The Present Value Model of Rational Commodity Pricing

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ABSTRACT

The present value model says that an asset's price equals the sum of current and future discounted expected future payoffs from ownership of the asset. I explore the limits of the present value model by testing its ability to explain the pricing of storable commodities. For commodities the payoff stream is the convenience yield that accrues from holding inventories, and it can be measured directly from spot and futures prices. The present value model imposes restrictions on the joint dynamics of spot and futures prices, which I test for four commodities. I find a close conformance to the model for heating oil, but not for copper or lumber, and especially not for gold. The pattern is the same when one looks at the serial dependence of excess returns. These results suggest that for three of the four commodities, prices at least temporarily deviate from fundamentals.

1. Introduction.

The present value model is the most basic description of rational asset pricing. It says that an asset's price, P_t, must equal the sum of current and discounted expected future payoffs, or benefits, from ownership of the asset:

$$P_t = \delta \sum_{i=0}^{\infty} \delta^i E_t \psi_{t+i} \tag{1}$$

Hence the present value model explains changes in asset prices in terms of "fundamentals," i.e., changes in expected future payoffs (ψ_{t+i}) or changes in discount rates (δ).

Most tests of the present value model use data for stocks, where the payoffs are dividends or earnings, or for bonds, where the payoffs are interest and principal payments. The outcomes of those tests have been mixed, reflecting in part statistical and data problems.[1] One problem, particularly for stocks, is that the flow of payoffs can be difficult to measure. Dividends, for example, are a choice variable of managers and true earnings are not observable, so that one has at best very noisy data for the payoff stream of a security.[2]

This paper explores the limits of the present value model by testing its ability to explain the pricing of storable commodities. Applying the present value model to commodities is useful for a number of reasons. First, the model is helpful in understanding price movements, and lets us test the rationality of commodity pricing in a way that is very different from earlier tests. Second, these tests provide evidence of the robustness of the present value model itself. (If the

[1]Most tests for stocks and bonds have been variance bounds tests or else attempts to show predictability of returns. For a discussion of the relationship between these two types of tests and a review of the literature, see Mankiw, Romer, and Shapiro (1991). Campbell and Shiller (1987) test restrictions implied by the model for the joint dynamics of P_t and ψ_t, and Pindyck and Rotemberg (1990b) develop tests based on the correlations of returns.

[2]There are also timing problems. Dividends and earnings are paid and announced quarterly, but firms often make statements about these variables well before the announcements.

model is valid, it should explain the pricing of *any* asset that yields a payoff stream.) Third, if the commodity is traded on a futures market, the model can be written entirely in terms of spot and futures prices, and provides a parsimonious description of rational price dynamics. In addition, the use of futures price data eliminates the problems of measuring or interpreting the payoff stream that arise with stocks.

For a storable commodity, the payoff stream ψ_t is the *convenience yield* that accrues from holding inventories. Convenience yield is the value of any benefits that inventories provide, including the ability to smooth production, avoid stockouts, and facilitate the scheduling of production and sales. It is the reason that firms hold inventories even when the expected capital gain on them is below the risk-adjusted rate, or negative.[3] While economists have debated the relative importance of these different benefits, for many commodities convenience yield is quantitatively important. As shown in Section 3, for example, firms sometimes incurred an expected cost of 5 to 10 percent per month - plus interest and direct storage costs - to maintain stocks of copper, lumber, and heating oil.[4]

The convenience yield that accrues to the owner of a commodity is directly analogous to the dividend on a stock. If the commodity is well defined and easily traded, and if aggregate storage is always positive, then eqn. (1) always holds, and the price of the commodity must

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[3]The concept of convenience yield was introduced by Working (1949) and further developed by Brennan (1958) and Telser (1958). They showed that convenience yield can be inferred from the relation between spot and futures prices, and illustrated its dependence on the aggregate level of inventories.

