Evaluation of a Cable-Stayed Roof Design

by

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Abstract

This thesis presents a tool for evaluating the performance of a cable-stayed roof, proposed for a new Civil and Environmental Engineering Department building. The CEE building (as designed by the author and the other members of his team as part of the 1998 M.Eng. High-Performance Group Project) is introduced, the relevant design criteria are listed, and a design of the roof is described. Then, the basis and implementation (in Matlab) of the analytical model are presented. Lastly, the model is used to evaluate the roof design and propose changes.

Thesis Supervisor: Prof. Jerome J. Connor
Title: Professor, Civil and Environmental Engineering

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Chapter 1
Introduction

This thesis is concerned with the analysis of a cable-stayed roof which is part of a design proposed for a new MIT Civil and Environmental Engineering Department building. The cable-stayed roof is the centerpiece of a creative and visible high performance structural system. This type of roof is relevant to a civil engineering building because it illustrates the power, elegance, and responsibility of civil engineering. Moreover, the resemblance to "half a cable-stayed bridge" is obvious and intentional.

The field of cable-stayed structures is an area of recent development in design and construction techniques. The use of this technology in the CEE Department's signature building would emphasize MIT's role in the development and use of state-of-the-art technology.

The cable-stayed roof represents a technical challenge, as the cables are required if the building's roof is to remain stable. Without the cables, the roof truss would need to be connected to the piers with a rigid connection which would need to resist a very large moment. The cables reduce displacements along the length of the roof by sharing the load that the space truss roof would otherwise carry alone.

Using cable-stays on the roof is also a construction challenge. One way to construct it would be to treat it like a bridge design, using the cantilever method. Construction would start at the mast or pier where some sections of the truss roof would be installed and connected to the innermost cables. Then construction would proceed moving outward from the pier in both directions as truss components and then cables were added. Cable tensions would be adjusted during this process to obtain a desired roof shape (flat or cambered) in the final product. Alternatively, the building underneath could be used as a staging area, where large sections of the truss could be assembled then hoisted into place and connected to the cables.

This thesis is concerned with evaluating the performance of the roof. Of primary interest is the deflection of the roof and the stiffness requirements for various vertical load resisting components of the system. Matlab code is provided which enables a designer to vary all aspects of the system's geometry, the shear and bending stiffness of the roof, and the cable stiffness and pre-tensioning.

1.1 The Building

MIT's Civil and Environmental Engineering Department is proposing the development of a new and unified building complex to house its academic, research, and administrative activities. A major goal for the CEE Department is to ensure that its visions and aspirations for the future directions of the profession are embodied and visible in the physical appearance of the complex. Thus the CEE Department is requesting a conceptual design for a showcase facility built with the most modern and advanced construction and technology to house the most modern educational and research technology.(Appendix A)

In answer to the Department's request, the Master of Engineering design team produced the design sketched below. (Sketch by Thu Nguyen.) [1]

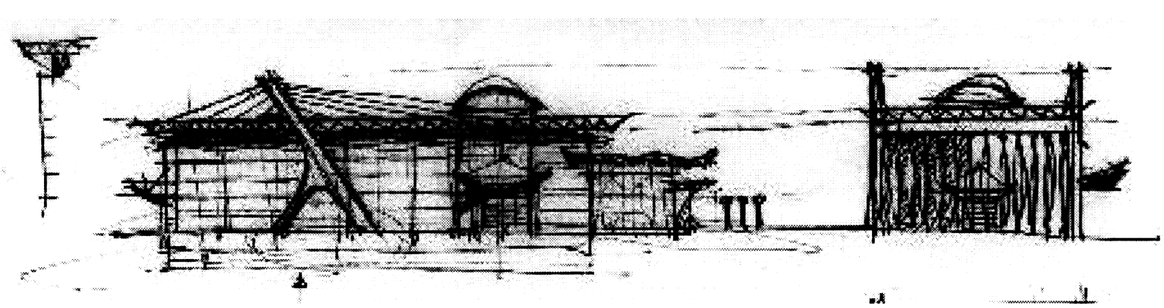

Figure 1.1: Proposal for the New CEE Building

The complex is comprised of two rectangular buildings. In this thesis we are concerned with the larger building in particular, with its cable-stayed roof. It is a six-story building reaching 90 ft. at the underside of the roof truss. The truss height is 10 ft. The tops of the pier masts rise to 50 ft. above the truss roof, or 150 ft. above the ground. The mast tops are located a horizontal distance of 80 ft. from the left end of the building in the above figure. The masts are inclined at 60° with respect to the ground. The lower half of the lambda-shaped pier is in the form of an equilateral triangle with its apex 75 ft. above the ground. There is one lambda-shaped pier on each of the long sides of the building. Beams connecting the piers through the volume of the building provide lateral resistance.

The roof is 240 ft. long and 75 ft. wide. A dome projects through a circular hole in the space truss to the right of the pier in the above sketch. The presence of this circular hole in the roof should be accounted for if this design is developed beyond the conceptual phase. The space truss projects beyond the edges of the building.

On each of the long sides of the building is a set of twelve cables in a vertical plane which run from the mast top to points along the 240 ft. edge of the building. Lateral bracing for the space truss is provided by other cables.

Chapter 2
Design Criteria

2.1 Loads

Snow (or live), dead, wind, and earthquake loads are relevant to the design of the entire roof system including the lateral bracing. The loads used to generate an initial design [1] are as follows: snow/live load of 30 psf, an assumed dead load of 75 psf, wind pressure from 21 to 26 psf, and earthquake loading of 382 kips in the long direction (191 kip/pier) and 220 kips in the short (transverse) direction. The factored load combinations from the ACI code are used because they are more conservative than the Massachusetts Building Code.[2,3]

Table 1: Factored Load Combinations for Determining Required Strength in ACI Code

Condition	Factored load or load effect U
Basic	U=1.4D+1.7L
Winds	U=0.75(1.4D+1.7L+1.7W)
	U=0.75(1.4D+1.7W)
	U=0.9D+1.3W
	U=1.4D+1.7L
Earthquake	U=0.75(1.4D+1.7L+1.87E)
	U=0.75(1.4D+1.87E)
	U=0.9D+1.43E
	U=1.4D+1.7L

For the evaluation contained in this thesis, the only loads of interest are the vertical loads applied to the flat rectangular roof, i.e. snow, wind, and dead loads. The vertical wind load applied at the rooftop is 20.8 psf calculated according to the codes. In the analysis, conceptually, the roof is split in half along its long axis so that we examine one half of the roof, one plane of cables, and one pier. Hence, we are interested in the values per half of the roof, per unit length along the roof. These values are: 1.125 kip/foot of snow, 0.78 kip/foot of wind, and 2.81 kip/foot of dead load. These values can be factored according to Table 1 when using the analysis program for design.

2.2 Strength

All structures are subjected to loads during their lifetimes which they must be able to resist in order to be safe. The roof must not fail under the loads given in section 2.1. While the focus of this thesis is on the deflection of cable-stayed roof designs due to various point and continuous loadings, the strength and stressing of the cables is discussed in some detail. The strength-based design of the piers is outlined in chapter 3 which concerns an initial design proposal. The strength of the roof truss is not addressed in this thesis. (See reference 1 for more details.)

2.3 Deflection

Serviceability is of utmost concern in this thesis. It is important to limit deflections at points along the roof to acceptable values. Deflection criteria for the roof are based on current engineering practice, according to the formulas below. [4]

$$\Delta_{live,\ service} \le \frac{L}{360} \qquad (2.3.1)$$

$$\Delta_{live\ +\ dead,\ service} \le \frac{L}{240} \qquad (2.3.2)$$

The maximum allowable deflection, found using equation 2.3.1, for the right- and leftmost points of the roof are 3.6" and 4.4" respectively. Equation 2.3.2 is not an issue since cable pre-tensioning is used to eliminate dead load deflection.

2.4 Stability

This thesis is concerned with overturning of the roof due to pivoting about its pinned connection to a beam running between the piers. Member buckling and global buckling of the truss are not treated.

Chapter 3
Initial Roof Design

What follows, in this chapter, is a description of the roof system designed for the new CEE Department building. [1] The performance of this design and some alternatives will be evaluated in chapter 5, using the analysis engine of chapter 4.

3.1 The Roof System

For the most part, the roof and the building it covers are structurally separate; indeed, the gravity load of the roof is not resisted by the framing of the six story building at all. This aspect of the design is discernible in three ways, as figures 3.1 and 3.2 show. First, the piers/masts stand just outside the 240' long exterior walls of the six-story building. Second, there is a bank of windows all around the sixth floor. Third, the top floor is free of columns. Lateral bracing of the piers might be integrated into frame of the six story building in a further design stage.

Massive, reinforced-concrete piers support the space truss roof, while steel cables connected to the mast resist a portion of the vertical loads acting on the roof. A lambda-shaped pier on each of the long sides of the building rises 150' vertically to the point where the cables are attached. The two piers support the entire weight of the roof system as well as the moments (in the x-y plane in all of the following figures) resulting from the non-symmetric cable layout and location of the point of attachment of the roof truss. The lateral bracing system of the masts consists of beams passing between the two lambda-shaped piers.

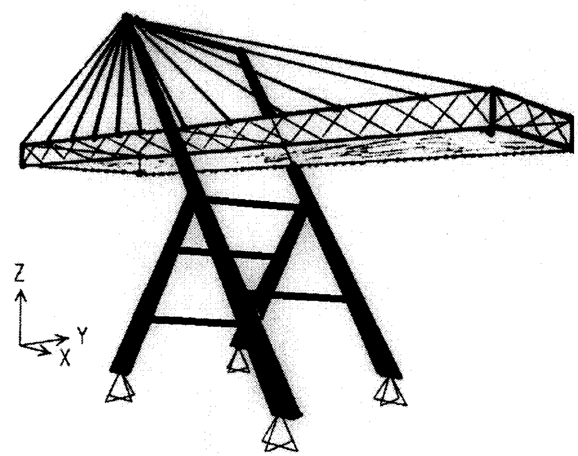

Figure 3.1: Perspective View of the Roof Truss, Cables, and Piers

As this is a conceptual design some loads are assumed and connections simplified. However, the design proves a useful test case for the analysis model in chapter 4.

The next *design* stage would involve the generation of a computer model for the *entire* building which would suggest changes beyond the conceptual design, such as linking the piers to the six story building for greater lateral reinforcement or altering the kinds of connections between elements.

