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Abstract

An electric field of sufficiently high frequency applied to electrons in a gas may deliver energy to the electrons without imparting to them any continous drift motion due to the field. The criterion for breakdown of a low-pressure gas at microwave frequencies is therefore that ionization by collision of electrons with neutral gas molecules replace loss by diffusion to the walls of the discharge tube. The condition is mathematically expressed as a simple boundary value problem. This breakdown principle is applied to converting microwave breakdown measurements into measurements of ionization rates as a function of the electric field strength, pressure, and frequency. A new ionization coefficient is introduced appropriate to the high-frequency discharge conditions, and its relation to the d-c Townsend coefficient is explained. The energy transfer from the electric field to the electrons at a given E/p is shown to be most efficient when the pressure is high enough or the frequency low enough to result in many collisions of electrons with gas molecules per cycle. This maximum efficiency is equal to the d-c energy transfer efficiency. When the pressure is lower or the frequency is higher, the electrons have an out-of-phase component motion and do not receive energy so efficiently; hence in this case lower ionization rates are experimentally observed.

BREAKDOWN OF A GAS AT MICROWAVE FREQUENCIES

The Townsend theory for breakdown of a low-pressure gas under the action of a d-c electric field postulates two sources of electrons. Most of the electrons are generated in the volume of the gas through ionization by collision. The original source of electrons at the cathode results from secondary emission due to positive ion or photon bombardment. Prediction of breakdown voltage requires numerical data on the efficiency of these processes. Thus attempts to determine ionization coefficients from breakdown data have been complicated by the operation of two electron-generation processes.

Breakdown caused by a high-frequency electric field is determined by the primary ionization process only; the electrons formed at the walls or in the gas by secondary emission have a negligible effect. It is therefore possible to predict the electric field for breakdown from a knowledge of the ionization coefficient only, or to measure the ionization coefficient from a breakdown experiment.

1. Motion of Electrons in a High-Frequency Field

Under the action of a d-c field an electron is accelerated by the field until it collides with a gas molecule. The direction of motion is then reoriented almost randomly. Most of the kinetic energy gained during the acceleration period is kept during the scattering process, since the mass of the molecule is large compared to that of the electron. After collision, the electron is accelerated or decelerated by the field, depending on the direction of the electron velocity relative to the field. The randomly directed velocity immediately after collision contributes nothing to the flow of electrons along the field direction. Only the component of velocity produced by acceleration in the field, which is destroyed on the average by random scattering at each collision, contributes to an electron current in the field direction. The kinetic energy of the electrons is built up through successive accelerations until the loss of energy by elastic and inelastic collisions and diffusion equals the gain of energy from the field. The motion consists of a large random and a small drift component. The energy transferred to the electrons is a function of E/p, where E is the electric field strength and p is the pressure. This quantity determines the energy gained between the collisions.

If an a-c field whose radian frequency is small compared to the collision frequency is applied to electrons, their motion will be identical in most respects with that of electron motion in a d-c field. The field-induced motion is interrupted by collisions, which occur often during an a-c cycle. The drift current of the electrons oscillates in phase with the field and the energy transfer will be the same as that in a d-c field, if the rms value of the a-c field strength is used.

As the frequency is increased or the pressure decreased, collisions no longer occur frequently enough to keep the electron drift current in phase with the field. The inertia of the electrons causes an out-of-phase component. The transfer of energy from the electric field to the electrons becomes less efficient. As the pressure goes to zero or the frequency becomes infinite, the electrons merely oscillate out of phase with the field and no energy is transferred on the time average. The energy transfer efficiency may be taken into account through an effective field,[1]

$$E_e^2 = E^2 \frac{(v/\ell)^2}{(v/\ell)^2 + \omega^2},$$

where E is the rms value of the applied field, v is the electron velocity, ℓ is the mean free path, v/ℓ is the collision frequency, ω is the radian frequency of the field, and E_e is the effective field which would produce the same energy transfer at zero frequency. The frequency variable may be introduced as the quantity $p\lambda$, where p is the gas pressure and λ is the free-space wavelength of the excitation energy.[2] This variable determines the ratio of the collision frequency to the field radian frequency for electrons of a given velocity.

