterize the profit-maximizing contract when the insurer knows the agent's perceived probability function  $\hat{\pi}(e)$  and the agent's outside option equals  $(w_0, \Delta_0) = (w, L)$ . The insurer expects to pay insurance coverage  $L - \Delta$  with probability  $1 - \pi(\hat{e}(w, \Delta))$ , whereas the agent expects to receive this coverage with probability  $1 - \hat{\pi}(\hat{e}(w, \Delta))$ . When acting as a monopolist, the profit-maximizing contract  $(w_m^*, \Delta_m^*)$  solves

$$\max_{(w, \Delta)} W - w - (1 - \pi(\hat{e}(w, \Delta)))(L - \Delta)$$

such that

$$u(w) - (1 - \hat{\pi}(\hat{e}(w, \Delta)))[u(w) - u(w - \Delta)] - \hat{e}(w, \Delta) \geq U(w_0, \Delta_0).$$

The competitive equilibrium  $(w_c^*, \Delta_c^*)$  solves the dual problem with the equilibrium profits equal to zero,

$$\max_{(w, \Delta)} u(w) - (1 - \hat{\pi}(\hat{e}(w, \Delta)))[u(w) - u(w - \Delta)] - \hat{e}(w, \Delta)$$

such that

$$W - w - (1 - \pi(\hat{e}(w, \Delta)))(L - \Delta) \geq 0.$$

This implies the following proposition.<sup>2</sup>

**Proposition 15** *The profit-maximizing contract  $(w^*, \Delta^*)$  is characterized by*

$$\frac{\frac{1-\hat{\pi}(\hat{e})}{1-\pi(\hat{e})} \frac{\pi(\hat{e})}{\hat{\pi}(\hat{e})} u'(w^* - \Delta^*) - u'(w^*)}{u'(w^*)} = \varepsilon_{1-\pi(\hat{e}), w-\Delta} \frac{L-\Delta^*}{w^*-\Delta^*},$$

with  $\hat{e} = \hat{e}(w^*, \Delta^*)$ . In monopoly, the perceived expected utility  $U(w^*, \Delta^*) = U(w_0, \Delta_0)$ .

In competition, the expected profit  $\Pi(w^*, \Delta^*) = 0$ .

The contracts optimally trade off the moral hazard cost of insurance and the consumption smoothing benefits of insurance, as perceived by the agent.<sup>3</sup> The moral hazard cost is determined by the elasticity of the probability that the bad state occurs with respect to the level of insurance coverage,  $\varepsilon_{1-\pi(\hat{e}), w-\Delta}$ . The perceived consumption smoothing benefits are determined by the wedge in marginal utilities in the good and the bad state, corrected for the baseline bias. When the agent is baseline-optimistic (i.e.  $\frac{1-\hat{\pi}(\hat{e})}{1-\pi(\hat{e})} \frac{\pi(\hat{e})}{\hat{\pi}(\hat{e})} < 1$ ), the perceived consumption smoothing benefit of actuarially fair insurance is lower than the true consumption smoothing benefit. Since a baseline optimist perceives insurance as less valuable than an unbiased agent, the insurance coverage offered to a baseline-optimistic agent is unambiguously lower. The optimal response to control optimism is ambiguous though. If an insuree becomes more control-optimistic, less risk is required to induce her to exert the same level of effort. Hence, the insurers substitute towards inducing more effort, but given the control optimism, could do so by giving at the same time more insurance. The insurance coverage can therefore be higher for either the more control-optimistic or the more control-pessimistic insuree in the full-information equilibrium. This is in stark contrast with Proposition 14. If beliefs are private and one insuree is more optimistic than the other (e.g. because she is more control-optimistic), she receives less insurance coverage in any incentive compatible equilibrium.

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<sup>2</sup>This proposition is a restatement of Proposition 3 in Chapter 1.

<sup>3</sup>I assume that the first order condition is sufficient for the characterization of the optimum, as in Chapter 1.

### 3.3.2 Binding Incentive Compatibility

A prime issue for characterizing optimal contracts with private information is determining which incentive compatibility (IC) constraints are binding. The difference in control beliefs determines which IC constraint is binding in the competitive equilibrium. The difference in baseline beliefs determines which IC constraint is binding at the monopolistic optimum.

**Control Beliefs and Zero Profit Contracts** A control-optimistic type chooses a higher effort level than a type with unbiased beliefs, when given the same contract. The insurer's profit for a contract is increasing in the effort choice of the agent. Hence, conditional on a contract being accepted by both types, an insurer generates more profit from the control-optimistic type than from the unbiased type. In a competitive equilibrium, the expected profit from any contract equals zero. A control-optimistic type can be offered better terms than an unbiased type. By revealed preference, the control-optimistic type always prefers her full information contract to the full information contract offered to the unbiased types. The latter contract would make non-negative profits on the control-optimistic type, but since it is not offered in equilibrium, it must be that the control-optimistic type prefers the former contract. However, the unbiased type may prefer the full information contract offered to the control-optimistic type. This implies the following lemma.

**Lemma 4** *If type  $i$  is more control-optimistic than type  $j$ , the IC constraint for type  $i$  is never binding in a separating competitive equilibrium.*

Since the true risk is the same function of effort for both types, the control beliefs need to differ and effort needs to have a non-negligible impact on the outcome for the zero-profit conditions not to coincide. If the zero-profit conditions coincide, types with different beliefs prefer different contracts that satisfy this zero-profit condition. Hence, if only beliefs differ and there is no moral hazard, the full-information contracts are separating. The presence of the one type does not distort the contract offered to another type.

**Baseline Beliefs and Outside Options** An insuree can always choose the outside option  $(w_0, \Delta_0)$ . The perceived utility increase from taking the contract  $(w_i, \Delta_i)$  rather than the outside option  $(w_0, \Delta_0)$  has to be non-negative,

$$\phi^i [(w_i, \Delta_i), (w_0, \Delta_0)] \geq 0 \text{ for } i = 1, 2.$$

With heterogeneity in beliefs, the perceived expected utility in the outside option  $U^i(w_0, \Delta_0)$  is type-dependent.<sup>4</sup> As for any other contract, this perceived expected utility is increasing in the agent's baseline optimism. The wedge between the expected utility levels as perceived by a baseline optimist and by a type with unbiased beliefs is greater the less insurance the outside option provides. The wedge disappears when the outside option provides full insurance. Baseline optimists require less compensation for an increase in the deductible, but value a decrease in the deductible less. If contracts provide more insurance than the outside option, the pessimistic type is tempted to take the favorable insurance contract offered to the optimistic type. If contracts provides less insurance than the outside option, the optimistic type is tempted to take the favorable incentive contract offered to the pessimistic type. Hence, it is the combination of the insurance provided in the outside option together with the difference in baseline beliefs that determines which incentive compatibility constraint will be binding for the monopolist.

**Lemma 5** *In a separating monopolistic optimum with type  $i$  more optimistic than type  $j$ , the IC constraint is binding for type  $i$  and the IR constraint is binding for type  $j$  if  $\Delta_0 = 0$ . The reverse is true if  $\Delta_0 = L$ .*

### 3.3.3 Competitive Equilibrium

I now further characterize the competitive equilibrium. I restrict the analysis to equilibrium contracts that provide more insurance than the outside option. The control beliefs are central. In this subsection, I assume that type 1 is more control-optimistic than

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<sup>4</sup>Jullien (2000) analyzes screening contracts when the utility of outside options is type-dependent.

