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of a Frequency-Stabilized Oscillator

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DISTORTION IN THE FREQUENCY-MODULATED OUTPUT
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by

W. C. Galloway

Abstract

The amount of distortion is determined in the frequency-modulated output of a microwave oscillator when the oscillator is frequency stabilized. Two stabilizing circuits are investigated. They are the i-f carrier type developed by R. V. Pound and a modification of this discriminator, called the equal-arm discriminator, developed by F. P. Zaffarano. The distortion is calculated for both types of stabilizers and the calculations are checked by measurements. Measured values for the distortion are somewhat higher than the predicted values, but there is a reasonable correlation between the two.



DISTORTION IN THE FREQUENCY-MODULATED OUTPUT
OF A FREQUENCY-STABILIZED OSCILLATOR

1. Introduction

Methods have been developed for stabilizing the frequency of microwave oscillators.^{1,2} The Pound i-f type of stabilizer and the modification of this stabilizer developed by Zaffarano can be used for controlling the frequency excursions of a frequency-modulated klystron as well as controlling changes in the average frequency of the klystron. For some communication systems it may be desirable to frequency-modulate a frequency-stabilized microwave oscillator. If this is so, two general methods of approach are possible.

(a) The stabilizing circuit can be designed so that it is not effective at the modulating frequencies and thus only the average frequency of the oscillator is controlled by the stabilizing circuit.

(b) The stabilizing circuit can be designed so that it controls the frequency of the oscillator during modulation cycles as well as the average frequency.

The latter type of control offers some desirable characteristics, particularly when any distortion in the oscillator output must be held below a given amount. When the gain of the stabilizing circuit is large, the output frequency of the oscillator depends primarily on the frequency-versus-voltage characteristic of the stabilizing circuit and no longer on the frequency-versus-voltage characteristic of the microwave generator. The stabilized oscillator can be thought of as a feedback amplifier in which the oscillator (klystron) is the original amplifier, or μ circuit, and the frequency discriminator is the β circuit. Since the output frequency depends primarily upon the characteristics of the microwave discriminator, a calculation of the non-linearity of the discriminator will be a measure of the distortion in the frequency-modulated output of the transmitter.

The purpose of this research is to investigate the distortion in the frequency-modulated output of a frequency-stabilized microwave oscillator. The distortion will be calculated and measured experimentally as a function of the peak deviation of the microwave oscillator. The modulating signal will be introduced at the repeller of the klystron.

2. Theoretical Considerations

A block diagram of the i-f stabilizer developed by Pound is shown in Fig. 1. The microwave part of the stabilizer is shown as well as the necessary lower-frequency components.

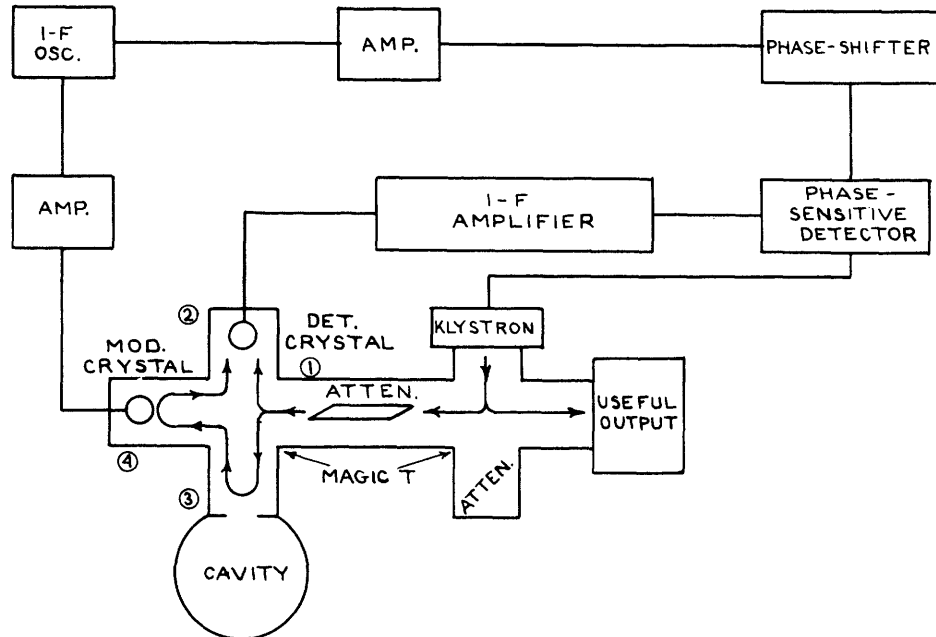


Figure 1. Block diagram of Pound i-f type stabilizer.