[4]Specifically, the expected capital loss was 5 to 10 percent per month. Most empirical studies of the use and value of inventories have been directed at manufactured goods; for a survey, see Blinder and Maccini (1990). Pindyck (1990) also surveys many of these studies and develops a model of inventory dynamics in commodity markets.

equal the present value of the flow of expected future convenience yields. The present value model thus provides a compact explanation for changes in a commodity's price; price changes are due to changes in expected future convenience yields. We usually try to explain commodity price movements in terms of changes in current and expected future demand and supply, but changes in demand and supply in turn cause changes in current and expected future convenience yields. Hence the present value model can be viewed as a highly reduced form version of a dynamic supply and demand model.

For some commodities, such as gold, the convenience yield is almost always very small, and often insignificantly different from zero. The reason is that inventories, which are largely held for "investment" purposes, are very large relative to production (for gold, about 50 times annual production). But the present value model also applies to such commodities, and provides a fundamentals-based explanation of why rational investors would hold them. Investors should hold these commodities if they think there is a large enough probability that convenience yield will rise substantially in the future. With gold, for example, this could occur if the metal were some day monetized, which would cause inventories to fall dramatically and convenience yield to rise.

For commodities traded on futures markets, convenience yield can be measured directly and (if the futures market is efficient in the sense that there are no arbitrage opportunities) without error from the relation between spot and futures prices. As a result, the present value model is also parsimonious in terms of data; tests can rely on data only for spot and futures prices. One does not, for example, need data on inventories, production costs, or other variables that affect supply, demand, or convenience yield.

I exploit futures price data to test the ability of the present value model to explain the prices of four commodities -- copper, lumber, heating oil, and gold. To do this, I draw extensively on work by Campbell and Shiller (1987), who showed that the present value model implies that the price of an asset and its payoff stream are cointegrated, and derived testable implications for the joint dynamics of the two. I show that the present value model imposes similar restrictions for the joint dynamics of the spot and futures prices of a storable commodity.

The basic theory is presented in the next section. I first review the arbitrage relation that determines a commodity's convenience yield from its spot and futures prices. I then discuss the restrictions on the joint dynamics of spot and futures prices that are implied by the present value relation of eqn. (1), and a set of tests that follow from those restrictions. Finally, I derive an alternative present value relation for the ratio of convenience yield to price (the commodity's percentage net basis), normalized relative to its mean value. This relation is similar to that derived by Campbell and Shiller (1989) for the log dividend-price ratio of a stock, and when combined with a model for the commodity's expected return, can be tested in the same way that (1) is.

Section 3 discusses the data set used in this study, and examines the behavior of prices and convenience yields for each of the four commodities. It also shows sample means and estimates of the expected excess return for each commodity, with the latter obtained from a cointegrating regression of futures and spot prices. Tests of the present value model are presented in Section 4, and the results are mixed. Heating oil prices conform closely to the model, and none of the constraints implied by (1) are rejected. Gold, however, does not conform to the model, and copper and lumber are in between.

Given these mixed results, it is useful to see whether other tests of market efficiency result in similar patterns across commodities. Section 5 examines the serial dependence of excess returns. Cutler, Poterba, and Summers (1990) recently studied the serial correlation of returns for a broad range of assets, including gold, silver, and an index of industrial metals. However, they ignored convenience yield when measuring returns. While this introduces only small errors for gold, it can lead to large measurement errors for industrial commodities, where convenience yield is often a large component of returns. I find that the extent of serial correlation in excess returns parallels conformance with the present value model; there is no significant serial correlation for heating oil, there is some for copper and lumber, and there is a considerable amount for gold.

2. The Present Value Model.

The present value model is given by eqn. (1), where ψ_t is the 1-period per unit net marginal convenience yield, i.e., the benefit that accrues from holding a marginal unit of the commodity from the beginning to the end of period t, net of storage and insurance costs over the period. Here, $\delta = 1/(1 + \mu)$, where μ is the commodity-specific 1-period discount rate, i.e., the expected rate of return that an investor would require to hold a unit of the commodity over period t.[5] For the time being I will assume that μ is constant, and can be written as $\mu = r + \rho$, where r is the 1-period risk-free rate, and ρ is a risk premium. In this section I first discuss the relationship of ψ_t to spot and futures prices, and then the implications of the present value model for the joint dynamics of spot and futures prices.