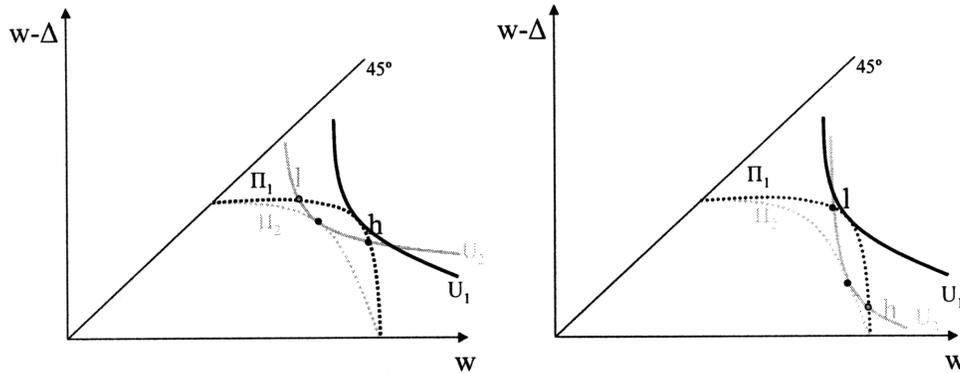


Figure 3-1: Competitive Equilibrium: Positive vs. Negative Correlation

type 2 and characterize the competitive equilibrium depending on whether type 1 is more optimistic or more pessimistic.

**Assumption 4'** *Type 1 is more control-optimistic than type 2.*

The contract offered under full information in the competitive equilibrium to type 1 would make negative profit if chosen by type 2. There are two exceptions. Two contracts always make zero profits, regardless of the beliefs of the agent: the *full insurance contract* with  $(w, \Delta) = (W - (1 - \pi(0))L, 0)$  and the *no insurance contract* with  $(w, \Delta) = (W, L)$ . I show this graphically in Figure 3-1. The respective zero-profit curves are denoted by  $\Pi_1$  and  $\Pi_2$ . Both curves connect the full insurance contract (on the 45°-line) and the no insurance contract (on the  $x$ -axis). However, the zero-profit curve for type 1 connects contracts that provide more consumption in the good and bad state than the zero-profit curve for type 2. The indifference curves are represented by  $U_1$  and  $U_2$ .  $U_1$  crosses  $U_2$  once by the single-crossing property: from above if type 1 is more optimistic, from below if type 1 is more pessimistic. The full-information equilibrium contract for a type is determined by the tangency point between the zero-profit curve and the indifference curve for that type.

The single-crossing property allows to fully characterize the separating equilibrium, if it exists. I first introduce the two contracts  $(w^h, \Delta^h)$  and  $(w^l, \Delta^l)$ .

**Definition 6** Contracts  $(w^h, \Delta^h)$  and  $(w^l, \Delta^l)$  such that for  $i = h, l$ ,

$$\begin{cases} (w^i, \Delta^i) \sim_2 (w_{c,2}^*, \Delta_{c,2}^*) \\ W - w^i = (1 - \pi(\hat{e}_i(w^i, \Delta^i)))(L - \Delta^i), \end{cases}$$

and

$$(w^h, \Delta^h) \triangleright (w^l, \Delta^l).$$

Both contracts satisfy the zero-profit condition of type 1 and leave type 2 indifferent with his full-information contract  $(w_{c,2}^*, \Delta_{c,2}^*)$ . The contract  $(w^h, \Delta^h)$  provides less insurance coverage at a lower insurance premium than  $(w^l, \Delta^l)$ . The contracts are indicated by  $h$  and  $l$  in Figure 3-1.

Proposition 16 characterizes the separating equilibrium when type 1 is both more optimistic and more control-optimistic than type 2. I assume that the perceived expected utility is concave in consumption in the good the and the bad state.

**Proposition 16** *If a separating equilibrium exists and type 1 is both more optimistic and more control-optimistic, the equilibrium contracts equal*

$$\begin{aligned} (w_{c,1}^{**}, \Delta_{c,1}^{**}) &= (w^h, \Delta^h) \\ (w_{c,2}^{**}, \Delta_{c,2}^{**}) &= (w_{c,2}^*, \Delta_{c,2}^*), \end{aligned}$$

*unless  $(w_{c,2}^*, \Delta_{c,2}^*) \succeq_2 (w_{c,1}^*, \Delta_{c,1}^*)$ , in which case the full-information contracts are separating.*

The presence of type 1 who is more control-optimistic has no impact on the contract offered to type 2, by Lemma 4. The presence of type 2 has no impact on the equilibrium contract offered to type 1 either if the full information equilibrium is separating. For instance, if type 1 is very optimistic, she will not be offered any insurance, regardless of the presence of a pessimistic type. However, if the full information equilibrium is not separating, it is because the more control-pessimistic type 2 prefers type 1's full information contract. Contracts  $h$  and  $l$  are natural alternatives, since type 2 is exactly indifferent between her full information contract  $(w_{c,2}^*, \Delta_{c,2}^*)$  and these contracts. Contract  $h$  will

be offered in equilibrium though, since the optimistic type 1 prefers the high deductible contract  $(w^h, \Delta^h) \triangleright (w_{c,1}^*, \Delta_{c,1}^*)$  to the low deductible contract  $(w^l, \Delta^l) \triangleleft (w_{c,1}^*, \Delta_{c,1}^*)$  by the single-crossing property. The two types are thus separated by decreasing the insurance coverage for the optimistic type 1. I show this graphically in the left panel of Figure 3-1. The correlation between the ex-post risk and insurance coverage will be positive by Corollary 10.

Type 1's contract is distorted in the opposite direction if she is more control-optimistic, but at the same time more pessimistic than type 2.

**Proposition 17** *If a separating equilibrium exists, utility is concave in consumption and type 1 is more control-optimistic, but more pessimistic than type 2, the equilibrium contracts are*

$$\begin{aligned}(w_{c,1}^{**}, \Delta_{c,1}^{**}) &= (w^l, \Delta^l) \\ (w_{c,2}^{**}, \Delta_{c,2}^{**}) &= (w_{c,2}^*, \Delta_{c,2}^*),\end{aligned}$$

*unless  $(w_{c,2}^*, \Delta_{c,2}^*) \succeq_2 (w_{c,1}^*, \Delta_{c,1}^*)$ , in which case the full-information equilibrium is separating.*

The separating equilibrium contract for the control-pessimistic type 2 is still  $(w_{c,2}^*, \Delta_{c,2}^*)$  as a consequence of Lemma 4. However, the pessimistic type 1 now prefers  $(w^l, \Delta^l)$  to  $(w^h, \Delta^h)$ , because of the reversed single-crossing property. Since  $(w^l, \Delta^l) \triangleleft (w_{c,2}^*, \Delta_{c,2}^*)$ , the two types are separated by increasing the insurance coverage for type 1. I show this graphically in the right panel of Figure 3-1. Type 1 now receives less insurance than type 2. If type 2 is sufficiently control-pessimistic compared to type 1, the correlation between ex post risk and insurance coverage is negative, in line with Corollary 11.

Propositions 16 and 17 together imply that the insurance coverage of the control-optimistic type 1 may be non-monotonic in the baseline beliefs of type 2. When type 2 is sufficiently baseline-pessimistic, type 1's equilibrium contract  $(w_{c,1}^*, \Delta_{c,1}^*)$  is separating. When type 2 becomes more baseline-optimistic, type 1's separating contract becomes  $(w^h, \Delta^h)$ , providing less insurance than  $(w_{c,1}^*, \Delta_{c,1}^*)$  by Proposition 16. The more baseline-optimistic type 2 becomes, the more type 1's contract needs to be distorted, providing even less insurance, to keep the contracts separated, i.e.  $\Delta^h$  increases. If type 2 becomes

so baseline-optimistic that despite her relative control-pessimism she becomes more optimistic than type 1, the equilibrium contract for type 1 jumps to  $(w^l, \Delta^l)$  with  $\Delta^l < \Delta^h$  by Proposition 17. Hence, the insurance coverage jumps up. This contract may provide more insurance than  $(w_{c,1}^*, \Delta_{c,1}^*)$ .<sup>5</sup> Moreover, if type 2's baseline optimism further increases, type 1's insurance coverage decreases again, i.e.  $\Delta^l$  increases, until eventually the full information contract  $(w_{c,1}^*, \Delta_{c,1}^*)$  becomes separating again.