The oscillatory drift current is not capable of transporting electrons from one point to another in the discharge tube. It cannot account for loss of electrons to the walls of the tube, which is a principal electron withdrawal mechanism in discharges. Electron losses to the walls must therefore be caused by diffusion, resulting from the large random motion. If the electric field is not uniform there may also be a flow of electrons due to an electron "temperature", or kinetic energy gradient. Both of these flow terms may be included in the expression,[3]

$$\vec{\Gamma} = -\nabla(Dn), \qquad (1)$$

where D is the diffusion coefficient for electrons, n the electron density, and $\vec{\Gamma}$ the electron current density in electrons per second per unit area. The product may be differentiated to yield two terms; $-D\nabla n$ is the usual diffusion current density, and $-n\nabla D$ is the current density due to the kinetic energy gradient. The diffusion coefficient is proportional to the average velocity of the electrons.

Since the current density vector is given as the gradient of the scalar quantity Dn in Eq. (1), this quantity may be referred to as the current density potential. It has nothing to do with energy, but is analogous to the velocity potential used in hydrodynamics. This potential,

$$\Psi = Dn, \qquad (2)$$

will be used in the diffusion computations for the breakdown condition.

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1. H. Margenau, Phys. Rev. 60,508 (1946).
2. H. Margenau, Phys. Rev. 73,326 (1948).
3. The formula $\vec{\Gamma} = -\nabla(Dn)$ may be obtained most directly from an extension of the work of P. M. Morse, W. P. Allis, and E. S. Lamar, Phys Rev. 48, 412 (1935). See also E. H. Kennard, "Kinetic Theory of Gases", McGraw-Hill Book Co., Inc., New York and London, 1938, pp. 204-205.

2. Breakdown Criterion: Balance of Ionization against Diffusion

The breakdown condition will be developed for a region in a resonant cavity bounded by walls which absorb electrons. A radioactive source near the discharge cavity provides a small amount of ionization in the cavity. The microwave field in the cavity is gradually increased until the gas suddenly begins to glow, becomes conducting, and the field drops to a much lower value. The field necessary to produce this phenomenon is called the breakdown field.

A detailed study of the buildup of the discharge is obtained from considering the continuity equation for electrons,

$$\partial n/\partial t = \nu n - \nabla \cdot \vec{\Gamma} \tag{3}$$

where ν is the net production rate of electrons per electron. It may be represented as the difference,

$$\nu = \nu_i - \nu_a ,$$

where ν_i is the ionization rate per electron and ν_a the attachment rate to neutral molecules per electron. At breakdown the electron density is low enough to neglect the effects of space charge and recombination of electrons with positive ions in the volume of the gas.[1] Using the current density given by Eq. (1) and putting Eq. (3) in terms of the potential Ψ, we obtain for the continuity equation,

$$\frac{1}{D}\frac{\partial \Psi}{\partial t} = \nabla^2 \Psi + \frac{\nu}{D}\Psi .$$

The time factor may be separated by putting

$$\Psi = \Psi_0(x,y,z)e^{-t/\tau}.$$

The resulting equation for Ψ_0 is

$$\nabla^2 \Psi_0 + \frac{\nu}{D}\Psi_0 + \frac{1}{D\tau}\Psi_0 = 0. \tag{4}$$

The time constant τ is determined by the boundary conditions.