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[5]The relation between price and convenience yield is typically written in the differential form of eqn. (1), i.e., $E_tP_{t+1} = (1 + \mu)P_t - \psi_t$, where again, ψ_t is net of storage costs.

Futures Prices, Spot Prices, and Convenience Yield.

For commodities with actively traded futures contracts, we can use futures prices to measure the net marginal convenience yield. Let $\psi_{t,T}$ be the (capitalized) flow of marginal convenience yield net of storage costs over the period t to $t+T$, per unit of commodity. Then, to avoid arbitrage opportunities, $\psi_{t,T}$ must satisfy:

$$\psi_{t,T} = (1 + r_T)P_t - f_{T,t} \tag{2}$$

where P_t is the spot price, $f_{T,t}$ is the *forward* price for delivery at $t+T$, and r_T is the risk-free T-period interest rate. To see why eqn. (2) must hold, note that the (stochastic) return from holding a unit of the commodity from t to $t+T$ is $\psi_{t,T} + (P_{t+T} - P_t)$. If one also shorts a forward contract at time t, one receives a total return of $\psi^*_{t,T} + f_{T,t} - P_t$. No outlay is required for the forward contract and this total return is non-stochastic, so it must equal $r_T P_t$, from which (2) follows.[6]

For most commodities, futures contracts are much more actively traded than forward contracts, and good futures price data are more readily available. A futures contract differs from a forward contract only in that it is "marked to market," i.e., there is a settlement and corresponding transfer of funds at the end of each trading day. As a result, the futures price will be greater (less) than the forward price if the risk-free interest rate is stochastic and is

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[6]Note that the *expected* future spot price, and thus the risk premium on a forward contract, will depend on the commodity's risk premium (its "beta" in the context of the CAPM). But because $\psi_{t,T}$ is capitalized over t to $t+T$, expected spot prices or risk premia do not appear in eqn. (2). Indeed, eqn. (2) depends in no way on the stochastic structure of price evolution or on any particular model of asset pricing.

positively (negatively) correlated with the spot price.[7] However, for most commodities the difference in the two prices is extremely small. In another paper (1990) I have estimated this difference for copper, lumber, and heating oil, using the sample variances and covariances of the interest rate and futures price, and shown that it is negligible.[8] Thus I use the futures price, $F_{T,t}$, in place of the forward price in eqn. (2). Also, I work with the 1-month convenience yield, $\psi_{t,1}$, which I denote as simply ψ_t, and the corresponding futures price $F_{1,t}$.

Note that for the present value model to hold, inventories must always be positive, i.e., it must not be the case that stockouts sometimes occur.[9] Although we never observe aggregate inventories falling to zero in the data, one could argue that stockouts still do occur.[10] First, stockouts might occur with very low probability (but at very high cost to the firm if and when they do occur), so they are simply not observed in a sample of 20 or so years. Second, the data aggregate inventories for different products and different firms, and it may be that stockouts do

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[7]If the interest rate is non-stochastic, the present value of the expected daily cash flows over the life of the futures contract equals the present value of the expected payment at termination of the forward contract, so the futures and forward prices must be equal. If the interest rate is stochastic and positively correlated with the price of the commodity (which is the case for most industrial commodities), daily payments from price increases will on average be more heavily discounted than payments from price decreases, so the initial futures price must exceed the forward price. For a rigorous proof of this result, see Cox, Ingersoll, and Ross (1981).

[8]Also, French (1983) compares the futures prices for silver and copper on the Comex with their forward prices on the London Metals Exchange and shows that the differences are very small (about 0.1% for 3-month contracts).