R. V. Pound has shown that the i-f voltage of the discriminator is of the form,¹

$$e_{i-f} = \frac{K\alpha a}{(\alpha+1)^2 + a^2} \cos \omega_2 t \quad (1)$$

where

K = a constant,

$\alpha = Q_o \left(\frac{1}{Q_L} - \frac{1}{Q_o} \right) = \frac{Q_o}{Q_L} - 1,$

$a = 2Q_o \left[\frac{f-f_o}{f} \right],$

ω_2 = the angular frequency of the i-f oscillator,

Q_o = unloaded cavity Q ,

Q_L = loaded cavity Q ,

f_o = resonant frequency of the cavity,

f = instantaneous frequency of the microwave source.

Equation (1) times a constant also represents the correction voltage, if one assumes that the output of the phase-sensitive detector is a linear function of the i-f signal, and the bandwidth of the i-f amplifier and phase-sensitive detector is large compared to the modulating frequencies of interest.

Since the frequency deviation from the center frequency is of interest, Eq. (1) can be rewritten showing the frequency deviation explicitly. Also, with the assumptions mentioned in the above paragraph in mind, Eq. (2) can be interpreted as being the correction voltage, e_2 .

$$e_2 = \frac{K' \alpha \frac{2Q_0 \delta}{f_0}}{(\alpha+1)^2 + \frac{4Q_0^2 \delta^2}{f_0^2}} = \frac{B \delta}{C^2 + A^2 \delta^2} = \frac{B \delta}{C^2} \left(1 + \frac{A^2 \delta^2}{C^2}\right)^{-1} \quad (2)$$

where

K' = K times the constant for the gain of the i-f amplifier and phase-sensitive detector,

δ = $f - f_0$,

A = $2Q_0/f_0$,

B = $K' \alpha C$,

C = $\alpha + 1$.

The frequency of the klystron must now be related to this correction voltage. A schematic diagram of the output circuit of the phase-sensitive detector and a method of introducing a modulating signal is shown in Fig. 2. The voltages appearing at the points of interest in the feedback circuit are defined in Fig. 2.

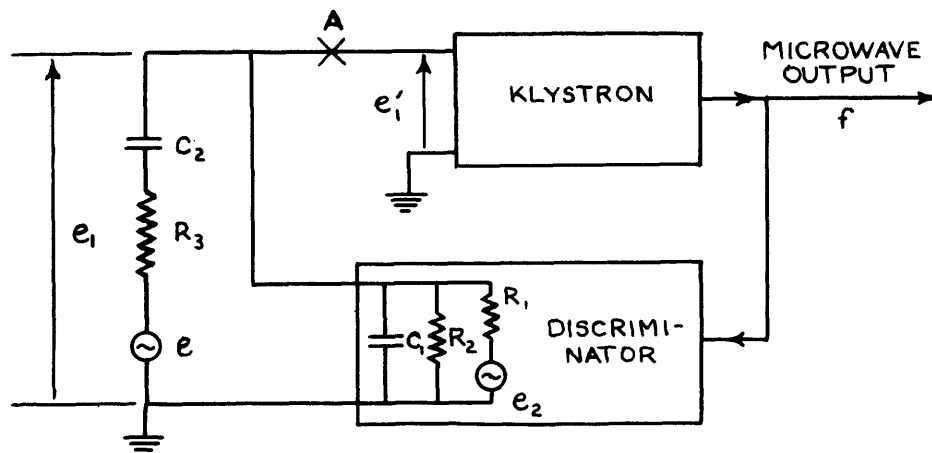


Figure 2. Schematic diagram of correction-voltage circuit.

If the modulating frequencies are limited to values so that the impedance of condenser C_1 is very high and the impedance of condenser C_2 is very small compared to the resistors involved, the voltage at the klystron repeller in terms of the voltages e_2 and e is,

$$e_1 = \frac{R_2(e_2 R_3 + e R_1)}{R_1 R_2 + R_1 R_3 + R_2 R_3} = \frac{R_2(e_2 R_3 + e R_1)}{R} \quad (3)$$

By using Eq. (2), e_2 can be expressed in terms of the frequency deviation, δ . Also, we can relate e_1 to the frequency deviation as,

$$e_1 = \frac{\delta}{\mu} \quad (4)$$

where μ = the function relating the frequency of the klystron to the repeller voltage in cycles per second per volt (in general, a function of the repeller voltage).