I have ignored pooling equilibria in this analysis, which may survive if the single-crossing property does not hold. If the single-crossing property does hold, a pooling contract can never be offered in an equilibrium because of cream skimming (Rothschild and Stiglitz 1976). However, they may inhibit the existence of a separating equilibrium if the ratio of type 2 agents is sufficiently low.

### 3.3.4 Monopolistic Optimum

I now further characterize the monopolistic optimum. I allow the outside option to be different than the contract that provides no insurance. The beliefs about the likelihood of the risk are central to the analysis. I again assume that type 1 is more optimistic than type 2 and I characterize the monopolistic optimum depending on the extent to which the agent is insured the outside option. I generalize the methodology in Jullien et al. (2007) who analyze monopolistic screening in the presence of moral hazard and adverse selection due to unobserved heterogeneity in CARA preferences.

**Assumption 4** *Type 1 is more optimistic than type 2.*

If the outside opportunity provides no insurance, the monopolist needs to pay a rent to the pessimist to induce her not to choose the contract offered to the optimist. Since the optimist needs to be compensated less than the pessimist for an increase in risk, the monopolist reduces the rent paid to the pessimist by imposing more risk on the optimist. The separating contract offered to the pessimist, however, is constrained efficient.

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<sup>5</sup>This holds with certainty if type 1's zero-profit curve is decreasing in  $(w, w-\Delta)$  - space.

If the outside opportunity provides full insurance, it is the optimist who needs to be paid a rent in order to be separated from the pessimist. The monopolist now imposes less risk on the pessimist to reduce the rent paid to the optimist. The separating contract offered to the optimist is constrained efficient. Proposition 18 summarizes these results.

**Proposition 18** *If the monopolist separates types, the optimal contract satisfies that*

$$\begin{aligned} (w_{m,1}^{**}, \Delta_{m,1}^{**}) &\triangleright (w_{m,1}^*, \Delta_{m,1}^*) \text{ when } \Delta_0 = L, \\ (w_{m,2}^{**}, \Delta_{m,2}^{**}) &\triangleleft (w_{m,2}^*, \Delta_{m,2}^*) \text{ when } \Delta_0 = 0. \end{aligned}$$

*In both cases, the contract offered to the other type is constrained efficient.*

The monopolist may exclude a type if the profit from this type does not compensate for the rents paid to the other type to induce her to select the contract proposed to her. In that case, the latter type is proposed the full-information contract. If  $\Delta_0 = L$  and the agent is optimistic with sufficiently low probability  $\kappa$ , the optimistic type is excluded. If  $\Delta_0 = 0$  and the agent is optimistic with sufficiently high probability  $\kappa$ , the pessimistic type is excluded. Finally, a pooling contract may dominate any separating contract. This happens when the solution to the profit-maximizing problem constrained to a binding participation constraint for one type and a binding incentive compatibility constraint for the other type gives more insurance to the optimistic type than to the pessimistic type.<sup>6</sup>

A sufficient condition under which these results generalize for outside opportunities that provide some but not full insurance is that the full-information problem is convex. One special case may arise when the outside opportunity provides partial insurance; if the full information contracts specify deductibles  $\Delta_2^* < \Delta_0 < \Delta_1^*$ , then these contracts are incentive compatible.<sup>7</sup>

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<sup>6</sup>Notice that with CARA preferences and monetary costs of efforts, as considered in Jullien et al. (2007) and in Section 3.5, the monopolist pays a rent to the potentially imitating agent  $i$  by increasing  $w_i$ , but keeping  $\Delta_i$  unchanged at  $\Delta_i^*$ . Hence, if for the full-information contracts  $\Delta_1^* \geq \Delta_2^*$ , then a pooling contract can never be optimal.

<sup>7</sup>If for a given outside opportunity  $(w_0, \Delta_0)$  the full information contracts are such that  $\Delta_1^* < \Delta_0 < \Delta_2^*$ , then the two types are likely to be pooled. This ‘irregular case’ is also treated by Jullien et al. (2007).

**Proposition 19** *If type 1 is more optimistic than type 2 and the full-information problem is convex, the optimal contract with types separated satisfies*

$$\begin{aligned} (w_{m,1}^{**}, \Delta_{m,1}^{**}) &\triangleright (w_{m,1}^*, \Delta_{m,1}^*) \text{ if } \Delta_0 > \max \{\Delta_1^*, \Delta_2^*\}, \\ (w_{m,2}^{**}, \Delta_{m,2}^{**}) &\triangleleft (w_{m,2}^*, \Delta_{m,2}^*) \text{ if } \Delta_0 < \min \{\Delta_1^*, \Delta_2^*\}. \end{aligned}$$

*The contract offered to the other type is constrained efficient in both cases. Also, if  $\Delta_2^* < \Delta_0 < \Delta_1^*$ ,  $(w_{m,i}^{**}, \Delta_{m,i}^{**}) = (w_{m,i}^*, \Delta_{m,i}^*)$  for  $i = 1, 2$ .*

If both contracts are incentive contracts (i.e.  $\Delta_i > \Delta_0$ ), the optimistic type receives a rent and the contract for the pessimistic type is distorted towards less incentives. If both contracts are insurance contracts (i.e.  $\Delta_i < \Delta_0$ ), the pessimistic type receives a rent and the contract of the optimistic type is distorted towards less insurance.

### 3.4 Welfare Analysis

The normative implications of heterogeneity in risk perceptions are twofold. First, the presence of one type imposes an externality on the other type if the insurer changes the terms of the contracts to separate types. Second, this externality may aggravate or reduce the distortion due to an agent's bias in beliefs.

I first focus on the informational externality biased agents impose on unbiased agents. In equilibrium, competing insurers offer contracts that maximize the insuree's perceived expected utility and make zero profits in expectation. Insurees with different perceptions have different preferences. Hence, if the set of contracts that make zero profits is the same for two types of insurees, the presence of the one does not affect the contract offered to the other. Moral hazard and disagreement about the returns to effort are therefore necessary ingredients for informational externalities to occur in the competitive equilibrium. If these ingredients are present, the insurance contract for the more control-optimistic type may be distorted compared to the full-information contract by Lemma 4. The contract offered to the more control-pessimistic type is unchanged. This has immediate welfare implications for an agent with unbiased beliefs, in line with Proposition 16 and 17.

**Corollary 12** *In a separating competitive equilibrium, an agent with unbiased beliefs never gains from the presence of an agent with biased beliefs and may strictly lose only if that agent is control-pessimistic.*

In a separating monopolistic optimum, the contracts offered to both types may change when the insurees' perceptions are not observable. If one IC constraint binds at the optimum, one type receives a rent not to switch to the other type's contract. The first type ends up strictly better off than with the full-information contract. The second type will still be indifferent about switching to the outside option, but her contract is distorted compared to the full-information contract to reduce the rent paid to the first type. The cases in which an agent with unbiased beliefs is paid a rent follow immediately from Proposition 19.

**Corollary 13** *In a separating monopolistic optimum, an agent with unbiased beliefs gains from the presence of an optimistic agent when offered incentive contracts and from the presence of a pessimistic agent when offered insurance contracts.*

These results generalize for an agent with biased beliefs in terms of her perceived expected utility, but not necessarily in terms of her true expected utility. Evaluated in terms of true expected utility, competing insurers offer too little insurance to baseline-optimistic or control-optimistic agents in an equilibrium without private information. Profit-maximizing insurers respond to the low perceived value of insurance for a baseline-optimistic insuree and do not correct the high incentives due to control-optimistic beliefs. Increasing the insurance coverage would increase the agent's true expected utility, when

$$\{\hat{\pi}(e) - \pi(e)\} \geq \{\pi'(e) - \hat{\pi}'(e)\} \frac{\hat{\pi}'(e)}{-\hat{\pi}''(e)}. \quad (3.1)$$

This is Corollary 3 in Chapter 1. Hence, heterogeneity in risk perceptions may aggravate or mitigate the distortion due to biases in beliefs. In a competitive equilibrium with optimistic agents for whom (3.1) holds, the insurance coverage and therefore the true expected utility decreases further due to heterogeneity in risk perception if types who are

more control-optimistic are also more optimistic. This increases the scope for government intervention through insurance mandates (Rothschild and Stiglitz 1976).