The boundary condition on Ψ is obtained by setting the diffusion current approaching the wall equal to the random current collected by the wall. This problem has been considered for neutron diffusion[2] with the

1. L. M. Hartman, Phys. Rev. 73, 316 (1948), has computed breakdown for the case of an infinite medium in which diffusion losses are zero and recombination is the controlling loss mechanism. Such a case would be realized experimentally with very large tubes or high pressures, but is not within the range of the present experiments.
2. G. Placzek, W. Seidel, Phys. Rev. 72, 550 (1947).

result that the extrapolated neutron density goes to zero at a distance of the order of a mean free path beyond the wall. For electrons the result should be modified to include the action of the image charge induced by the approaching electrons. Because diffusion theory holds only when the mean free path is small compared to the dimensions of the discharge tube, it is sufficiently accurate to apply the condition that the electron density and therefore ψ vanish at the walls.

The condition that ψ is zero on the boundary results in a set of characteristic values for $1/\tau$, which are denoted by $1/\tau_a$, and in a corresponding set of orthogonal functions ψ_a. The function corresponding to the lowest of the $1/\tau_a$'s is positive everywhere, while the others all change sign at some point in the discharge tube. The background electron density at the instant the field is applied may be expanded in a series of the functions ψ_a. Each term thereafter rises or decays exponentially in time at its own characteristic rate, depending on whether τ_a is negative or positive. If the electric field is zero, ν is zero or negative, and all of the time constants are positive, because the discharge must decay without ionization. As the electric field is raised the $1/\tau_a$'s become smaller until the lowest goes through zero and the corresponding function builds up in time. The other terms do not build up, as their time constants are still positive. At this point the ionization rate produced by the field has reached the value where it is replacing diffusion losses. A slightly higher field then causes breakdown.

The breakdown field may now be defined as the field necessary to maintain a steady discharge in the limit of zero electron density. Although this field will not produce breakdown, any greater field will do so. It may be computed as the lowest characteristic value of the equation,

$$\nabla^2\psi + \frac{\nu}{D}(E)\psi = 0. \tag{5}$$

The ratio ν/D is indicated as a function of the electric field, which in turn is a function of the space coordinates. The field may be represented as

$$E = E_o f(x,y,z),$$

where E_o is the field at a reference point P_o, and f is a space factor determined from Maxwell's equations. The value of f at P_o is unity. The electric field at any point may be specified by the field at the reference point and the function f. The characteristic value, obtained from Eq. (5) and the boundary conditions, thus specifies the breakdown field in terms of E_o.

3. The Effect of Secondary Processes

The effect of secondary emission of electrons from the walls of the discharge tube may be included by appropriate changes in the boundary conditions. These should be modified to make the diffusion current approaching the wall equal to the difference between the random current absorbed by the wall and the secondary emission current emitted from the wall. The secondary emission term is appreciable only when the number of secondary electrons per positive ion is of the order of unity, and this is not the case.

In the d-c discharge the electrons leave the cathode and multiply as they proceed toward the anode, but they cannot return to the cathode once they have lost energy through an inelastic collision. The discharge is therefore entirely dependent upon the emission properties of the cathode. On the other hand, the a-c discharge may be maintained entirely from electrons obtained from ionization by collision in the gas. If an electron leaves the wall, one more electron must return to the wall to satisfy the continuity requirement. The only effect of this emission is to move the equivalent wall position slightly farther from the center of the discharge, and thus reduce the breakdown field by an equally slight amount. This effect is entirely negligible owing to the small emission probability.

4. High-Frequency Ionization Coefficient

Equation (5) indicates that breakdown calculations depend on a knowledge of the ratio ν/D as a function of the gas used, the electric field, the pressure, and the frequency. This quantity is analogous to the Townsend coefficient α. The number of ionizations per centimeter of electron drift is $\nu/\bar{v}$, where $\bar{v}$ is the drift velocity. A d-c field produces a drift velocity much larger than that due to diffusion. If diffusion is neglected $\bar{v} = \mu E$, where μ is the electron mobility. Then, $\alpha = \nu/\mu E$. The Townsend coefficient refers ionization to a drift motion produced by electron mobility, while ν/D refers it to a drift motion produced by diffusion.