[9]Deaton and Laroque (1989) developed a model of commodity prices in which stockouts play a key role. In their model, prices are relatively stable, with sudden price flares accompanied by inventory falling to near zero. People hold inventory in normal times because of a convex price function; price goes up more when there is a shortfall than when there is a glut, making storage profitable. But there is no convenience yield at all in their model; inventories are held only as a speculation against price shocks.

[10]Kahn (1991) makes this argument in the case of inventories of manufactured goods.

occur for some products and/or some firms. But these are not likely to be problems for the commodities that are studied here. First, the products are homogeneous and very clearly defined. Second, futures (and forward) markets are extremely liquid and have low transactions costs; any firm can easily buy or sell inventories through these markets, and therefore need never experience a stockout. Finally, there is good evidence that convenience yield is highly convex in the aggregate level of inventories, and becomes very large as that level becomes small, so that firms would never allow stockouts to occur.[11]

Implications of the Present Value Model for Spot and Futures Prices.

As Campbell and Shiller (1987) have shown, if P_t and ψ_t are both integrated of order 1, the present value relation of eqn. (1) implies that they are cointegrated, and the cointegrating vector is $(1 \;\; -1/\mu)'$. One can therefore define a "spread,"

$$S_t' = P_t - (1/\mu)\psi_t \,, \tag{3}$$

which will be stationary. Hence, in principle, one could estimate the expected return on a commodity, μ, by running a cointegrating regression of P_t and ψ_t.

In addition, it is easily shown that (1) and (3) imply that:

$$S_t' = (1/\mu)E_t\Delta P_{t+1} \tag{4}$$

Hence P_t and ψ_t contain all information necessary to optimally forecast P_{t+1}. If the futures market is efficient, this is equivalent to saying that P_t and $F_{1,t}$ are sufficient to optimally forecast P_{t+1}. Substituting (2) and (3) into (4) gives the standard result:

$$E_tP_{t+1} = F_{1,t} + (\mu - r)P_t \,, \tag{5}$$

[11]The earliest evidence is by Brennan (1958) and Telser (1958). Also, Pindyck (1990) models the convenience yields for copper, heating oil, and lumber and shows they are highly convex in the level of inventories.

i.e., the futures price is a biased predictor of the future spot price, and the bias is equal to the commodity's expected excess return. Thus either (4) or (5) can be used to forecast P_{t+1} if μ is known.

Campbell and Shiller also show that (1) and (3) together imply that:

$$\mu S_t' = E_t \sum_{i=1}^{\infty} \delta^i \Delta\psi_{t+i} \tag{6}$$

so that $\mu S_t'$ is the present value of expected future changes in the convenience yield. We can use (4) and (6) to see how the futures and spot prices describe the market's expectation of how ψ_t and P_t will evolve.

Assume for simplicity that $\mu = r$, so that $S_t' = (1/r)(F_{1,t} - P_t)$.[12] First, suppose that the futures are in full carry, i.e., $F_{1,t} = (1+r)P_t$. Then $\psi_t = 0$, and $S_t' = P_t$. Also $E_t(P_{t+1}) = (1+r)P_t$. Although convenience yield is currently zero in this case, people hold stocks of the commodity and rationally expect price to rise at the rate of interest because they expect the convenience yield to rise in the future. (In fact, $P_t = E_t\Sigma_i\delta^i\Delta\psi_{t+i}$, i.e., the value of a unit of the commodity is just the present value of expected future increases in convenience yield.) This is usually the case for gold, where stocks are very large relative to production, and the futures are often close to full carry. If holdings of gold are based on "rational fundamentals" (as opposed to a rational bubble, in which eqn. (1) includes a term b_t satisfying $b_t = \delta E_t b_{t+1}$), it must be because there is some probability that gold's convenience yield will rise sharply in the future (perhaps as a result of economic instability that leads to its monetization).

Now suppose the futures are at less than full carry, but in contango, i.e., $P_t < F_{1,t} <$

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[12]This is approximately the case for most agricultural commodities, as well as gold. See Dusak (1973).