Substituting Eq. (2) and Eq. (4) into Eq. (3) we have,

$$\frac{\delta}{\mu} = \frac{R_2}{R} \left(\frac{B\delta}{C^2 + A^2\delta^2} R_3 + e R_1 \right), \quad (5)$$

or

$$\delta = \frac{e R_1 R_2 \frac{\mu}{R}}{1 - \frac{B R_2 R_2}{(C^2 + A^2\delta^2) R}} \quad (6)$$

A measure of the stabilizing action of the system can be obtained by opening the feedback circuit at the point A indicated in Fig. 2 and measuring the ratio of e_1 to e_1' . Doing this analytically, we have from Eq. (3) and Eq. (4), with the modulating voltage e short-circuited and a small voltage inserted for e_1' ,

$$e_1 = \frac{R_2 R_3 e_2}{R} \quad (7)$$

and

$$\delta = \mu e_1' \quad (8)$$

Substituting Eq. (7) and Eq. (8) into Eq. (2), we have

$$\frac{e_1}{e_1'} = \frac{BR_2R_3}{(C^2+A^2\delta^2)} \frac{\mu}{R} \quad (9)$$

For the klystron to be well stabilized, this ratio of voltages, which can be called the open-circuit loop gain, must be much larger than unity. Thus, to a good approximation, Eq. (6) may be written as,

$$\delta \approx - \frac{\frac{eR_1R_2}{BR_3R_2} \frac{\mu}{R}}{\frac{(C^2+A^2\delta^2)}{R}} = -e \frac{R_1}{R_3} \frac{(C^2+A^2\delta^2)}{B} \quad (10)$$

or in terms of δ ,

$$e = - \frac{R_2}{R_1} \frac{B\delta}{C^2+A^2\delta^2} \quad (11)$$

Eq. (11) shows that δ depends only on e and not on μ .

The problem is to determine the distortion in δ when e is varied as a given function of time. From Eq. (11) an expression can be obtained for δ in terms of e . If this is done, however, the result will contain the distortion in the output as a function of the modulating voltage and not as a function of the frequency deviation.

Since it is more expedient to obtain the distortion in terms of the frequency deviation, the problem will be examined in a slightly different manner. Assume that a frequency-modulated output signal containing no harmonic distortion is desired. When this pure signal is applied to the stabilizing circuit, e_2 will not be a simple harmonic wave, but will contain some harmonic distortion terms. The relative magnitude of these terms can be obtained by analyzing Eq. (2). The harmonic distortion in e_2 can be removed by applying small voltage generators in series with e_2 that counteract the harmonic distortion. Or, these small generators can be put in the frequency-modulated wave rather than in e_2 and again a pure sine wave is obtained for e_2 . Since μ does not appear in Eq. (11), consider it to be a constant. Then, since both e_2 and e are sinusoidal, to get the required frequency deviation, e has only to be adjusted to the correct value.

If the distortion introduced in the harmonics coming through the stabilizing circuit is neglected, the harmonic terms in the frequency-modulated output will bear the same relationship to the fundamental as the harmonic terms in e_2 bear to the fundamental of e_2 when a pure frequency-modulated signal is applied to the discriminator. In other words, a calculation of the distortion produced in the output of the phase-sensitive detector, when a pure frequency-modulated wave is applied to the stabilizer, will give the distortion to be expected in the frequency-modulated output

of the klystron when the stabilizing circuit is connected and a sinusoidal modulating voltage is applied. This reasoning neglects the distortion introduced in the harmonic distortion terms by the discriminator characteristic, but as long as the harmonic distortion is small, this omission will not cause an appreciable error. Also, this argument no longer holds when μ is non-linear enough to produce harmonic distortions in the output greater by a factor of the open-circuit loop gain than those caused by the stabilizer characteristic.

The harmonic distortion in the output, with the approximation of a linear amplifier and phase-sensitive detector, may be obtained by expanding Eq. (2) and collecting the Fourier coefficients. W. G. Tuller has done this for the Pound stabilizer and plotted the distortion as a function of the normalized frequency deviation.^{3,4} His work shows that the fractional harmonic distortion terms are:

$$\begin{aligned}
 \text{3rd Harmonic} &= \frac{16\Delta^2 - 20\Delta^4 + 21\Delta^6}{64 - 48\Delta^2 + 40\Delta^4 - 35\Delta^6}, \\
 \text{5th Harmonic} &= \frac{4\Delta^4 - 7\Delta^6}{64 - 48\Delta^2 + 40\Delta^4 - 35\Delta^6}, \\
 \text{7th Harmonic} &= \frac{\Delta^6}{64 - 48\Delta^2 + 40\Delta^4 - 35\Delta^6},
 \end{aligned} \tag{12}$$

where

$$\begin{aligned}
 \Delta &= \text{normalized frequency deviation} = \frac{2Q_L}{f_0} \delta_m, \\
 \delta_m &= \text{peak frequency deviation.}
 \end{aligned}$$