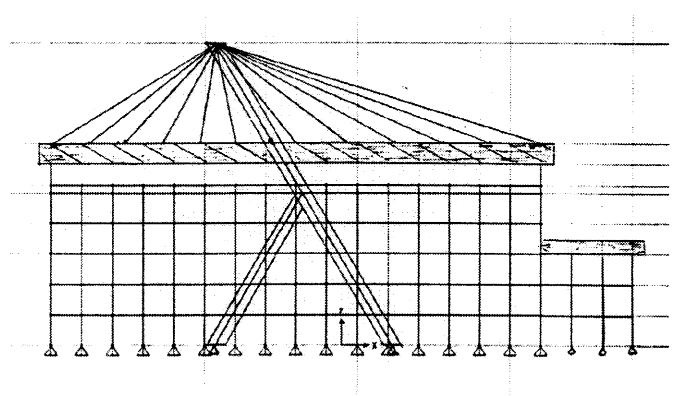

Figure 3.2: Elevation View of the New CEE Building Drawn to Scale

3.1.1 Cables

Twenty-four 50 ksi steel cables counteract deflection and resist a portion of the load applied to the roof truss. Twelve cables are attached to the top of each mast and connect to points along the 240' edge of the roof adjacent to the mast. As these cables are arranged in a vertical plane, they only resist vertical forces applied to the roof. The cables are prestressed so that there is no deflection in the roof due to dead load.

The mast top is located 50 ft. above the roof (or 150 ft. above the ground) and 80 ft. from the left-hand side of the roof truss. (Figure 3.2) Since the roof truss modules are 6 ft., the cables are attached to the truss at intervals which are multiples of 6 ft.. For aesthetic reasons, the attachment points of the leftmost six cables are spaced at 18 ft., while for the other six the spacing is 24 ft.

The means of calculating the vertical deflection and vertical stiffness of the cables (vertical stiffness is the force required for a unit vertical deflection at the cable connection point) are illustrated in figures 3.3 and 3.4. Equations 3.1.1 through 3.1.8, developed from these figures, are used in the cable deflection data spreadsheet (Figure 3.5) and in the analysis model of chapter 4.

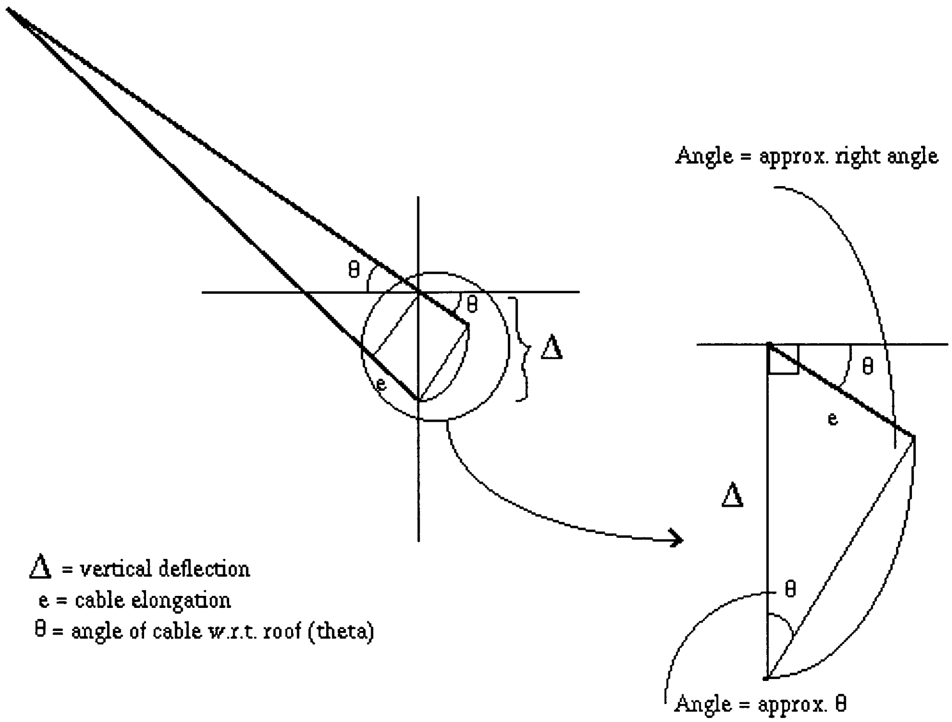

Figure 3.3: Calculation of Vertical Deflection from Cable Elongation

$$\frac{e}{\sin\theta} \cong \Delta = VerticalDeflection \qquad (3.1.1)$$

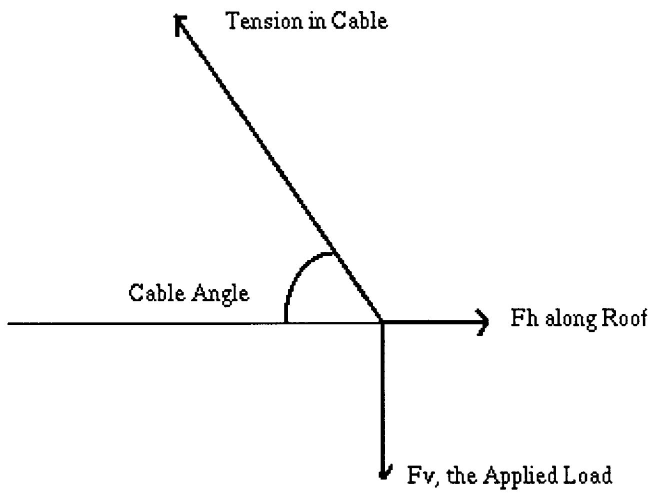

Figure 3.4: Calculation of Cable Elongation

$$Tension \cdot \sin\theta = F_{vert} \tag{3.1.2}$$

$$T = \frac{F_{vert}}{\sin\theta} \tag{3.1.3}$$

$$k = \frac{EA}{L} \tag{3.1.4}$$

$$e = \frac{T}{k} = \frac{L}{EA} \cdot \frac{F_{vert}}{\sin\theta} \tag{3.1.5}$$

$$k_{vert} \cdot \Delta = F_{vert} \tag{3.1.6}$$

$$\Delta \cong \frac{e}{\sin\theta} = \frac{L}{EA} \cdot \frac{F_v}{(\sin\theta)^2} \tag{3.1.7}$$

$$k_{vert} = \frac{EA}{L} \cdot (\sin\theta)^2 \qquad (3.1.8)$$

The cable deflection data spreadsheet (Figure 3.5) shows the cable locations and calculates the length, stiffness, and sine of the angle (with respect to the roof) of each cable given its area and material properties. Tension, stress, elongation, and vertical deflection of the point of attachment to the roof truss are also determined, given the vertical force which each cable is to resist. The magnitude of the vertical forces result is determined by distributing the roof loads to the cables using tributary areas and neglecting irregularities in uniform loads caused by the presence of the dome.

One can see that the spreadsheet in figure 3.5 does not account for a strength reduction factor, thus a factored cable stress greater than 45 ksi (or 0.9 * 50 ksi) would be unacceptable. For example, if we wanted the cable system to resist all of the factored live and dead load, we see from the data that we would need to redesign the twelfth cable.

The prestress of the cable system is designed to equilibrate the unfactored roof dead load, hence roof deflection due to service dead load is eliminated. (See Table 2) The cables must also be strong enough to bear all of the factored dead load and at least a portion of the factored live load.

It is important to decide how much of the live load will be born by the cables and how much by the roof truss, keeping in mind that both sides of the roof must be loaded for the roof truss to carry any load. That is, the truss contributes stiffness when loads are applied to both sides of the pin connection of the truss to the piers, such that the resulting moments are balanced.

Cable deflection data

Final cable geometry

If load is taken as 1.4D+1.7L (2808 kip/roof)

Cable	x	mast x	mast y	length (ft)	sin theta	E (ksi)	A (in^2)	EA/L	F vert	Tension	elong (ft)	defl vert (in)	Cable stress (ksi)	F horiz at mast
1	0	80	50	94.3	0.53	29000	4.9	1506	105	199	0.13	2.99	40.55	-168.48
2	18	80	50	79.6	0.63	29000	4.9	1784	105	168	0.09	1.80	34.23	-130.57
3	36	80	50	66.6	0.75	29000	4.9	2134	105	140	0.07	1.05	28.63	-92.66
4	54	80	50	56.4	0.89	29000	4.9	2521	105	119	0.05	0.64	24.22	-54.76
5	72	80	50	50.6	0.99	29000	4.9	2806	105	107	0.04	0.46	21.76	-16.85
6	90	80	50	51.0	0.98	29000	4.9	2787	105	107	0.04	0.47	21.92	21.06
7	120	80	50	64.0	0.78	29000	4.9	2219	70	90	0.04	0.62	18.29	56.00
8	144	80	50	81.2	0.62	29000	9.6	3428	140	228	0.07	1.30	23.76	179.71
9	168	80	50	101.2	0.49	29000	9.6	2751	140	284	0.10	2.51	29.60	247.10
10	192	80	50	122.7	0.41	29000	9.6	2270	140	344	0.15	4.47	35.88	314.50
11	216	80	50	144.9	0.35	29000	9.6	1921	140	407	0.21	7.36	42.38	381.89
12	240	80	50	167.6	0.30	29000	9.6	1661	140	471	0.28	11.40	49.03	449.28
									1404					1186.22

If load is taken as D+L (1890 kip/roof)

Cable	x	mast x	mast y	length (ft)	sin theta	E (ksi)	A (in^2)	EA/L	F vert	Tension	elong (ft)	defl vert (in)	Cable stress (ksi)	F horiz at mast
1	0	80	50	94.3	0.53	29000	4.9	1506	71	134	0.09	2.01	27.34	-113.60
2	18	80	50	79.6	0.63	29000	4.9	1784	71	113	0.06	1.21	23.08	-88.04
3	36	80	50	66.6	0.75	29000	4.9	2134	71	95	0.04	0.71	19.30	-62.48
4	54	80	50	56.4	0.89	29000	4.9	2521	71	80	0.03	0.43	16.33	-36.92
5	72	80	50	50.6	0.99	29000	4.9	2806	71	72	0.03	0.31	14.67	-11.36
6	90	80	50	51.0	0.98	29000	4.9	2787	71	72	0.03	0.32	14.78	14.20
7	120	80	50	64.0	0.78	29000	4.9	2219	44	56	0.03	0.39	11.50	35.20
8	144	80	50	81.2	0.62	29000	9.6	3428	95	154	0.05	0.88	16.07	121.60
9	168	80	50	101.2	0.49	29000	9.6	2751	95	192	0.07	1.70	20.03	167.20
10	192	80	50	122.7	0.41	29000	9.6	2270	95	233	0.10	3.02	24.28	212.80
11	216	80	50	144.9	0.35	29000	9.6	1921	95	275	0.14	4.98	28.68	258.40
12	240	80	50	167.6	0.30	29000	9.6	1661	95	318	0.19	7.72	33.18	304.00
									945					801.00