In general, the heterogeneity in risk perceptions creates screening opportunities for the insurers. This is in contrast with the conclusions in Sandroni and Squintani (2007). Central to their analysis is that some types of agents perceive their risk to be the same, although their true risk is different. These agents are necessarily pooled in any equilibrium. This is exactly opposite to the analysis here where agents with the same risk are separated based on their heterogeneity in beliefs. In particular, Sandroni and Squintani (2007) analyze the case where some of the high-risk type agents are optimistic about being a low-risk type agent and always choose the contract designed for the latter. The low-risk type therefore necessarily subsidizes the optimistic high-risk type in equilibrium. Given the higher insurance premium, it may be that the low-risk type agent does not prefer to have more insurance than what is offered in a separating equilibrium. Since the incentive compatibility constraint is not binding, mandating insurance is detrimental to low-risk agents in their analysis.

### 3.5 Example: Continuous Output and Linear Contracts

In this section, I consider an example with a continuum of states. I characterize the optimal linear screening contracts in the framework considered by Holmström and Milgrom (1987).

Output  $q = \theta e + \varepsilon$  is additive in effort and a normal noise term  $\varepsilon$  with variance  $\sigma^2$ . The return to effort depends on the agent's ability  $\theta$ . However, the agent may perceive her ability differently. I consider two types. Type 1 perceives the return to effort to be  $\hat{\theta}_1$ . Type 2 perceives this return to be  $\hat{\theta}_2$ , with  $\hat{\theta}_1 > \hat{\theta}_2$ . Hence, type 1 is both more baseline-optimistic and control-optimistic, and thus more optimistic than type 2 by Lemma 1. The agent is of type 1 with probability  $\kappa$ . Both types have the same constant absolute risk aversion  $\eta$  and face the same monetary costs of effort  $\psi \frac{e^2}{2}$ . The outside option is the same,

but the perceived expected utility of the outside option depends on the ability perception. Denote the respective expected utility levels by  $\hat{u}_1$  and  $\hat{u}_2$ .

Given a linear contract  $t + sq$ , the certainty equivalent of type  $i$ 's perceived utility equals

$$CE_i(s, t) = t + s\hat{\theta}_i e - \psi \frac{e^2}{2} - \frac{\eta}{2} s^2 \sigma^2.$$

Type  $i$ 's effort choice is

$$\hat{e}_i(s) = \frac{\hat{\theta}_i}{\psi} s.$$

The expected change in output in response to a change in the sharing rule,  $\frac{dEq_i}{ds_i}$ , equals  $\theta \frac{\hat{\theta}_i}{\psi}$  and thus depends both on the true and perceived ability. The expected profit for the insurer equals

$$\Pi_i(s, t) = (1 - s)\theta \hat{e}_i(s) - t.$$

I first characterize the optimal linear sharing rule with full information as a benchmark.

**Result 1** *With  $\theta$  and  $\hat{\theta}_i$  the true and perceived ability, the linear sharing contract offered by a monopolist or competing insurers under full information determines a linear sharing rule*

$$s_i^* = \left[ 1 + \frac{1}{\hat{\theta}_i \theta} \psi \eta \sigma^2 - \frac{\hat{\theta}_i - \theta}{\theta} \right]^{-1} \equiv \left[ \Xi_i - \frac{\hat{\theta}_i - \theta}{\theta} \right]^{-1}.$$

The contract is more performance-dependent the more optimistic the agent is about her ability. An increase in  $\hat{\theta}_i$  increases the agent's perceived ability to increase output and thus the responsiveness to incentives  $\frac{dEq_i}{ds_i}$ . An increase in  $\hat{\theta}_i$  also increases the probabilistic weight the agent puts on states with high output. Private insurers respond to this baseline optimism by decreasing  $[s_i^*]^{-1}$  by  $\frac{\hat{\theta}_i - \theta}{\theta}$ . The linear sharing rule that maximizes the true expected utility is not affected by baseline optimism, but corrects for control optimism. The socially optimal sharing rule equals  $\left[ \Xi_i + \frac{\hat{\theta}_i - \theta}{\theta} \right]^{-1}$ , providing less incentives (and more insurance) than the competitive sharing rule when  $\hat{\theta}_i > \theta$ .

When types cannot be distinguished, the full information contracts may not be incentive compatible. In any equilibrium, the optimist's contract is at least as performance-dependent as the pessimist's contract. This result is in line with Proposition 14.

**Result 2** *In any incentive compatible contract, type 1's contract is more performance-dependent than type 2's contract,*

$$s_1^{**} \geq s_2^{**} \text{ and } t_1^{**} \leq t_2^{**}.$$

In competition, the pessimist prefers the full information contract offered to the optimist. The profit for a given contract is higher when it is accepted by an agent who is optimistic about her ability and therefore exerts more effort. For the insurer to make zero profit, the fixed transfer  $t$  given to the optimist will thus be relatively high. Hence, with asymmetric information about the agent's beliefs, the contract of the optimist is distorted more towards performance to discourage the pessimist from switching to the optimist's contract. This result is in line with Proposition 16.

**Result 3** *In any competitive equilibrium, the performance-dependence is distorted upward for type 1,*

$$s_{c,1}^{**} \geq s_1^* \text{ and } s_{c,2}^{**} = s_2^*.$$

With a monopolist, the outside opportunities are relevant. I denote by  $\hat{u}_i$  the perceived expected utility of the outside option for type  $i$ . If the perceived expected utility from the outside option is the same for the optimist and the pessimist (i.e.  $\hat{u}_1 = \hat{u}_2$ ), the monopolist can pay a relatively low transfer  $t$  to the optimist to convince her to take the contract. The optimist prefers the monopolistic full information contract offered to the pessimist. In the separating optimum, the optimist receives a rent and the linear sharing rule for the pessimist is distorted downwards, providing less incentives. If the optimist's perceived expected utility of the outside option is higher than some upper bound  $h$ , the monopolist needs to pay a relatively high transfer  $t$  to convince the optimist to take the contract. The pessimist then prefers the full information contract offered to the optimist. In the separating optimum, the linear sharing rule of the optimist's contract is distorted upward, providing more incentives, unless the optimists are excluded. If the optimist's perceived expected utility of the outside option is below this upper bound  $h$ , but above

some lower bound  $l$  ( $> \hat{u}_2$ ), the full-information contracts are incentive compatible.<sup>8</sup> This is summarized in the following result.

**Result 4** *If the monopolist separates type 1 and type 2 with  $\hat{\theta}_1 > \hat{\theta}_2$ , the linear sharing rules of the separating contracts equal*

$$\{s_{m,1}^{**}, s_{m,2}^{**}\} = \begin{cases} \{[\Xi_1 - \frac{\hat{\theta}_1 - \theta}{\theta}]^{-1}, [\Xi_2 - \frac{\hat{\theta}_2 - \theta}{\theta} + \frac{\kappa}{1-\kappa} \frac{(\hat{\theta}_1)^2 - (\hat{\theta}_2)^2}{\theta \hat{\theta}_2}]^{-1}\} & \text{for } \hat{u}_1 < l \\ \{[\Xi_1 - \frac{\hat{\theta}_1 - \theta}{\theta} - \frac{1-\kappa}{\kappa} \frac{(\hat{\theta}_1)^2 - (\hat{\theta}_2)^2}{\theta \hat{\theta}_1}]^{-1}, [\Xi_2 - \frac{\hat{\theta}_2 - \theta}{\theta}]^{-1}\} & \text{for } \hat{u}_1 > h. \end{cases}$$

The sharing rules are the same as in the full-information contracts  $(s_{m,1}^*, s_{m,2}^*)$  for  $\hat{u}_1 \in [l, h]$ .