The microwave circuit of the equal-arm discriminator is shown in Fig. 3. In this discriminator, the energy entering the magic T splits equally between the reference cavity and the modulator crystal. Let E_0 be the magnitude of the voltage of the microwave signal incident on the cavity and modulator crystal. From the same considerations as were used to derive Eq. (1), the voltage reflected, E_1 , from the cavity can be shown to be,

$$E_1 = \frac{2E_0 A \alpha \delta}{C^2 + A^2 \delta^2} \tag{13}$$

After one traverse through the magic T, the microwave signal incident on the detector crystal is,

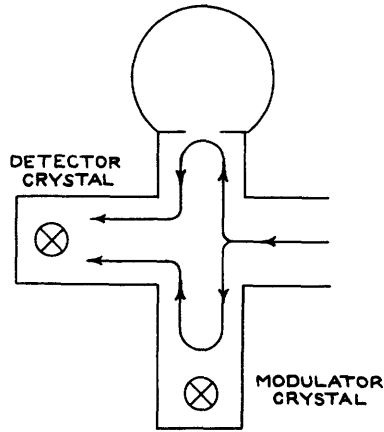


Figure 3. Microwave circuit of the equal-arm discriminator.

$$E_2 = \frac{2E_0 A \alpha \delta}{\sqrt{2}(c^2 + A^2 \delta^2)} \quad (14)$$

A convenient way of defining and measuring the efficiency of the modulating crystal is to define the efficiency,

$$\text{efficiency} = c = \frac{\text{total sideband power reflected}}{\text{total incident power}} .$$

Thus, the voltage reflected in each sideband, E_3 , from the modulating crystal is,

$$E_3 = E_0 \sqrt{\frac{c}{2}} \quad (15)$$

and the voltage, E_4 , in each sideband incident on the detector crystal is,

$$E_4 = E_0 \frac{\sqrt{c}}{2} \quad (16)$$

It has been shown that the envelope of the signal incident on the detector crystal is,²

$$\text{Envelope} = |E_2 + 2E_4 \cos \omega_2 t| \quad (17)$$

Since the carrier of the wave that is incident on the detector crystal varies from essentially zero magnitude, the assumption that the detector crystal will operate as a linear detector is questionable. For this reason, the form of the correction voltage will be investigated assuming both a square-law and a linear detector.

First, if it is assumed that the detector crystal follows a square law, the i-f output voltage can be formulated by squaring Eq. (17).

$$(\text{Envelope})^2 = E_2^2 + 4E_2E_4 \cos \omega_2 t + 4E_4^2 \cos^2 \omega_2 t \quad (18)$$

Since the i-f amplifier only accepts signals of frequency $\omega_2/2\pi$, the i-f signal is

$$e_{i-f} = 4E_2E_4 \cos \omega_2 t = \frac{2\sqrt{2}E_0^2 \sqrt{cA\alpha\delta}}{c^2 + A^2\delta^2} \cos \omega_2 t \quad (19)$$

Except for the constant multiplier, this expression is identical in form to Eq. (1) which is the expression for the i-f voltage of the Pound discriminator. Since the constant multiplier does not change the relative sizes of the harmonic distortion terms, the distortion for the equal-arm discriminator with a square-law detector will be the same as that of the Pound type.

When the detector crystal operates as a linear detector, it has been shown that the i-f voltage is of the form,²

$$e_{i-f} = \frac{4E_4}{\pi} \left[\frac{E_2}{2E_4} \sqrt{1 - \left(\frac{E_2}{2E_4}\right)^2} + \sin^{-1} \frac{E_2}{2E_4} \right] \cos \omega_2 t \quad (20)$$

For small values of $E_2/2E_4$, corresponding to reasonably small deviations, this equation reduces to

$$e_{i-f} \approx \frac{4E_4}{\pi} \left[\frac{E_2}{2E_4} - \frac{1}{2} \left(\frac{E_2}{2E_4}\right)^3 - \frac{1}{8} \left(\frac{E_2}{2E_4}\right)^5 - \frac{1}{16} \left(\frac{E_2}{2E_4}\right)^7 - \dots \right. \\ \left. + \frac{E_2}{2E_4} + \frac{1}{6} \left(\frac{E_2}{2E_4}\right)^3 + \frac{3}{40} \left(\frac{E_2}{2E_4}\right)^5 + \frac{15}{336} \left(\frac{E_2}{2E_4}\right)^7 + \dots \right] \cos \omega_2 t \quad (21)$$