If load is taken as L (540 kip/roof)

Cable	x	mast x	mast y	length (ft)	sin theta	E (ksi)	A (in^2)	EA/L	F vert	Tension	elong (ft)	defl vert (in)	Cable stress (ksi)	F horiz at mast
1	0	80	50	94.3	0.53	29000	4.9	1506	20	38	0.03	0.57	7.70	-32.00
2	18	80	50	79.6	0.63	29000	4.9	1784	20	32	0.02	0.34	6.50	-24.80
3	36	80	50	66.6	0.75	29000	4.9	2134	20	27	0.01	0.20	5.44	-17.60
4	54	80	50	56.4	0.89	29000	4.9	2521	20	23	0.01	0.12	4.60	-10.40
5	72	80	50	50.6	0.99	29000	4.9	2806	20	20	0.01	0.09	4.13	-3.20
6	90	80	50	51.0	0.98	29000	4.9	2787	20	20	0.01	0.09	4.16	4.00
7	120	80	50	64.0	0.78	29000	4.9	2219	15	19	0.01	0.13	3.92	12.00
8	144	80	50	81.2	0.62	29000	9.6	3428	27	44	0.01	0.25	4.57	34.56
9	168	80	50	101.2	0.49	29000	9.6	2751	27	55	0.02	0.48	5.69	47.52
10	192	80	50	122.7	0.41	29000	9.6	2270	27	66	0.03	0.86	6.90	60.48
11	216	80	50	144.9	0.35	29000	9.6	1921	27	78	0.04	1.42	8.15	73.44
12	240	80	50	167.6	0.30	29000	9.6	1661	27	91	0.05	2.19	9.43	86.40
									270					230.40

Figure 3.5: Cable Layout & Cable Deflection Data Spreadsheet

Table 2: Cable Pre-Tensioning

Cable	X Coordinate	Pre-Tension (kips)
1	0	96
2	18	81
3	36	68
4	54	57
5	72	52
6	90	52
7	120	37
8	144	110
9	168	138
10	192	167
11	216	197
12	240	228

Chapter 4 describes an *analysis* model for the cable-truss interaction, but for design purposes the vertical load is divided between the truss and cables as follows: the cables carry all the factored dead load plus half the factored live load, while the truss supports half the live load. One can see from the spreadsheet in figure 3.5 that the cables deflect 0.57 in. and 2.19 in. under service live load at the left- and right-hand sides respectively; even without the bending stiffness of the space truss this is more than adequate. (The limiting values are 3.6 in. and 4.4 in.. Section 2.3) Looking at Case II. of the mast loading data (Appendix D) it can be seen that the greatest cable stress is 41 ksi, which is below the reduced cable strength of 45 ksi (0.9 * 50 ksi). Thus the cable system of figure 3.2 (Described in figure 3.5) meets both strength and deflection criteria.

3.1.2 Piers

The first step in designing the masts supporting the cables and truss is to decide on its exact shape and location with respect to the building. The arrangement selected involves raising the mast to 150' (in order to generate reasonably steep cable angles) and angle it at 60 degrees (to reduce moments in the mast due cable-stays pulling at point A) and placing the mast such that point A has a horizontal coordinate of 80' from the left-hand side of the building. A Matlab program, muse.m, is used to generate coordinates of points on the masts as the angle and height of the elements are varied (Appendix B, Fig. 3.6, & Table 3); this information is used to enter the mast geometry in SAP2000 (Figs. 3.1 & 3.2). The pier footing locations are shown in figure 3.7. One can evaluate the structural effects of changes in the geometry of the mast and roof truss easily, with the arrangement of spreadsheets in figure 3.5 and appendix D.

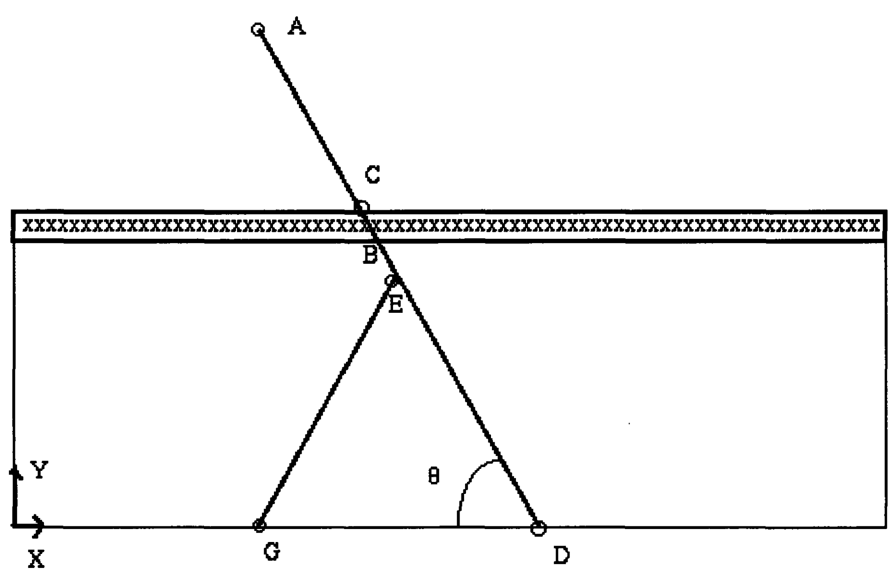

Figure 3.6: Points Establishing Pier Geometry (Key for muse.m Program)

Table 3: Coordinates of Points Establishing Pier Geometry

Point	X	Y	Z
A	80	150	0
B	114.6	90	0
C	108.9	100	0
D	166.6	0	0
E	123.3	75	0
G	80	0	0

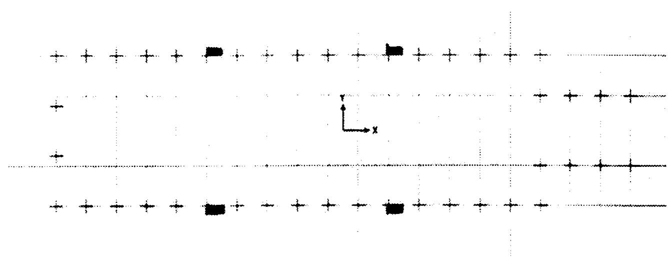

Figure 3.7: Plan View Showing Column & Pier Locations

The loads acting on one of the two lambda-shaped piers can be found in appendix D (Mast Loading Data). The cables and space truss (attached via pinned connections to the underside of a beam between the two piers, point c) transmit horizontal and vertical forces to the masts. The moments, shear forces, and axial forces which result are calculated on this spreadsheet.

Three gravity load cases are examined: A. Live load applied uniformly over the roof, B. Live load applied only to the cables on the right-hand side which pull the mast top in that direction, and C. Live load applied only to the cables that pull the mast top to the left. All member forces, moments, and reactions in the mast are calculated for Case II. A.B.C., where it is assumed that the cables resist all the dead load and half the live load on the structure. (Appendix D: Mast Design Data & Figs. 3.8a,b)

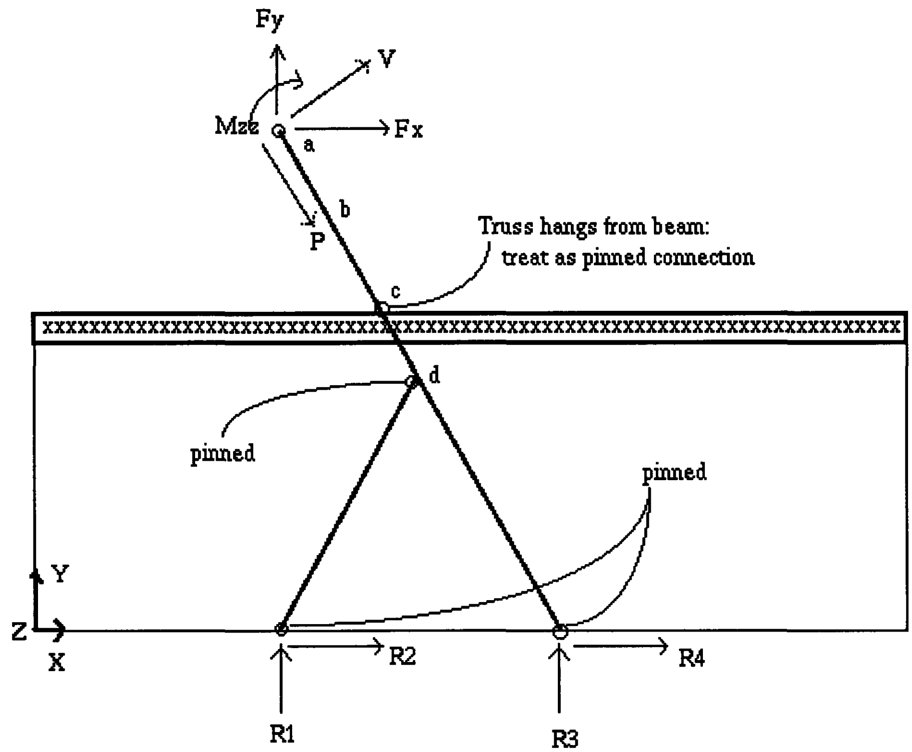

Figure 3.8a: Forces on the Pier (Key for "Mast Design Data" Appendix D)

These values are used to calculate an estimate of the pier dimensions. [1] The resulting section has an effective depth of 7' and a width of 4', f'c = 5000 psi and fy = 60,000. This section would require both negative and positive reinforcement to resist changes in direction of moment that result from different load scenarios.

Similar means are employed using the earthquake load to estimate the dimensions of the five or six beams that join the two piers and act as lateral bracing. Approximate dimensions for those members are 3.5'x3.5'.