The analogy is more apparent when the Townsend coefficient η is compared with the corresponding high-frequency coefficient, ζ. Since $\eta = \alpha/E$,

$$\eta = \nu/\mu E^2. \qquad (6)$$

The units are 1/volts. The units of ν/D are $1/cm^2$, may be changed to $1/volts^2$ by dividing by the square of the field:

$$\zeta = \nu/DE^2 \ . \qquad (7)$$

The coefficients are related by

$$\zeta = \eta \frac{\mu}{D} \ .$$

Putting the high-frequency coefficient into the dimensions of volts has the advantage of making it a function of E/P and $p\lambda$ for a given gas. Thus, if the function

$$\zeta = \zeta(E/p,\ p\lambda)$$

is known, breakdown may be computed by using Eq. (5). The inverse problem of determining ζ from breakdown experiments will now be considered.

5. Breakdown between Parallel Plates

Measurements of ζ from breakdown data were made by using TM_{010}-mode cylindrical cavities. The end plate separations were small compared to the diameter of the region where the field is substantially uniform, in the vicinity of the center of the cavity. These cavities therefore approximated the conditions of infinite parallel plates with a uniform electric field. The solution of Eq. (5) for this case is

$$\Psi = A \sin \frac{z}{\Lambda} ,$$

where $\Lambda = L/\pi$, L is the plate separation distance, z is the distance from one plate to an arbitrary point in the cavity, and A is a constant. The breakdown condition is

$$\zeta = 1/\Lambda^2 E^2 . \qquad (8)$$

The electric field for breakdown may be measured as a function of pressure, frequency, and plate separation. From these data the ionization coefficient ζ may be computed.

6. Experimental Apparatus

A block diagram of the apparatus used in the experiment is shown in Fig. 1. A continuous-wave tunable magnetron in the 10-cm wavelength region supplies up to 150 watts of power into a coaxial line connecting to the measuring equipment. A probe in the line samples a small signal for the cavity wavemeter. A power divider provides a means of varying the power. A known fraction of the incident wave is sampled by a directional coupler and sent to a thermistor element whose resistance change measured by a sensitive bridge provides a measure of the power incident on the cavity. A standing-wave detector consisting of a slotted section of line and a movable probe is used to measure the standing-wave ratio and position of minimum voltage in the line leading to the discharge cavity. The probe signal is carried through a flexible cable to a detector, which is equipped with a waveguide-beyond-cutoff type attenuator. With this arrangement one can measure the cavity input impedance and the fraction of incident power absorbed by the cavity.[1] The cavities are designed to resonate in the TM_{010}-mode at 10-cm wavelength. They are coupled to the input transmission line by a coupling loop. A second coupling loop provides a trans-

1. For details of experimental technique, reference should be made to such sources as: C. G. Montgomery, "Technique of Microwave Measurements", McGraw Hill Book Company, New York, 1947.

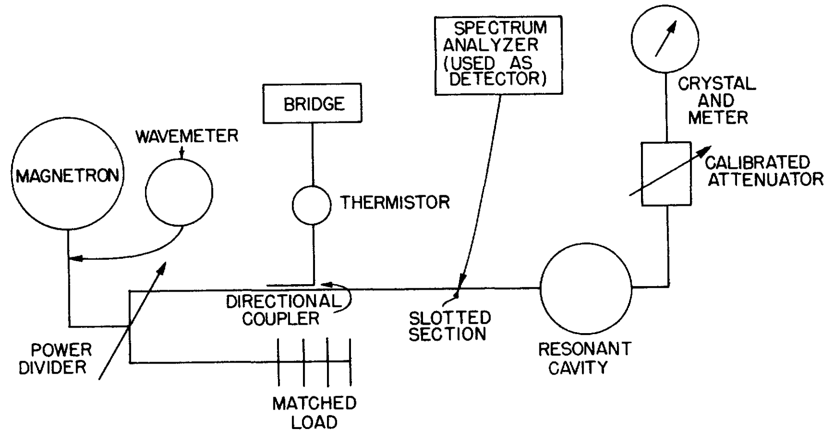

Fig. 1. Block diagram of experimental apparatus.

mitted signal to a cutoff attenuator terminated in a crystal rectifier and microammeter.