After collecting terms Eq. (21), in terms of the circuit constants, is:

$$e_{i-f} \approx \frac{2E_0}{\pi} \sqrt{c} \left[\frac{2\sqrt{2}A\alpha\delta}{\sqrt{c}(c^2 + A^2\delta^2)} - \frac{1}{3} \left(\frac{\sqrt{2}A\alpha\delta}{\sqrt{c}(c^2 + A^2\delta^2)} \right)^3 \right. \\ \left. - \frac{1}{20} \left(\frac{\sqrt{2}A\alpha\delta}{\sqrt{c}(c^2 + A^2\delta^2)} \right)^5 - \frac{1}{56} \left(\frac{\sqrt{2}A\alpha\delta}{\sqrt{c}(c^2 + A^2\delta^2)} \right)^7 \right] \cos \omega_2 t \quad (22)$$

Equation (22), if we again assume a linear i-f amplifier and phase-sensitive detector, represents the correction voltage. To get the distortion we must

expand the magnitude of Eq. (22). Expanding $(1 + (A^2\delta^2)/c^2)^{-1}$, for $\delta = \delta_m \sin pt$, and collecting terms, we find the correction voltage e_2 to be:

$$\begin{aligned}
 e_2 = KAc\delta_m & \left[\sin pt - \left(\frac{A\delta_m}{c}\right)^2 \left(1 + \frac{\alpha^2}{3c^2}\right) \sin^3 pt \right. \\
 & + \left(\frac{A\delta_m}{c}\right)^4 \left(1 + \frac{\alpha^2}{c^2} - \frac{\alpha^4}{10c^2c^4}\right) \sin^5 pt \\
 & \left. - \left(\frac{A\delta_m}{c}\right)^6 \left(1 + \frac{2\alpha^2}{c^2} - \frac{\alpha^4}{2c^2c^4} + \frac{\alpha^6}{14c^3c^6}\right) \sin^7 pt \right].
 \end{aligned} \tag{23}$$

Expanding the powers of $\sin pt$ and collecting terms, the fractional harmonic distortion terms can be shown to be

$$\begin{aligned}
 \text{3rd Harmonic} & = \frac{16a_2 - 20a_3 + 21a_4}{64a_1 - 48a_2 + 40a_3 - 35a_4}, \\
 \text{5th Harmonic} & = \frac{4a_3 - 7a_4}{64a_1 - 48a_2 + 40a_3 - 35a_4}, \\
 \text{7th Harmonic} & = \frac{a_4}{64a_1 - 48a_2 + 40a_3 - 35a_4},
 \end{aligned} \tag{24}$$

where

$$\begin{aligned}
 a_1 & = 1, \\
 a_2 & = -\Delta^2 \left(1 + \frac{\alpha^2}{3c^2}\right), \\
 a_3 & = \Delta^4 \left(1 + \frac{\alpha^2}{c^2} - \frac{\alpha^4}{10c^2c^4}\right), \\
 a_4 & = -\Delta^6 \left(1 + \frac{2\alpha^2}{c^2} - \frac{\alpha^4}{2c^2c^4} + \frac{\alpha^6}{14c^3c^6}\right).
 \end{aligned}$$

It is not possible to normalize this result because of the dependence of the distortion on the modulator-crystal efficiency. Some of these distortion components will be calculated and plotted for the circuit tested

experimentally and compared to the case for the square-law detector. The validity of assuming that the detector crystal follows the envelope is very questionable. However, these results above may be considered as an upper limit for the distortion.

Although the normalized deviation, Δ , was used in expressing the above distortion terms, f_o/Q_L is not necessarily the frequency difference between the peaks of the discriminator curve for the equal-arm discriminator. The peak of the discriminator curve occurs either at the frequency deviation, $f_o/2Q_L$, or the frequency when E_2 of Eq. (14) equals $2E_4$ of Eq. (16), depending on which of the above conditions is met first.

3. Experimental Procedure

The experimental arrangement is shown in the block diagram of Fig. 4. The upper part of Fig. 4 shows the Pound stabilizing system and the lower part shows a receiving circuit for detecting the modulated output