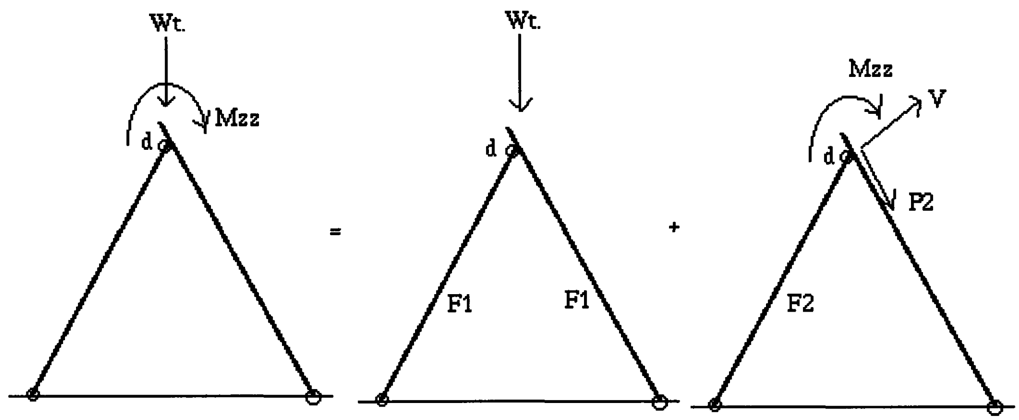

Figure 3.8b: Forces on the Pier (Use with "Mast Design Data" Appendix D)

3.1.3 Roof Truss

The roof truss is connected with simple connections to the underside of a beam passing between the two masts and to the 24 cables. Additional tension members running from the foundation to the corners of the roof and/or from the top of the six story building laterally brace and stabilize the roof. A circular opening in the roof truss is intended to accommodate a dome. The truss forms a flat 240'x75' roof. It is assumed to weigh 75 psf of roof surface. As mentioned in section 3.1.1, the roof truss is intended to carry half of the roof live load, while the cables carry resist the rest of the live and other vertical loads.

3.2 Design Improvements

As the building is designed, the roof and piers do not carry much of the load, compared to the six-story building frame. In the next design phase, it is recommended that one or more floors be suspended from the roof truss. This will make tension elements of the vertical elements in those floors, and allow the lower level columns to be more slender. The additional cost of suspending the upper stories from the roof truss and cables may be offset by two benefits gained by reducing the number and size of vertical members: Costs are reduced since there are fewer beam-column connections in the six story building and flexibility results from the larger bay sizes.

Given that the lateral bracing system of the masts consists of beams passing though the volume covered by the roof truss, the beams would be integrated with the floors of the six story building in a more detailed design. It is also likely that pin-connections between the concrete pier components would really be constructed as rigid joints, although this is more difficult to design. Furthermore, we assume for our conceptual design that each set of twelve cables forms a vertical plane and is connected along an edge of the space truss. In reality, the space truss would project beyond the exterior walls of the building, the cables would be attached some distance in from the edge, and the plane could be non-vertical (inducing biaxial bending in the masts).

3.3 Construction

One way to construct the cable-stayed roof truss would be to treat it like a bridge design, and use the cantilever method. Construction would start at the mast or pier where some sections of the truss roof would be installed and connected to the innermost cables. Then construction would proceed moving outward from the pier in both directions as truss components and then cables were added. Cable tensions would be adjusted during this process to obtain a desired roof shape (flat or cambered) in the final product.

Alternatively, the six-story building frame could be used as a staging area, where large sections of the truss could be assembled then hoisted into place and connected to the cables.

The effects of loads resulting from the cantilever method of construction can be evaluated using the analysis model presented in this thesis. (Chapters 4 & 5, Appendix C) One can create an input file for the cable and truss geometry and erection loads of each construction stage and run the program to see the resulting tensions and deflections.

Chapter 4
Analytical Model

This chapter describes the analytical model of the roof system. The model is implemented as a Matlab program that reads information about the roof geometry and materials from input files. The roof is then modeled as a beam with constant shear (GA) and bending (EI) stiffness supported at one point by the pier and at other points by the cables. The roof is simply connected to the pier, hence at least two cables are required to make the structure stable under all load scenarios. Any number of vertical load combinations can be applied to the roof simultaneously; these load combinations can have different locations and magnitudes. The deflection response of the roof is then calculated and plotted. The resulting cable tensions are determined as well.

The geometric and material properties of the cable-stayed roof system can all be changed by simply modifying the input file. Scenarios in which location, number, cross-sectional area, and pre-stress of the cables are different can be analyzed. The dimensions of the roof truss and the location of its connection to the pier can be varied. The pier position, shape, and dimensions can be changed as well.

Finally a plot of the pier can be superimposed on the plot of the deflected shape of the roof, for a sense of the scale of the deflections.

4.1 Basis and Implementation of the Model

The model evaluates the roof behavior by considering it to be a long beam with a simple support at its connection to the pier, and vertical tension-only springs where the cables are connected. The Matlab code for this analysis engine and a sample input file are found in appendix C.

The roof is divided into segments, with nodes as specified in the input file. Nodes are located at cable connection points. By specifying additional "zero-area cables", the user causes more nodes to be generated. Doing this results in a better plot of the deformed shape of the roof and a more accurate solution of the roof deformation, because uniform loads are distributed to the node points in the Matlab implementation; in the case where we have many cables, as with the roof designed in chapter 3, the addition of additional nodes causes only slight improvements in the solution

Figure 4.1: Beam Segment Forces and Displacements

The stiffness of the roof beam segments is based on figure 4.1, and results in equations 4.1.1. [5] The two columns and rows of the stiffness matrix associated with F and u are not used, as the axial forces and displacements (*along* the roof truss) are not considered in this evaluation.

$$\begin{bmatrix} \frac{D_s}{L} & 0 & 0 & \frac{-D_s}{L} & 0 & 0 \\ 0 & \frac{12D_B*}{L^3} & \frac{6D_B*}{L^2} & 0 & \frac{-12D_B*}{L^3} & \frac{6D_B*}{L^2} \\ 0 & \frac{6D_B*}{L^2} & \frac{(4+a)D_B*}{L} & 0 & \frac{-6D_B*}{L^2} & \frac{(2-a)D_B*}{L} \\ \frac{-D_s}{L} & 0 & 0 & \frac{D_s}{L} & 0 & 0 \\ 0 & \frac{-12D_B*}{L^3} & \frac{-6D_B*}{L^2} & 0 & \frac{12D_B*}{L^3} & \frac{-6D_B*}{L^2} \\ 0 & \frac{6D_B*}{L^2} & \frac{(2-a)D_B*}{L} & 0 & \frac{-6D_B*}{L^2} & \frac{(4+a)D_B*}{L} \end{bmatrix} \cdot \begin{bmatrix} u_A \\ v_A \\ \beta_A \\ u_B \\ v_B \\ \beta_B \end{bmatrix} = \begin{bmatrix} F_A \\ V_A \\ M_A \\ F_B \\ V_B \\ M_B \end{bmatrix} \tag{4.1.1a}$$

where:

$$a = \frac{12D_B}{L^2 D_T}, \tag{4.1.1b}$$

$$D_B* = \frac{1}{1+a} \cdot D_B, \tag{4.1.1c}$$

and

$$D_S = EA,\ D_B = EI,\ \text{and}\ D_T = GA. \tag{4.1.1d}$$

The roof stiffnesses, EI and GA, are read from an input file. EA is not used.

These element stiffness matrices are combined in a global stiffness matrix, then cable vertical stiffnesses (Section 3.1.1) are added along the diagonal, and the vertical deflection at the simple support (point c of Fig. 3.8a) is set to zero. Iteration in the program ensures that cables only provide stiffness when they are in tension. Because of the preloading of the cables, the cables provide stiffness even when the roof is deflected above the flat position--up to the point where there is still some tension in the cable. That the prestressed cables operate in this fashion, can be verified by running the analysis engine using an input file with no loads applied to the roof, but with prestressed cables. The roof will deflect upwards such that the resulting cable tensions are zero.

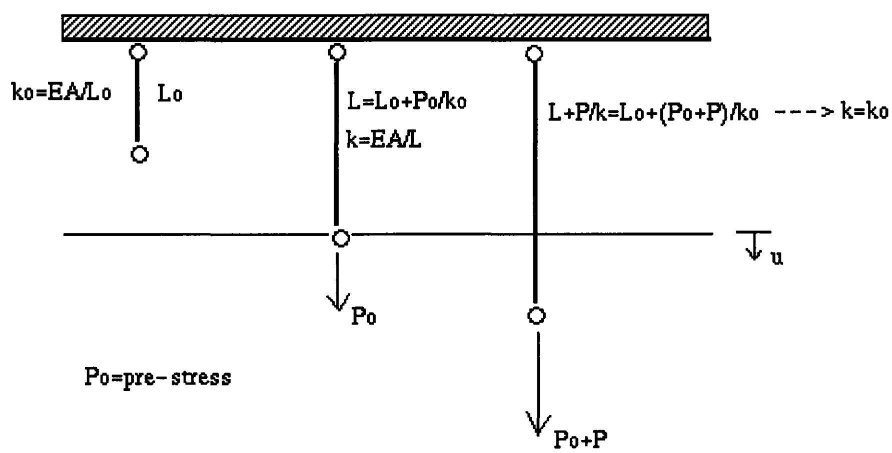

Figure 4.2: Cables with Pre-Stress

The cables stiffnesses are a function of area, material stiffness, and pre-tensioning. Figure 4.2 is used to derive the stiffness of a prestressed cable, for use in the analysis engine, below:

$$L = L_0 + \frac{P_0}{k_0} = L_0 + \left(\frac{P_0}{EA}\right)L_0 \qquad (4.1.2)$$

$$L_0 = \frac{L}{1 + \frac{P_0}{EA}} \tag{4.1.3}$$

$$\frac{1}{k_0} = \frac{L_0}{EA} = \frac{L}{EA + P_0} \tag{4.1.4}$$

$$k = k_0 = \frac{EA + P_0}{L} \tag{4.1.5}$$

The applied loads are distributed to the nodes, that is, a vector with nodal shear forces and moments is constructed. This vector--VM in the code--is adjusted for the cable pre-stress before displacements are calculated. The nodal rotations and vertical displacements along the roof are calculated, as are cable tensions and stresses.

The vertical displacements and allowable vertical displacements are plotted. In the plot, node points (where there are cables or zero-area "cables") appear as circles connected by a curve (generated by cubic spline interpolation) while the deflection limits are shown as straight lines, crossing where the space truss roof is pin-connected to the pier.

There are two items which must be manually verified by the user to ensure that the solution of the analysis engine is correct. First, the Matlab program will pause for the user to verify that the two columns of the matrix check show the same values; if they do not, then the structure is unstable. Second, after the program finishes iterating to ensure that cables contribute no stiffness to the roof unless they are in tension, the user must check that preloops (the number of iterations completed) is less than maxloops (the number of iterations before the program stops iterating). The variable maxloops must be increased.