If  $\hat{\theta}_1 > \hat{\theta}_2 > \theta$ , private insurers provide too little insurance to the optimistic workers. The screening distortion aggravates this distortion in the competitive equilibrium. In the monopolistic case, this is only true if the optimistic type perceives the expected utility of the outside option to be sufficiently higher than the pessimistic type.

### 3.6 Conclusion

People often face the same risk, but may have very different perceptions about the likelihood and the controllability of the risk. I analyze how insurance companies separate people based on the heterogeneity in their perceptions. People with heterogeneous abilities or risks, but identical perceptions cannot be separated. People with different perceptions can be separated with a menu of screening contracts, even when the true abilities or risks are the same.

The differences in perceptions are at the heart of adverse selection and the distinction between baseline and control beliefs is essential. First, I show that contracts offer less insurance and thus provide more incentives to optimists than to pessimists. This very general result does not depend on the nature of competition. Second, I analyze which type's contract is distorted due to the heterogeneity in beliefs. This analysis crucially

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<sup>8</sup>The bounds are  $l \equiv \hat{u}_2 + \frac{(\hat{\theta}_1)^2 - (\hat{\theta}_2)^2}{2\psi} (s_2^*)^2$  and  $h \equiv \hat{u}_2 + \frac{(\hat{\theta}_1)^2 - (\hat{\theta}_2)^2}{2\psi} (s_1^*)^2$ , as derived the appendix.

depends on the nature of competition. A monopolistic insurer distorts the contract offered to the type with low willingness to pay. Competing insurers distort the contract offered to the type to whom insurance can be provided at low cost. Heterogeneity in beliefs, in contrast with heterogeneity in risks, drives a wedge between the insuree's willingness to pay for insurance and the insurer's cost of providing insurance.

Heterogeneity in beliefs can explain why contracts offer too little insurance in some markets (e.g., health insurance, car insurance) and in other markets provide no incentives at all, although a small risk suffices to induce effort (e.g., no-limit contracts on rented cars mileage, cell phone usage). However, unobserved differences in risk or ability could have the same positive implications. Heterogeneity in beliefs can also explain why in many insurance markets the correlation between risk occurrence and insurance coverage is negative. However, unobserved differences in preferences could explain this as well. Identifying to what extent results are driven by true differences or misperceived differences, or by heterogeneity in beliefs or heterogeneity in preferences is clearly a challenge (Manski 2004). The distinction is however essential for policy and welfare analysis. It seems that the natural approach to quantify the importance of heterogeneity in beliefs is to identify beliefs directly, either by eliciting expectations through surveys, as in Chapter 1, or by relating anomalies in behavior to biases in perceptions (Kőszegi and Rabin 2007). These challenging approaches have been followed recently in the literature on CEO compensation (Landier and Thesmar 2009, Malmendier, Tate and Yan 2007). Extending these empirical approaches to the analysis of private insurance markets and social insurance schemes seems a promising avenue for future research.

### 3.7 Appendix: Proofs

#### Proof of Lemma 1

If  $\hat{\pi}_i(e) \geq \hat{\pi}_j(e)$  for any  $e$  and  $\hat{e}_i(c) = \hat{e}_j(c)$ , then  $\hat{\pi}_i(\hat{e}_i(c)) \geq \hat{\pi}_j(\hat{e}_j(c))$  for any  $c$ . If also  $\hat{\pi}'_i(e) \geq \hat{\pi}'_j(e)$  for any  $e$ , then  $\hat{e}_i(c) \geq \hat{e}_j(c)$  for any  $c$ . The Lemma follows, since  $\hat{\pi}'(e) > 0$ .  $\square$

#### Proof of Lemma 2

The marginal rate of substitution (MRS) between  $\Delta$  and  $w$  equals

$$\begin{aligned} \left. \frac{d\Delta}{dw} \right|_{\hat{\pi}_i} &= \frac{\hat{\pi}_i(\hat{e}_i(c)) u'(w) + (1 - \hat{\pi}_i(\hat{e}_i(c))) u'(w - \Delta)}{(1 - \hat{\pi}_i(\hat{e}_i(c))) u'(w - \Delta)} \\ &= \frac{\hat{\pi}_i(\hat{e}_i(c))}{(1 - \hat{\pi}_i(\hat{e}_i(c)))} \frac{u'(w)}{u'(w - \Delta)} + 1. \end{aligned}$$

Since  $\frac{u'(w)}{u'(w-\Delta)} > 0$ , the MRS is increasing in  $\hat{\pi}_i(\hat{e}_i(c))$ . The lemma follows, since  $\hat{\pi}_1(\hat{e}_1(c)) \geq \hat{\pi}_2(\hat{e}_2(c))$  for any  $c$ .  $\square$

#### Proof of Lemma 3

Given  $\tilde{w}(\Delta) = w_j + (\Delta - \Delta_j) \frac{w_i - w_j}{\Delta_i - \Delta_j}$ ,

$$\begin{aligned} U_w^i(\tilde{w}(\Delta), \Delta) \tilde{w}'(\Delta) + U_\Delta^i(\tilde{w}(\Delta), \Delta) = \\ \hat{\pi}_i(\hat{e}_i(\tilde{w}(\Delta), \Delta)) u'(\tilde{w}(\Delta)) \frac{w_1 - w_2}{\Delta_1 - \Delta_2} - (1 - \hat{\pi}_i(\hat{e}_i(\tilde{w}(\Delta), \Delta))) u'(\tilde{w}(\Delta) - \Delta) \frac{[w_2 - \Delta_2] - [w_1 - \Delta_1]}{\Delta_1 - \Delta_2}. \end{aligned}$$

Hence,

$$\begin{aligned} \phi^1[(w_1, \Delta_1), (w_2, \Delta_2)] - \phi^2[(w_1, \Delta_1), (w_2, \Delta_2)] = \\ \int_{\Delta_2}^{\Delta_1} \{ [\hat{\pi}_1(\hat{e}_1(\tilde{w}(\Delta), \Delta)) - \hat{\pi}_2(\hat{e}_2(\tilde{w}(\Delta), \Delta))] \times \\ [u'(\tilde{w}(\Delta)) \frac{w_1 - w_2}{\Delta_1 - \Delta_2} + u'(\tilde{w}(\Delta) - \Delta) \frac{[w_2 - \Delta_2] - [w_1 - \Delta_1]}{\Delta_1 - \Delta_2}] \} d\Delta. \end{aligned}$$

Since  $\frac{w_1 - w_2}{\Delta_1 - \Delta_2} > 0$  and  $\frac{[w_2 - \Delta_2] - [w_1 - \Delta_1]}{\Delta_1 - \Delta_2} > 0$ , given  $(w_1, \Delta_1) \triangleright (w_2, \Delta_2)$ , the term in the integral between squared brackets is greater than zero. Since the integration is from

$\Delta_2$  to  $\Delta_1$  with  $\Delta_1 > \Delta_2$ ,

$$\phi^1 [(w_1, \Delta_1), (w_2, \Delta_2)] - \phi^2 [(w_1, \Delta_1), (w_2, \Delta_2)] > 0,$$

if  $\hat{\pi}_1(\hat{e}_1(\tilde{w}(\Delta), \Delta)) \geq \hat{\pi}_2(\hat{e}_2(\tilde{w}(\Delta), \Delta))$  for all  $\Delta \in [\Delta_2, \Delta_1]$  and  $\hat{\pi}_1(\hat{e}_1(\tilde{w}(\Delta), \Delta)) > \hat{\pi}_2(\hat{e}_2(\tilde{w}(\Delta), \Delta))$  for some  $\Delta \in [\Delta_2, \Delta_1]$ .  $\square$