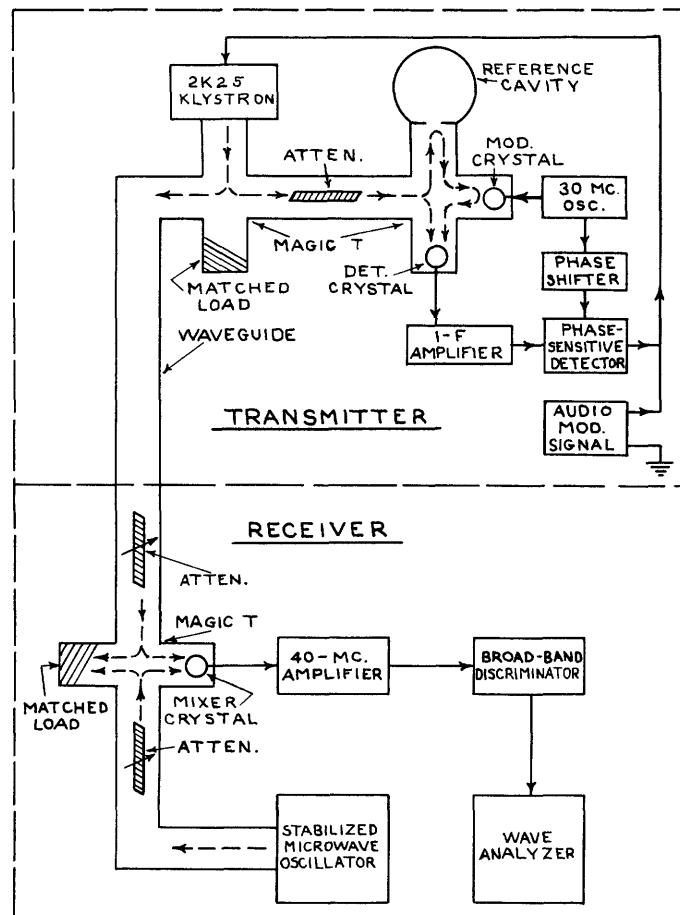


Figure 4. Block diagram of experimental equipment.

of the stabilized-oscillator transmitter. The transmitter was constructed as a bench set-up to allow adjustments and changes to be made. The modulating signal was introduced in the manner shown in the figure.

In the receiver, the modulated output of the stabilized oscillator was mixed in the mixer crystal with an unmodulated signal from a second stabilized oscillator. The frequency of the signal on the output of this crystal was adjusted to 40 Mc/sec, amplified in the 40-Mc amplifier, and detected in the discriminator. Harmonic distortion in the detected output was measured on the wave analyzer.

The method of connecting the phase-sensitive detector output to the klystron is shown in more detail in Fig. 5. This method of connection

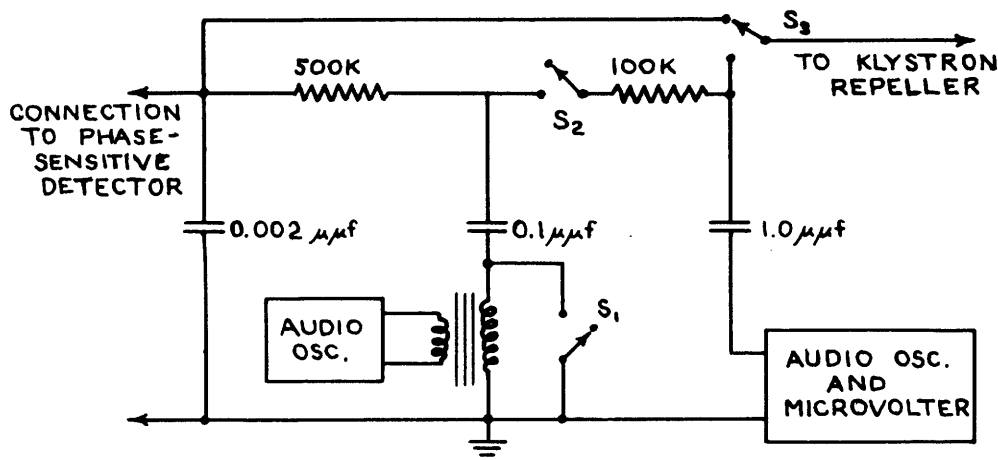


Figure 5. Schematic diagram of circuit connecting phase-sensitive detector to the klystron repeller.

allows measurements to be made with or without stabilization for the modulating frequencies. With the switches in the positions shown in Fig. 5, the klystron is stabilized for all frequencies of interest. With switches S₁ and S₂ closed and switch S₃ in the down position, the klystron is stabilized only at the center frequency and not for modulating frequencies. With this connection, the open-circuit loop gain can be measured by measuring the ratio of the voltage across the terminals of the phase-sensitive detector to the voltage applied by the microvolter. An audio transformer of the line-to-grid type was employed to match the audio oscillator to the klystron circuit.

The 0.002-microfarad condenser across the phase-sensitive detector terminals is necessary to keep the stabilizing circuit from oscillating at high frequencies. The half-power frequency of the stabilizing circuit, determined primarily by the output impedance of the phase-sensitive detector and the 0.002-microfarad condenser, was 4,000 cycles/sec.