Chapter 5
Case Study Using the Analysis Engine

The following case study of the roof designed in chapter 3 demonstrates the usefulness of the analysis model in the evaluation of a cable-stayed roof design. The roof is evaluated and altered when necessary, to meet performance criteria. The example file, infile_r1.m (Appendix C) contains all the relevant data for the roof including the three, service (unfactored) loads: dead, snow, and live. The matrix containing cable data includes several zero-area "cables" as described in section 4.1

5.1 Deflection Under Service Dead Load

In section 3.1.1, cable prestresses were selected with the goal of eliminating deflections due to service dead load. The deflected shape of the roof with only dead loads present is shown in the following plot from Matlab:

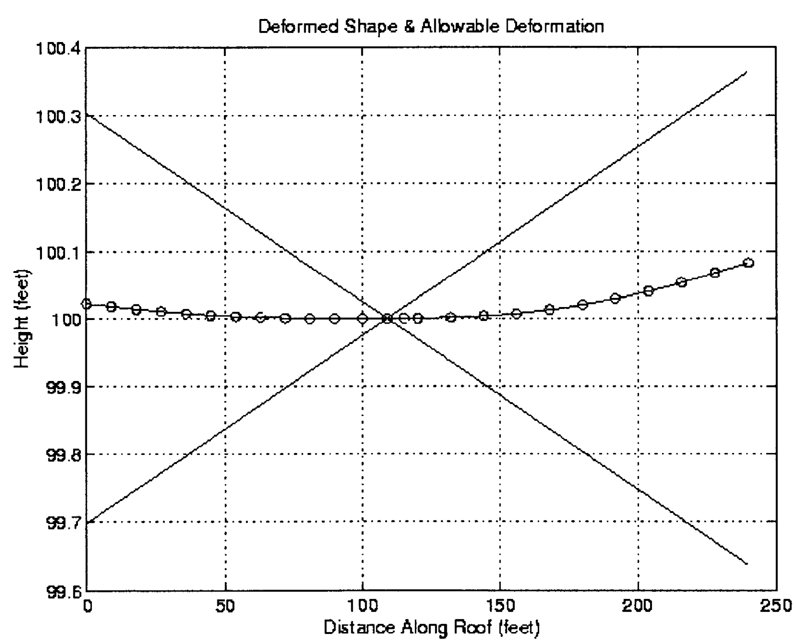

Figure 5.1: Deformation Due to Service Dead Load

It is clear from the figure, that while the roof is well within the deflection limit, the prestress towards the ends is overcompensating. Reducing by 25% the pretensioning of a few cables near the ends, leads to a shape which is almost a straight line, with a deflection of at most 0.09 inches from level.

5.2 Deflection Under Snow (Live), Wind, and Dead Load

Testing the roof with the new cable prestresses and maximum service loads (snow, wind, and dead) leads to the deformed shape in figure 5.2:

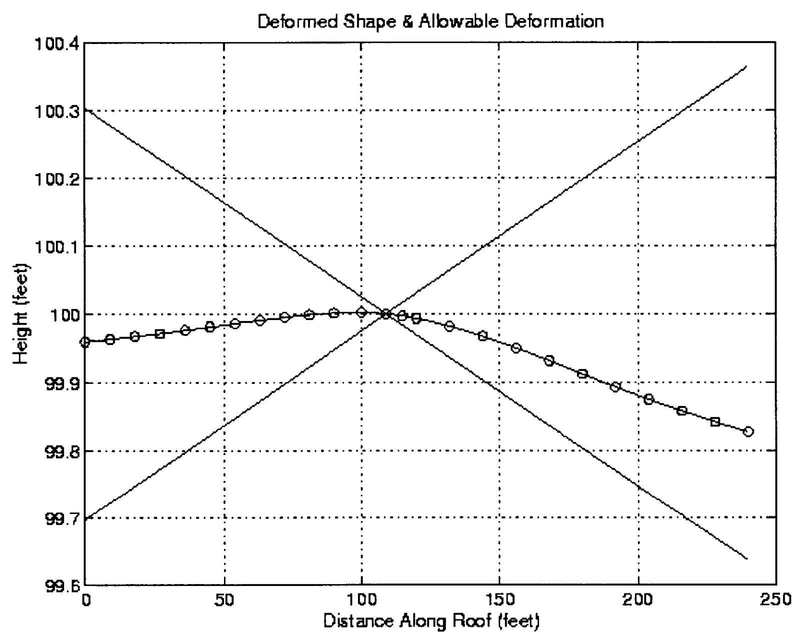

Figure 5.2: Deformation Due to Full, Service Load

It can be seen from figure 5.2 and table 4 that the deflection is kept within acceptable limits. The deflection criteria are also met for the cases when snow and wind forces (in addition to dead load, which is always applied along the entire length of the roof) are applied to only the right or left side of the roof. When the wind load is directed upward (along the entire roof or either side) with no other live/snow load the roof deflects less than the allowable.

Table 4: Displacements at Cable Connection Points (Full, Service Load--Both Sides)

Cable	X Coordinate	Vertical Displacement (ft)	Rotation (ft/ft)
1	0	-0.0424	0.0005
2	18	-0.0337	0.0005
3	36	-0.0241	0.0006
4	54	-0.0142	0.0005
5	72	-0.0052	0.0004
6	90	0.0009	0.0002
7	120	-0.0066	-0.0008
8	144	-0.0324	-0.0013
9	168	-0.0681	-0.0016
10	192	-0.1075	-0.0016
11	216	-0.1448	-0.0015
12	240	-0.1786	-0.0014

5.3 Cable Stresses

The governing factored load case for the cables is 1.4D+1.7L (5.85 kip/foot downward) from table 1. (Even under the factored loads the roof meets the deflection criteria set section 2.3 for service load alone.) Table 5 shows that the cable stresses are less than the allowable 45 ksi. (Section 3.1.1) In cases when load types are applied along the whole length of the truss, and in cases when they are only applied to one side of the support, the most highly-stressed cable never exceeds 33 ksi.

Table 5: Cable Forces and Stresses

Cable	1	2	3	4	5	6	7	8	9	10	11	12
Tension (kips)	128	123	131	109	76	49	54	214	281	280	299	313
Stress (ksi)	26	25	27	22	16	10	11	22	29	29	31	33

5.4 Correcting the Design

The evaluation tool, engine.m, showed that the prestressing of some cables was higher than necessary and lead to a change in the design. As the tool is used to evaluate other designs, the results tend to suggest improvements--such as changing the cross-sectional areas of cables, cable strength, or the stiffness properties of the roof truss. Re-evaluating an altered design becomes a simple matter of editing the parameters of the input file and running the program.

Chapter 6
Conclusion

In the design of a showcase structure intended to attract attention to civil and environmental engineering, a structural engineer has a tremendous opportunity for creativity. For that creativity to become a structure that can be built, there must be a means of evaluating its performance before construction. The evaluation tool described in this thesis helps respond to the technical challenges of a cable-stayed roof, in hopes that a simple tool might determine whether to take the design beyond the conceptual stage and into the new CEE building.

The design of the roof for the new CEE building is examined in chapter 5. It is found that, while the roof meets the deflection requirements in section 2.3, it does not meet the desired goal of zero deflection under service dead load because the prestressed cables lift the ends of the roof.

With the level of prestress reduced such that the roof is flat under service dead load, the design behaves well, meeting deflection requirements even when factored load combinations are used to evaluate cable stresses. Under the expected load scenarios, the roof typically deforms with a shape similar to what is shown in figure 5.2. Table 4 gives the vertical displacements and rotations at each point along the truss where cables are attached (when full service load is applied to both sides). When snow/live and wind loads are applied to one side of the truss only, some of nodes on the other side deflect upward as one might expect, but do not exceed the deflection limits.

Cables are found to be stressed at 75% (or less) of the allowable amount. This information, and the fact that the deflection limits are being met, indicates a way to save material. By reducing cable areas and increasing prestress, it is possible to design a roof that is more material-efficient.

This thesis indicates that the proposed conceptual design is adequate (with a reduction of prestress values). However, greater efficiency may be achieved through further changes--evaluating each successive design with the analysis tool provided.

References

[1] *1998 MIT M.Eng. Group Project Report.* Unpublished.

[2] American Concrete Institute. Building Code Requirements for Structural Concrete (ACI 318-95) and Commentary (ACI 318R-95). Farmington Hills, MI, 1995.

[3] State Board of Building Regulations and Standards. Massachusetts State Building Code (780 CMR). 6th ed. Boston: William F. Galvin, Secretary of the Commonwealth, 1997.

[4] Nilson, A. H. and Winter, G. Design of Concrete Structures. 11th ed. New York: McGraw-Hill, Inc., 1991.

[5] Connor, Jerome J. *Course notes from 1.571: Structural Analysis and Control.* Unpublished.

[6] Englekirk, Robert. Steel Structures: Controlling Behavior Through Design. New York: John Wiley and Sons, 1994, pp. 473-478.

[7] Connor, J. J. and Klink, B. S. A. Introduction to Motion Based Design. Boston: Computational Mechanics Publications, 1996.

Appendix A: Purpose & Guiding Principles for the New CEE Building

THE CIVIL & ENVIRONMENTAL ENGINEERING BUILDING COMPLEX 12/18/97

Purpose and Guiding Principles

In keeping with the unique and defining culture of the Massachusetts Institute of Technology, the Department of Civil and Environmental Engineering is committed to maintaining the excellence of its civil and environmental engineering education and research activities and dedicated to shaping the future of the profession. The forces that are driving change in the discipline include:

-Increasing emphasis on improved methods for the conceptualization, design, construction, and operation of both new and revitalized buildings and infrastructure systems in order to improve economic productivity, competitiveness, and quality of life for the citizens of the world.

-Growing commitment to environmental stewardship and sustainable development which is affording higher priority to compatibility between built and natural environments and providing impetus to the continuing evolution of synergistic partnerships among civil and environmental engineering disciplines.

-Rapid advancements in technology, particularly information technology, which hold great potential for dramatically changing and enhancing the processes for realizing civil and environmental facilities.

The CEE Department, to enhance its mission of educating future leaders of the profession and providing them with the skills and systems for life-long learning, personal growth, and service to society, proposes the development of a new and unified building complex to house relevant academic, research, and administrative activities. The complex will consist of one of the following two alternatives:

1. A new building located adjacent to and connected to a renovated Parsons Laboratory (Building 48).
2. A totally new building on the site of a demolished Parsons Laboratory and the adjacent parking lot.

A major goal of the CEE Department's leadership, is to ensure that its visions and aspirations for the future directions of the profession are embodied and visible in the physical appearance and operating characteristics of the complex. Thus the complex should serve as a showcase facility built with the most modern and advanced construction and environmental technology to house the most modern educational and research technology. Some possible aspects to showcase include: a creative and visible high performance structural system; information technology systems for both building control systems and educational activities; HVAC systems and building materials that contribute to sustainable development; and alternative project delivery systems.