#### Proof of Lemma 4

If type 1 is (strictly) more control-optimistic than type 2,  $\Pi_1(w, \Delta) > \Pi_2(w, \Delta)$ . In any separating equilibrium,

$$\Pi_1(w_1, \Delta_1) = \Pi_2(w_2, \Delta_2) = 0.$$

Assume by contradiction that  $(w_2, \Delta_2) \sim_1 (w_1, \Delta_1)$  in a separating equilibrium, but  $\Pi_2(w_2, \Delta_2) = 0$ . Hence, if preferences are continuous and the single-crossing property is satisfied, an insurer can change the contract  $(w_2, \Delta_2)$  to  $(w'_2, \Delta'_2)$  such that  $\Pi_2(w'_2, \Delta'_2) = 0$ , but  $(w'_2, \Delta'_2) \succ_1 (w_1, \Delta_1)$  and  $\Pi_1(w'_2, \Delta'_2) > 0$ . This is a profitable deviation.  $\square$

#### Proof of Lemma 5

If  $\Delta_0 = L$ , then for any interior solution  $\Delta_1 < \Delta_0$  by the risk aversion of the agent and the risk neutrality of the principal. This implies  $\phi^2 [(w_1, \Delta_1), (w_0, \Delta_0)] > \phi^1 [(w_1, \Delta_1), (w_0, \Delta_0)]$ . Given the binding IC/IR constraints, this implies

$$\phi^2 [(w_2, \Delta_2), (w_0, \Delta_0)] = \phi^2 [(w_1, \Delta_1), (w_0, \Delta_0)] > 0.$$

Moreover, since  $(w_1, \Delta_1) > (w_2, \Delta_2)$  if the contracts are separating,  $\phi^1 [(w_1, \Delta_1), (w_0, \Delta_0)] > \phi^1 [(w_2, \Delta_2), (w_0, \Delta_0)]$  by Lemma 3. This in turn implies that  $\phi^1 [(w_1, \Delta_1), (w_0, \Delta_0)] = 0$ .

If  $\Delta_0 = 0$ , then for any interior solution  $\Delta_2 > \Delta_0$ , since the agent cannot overinsure. This implies  $\phi^1 [(w_2, \Delta_2), (w_0, \Delta_0)] > \phi^2 [(w_2, \Delta_2), (w_0, \Delta_0)]$ . Given the binding IC/IR constraints, this implies

$$\phi^1 [(w_1, \Delta_1), (w_0, \Delta_0)] = \phi^1 [(w_2, \Delta_2), (w_0, \Delta_0)] > 0.$$

Moreover, since  $(w_1, \Delta_1) > (w_2, \Delta_2)$  if the contracts are separating,  $\phi^2 [(w_2, \Delta_2), (w_0, \Delta_0)] > \phi^2 [(w_1, \Delta_1), (w_0, \Delta_0)]$  by Lemma 3. This in turn implies that  $\phi^2 [(w_2, \Delta_2), (w_0, \Delta_0)] = 0$ .  $\square$

### Proof of Proposition 14

Assume, by contradiction, that  $(w_2, \Delta_2) \triangleright (w_1, \Delta_1)$ , although type 1 is more optimistic than type 2. In that case,

$$\phi^1 [(w_2, \Delta_2), (w_1, \Delta_1)] \geq \phi^2 [(w_2, \Delta_2), (w_1, \Delta_1)] \geq 0,$$

by Lemma 3. But this contradicts  $\phi^1 [(w_1, \Delta_1), (w_2, \Delta_2)] \geq 0$ , since  $\phi^1 [x, y] = -\phi^1 [y, x]$ .  $\square$

### Proof of Proposition 15

The competing insurers and the monopolist solve

$$\max \hat{\pi}(e) [u(w) - u(w - \Delta)] + u(w - \Delta) - e$$

such that

$$\begin{aligned} \hat{\pi}'(e) [u(w) - u(w - \Delta)] &= 1 \\ W - w - (1 - \pi(e)) [L - \Delta] &\geq \bar{\Pi}, \end{aligned}$$

where  $\bar{\Pi}$  equals 0 in the competitive equilibrium and  $\bar{\Pi}$  equals the expected profits such that  $U(w, \Delta) = U(w_0, \Delta_0)$  in the monopolistic optimum.

I define the insurance coverage  $b = w - \Delta$  and the tax on the good state  $\tau = W - w$ , similarly as in Chapter 1. Denote by  $\hat{e}_i(b)$  the level of effort and  $\hat{\tau}_i(b)$  the tax that solve the *IC* constraint and the respective profit constraints for a given level of insurance coverage  $b$  with  $i = c, m$ . The change in the required tax  $\tau$  when the insurance coverage increases equals

$$\tau'_i(b) = \frac{1 - \pi(\hat{e}_i(b))}{\pi(\hat{e}_i(b))} \left[ 1 + \frac{\tau_i(b) + b - W + L}{b} \varepsilon_{1 - \pi(\hat{e}_i(b)), b} \right].$$

Notice that  $\bar{\Pi}$  does not enter the expression for this derivative directly.

The first order condition with respect to  $b$  gives

$$(1 - \hat{\pi}(\hat{e}_i(b))) u'(b) - \hat{\pi}(\hat{e}_i(b)) u'(w - \tau_i(b)) \tau'_i(b) = 0.$$

The effect through effort is of second order by the envelope condition. Plugging in for  $\tau'_i(b)$  and rewriting the expression, one finds

$$\frac{\frac{1 - \hat{\pi}(\hat{e}_i(b))}{1 - \pi(\hat{e}_i(b))} u'(b) - \frac{\hat{\pi}(\hat{e}_i(b))}{\pi(\hat{e}_i(b))} u'(w - \tau_i(b))}{\frac{\hat{\pi}(\hat{e}_i(b))}{\pi(\hat{e}_i(b))} u'(w - \tau_i(b))} = \frac{\tau_i(b) + b - W + L}{b} \varepsilon_{1 - \pi(\hat{e}_i(b)), b}.$$

With  $b = w - \Delta$  and  $\tau = W - w$ , the proposition immediately follows.  $\square$

### Proof of Proposition 16

If the full-information contracts are incentive compatible, the competitive equilibrium coincides with the full-information equilibrium. If the full-information contracts are not incentive compatible, then the IC constraint of type 1 is still not binding in a separating equilibrium, by Lemma 4. Hence, if a separating equilibrium exists, type 2's equilibrium contract equals the full-information contract. The actuarial contract that maximizes the perceived expected utility of type 1, but is not strictly preferred by type 2 to its full-information contract is  $(w^h, \Delta^h)$ . Since  $(w^h, \Delta^h) \sim_2 (w^l, \Delta^l)$  and  $(w^h, \Delta^h) \triangleright (w^l, \Delta^l)$ ,  $(w^h, \Delta^h) \succ_1 (w^l, \Delta^l)$  by Lemma 3. Then, since utility is concave in consumption,  $(w^h, \Delta^h) \succ_1 (w, \Delta)$  for any actuarial contract for which  $(w, \Delta) \triangleright (w^h, \Delta^h)$  or  $(w, \Delta) \triangleleft (w^l, \Delta^l)$ . Hence, type 1's equilibrium contract equals  $(w^h, \Delta^h)$ .  $\square$

### Proof of Proposition 17

The proof is analogue to the proof of Proposition 16. Since the single-crossing property is reversed now,  $(w^h, \Delta^h) \sim_2 (w^l, \Delta^l)$  and  $(w^h, \Delta^h) \triangleright (w^l, \Delta^l)$  imply that  $(w^l, \Delta^l) \succ_1 (w^h, \Delta^h)$ . Type 1's equilibrium contract equals  $(w^l, \Delta^l)$ .  $\square$