The frequency deviation of the oscillator, when modulated, can be determined by measuring the magnitude of the alternating voltage from the klystron repeller to ground. As long as the distortion in this voltage is small, the peak value will be a measure of the peak value of the frequency deviation. The relationship between the voltage at the klystron repeller and output frequency was measured over a range of frequencies around the center frequency. The measurement was made by changing the direct voltage applied to the repeller and measuring the change in frequency with a wavemeter. For the particular klystron used this constant relating frequency to voltage was 1.60 Mc/volt.

To calibrate the equipment a frequency-modulated signal free from distortion was needed. No standard signal generator with the desired linearity was available and a point-by-point calibration did not seem feasible as the calibration had to be accurate enough to measure distortions added by the receiver of the order of one-tenth of one per cent. The stabilizing circuit under test had been constructed with a high-Q cavity so that only small frequency deviations of the klystron would be necessary to produce a measurable amount of distortion. This narrow bandwidth was used so that the effects of the stabilizing circuit could be easily studied, and not necessarily so that it would be more linear than the klystron characteristic. Since the deviations necessary to produce measurable distortions were small compared to the mode of the klystron, it seemed reasonable to expect that the klystron could be used as the standard signal generator.

An analysis was made of the frequency of the klystron as a function of the repeller voltage. The results of the analysis showed that there should be no even-harmonic distortion on the output of the klystron when operating around the center of the mode and that for the particular tube used the third-harmonic distortion could be expected to be as shown in Table I.

TABLE I.

| Peak Deviation in Mc/sec | Peak Voltage Applied to Repeller Electrode in Mv. | Third-Harmonic Distortion Db Below Fundamental |
|-----------------------------|---|--|
| 0.032 | 20 | 131 |
| 0.064 | 40 | 120 |
| 0.096 | 60 | 113 |
| 0.128 | 80 | 108 |
| 0.16 | 100 | 104 |

Since this distortion caused by the klystron itself is smaller than could be measured with the equipment available, this tube can be used as the standard signal generator for checking the receiver.

Because the frequency-versus-voltage characteristic of the klystron is linear, the distortion measurements can be simplified by not using the receiver. If no distortion is introduced in the tube itself, the voltage at the repeller electrode of the tube will be an exact measure of the frequency on the output of the tube. Thus, to take measurements, it is only necessary to measure the voltage at the klystron repeller. Some measurements were made to check this reasoning. A modulating signal was applied and the third-harmonic distortion was measured in the receiver output and at the klystron repeller. The third-harmonic distortion measured at these two places checked within five per cent. Thus, since measurements were easier to make at the klystron repeller, all of the measurements were made of this voltage.

Although the calculations indicate that no even-harmonic distortion should be caused by the stabilizing circuit, some second-harmonic distortion was present. The amount of second-harmonic distortion could be reduced by readjusting the stabilizing circuit so that the point of operation was more nearly at the center of the discriminator curve. The effect on the third-harmonic distortion of the presence of second-harmonic distortion was investigated experimentally and it was determined that even appreciable amounts of second harmonic did not change the amount of third-harmonic measured.

The theoretical and experimental data for the circuit tested are plotted in Fig. 6. The theoretical distortions for the Pound discriminator and equal-arm discriminator (with a square-law detector) are given in the solid curves of Fig. 6. The theoretical distortions for the equal-arm discriminator with a linear detector are the dashed curves of Fig. 6. These data were taken and the curves were calculated using the following constants of the experimental equipment.

| | |
|------------------------------|------------------|
| Modulating frequency | = 500 cycles/sec |
| Microwave frequency | = 9375 Mc/sec |
| Cavity Q, loaded | = 11,100 |
| Cavity Q, unloaded | = 15,950 |
| Open-circuit loop gain, | |
| Pound Stabilizer | = 153 |
| Equal-arm stabilizer | = 217 |
| Bias of i-f amplifier | = -2 volts |
| α | = 0.437 |
| μ | = 1.60 Mc/volt |
| Modulator crystal efficiency | = 0.40 |