Appendix B: Matlab Code for Pier Geometry (muse.m & Ww.m)

What follows is the code used to generate coordinates of the points shown in figure 3.5. The code also draws the pier.

muse.m

```
% FILE NAME: muse.m
% Pier Geometry generation file
clear
hold on

% Get input:
Ww

ss = sin(angle_xy_mast*(pi/180));
cc = cos(angle_xy_mast*(pi/180));
tt = tan(angle_xy_mast*(pi/180));

ht_rooftop = (ht_to_roof + t_roof);
ht_mast = (ht_to_roof + t_roof + ht_mast_above);
Mast = [
xcoord_A                                      ht_mast       0
xcoord_A+(ht_mast_above+t_roof)/tt            ht_to_roof    0
xcoord_A+ht_mast_above/tt                     ht_rooftop    0
xcoord_A+ht_mast/tt                           0             0
xcoord_A+(ht_mast-ycoord_E)/tt                ycoord_E      0
xcoord_A+(ht_mast-ycoord_E)/tt-(ycoord_E/tt)  0             0
]

plot(Mast(:,1),Mast(:,2)), grid on
```

Ww.m

```
% FILE NAME: Ww.m
% Input file for muse.m
% Units = feet, kips

% Axes defined:
% X length
% Y height
% Z width

% Points defined:
% A top of long mast element
% B where long mast element intersects underside of roof
```

```
% C where long mast element intersects top of roof
% D bottom of long mast element
% E top of short mast element
% G bottom of short mast element

% Building geometry
ht_to_roof        = 90   % feet
t_roof            = 10   % feet
ht_mast_above     = 50   % feet
len_roof          = 240  % feet
wid_roof          = 75   % feet

% Tower geometry
angle_xy_mast     = 60   % wrt ground, value between 0 and 90
xcoord_A          = 80   % feet
ycoord_E          = 75   % feet (max = ht_to_roof)
```

Appendix C: Matlab Code for Roof Evaluation (engine.m & infile_r1.m)

What follows is the code used evaluate the performance of a cable-stayed roof and an example input file. The input file, infile_r1.m, contains the data for the design described in chapter 3.

engine.m

```
% FILE NAME: engine.m
% Analysis engine for cable-stayed roof evaluation
clear

% Get input:
infile_r1

ss = sin(angle_xy_mast*(pi/180));
cc = cos(angle_xy_mast*(pi/180));
tt = tan(angle_xy_mast*(pi/180));

mast_x = xcoord_a;
mast_y = ycoord_a-100;

num_cab   = size(cable,1)
data      = zeros(num_cab,5);
data(:,1) = cable(:,1);

% cable length
data(:,2) = sqrt((mast_x-cable(:,2)).^2 + mast_y^2);

% cable sin_theta
data(:,3) = mast_y./data(:,2);

% cable EA/L adjusted for prestress
data(:,4) = (E_cable*cable(:,3)+cable(:,4))./data(:,2);

% Cable Vertical stiffness: EA/L *sin^2 theta
data(:,5) = data(:,4).*data(:,3).^2
kiter = data(:,5);

% roof stiffness matrix
Db        = EIxx_roof;
Dt        = GA_roof;
a1        = 12*Db/Dt;

% loop for no compression in cable
maxloops = 12;
preloops = 0;
```

```
for ggg=1:maxloops

% number of uniformly distributed load cases
num_load = size(BB,1);

K        = zeros(2*(num_cab+1),2*(num_cab+1));
VM       = zeros(2*(num_cab+1),1);
found    = 0;

% FOR LOOP to create global K matrix
for  r=2:1:num_cab
      % beam member length
      if cable(r,2) < support  |  found == 1
            anode = cable(r-1,2);
            bnode = cable(r,2);
            L = bnode-anode;
            a = a1/L^2;
            Db_star = Db/(1+a);
            k1 =    12*Db_star/L^3;
            k2 =     6*Db_star/L^2;
            k3 = (4+a)*Db_star/L;
            k4 = (2-a)*Db_star/L;
            member = r-1+found;
            index  = 2*member-1;
            K(index, index) = K(index, index) + k1;
            K(index, index+1) = K(index, index+1) + k2;
            K(index, index+2) = K(index, index+2) - k1;
            K(index, index+3) = K(index, index+3) + k2;

            K(index+1, index) = K(index+1, index) + k2;
            K(index+1, index+1) = K(index+1, index+1) + k3;
            K(index+1, index+2) = K(index+1, index+2) - k2;
            K(index+1, index+3) = K(index+1, index+3) + k4;

            K(index+2, index) = K(index+2, index) - k1;
            K(index+2, index+1) = K(index+2, index+1) - k2;
            K(index+2, index+2) = K(index+2, index+2) + k1;
            K(index+2, index+3) = K(index+2, index+3) - k2;

            K(index+3, index) = K(index+3, index) + k2;
            K(index+3, index+1) = K(index+3, index+1) + k4;
            K(index+3, index+2) = K(index+3, index+2) - k2;
            K(index+3, index+3) = K(index+3, index+3) + k3;

            % cable stiffness
            K(index+2*found, index+2*found) = K(index+2*found, index+2*found) +
```

```
kiter(member);

        % Shear and moment *1*
        for ff = 1:1:num_load
            if anode >= BB(ff,3) | bnode <= BB(ff,2)
            elseif anode >= BB(ff,2) & bnode <= BB(ff,3)
                VM(index)   = VM(index)+L*BB(ff,1)/2;
                VM(index+2) = VM(index+2)+L*BB(ff,1)/2;
            elseif anode >= BB(ff,2) & bnode > BB(ff,3)
                loadb  = (BB(ff,3)-anode)*BB(ff,1);
                dist1 = (BB(ff,3)-anode)/2;
                dist2 = L - dist1;
                VM(index)   = VM(index) + loadb*dist2/L;
                VM(index+2) = VM(index+2)+ loadb*dist1/L;
            elseif anode < BB(ff,2) & bnode <= BB(ff,3)
                loadb  = (bnode-BB(ff,2))*BB(ff,1);
                dist2 = (bnode-BB(ff,2))/2;
                dist1 = L - dist2;
                VM(index)   = VM(index) + loadb*dist2/L;
                VM(index+2) = VM(index+2)+ loadb*dist1/L;
            elseif anode < BB(ff,2) & bnode > BB(ff,3)
                loadb  = (BB(ff,3)-BB(ff,2))*BB(ff,1);
                dist1 = (BB(ff,3)-BB(ff,2))/2 + (BB(ff,2)-anode);
                dist2 = L - dist1;
                VM(index)   = VM(index) + loadb*dist2/L;
                VM(index+2) = VM(index+2)+ loadb*dist1/L;
            end
        end
    else
        % if the program reaches a segment with the support in it
        anode = cable(r-1,2);
        bnode = support;
        L = bnode-anode;                    % bug: if support = 0
        a = a1/L^2;
        Db_star = Db/(1+a);
        k1 =   12*Db_star/L^3;
        k2 =    6*Db_star/L^2;
        k3 = (4+a)*Db_star/L;
        k4 = (2-a)*Db_star/L;
        member = r-1+found;
        index  = 2*member-1;
        K(index, index) = K(index, index) + k1;
        K(index, index+1) = K(index, index+1) + k2;
        K(index, index+2) = K(index, index+2) - k1;
        K(index, index+3) = K(index, index+3) + k2;
```

```
        K(index+1, index) = K(index+1, index) + k2;
        K(index+1, index+1) = K(index+1, index+1) + k3;
        K(index+1, index+2) = K(index+1, index+2) - k2;
        K(index+1, index+3) = K(index+1, index+3) + k4;

        K(index+2, index) = K(index+2, index) - k1;
        K(index+2, index+1) = K(index+2, index+1) - k2;
        K(index+2, index+2) = K(index+2, index+2) + k1;
        K(index+2, index+3) = K(index+2, index+3) - k2;

        K(index+3, index) = K(index+3, index) + k2;
        K(index+3, index+1) = K(index+3, index+1) + k4;
        K(index+3, index+2) = K(index+3, index+2) - k2;
        K(index+3, index+3) = K(index+3, index+3) + k3;

        % cable stiffness
        K(index+2*found, index+2*found) = K(index+2*found, index+2*found) +
kiter(member);

        % Shear and moment *2*
        for ff = 1:1:num_load
            if anode >= BB(ff,3) | bnode <= BB(ff,2)
            elseif anode >= BB(ff,2) & bnode <= BB(ff,3)
                VM(index)   = VM(index)+L*BB(ff,1)/2;
                VM(index+2) = VM(index+2)+L*BB(ff,1)/2;
            elseif anode >= BB(ff,2) & bnode > BB(ff,3)
                loadb  = (BB(ff,3)-anode)*BB(ff,1);
                dist1 = (BB(ff,3)-anode)/2;
                dist2 = L - dist1;
                VM(index)   = VM(index) + loadb*dist2/L;
                VM(index+2) = VM(index+2)+ loadb*dist1/L;
            elseif anode < BB(ff,2) & bnode <= BB(ff,3)
                loadb  = (bnode-BB(ff,2))*BB(ff,1);
                dist2 = (bnode-BB(ff,2))/2;
                dist1 = L - dist2;
                VM(index)   = VM(index) + loadb*dist2/L;
                VM(index+2) = VM(index+2)+ loadb*dist1/L;
            elseif anode < BB(ff,2) & bnode > BB(ff,3)
                loadb  = (BB(ff,3)-BB(ff,2))*BB(ff,1);
                dist1 = (BB(ff,3)-BB(ff,2))/2 + (BB(ff,2)-anode);
                dist2 = L - dist1;
                VM(index)   = VM(index) + loadb*dist2/L;
                VM(index+2) = VM(index+2)+ loadb*dist1/L;
            end
        end
```

```
        support_node = member+1;
        found = 1;

        anode = support;
        bnode = cable(r,2);
        L = bnode-anode;
        a = a1/L^2;
        Db_star = Db/(1+a);
        k1 =    12*Db_star/L^3;
        k2 =     6*Db_star/L^2;
        k3 = (4+a)*Db_star/L;
        k4 = (2-a)*Db_star/L;
        member = r-1+found;
        index  = 2*member-1;
        K(index, index) = K(index, index) + k1;
        K(index, index+1) = K(index, index+1) + k2;
        K(index, index+2) = K(index, index+2) - k1;
        K(index, index+3) = K(index, index+3) + k2;

        K(index+1, index) = K(index+1, index) + k2;
        K(index+1, index+1) = K(index+1, index+1) + k3;
        K(index+1, index+2) = K(index+1, index+2) - k2;
        K(index+1, index+3) = K(index+1, index+3) + k4;

        K(index+2, index) = K(index+2, index) - k1;
        K(index+2, index+1) = K(index+2, index+1) - k2;
        K(index+2, index+2) = K(index+2, index+2) + k1;
        K(index+2, index+3) = K(index+2, index+3) - k2;

        K(index+3, index) = K(index+3, index) + k2;
        K(index+3, index+1) = K(index+3, index+1) + k4;
        K(index+3, index+2) = K(index+3, index+2) - k2;
        K(index+3, index+3) = K(index+3, index+3) + k3;

        % cable stiffness
        K(index+2*found, index+2*found) = K(index+2*found, index+2*found) +
kiter(member);

        % Shear and moment *3*
        for ff = 1:1:num_load
            if anode >= BB(ff,3) | bnode <= BB(ff,2)
            elseif anode >= BB(ff,2) & bnode <= BB(ff,3)
                VM(index)   = VM(index)+L*BB(ff,1)/2;
                VM(index+2) = VM(index+2)+L*BB(ff,1)/2;
            elseif anode >= BB(ff,2) & bnode > BB(ff,3)
                loadb  = (BB(ff,3)-anode)*BB(ff,1);
```

```
                    dist1 = (BB(ff,3)-anode)/2;
                    dist2 = L - dist1;
                    VM(index)   = VM(index) + loadb*dist2/L;
                    VM(index+2) = VM(index+2)+ loadb*dist1/L;
               elseif anode < BB(ff,2) & bnode <= BB(ff,3)
                    loadb  = (bnode-BB(ff,2))*BB(ff,1);
                    dist2 = (bnode-BB(ff,2))/2;
                    dist1 = L - dist2;
                    VM(index)   = VM(index) + loadb*dist2/L;
                    VM(index+2) = VM(index+2)+ loadb*dist1/L;
               elseif anode < BB(ff,2) & bnode > BB(ff,3)
                    loadb  = (BB(ff,3)-BB(ff,2))*BB(ff,1);
                    dist1 = (BB(ff,3)-BB(ff,2))/2 + (BB(ff,2)-anode);
                    dist2 = L - dist1;
                    VM(index)   = VM(index) + loadb*dist2/L;
                    VM(index+2) = VM(index+2)+ loadb*dist1/L;
               end
          end
     end
end

% adjust shear for prestress forces
% (similar means to adjust VM for reaction at support--don't bother)

VM;          % This VM represents the applied loads
%pause

presin = (cable(:,4).*data(:,3))';
VM(1:2:2*(num_cab+1)) = VM(1:2:2*(num_cab+1)) + (presin(1:support_node-1) 0
presin(support_node:num_cab) )';

% now VM is changed for calculations

K2 = zeros(2*(num_cab+1)-1,2*(num_cab+1)-1);
K2(1:2*support_node-2, 1:2*support_node-2) = K(1:2*support_node-2,
1:2*support_node-2);
K2(2*support_node-1:2*(num_cab+1)-1, 2*support_node-1:2*(num_cab+1)-1) =
K(2*support_node:2*(num_cab+1), 2*support_node:2*(num_cab+1));
K2(1:2*support_node-2, 2*support_node-1:2*(num_cab+1)-1) =
K(1:2*support_node-2, 2*support_node:2*(num_cab+1));
K2(2*support_node-1:2*(num_cab+1)-1, 1:2*support_node-2) =
K(2*support_node:2*(num_cab+1), 1:2*support_node-2);

VM2 = zeros(2*(num_cab+1)-1,1);
VM2(1:2*support_node-2)          = VM(1:2*support_node-2);
VM2(2*support_node-1:2*(num_cab+1)-1) =
```

```
VM(2*support_node:2*(num_cab+1));

U2 = K2\VM2;

% check if K2*U2 = VM2
check = (VM2 K2*U2);

U = zeros(2*(num_cab+1),1);
U(1:2*support_node-2) = U2(1:2*support_node-2);
U(2*support_node-1) = 0;
U(2*support_node:2*(num_cab+1)) = U2(2*support_node-1:2*(num_cab+1)-1);
U;
Udispl = U(1:2:2*(num_cab+1));

hold off
x = (cable(1:support_node-1,2)' support cable(support_node:num_cab,2)');
y = 100+Udispl;
lastpt = cable(num_cab,2);
xi = 0:4:lastpt;
yi = spline(x,y,xi);
plot(xi,yi,x,y,'o'), grid on

hold on
plot((0 support lastpt),(100-(support)/360 100 100-(lastpt-support)/360), `w');
plot((0 support lastpt),(100+(support)/360 100 100+(lastpt-support)/360), `w');
title(`Deformed Shape & Allowable Deformation')
xlabel(`Distance Along Roof (feet)')
ylabel(`Height (feet)')
hold off

Tension = cable(:,4)-data(:,4).*data(:,3).*(Udispl(1:support_node-1)'
Udispl(support_node+1:num_cab+1)')';

changed = 0;
Tens_kiter = (Tension kiter)
for jj=1:size(Tension,1)
	if Tension(jj) < 0 & kiter(jj) > 0
		kiter(jj) = 0;
		changed = 1;
	elseif Tension(jj) > 0 & kiter(jj) <= 0 & data(jj,5) > 0
		kiter(jj) = data(jj,5);
		changed = 1;
	end
end
clear jj
```

```
preloops = preloops +1;

if changed ~= 1
      break
end
end % for no compression in cable

pause

% check if K2*U2 = VM2
check
pause

for hgh=1:size(Tension)
      if Tension(hgh) < 0
            Tension(hgh) = 0;
      end
end
Tension
% (neg. stress values should be viewed as zero stress)
%cable_stress = Tension./cable(:,3)  % problem when cable area = zero
U
Udispl

preloops
maxloops
```