The unloaded Q of each cavity is determined by means of standing-wave measurements. The power incident on the cavity and the standing-wave ratio in the line provide the information necessary to determine the power dissipated in the cavity. The dissipated power with no discharge is related to the stored energy through the unloaded Q. The electric field can be determined from power and standing-wave measurements, and from the known electromagnetic field configuration of the cavity. It proves convenient to use a transmission coupling loop and crystal current meter to measure field, after the incident power measurements have been used to calibrate the crystal, meter, and attenuator.

7. Experimental Results

The breakdown experiment consists of filling the cavity with gas at a certain pressure, increasing the magnetron power while watching the transmission crystal current until this current reaches a maximum value and drops suddenly to a lower value. This drop indicates that the gas has broken down, and the maximum crystal current indicates the breakdown field. This operation is repeated for a variety of experimental conditions.

Results of experiments on three thin parallel-plate cavities are given in Fig. 2. The gas used was air from which impurities were removed by a liquid nitrogen trap. The rms breakdown field is plotted against pressure for each cavity. The field is lower for the larger cavities, because the diffusion losses are lower. For each curve there is a pressure at which the breakdown field is a minimum.

Figure 3 shows ζ plotted against E/p for various values of constant $p\lambda$, as determined from Fig. 2. The dotted curves show the course of the

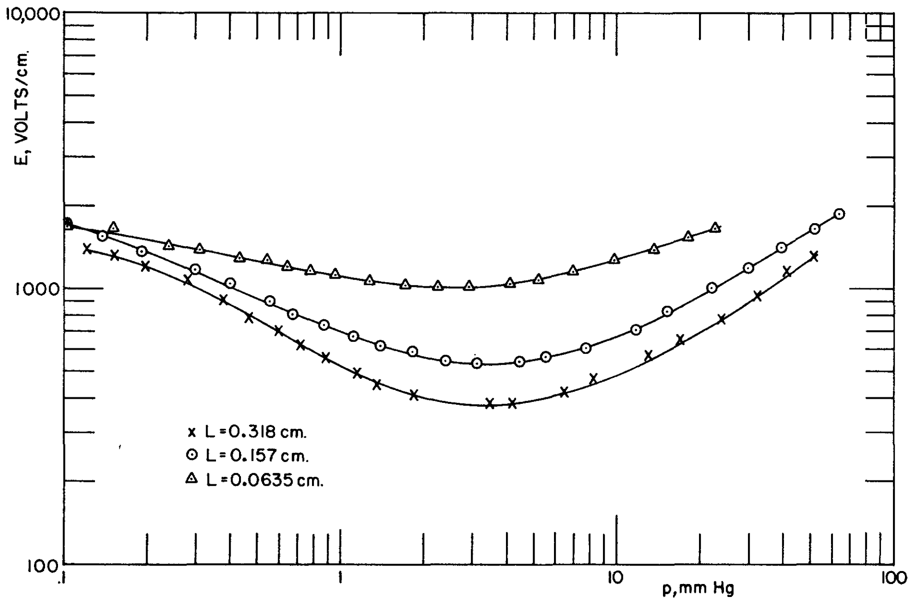

Fig. 2. Breakdown field(volts/cm) as a function of pressure (mm Hg) for three plate-separation distances (L cm). Cavity resonant wavelengths were all near 9.6 cm.