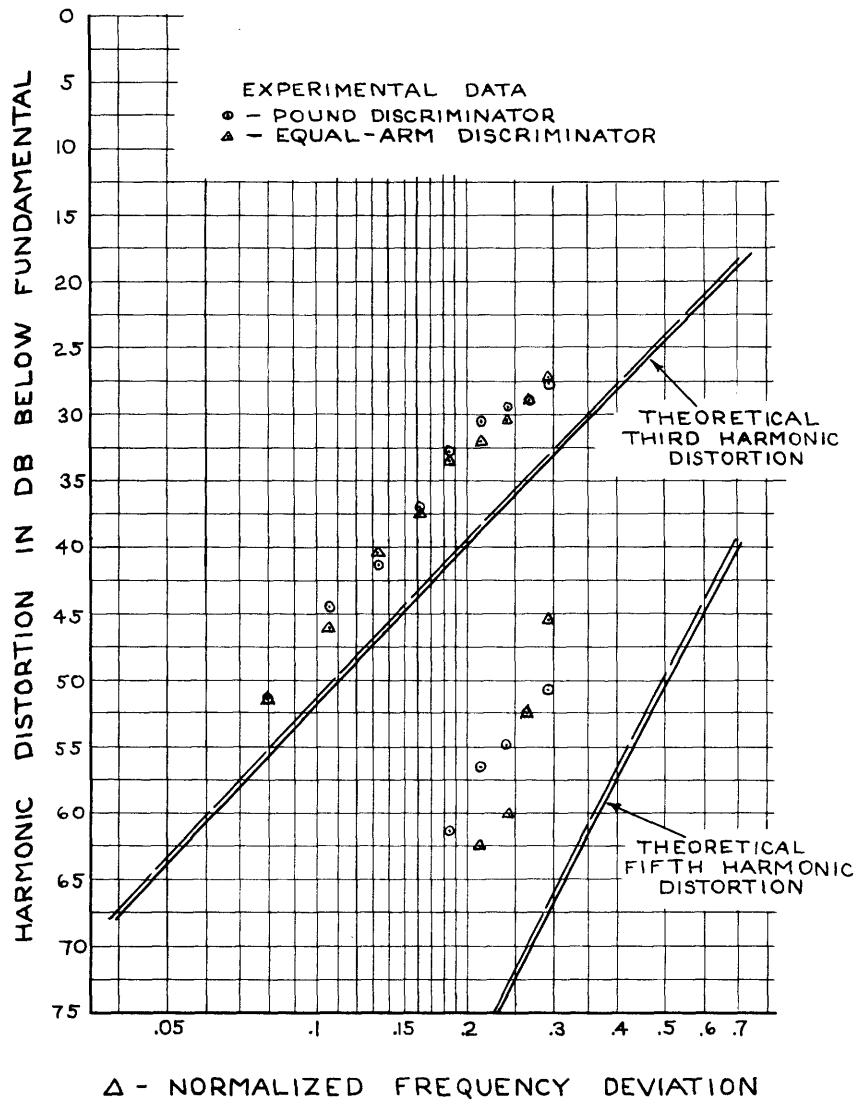


Figure 6. Third- and fifth-harmonic distortions.

The dashed curves of Fig. 6 do not differ appreciably from the solid curves. However, the dashed curves are a function of the modulator crystal efficiency, and for lower efficiency crystals the dashed curves will depart more from the solid curves.

The measured third-harmonic distortion is the same for the Pound and equal-arm discriminators. The distortion measured is approximately 6 db greater than the calculated values. The measured fifth-harmonic distortion for the stabilizers is approximately 15 db greater than the theoretical values, and approximately the same for both discriminators. Although the measured distortion is greater than the calculated values,

the measured distortion varies as a function of the deviation approximately as the calculations predict. The measured values of distortion could be greater than the calculated values for any one of the following reasons:

- a. The detector crystal may not act as the type of detector assumed.
- b. The i-f amplifier is non-linear.
- c. The phase-sensitive detector may not give an output voltage proportional to the magnitude of the input voltage.

If the high gain of the stabilizing systems and the number of non-linear devices employed are considered, the correlation between the measured and calculated values of distortion is good.

4. Conclusion

The distortion has been calculated for the equal-arm and Pound i-f discriminators. Distortion in the equal-arm discriminator is the same as that in the Pound discriminator if a square-law detector crystal is assumed. The theoretical distortion has been expressed as a function of the normalized frequency deviation.

Distortion present in the frequency-modulated output of the stabilized microwave oscillator has been related to the distortion in the output of the discriminator. The theoretical distortion has been obtained by making a calculation of the distortion produced in the output of the discriminator when a frequency-modulated wave (with no distortion) is applied at the input of the discriminator.

The theory has been checked by measurements. Measured third- and fifth-harmonic distortion was approximately the same for the Pound and equal-arm discriminators. The measured distortion was somewhat higher than the calculations predicted. The third harmonic was approximately 6 db too great and the fifth harmonic, approximately 15 db too great. The measured distortions varied with frequency deviation in the same manner as the calculated values.

The experimental data show that the calculated values of distortion are indicative of the amount of distortion to be expected when a stabilized oscillator is constructed. The data also show that, with the method of introducing modulation used in these tests, the actual distortion is somewhat higher than the predictions indicate. It is not unreasonable that the measured values of distortion are somewhat higher than the calculated values when one considers the amount of gain and the number of detectors in the stabilizing circuit.

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