infile_r1.m

```
% FILE NAME: infile_r1.m
% Input file for engine.m

% Units = feet, kips

% Axes defined:
% X length
% Y height
% Z width

% Points defined:
% a top of long mast element
% c point of connection with roof (pinned connection, top of roof)
% d top of short mast element (pinned connection w/ long element)

% Building geometry
ht_to_roof      = 90;          % feet
```

```
t_roof         = 10;          % feet  % distance between underside of roof & top
ht_mast_above  = 50;          % feet  % height of the mast above the roof
len_roof       = 240;         % feet
wid_roof       = 75;          % feet

% Tower geometry
angle_xy_mast  = 60;          % wrt ground, value between 0 and 90
xcoord_a       = 80;          % feet
ycoord_a       = 150;         % feet
ycoord_d       = 75;          % feet (max = ht_to_roof)

% Stiffnesses
E_cable        = (29000);     % ksi
EIxx_roof      = 650e5        % kip x ft^2 % EI for half of roof
GA_roof        = 9.3e6        % kip       % GA for half of roof

% Roof geometry
support   = 109;              % feet  % x coord of point c

% Strengths
cable_str      = 50           % ksi

% (1)      (2)  (3)     (4)
% CABLE  X      AREA    PRE-TENSION
cable = (
1     0     4.9    96
22    9     0      0     % "zero-area cable"
2     18    4.9    81
22    27    0      0
3     36    4.9    68
22    45    0      0
4     54    4.9    57
22    63    0      0
5     72    4.9    52
22    81    0      0
6     90    4.9    52
22    100   0      0
22    115   0      0
7     120   4.9    37
22    132   0      0
8     144   9.6    110
22    156   0      0
9     168   9.6    138
22    180   0      0
10    192   9.6    167
22    204   0      0
```

```
11    216   9.6    197
22    228   0      0
12    240   9.6    228
);

% Uniform Load (unfactored dead for half roof =  -2.81  k/ft)
% Uniform Load (unfactored snow for half roof =  -1.125 k/ft)
% Uniform Load (unfactored wind for half roof = +/-0.78  k/ft)
% (1)                    (2)       (3)
% MAGNITUDE(kip/ft) START(ft) END(ft)
BB = (
-2.81      0       240
-1.125      0       240
-0.78      0       240
);
```

Appendix D: Mast Loading Data & Mast Design Data

The following Excel spreadsheets contain data about the pier design of chapter 3.

Mast Loading Data

Mast loading data
Final cable geometry

Case I. Assume cables take entire load
Load is taken as 1.4D+1.7L (2808 kip/roof) Hence, Load=67% dead load effect & 33% live load effects.

A. Live load applied to both sides

Cable	x	mast x	mast y	length (ft)	sin theta	E (ksi)	A (in^2)	EA/L	F vert	Tension	elong (ft)	defl vert (in)	Cable stress (ksi)	F horiz at mast
1	0	80	50	94.3	0.53	29000	4.9	1506	105	199	0.13	2.99	41	-168
2	18	80	50	79.6	0.63	29000	4.9	1784	105	168	0.09	1.80	34	-131
3	36	80	50	66.6	0.75	29000	4.9	2134	105	140	0.07	1.05	29	-93
4	54	80	50	56.4	0.89	29000	4.9	2521	105	119	0.05	0.64	24	-55
5	72	80	50	50.6	0.99	29000	4.9	2806	105	107	0.04	0.46	22	-17
6	90	80	50	51.0	0.98	29000	4.9	2787	105	107	0.04	0.47	22	21
7	120	80	50	64.0	0.78	29000	4.9	2219	70	90	0.04	0.62	18	56
8	144	80	50	81.2	0.62	29000	9.6	3428	140	228	0.07	1.30	24	180
9	168	80	50	101.2	0.49	29000	9.6	2751	140	284	0.10	2.51	30	247
10	192	80	50	122.7	0.41	29000	9.6	2270	140	344	0.15	4.47	36	314
11	216	80	50	144.9	0.35	29000	9.6	1921	140	407	0.21	7.36	42	382
12	240	80	50	167.6	0.30	29000	9.6	1661	140	471	0.28	11.40	49	449
									1404					1186