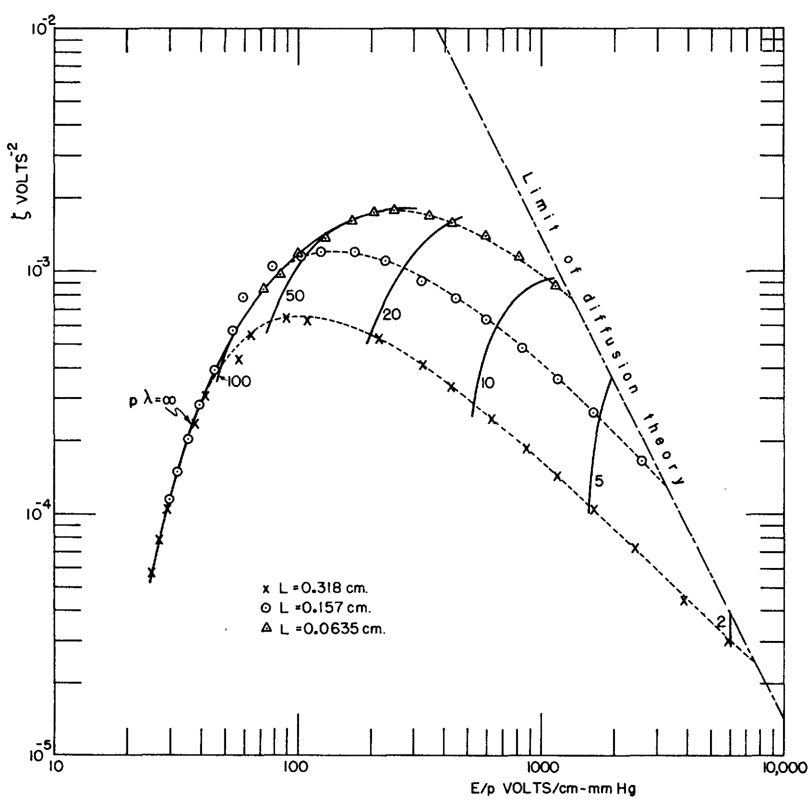

Fig. 3. High-frequency ionization coefficient $\zeta = \nu/DE^2$ (ionizations/volt^2) as a function of E/p (volts/cm-mm Hg) and pλ (mm Hg-cm), determined from data shown in Fig. 2.

data for a given cavity with variations of pressure. On the low E/p side they come together to form a common envelope, but on the high side they depart from the envelope at various values of E/p depending on the size of the cavity.

The dot-dash line sloping down to the right indicates the limit of validity of diffusion theory, where the mean free path is small compared to the tube dimensions. The line is drawn so that $\ell/\Lambda = 1$, where ℓ is the mean free path. This relation, with Eqs. (5) and (7), results in

$$\zeta = \frac{P_c^2}{(E/p)^2}$$

where P_c is the collision probability per centimeter per millimeter of pressure ($\ell = \frac{1}{P_c p}$) and is taken as 38 for air.

8. Discussion

The ionization produced by a given value of E/p is seen to be maximum when pλ is large. As pλ decreases, the ionization drops, slowly at first and then more rapidly. A simple calculation will show that the experimental values of pλ at which the ionization begins to drop correspond to the condition when the field radian frequency approaches the collision frequency.

The transition from the region of many oscillations per collision to many collisions per oscillation is marked also by the minimum breakdown voltage point in the field-versus-pressure curves. The breakdown voltage decreases as the pressure is lowered because the energy gained between collisions is increased and there is more ionization available to compensate for the increasing diffusion losses. When the transition region of pλ is reached, the energy transfer is no longer at its maximum efficiency, and a higher field is required to replace the diffusion losses. The minimum is therefore not a Paschen minimum and would not be expected to follow the Paschen law.

The present paper has postulated a condition for breakdown which balances electron generation through ionization by collision against electron loss through diffusion. Application of the diffusion equation and boundary conditions provides a means of computing breakdown fields, if the high-frequency ionization coefficient, which has been introduced, is known. The procedure has been reversed for the special case of parallel-plate and uniform-field breakdown, with breakdown data used to infer the ionization coefficients. These data may then be used to compute breakdown fields for other geometries and field configurations. Thus parallel-plate and uniform-field breakdown curves constitute a basic set of data from which other cases may be treated. Although the experimental work has been performed for air, the method is generally applicable.