### Proof of Proposition 18

The monopolist solves

$$\max \kappa \{W - w_1 - (1 - \pi(\hat{e}_1(1))) [L - \Delta_1]\} + (1 - \kappa) \{W - w_2 - (1 - \pi(\hat{e}_2(2))) [L - \Delta_2]\}$$

such that

$$\begin{aligned}\phi_1((w_1, \Delta_1), (w_0, \Delta_0)) &= \max\{0, \phi_1((w_2, \Delta_2), (w_0, \Delta_0))\} \\ \phi_2((w_2, \Delta_2), (w_0, \Delta_0)) &= \max\{0, \phi_2((w_1, \Delta_1), (w_0, \Delta_0))\}.\end{aligned}$$

If  $\Delta_0 = L$ , then by Lemma 5, the IC/IR constraints for a separating optimum simplify to<sup>9</sup>

$$\begin{aligned}\phi_1((w_1, \Delta_1), (w_0, \Delta_0)) &= 0 \\ \phi_2((w_2, \Delta_2), (w_1, \Delta_1)) &= 0.\end{aligned}$$

Assume  $(w_1, \Delta_1) < (w_1^*, \Delta_1^*)$ , then  $\phi_2((w_1, \Delta_1), (w_1^*, \Delta_1^*)) > 0$  by Lemma 3, so the utility rent paid to type 2 implied by the binding IC constraint is higher if type 1 receives  $(w_1, \Delta_1)$  rather than  $(w_1^*, \Delta_1^*)$ . Since the profit made on type 1 is higher in  $(w_1^*, \Delta_1^*)$  as well,  $(w_1, \Delta_1)$  can never be optimal. Hence,  $(w_1, \Delta_1) \geq (w_1^*, \Delta_1^*)$ .

Assume  $(w_1, \Delta_1) = (w_1^*, \Delta_1^*)$ , one can find a contract  $(w'_1, \Delta'_1)$  such that  $(w'_1, \Delta'_1) \sim_1 (w_1^*, \Delta_1^*)$  and  $(w'_1, \Delta'_1) > (w_1^*, \Delta_1^*)$  (and thus,  $(w'_1, \Delta'_1) \prec_2 (w_1^*, \Delta_1^*)$ ), but sufficiently close to  $(w_1^*, \Delta_1^*)$  such that the loss in profit on type 1 is of second order, but the reduction in the rent paid to type 2 is of first order. Hence,  $(w_1, \Delta_1) > (w_1^*, \Delta_1^*)$ .

Finally, if the optimum is separating, the incentive compatibility constraint for type 1 is slack. Hence, the problem becomes separable for type 2. The contract  $(w_2, \Delta_2)$  solves the full information problem with type 2's outside opportunity equal to  $(w_1, \Delta_1)$ . Clearly, this contract is efficient.

If  $\Delta_0 = 0$ , then by Lemma 5, the IC/IR constraints for a separating optimum simplify

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<sup>9</sup>Notice that the optimum is only separating if the solution of this simplified constrained problem satisfies monotonicity, i.e.  $(w_1, \Delta_1) \geq (w_2, \Delta_2)$ . If not, the two types are pooled.

to<sup>10</sup>

$$\phi_1((w_1, \Delta_1), (w_2, \Delta_2)) = 0$$

$$\phi_2((w_2, \Delta_2), (w_0, \Delta_0)) = 0.$$

Assume  $(w_2, \Delta_2) > (w_2^*, \Delta_2^*)$ , then  $\phi_1((w_2, \Delta_2), (w_2^*, \Delta_2^*)) > 0$  by Lemma 3, so the utility rent paid to type 1 implied by the binding IC constraint is higher if type 2 receives  $(w_2, \Delta_2)$  rather than  $(w_2^*, \Delta_2^*)$ . Since the profit made on type 2 is higher in  $(w_2^*, \Delta_2^*)$  as well,  $(w_2, \Delta_2)$  can never be optimal. Hence,  $(w_2, \Delta_2) \leq (w_2^*, \Delta_2^*)$ .

Assume  $(w_2, \Delta_2) = (w_2^*, \Delta_2^*)$ , one can find a contract  $(w'_2, \Delta'_2)$  such that  $(w'_2, \Delta'_2) \sim_2 (w_2^*, \Delta_2^*)$  and  $(w'_2, \Delta'_2) < (w_2^*, \Delta_2^*)$  (and thus,  $(w'_2, \Delta'_2) \prec_1 (w_2^*, \Delta_2^*)$ ), but sufficiently close to  $(w_2^*, \Delta_2^*)$  such that the loss in profit on type 2 is of second order, but the reduction in the rent paid to type 1 is of first order. Hence,  $(w_2, \Delta_2) < (w_2^*, \Delta_2^*)$ .

Finally, if the optimum is separating, the incentive compatibility constraint for type 2 is slack. Hence, the contract  $(w_1, \Delta_1)$  solves the full information problem with type 1's outside opportunity equal to  $(w_2, \Delta_2)$ . Clearly, this contract is efficient.  $\square$

### Proof of Proposition 19

If  $\Delta_0 < \min\{\Delta_1^*, \Delta_2^*\}$ , then any contract  $(w_2, \Delta_2) < (w_0, \Delta_0)$  such that  $(w_2, \Delta_2) \succeq_2 (w_0, \Delta_0)$ , is dominated by offering  $(w_0, \Delta_0)$  to type 2 and  $(w_1^*, \Delta_1^*)$  to type 1. These contracts are incentive compatible by Lemma 3. Given that the full-information problem is convex and  $(w_2^*, \Delta_2^*) > (w_0, \Delta_0)$ , the insurer makes lower profit when offering  $(w_2, \Delta_2) < (w_0, \Delta_0)$ . Moreover, the insurer cannot make higher profits than the full-information profits on type 1. Any contract  $(w_1, \Delta_1) < (w_0, \Delta_0)$  such that  $(w_1, \Delta_1) \succeq_1 (w_1^*, \Delta_1^*)$  is also dominated by offering  $(w_0, \Delta_0)$  to type 2 and  $(w_1^*, \Delta_1^*)$  to type 1. Again, the profit on type 1 cannot be higher than in  $(w_1^*, \Delta_1^*)$ . Moreover, by Lemma 3,  $(w_1, \Delta_1) \succeq_2 (w_0, \Delta_0)$  and thus incentive compatibility requires  $(w_2, \Delta_2) \succeq_2 (w_0, \Delta_0)$ . Again, given that the full-information problem is convex and  $(w_2^*, \Delta_2^*) > (w_0, \Delta_0)$ , the insurer makes lower profit when offering a contract  $(w_2, \Delta_2) < (w_0, \Delta_0)$  rather than  $(w_0, \Delta_0)$ . Hence, both  $(w_1, \Delta_1)$

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<sup>10</sup>Again, the optimum is only separating if the solution of this problem satisfies monotonicity, i.e.  $(w_1, \Delta_1) \geq (w_2, \Delta_2)$ .

and  $(w_2, \Delta_2)$  need to be greater than the outside option  $(w_0, \Delta_0)$  in order to be optimal. If  $\Delta_0 > \max\{\Delta_1^*, \Delta_2^*\}$ , the argument is exactly the same, *mutatis mutandum*. In this case, both  $(w_1, \Delta_1)$  and  $(w_2, \Delta_2)$  need to be smaller than the outside option  $(w_0, \Delta_0)$  in order to be optimal.

Now, if either

$$(w_1, \Delta_1) > (w_0, \Delta_0) \text{ and } (w_2, \Delta_2) > (w_0, \Delta_0)$$

or

$$(w_1, \Delta_1) < (w_0, \Delta_0) \text{ and } (w_2, \Delta_2) < (w_0, \Delta_0),$$

Lemma 5 and exactly the same argument as in Proposition 18 applies. This proves the first part of the proposition.