B. Live load applied to right side only I.e. remove LL effects from cables that pull mast to the left

Cable	x	mast x	mast y	length (ft)	sin theta	E (ksi)	A (in^2)	EA/L	F vert	Tension	elong (ft)	defl vert (in)	Cable stress (ksi)	F horiz at mast
1	0	80	50	94.3	0.53	29000	4.9	1506	70	133	0.09	2.00	27	-113
2	18	80	50	79.6	0.63	29000	4.9	1784	70	112	0.06	1.20	23	-87
3	36	80	50	66.6	0.75	29000	4.9	2134	70	94	0.04	0.70	19	-62
4	54	80	50	56.4	0.89	29000	4.9	2521	70	79	0.03	0.43	16	-37
5	72	80	50	50.6	0.99	29000	4.9	2806	70	71	0.03	0.31	15	-11
6	90	80	50	51.0	0.98	29000	4.9	2787	105	107	0.04	0.47	22	21
7	120	80	50	64.0	0.78	29000	4.9	2219	70	90	0.04	0.62	18	56
8	144	80	50	81.2	0.62	29000	9.6	3428	140	228	0.07	1.30	24	180
9	168	80	50	101.2	0.49	29000	9.6	2751	140	284	0.10	2.51	30	247
10	192	80	50	122.7	0.41	29000	9.6	2270	140	344	0.15	4.47	36	314
11	216	80	50	144.9	0.35	29000	9.6	1921	140	407	0.21	7.36	42	382
12	240	80	50	167.6	0.30	29000	9.6	1661	140	471	0.28	11.40	49	449
									1229					1340

C. Live load applied to left side only I.e. remove LL effects from cables that pull mast to the right

Cable	x	mast x	mast y	length (ft)	sin theta	E (ksi)	A (in^2)	EA/L	F vert	Tension	elong (ft)	defl vert (in)	Cable stress (ksi)	F horiz at mast
1	0	80	50	94.3	0.53	29000	4.9	1506	105	199	0.13	2.99	41	-168
2	18	80	50	79.6	0.63	29000	4.9	1784	105	168	0.09	1.80	34	-131
3	36	80	50	66.6	0.75	29000	4.9	2134	105	140	0.07	1.05	29	-93
4	54	80	50	56.4	0.89	29000	4.9	2521	105	119	0.05	0.64	24	-55
5	72	80	50	50.6	0.99	29000	4.9	2806	105	107	0.04	0.46	22	-17
6	90	80	50	51.0	0.98	29000	4.9	2787	70	72	0.03	0.32	15	14
7	120	80	50	64.0	0.78	29000	4.9	2219	47	60	0.03	0.42	12	38
8	144	80	50	81.2	0.62	29000	9.6	3428	94	152	0.04	0.87	16	120
9	168	80	50	101.2	0.49	29000	9.6	2751	94	190	0.07	1.68	20	165
10	192	80	50	122.7	0.41	29000	9.6	2270	94	230	0.10	2.98	24	210
11	216	80	50	144.9	0.35	29000	9.6	1921	94	272	0.14	4.92	28	255
12	240	80	50	167.6	0.30	29000	9.6	1661	94	314	0.19	7.62	33	300
									1113					639

Case II. Assume cables take entire DL but only half LL
Load is taken as 1.4D+1.7L/2 (2350 kip/roof) Hence, Load=80% dead load effect & 20% live load effects.

A. Live load applied to both sides

Cable	x	mast x	mast y	length (ft)	sin theta	E (ksi)	A (in^2)	EA/L	F vert	Tension	elong (ft)	defl vert (in)	Cable stress (ksi)	F horiz at mast
1	0	80	50	94.3	0.53	29000	4.9	1506	88	166	0.11	2.50	34	-141
2	18	80	50	79.6	0.63	29000	4.9	1784	88	140	0.08	1.50	29	-109
3	36	80	50	66.6	0.75	29000	4.9	2134	88	117	0.06	0.88	24	-78
4	54	80	50	56.4	0.89	29000	4.9	2521	88	99	0.04	0.53	20	-46
5	72	80	50	50.6	0.99	29000	4.9	2806	88	89	0.03	0.39	18	-14
6	90	80	50	51.0	0.98	29000	4.9	2787	88	90	0.03	0.39	18	18
7	120	80	50	64.0	0.78	29000	4.9	2219	59	76	0.03	0.52	15	47
8	144	80	50	81.2	0.62	29000	9.6	3428	118	191	0.06	1.09	20	150
9	168	80	50	101.2	0.49	29000	9.6	2751	118	238	0.09	2.10	25	207
10	192	80	50	122.7	0.41	29000	9.6	2270	118	288	0.13	3.74	30	263
11	216	80	50	144.9	0.35	29000	9.6	1921	118	341	0.18	6.16	35	320
12	240	80	50	167.6	0.30	29000	9.6	1661	118	394	0.24	9.54	41	376
									1175					993

B. Live load applied to right side only I.e. remove LL effects from cables that pull mast to the left

Cable	x	mast x	mast y	length (ft)	sin theta	E (ksi)	A (in^2)	EA/L	F vert	Tension	elong (ft)	defl vert (in)	Cable stress (ksi)	F horiz at mast
1	0	80	50	94.3	0.53	29000	4.9	1506	71	133	0.09	2.00	27	-113
2	18	80	50	79.6	0.63	29000	4.9	1784	71	112	0.06	1.20	23	-87
3	36	80	50	66.6	0.75	29000	4.9	2134	71	94	0.04	0.70	19	-62
4	54	80	50	56.4	0.89	29000	4.9	2521	71	79	0.03	0.43	16	-37
5	72	80	50	50.6	0.99	29000	4.9	2806	71	71	0.03	0.31	15	-11
6	90	80	50	51.0	0.98	29000	4.9	2787	88	90	0.03	0.39	18	18
7	120	80	50	64.0	0.78	29000	4.9	2219	59	76	0.03	0.52	15	47
8	144	80	50	81.2	0.62	29000	9.6	3428	118	191	0.06	1.09	20	150
9	168	80	50	101.2	0.49	29000	9.6	2751	118	238	0.09	2.10	25	207
10	192	80	50	122.7	0.41	29000	9.6	2270	118	288	0.13	3.74	30	263
11	216	80	50	144.9	0.35	29000	9.6	1921	118	341	0.18	6.16	35	320
12	240	80	50	167.6	0.30	29000	9.6	1661	118	394	0.24	9.54	41	376
									1087					1071

C. Live load applied to left side only I.e. remove LL effects from cables that pull mast to the right

Cable	x	mast x	mast y	length (ft)	sin theta	E (ksi)	A (in^2)	EA/L	F vert	Tension	elong (ft)	defl vert (in)	Cable stress (ksi)	F horiz at mast
1	0	80	50	94.3	0.53	29000	4.9	1506	88	166	0.11	2.50	34	-141
2	18	80	50	79.6	0.63	29000	4.9	1784	88	140	0.08	1.50	29	-109
3	36	80	50	66.6	0.75	29000	4.9	2134	88	117	0.06	0.88	24	-78
4	54	80	50	56.4	0.89	29000	4.9	2521	88	99	0.04	0.53	20	-46
5	72	80	50	50.6	0.99	29000	4.9	2806	88	89	0.03	0.39	18	-14
6	90	80	50	51.0	0.98	29000	4.9	2787	71	72	0.03	0.32	15	14
7	120	80	50	64.0	0.78	29000	4.9	2219	47	60	0.03	0.42	12	38
8	144	80	50	81.2	0.62	29000	9.6	3428	94	153	0.04	0.87	16	120
9	168	80	50	101.2	0.49	29000	9.6	2751	94	190	0.07	1.68	20	165
10	192	80	50	122.7	0.41	29000	9.6	2270	94	231	0.10	2.99	24	211
11	216	80	50	144.9	0.35	29000	9.6	1921	94	272	0.14	4.93	28	256
12	240	80	50	167.6	0.30	29000	9.6	1661	94	315	0.19	7.63	33	301
									1028					717

For case II	F vert carried by truss
1	229
2	142
3	85

Mast Design Data

(Refer to figures 3.7a & 3.7b.)

Mast design				
ht_to_roof	90			
t_roof	10			
ht_mast_above	50			
len_roof	240			
wid_roof	75		sin	cos
angle_xy_mast	**60**	**1.047**	**0.866**	**0.5**
xcoord_a	80			
ycoord_d	75			

Roof loads	D	L	1.4D+1.7L
(kips)	1350	540	2808

Case I. Assume cables take entire load

A. Live load applied to both sides

Point	Fx	Fy	Fz	P	V	Mzz at c	Mzz at d	Mvv at d
a	1186	-1404	0	1809	325	18770	28155	0
b	0	0	0	0	0	0	0	0
c	-1186	0	0	-593	-1027	0	-29650	0
d	0	0	0	0	0		0	0
				1216	-702	18770	-1495	

B. Live load applied to right side only

Point	Fx	Fy	Fz	P	V	Mzz at c	Mzz at d	Mvv at d
a	1340	-1229	0	1734	546	31522	47283	0
b	0	0	0	0	0	0	0	0
c	-1340	0	0	-670	-1160	0	-33500	0
d	0	0	0	0	0		0	0
				1064	-615	31522	13783	

C. Live load applied to left side only

Point	Fx	Fy	Fz	P	V	Mzz at c	Mzz at d	Mvv at d
a	639	-1113	0	1283	-3	-180	-269	0
b	0	0	0	0	0	0	0	0
c	-639	0	0	-320	-553	0	-15975	0
d	0	0	0	0	0		0	0
				964	-557	-180	-16244	

Case II. Assume cables take entire DL but only half LL

A. Live load applied to both sides

Point	Fx	Fy	Fz	P	V	Mzz at c	Mzz at d	Mvv at d
a	993	-1175	0	1514	272	15731	23596	0
b	0	0	0	0	0	0	0	0
c	-993	-229	0	-298	-974	0	-28130	0
d	0	0	0	0	0		0	0
		-1404		1216	-702	15731	-4534	

B. Live load applied to right side only

Point	Fx	Fy	Fz	P	V	Mzz at c	Mzz at d	Mvv at d
a	1071	-1087	0	1477	384	22171	33257	0
b	0	0	0	0	0	0	0	0
c	-1071	-142	0	-413	-999	0	-28825	0
d	0	0	0	0	0		0	0
		-1229		1064	-615	22171	4432	

C. Live load applied to left side only

Point	Fx	Fy	Fz	P	V	Mzz at c	Mzz at d	Mvv at d
a	717	-1028	0	1249	107	6174	9261	0
b	0	0	0	0	0	0	0	0
c	-717	-85	0	-285	-663	0	-19152	0
d	0	0	0	0	0		0	0
		-1113		964	-557	6174	-9891	

	Wt. at d	F1	Mzz at d	V	F2	P2	-F1+P2	F1+F2	R1	R2	R3	R4
Case II. A.	-1404	-811	-4534	52	-60	-30	780	-871	754	436	650	-436
Case II. B.	-1229	-710	4432	-51	59	30	739	-650	563	325	666	-325
Case II. C.	-1113	-643	-9891	114	-132	-66	577	-774	671	387	442	-387