If  $\Delta_2^* < \Delta_0 < \Delta_1^*$ ,  $\phi_1[(w_1^*, \Delta_1^*), (w_0, \Delta_0)] = 0$  implies that  $\phi_2[(w_1^*, \Delta_1^*), (w_0, \Delta_0)] < 0$  and  $\phi_2[(w_2^*, \Delta_2^*), (w_0^*, \Delta_0^*)] = 0$  implies that  $\phi_1[(w_2^*, \Delta_2^*), (w_0^*, \Delta_0^*)] < 0$ , by Lemma 3. Hence, both  $\phi_1[(w_1^*, \Delta_1^*), (w_2^*, \Delta_2^*)] \geq 0$  and  $\phi_2[(w_2^*, \Delta_2^*), (w_1^*, \Delta_1^*)] \geq 0$ . The incentive compatibility constraints are satisfied for the full-information contracts.  $\square$

### Proof of Result 1

The optimal linear sharing rule maximizes the total surplus, that is the sum of the perceived certainty equivalent for the agent and the expected profit for the firm. With  $\hat{e}_i(s) = \frac{\hat{\theta}_i}{\psi}s$ , this simplifies to

$$\max_s (1-s)\theta\hat{\theta}_i\frac{s}{\psi} + s\hat{\theta}_i^2\frac{s}{\psi} - \frac{\hat{\theta}_i^2}{\psi}\frac{s^2}{2} - \frac{\eta}{2}s^2\sigma^2.$$

The derivation of this problem yields

$$s_i^* = \frac{1}{1 + \frac{1}{\hat{\theta}_i\theta}\psi\eta\sigma^2 - \frac{\hat{\theta}_i - \theta}{\theta}},$$

for both the competitive and the monopolistic contract. The performance-independent transfer  $t$  is such that the profit of the firms is zero in competition and such that the perceived expected utility of the agent equals  $\hat{u}_i$  with a monopolist.  $\square$

### Proof of Result 2

For an agent with belief  $\hat{\theta}$ , the marginal rate of substitution between  $s$  and  $t$  equals

$$\left. \frac{ds}{dt} \right|_{\hat{\theta}} = \hat{\theta} \hat{e}(s) - \eta \sigma^2.$$

Hence,  $\left. \frac{ds}{dt} \right|_{\hat{\theta}_1} \geq \left. \frac{ds}{dt} \right|_{\hat{\theta}_2}$ , since  $\hat{\theta}_1 \geq \hat{\theta}_2$ . Given this single-crossing property, any competitive equilibrium must satisfy  $s_1 > s_2$  and  $t_1 < t_2$ .  $\square$

### Proof of Result 3

Since type 1 exerts more effort for any given contract, any contract making zero profit from type 2 makes non-negative profits from type 1. Hence, by revealed preference, type 1 prefers his full information contract. If type 2 prefers type 1's full information contract as well, type 1 will be given the contract that makes zero profit from type 1 and makes type 2 indifferent between the two contracts. Hence,

$$t^h = (1 - s^h) \theta \hat{\theta}_1 \frac{s^h}{\psi}$$

and

$$(1 - s_2^*) s_2^* \frac{\theta \hat{\theta}_2}{\psi} + (s_2^*)^2 \left[ \frac{(\hat{\theta}_2)^2}{2\psi} - \eta \sigma^2 \right] = (1 - s^h) s^h \frac{\theta \hat{\theta}_1}{\psi} + (s^h)^2 \left[ \frac{(\hat{\theta}_2)^2}{2\psi} - \eta \sigma^2 \right].$$

The result follows.  $\square$

### Proof of Result 4

The monopolist solves

$$\max \kappa [(1 - s_1) \theta e_1 - t_1] + (1 - \kappa) [(1 - s_2) \theta e_2 - t_2]$$

such that

$$t_i + s_i \hat{\theta}_i e_i - \psi \frac{e_i^2}{2} - \frac{\eta}{2} s_i^2 \sigma^2 \geq t_j + s_j \hat{\theta}_i e_i^j - \psi \frac{(e_i^j)^2}{2} - \frac{\eta}{2} s_j^2 \sigma^2 \quad (IC_i)$$

$$t_i + s_i \hat{\theta}_i e_i - \psi \frac{e_i^2}{2} - \frac{\eta}{2} s_i^2 \sigma^2 \geq \hat{u}_i \quad (IR_i)$$

and

$$e_i = \frac{\hat{\theta}_i}{\psi} s_i \text{ and } e_i^j = \frac{\hat{\theta}_i}{\psi} s_j \text{ for } i, j = 1, 2.$$

First, consider the case that  $IC_2$  and  $IR_1$  are binding, then type 2 is given a rent

$$R = s_1 \left( \hat{\theta}_2 e_2^1 - \hat{\theta}_1 e_1 \right) - \psi \frac{\left( (e_2^1)^2 - (e_1)^2 \right)}{2}$$

such that

$$t_2 + s_2 \hat{\theta}_2 e_2 - \psi \frac{e_2^2}{2} - \frac{\eta}{2} s_2^2 \sigma^2 = \hat{u}_1 + R.$$

The monopolist's problem simplifies to

$$\begin{aligned} \max \kappa & \left[ \theta e_1 + s_1 \left( \hat{\theta}_1 - \theta \right) e_1 - \psi \frac{(e_1)^2}{2} - \frac{\eta}{2} s_1^2 \sigma^2 \right] \\ & + (1 - \kappa) \left[ \theta e_2 + s_2 \left( \hat{\theta}_2 - \theta \right) e_2 - \psi \frac{(e_2)^2}{2} - \frac{\eta}{2} s_2^2 \sigma^2 - R \right] \end{aligned}$$

with

$$e_i = \frac{\hat{\theta}_i}{\psi} s_i \text{ for } i = 1, 2.$$

Except for the rent  $R$  paid to type 2, this problem is the same as in the full-information problem. Since  $R$  only depends on  $s_1$ , the optimal sharing rule for type 2 is the same as in the full-information problem. The monopolist reduces the rent by the distorting the sharing rule for type 1. The result in the proposition follows immediately from differentiating with respect to  $s_1$ .

Second, consider the case that  $IC_1$  and  $IR_2$  are binding. The problem is the same, mutatis mutandum. A rent is paid to type 1 now, so only the sharing rule for type 2 will

be different from the full-information problem. The sharing rule for type 2 is distorted compared to the full-information problem.

Finally, from the *IC* constraints, we find that the full-information sharing rules for type 1 and type 2 are optimal if respectively

$$\hat{u}_1 \geq \hat{u}_2 + \frac{(\hat{\theta}_1)^2 - (\hat{\theta}_2)^2}{2\psi} (s_2^*)^2$$

and

$$\hat{u}_2 \geq \hat{u}_1 + \frac{(\hat{\theta}_2)^2 - (\hat{\theta}_1)^2}{2\psi} (s_1^*)^2.$$

Since  $\hat{\theta}_1 > \hat{\theta}_2$  and  $s_1^* > s_2^*$ , it immediately follows that for  $\hat{u}_1 < l \equiv \hat{u}_2 + \frac{(\hat{\theta}_1)^2 - (\hat{\theta}_2)^2}{2\psi} (s_2^*)^2$ ,  $IC_1$  and  $IR_2$  are binding. For  $\hat{u}_1 \in [l, h]$  with  $h \equiv \hat{u}_2 + \frac{(\hat{\theta}_1)^2 - (\hat{\theta}_2)^2}{2\psi} (s_1^*)^2$ , the full-information contracts are optimal. For  $\hat{u}_1 > h$ ,  $IC_2$  and  $IR_1$  are binding.  $\square$

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