$(1+r)P_t$. Then $S_t' > 0$, and both price and convenience yield are expected to rise. Note that $S_t' < 0$ only if the futures are in backwardation, i.e., ψ_t is large enough so that $F_{1,t} < P_t$. Then the present value of expected future changes in ψ_t is negative. This would typically mean that price and convenience yields are expected to fall, at least initially, as supply and demand adjust towards long-run equilibrium levels and inventories rise.[13] These patterns for P_t and ψ_t can be seen in the data for copper, where sharp increases in the spot price occurred in 1974, 1979-80, and 1988-89 as a result of strikes and other disruptions to supply that were expected to be temporary. Hence inventories fell and convenience yields rose sharply, falling again only as contemporaneous supplies rose and/or demands fell. Finally, if $\mu > r$, $S_t' < 0$ if $F_{1,t} < (1+r-\mu)P_t$. Now the expected future spot price exceeds the futures prices, so price and convenience yield can be expected to rise even when the futures are in backwardation.

As Campbell and Shiller (1987) have shown, eqns. (4) and (6) can be used to test the present value model. First, suppose μ has been estimated (e.g., from the cointegrating regression), and consider a vector of variables $\mathbf{z}_t$ (e.g., production, inventories, etc.) that might be expected to affect future spot prices. Then (4) implies that in regressions of the form:

$$\Delta P_t = \alpha_0 + \alpha_1 S_{t-1}' + \Sigma_i b_i z_{i,t-1} + \epsilon_t \tag{7}$$

the β_i's should be groupwise insignificant. Second, eqn. (6) implies that Granger causality tests should show causality from S_t' to future $\Delta\psi_{t+i}$'s. Finally, eqn. (6) also implies a set of cross-

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[13]When $S_t' < 0$ one usually observes this pattern of declining future expected convenience yields in the futures of different maturities, i.e., removing seasonal factors, we observe $P_t - F_{1,t} < F_{1,t} - F_{2,t} < F_{2,t} - F_{3,t}$, etc. Also, we should observe that spot prices are more volatile than futures prices, particularly when the futures are in backwardation. As Fama and French (1988b) show, this is indeed the case.

equation restrictions on a vector autoregression of S_t' and $\Delta\psi_t$.

One problem is that if P_t and ψ_t are in nominal terms, the nominal expected return μ will fluctuate, even if the underlying real expected return is constant. Campbell and Shiller deal with this when testing the present value model for stocks and bonds by deflating the variables, but this can introduce measurement noise. With futures market data, however, we can avoid this problem altogether by using eqn. (2), with the futures price replacing the forward price. Define a new spread $S_t = \mu S_t'$, and substitute (2) for ψ_t:

$$S_t = F_{1,t} - (1-\rho)P_t \tag{8}$$

where $\rho = \mu - r$ is the expected excess return on the commodity. Thus S_t is the futures-spot spread, adjusted for the forecast bias in the futures price. Also, (8) implies that the futures and spot prices are cointegrated, and the cointegrating vector is $(1 \quad \rho\text{-}1)'$. Hence a simple regression of the futures price on the spot price can be used to estimate the expected excess return, ρ. If real expected returns are constant, the expected excess return should likewise be constant, and can be estimated from this regression without recourse to the CAPM or some related model of asset pricing.[14]

Eqn. (4) can also be written in terms of S_t, and then becomes:

$$S_t = E_t\Delta P_{t+1} \tag{9}$$

i.e., the spread S_t is an <u>unbiased</u> forecast of the change in the spot price. Note that this

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[14]One could also estimate ρ from the error correction representation of eqn. (8), i.e., by running the regression:

$$\Delta F_{1,t} = \alpha_0 + \alpha_1 F_{1,t-1} + \alpha_2 P_{t-1} + \alpha_3 \Delta F_{1,t-1} + \alpha_4 \Delta P_{t-1} + u_t$$

Then $\hat{\rho} = 1 - \hat{\rho}_1/\hat{\rho}_2$.

condition can also be derived directly from eqn. (5). Again, the current futures and spot prices must be sufficient for the optimal prediction of future spot prices. This condition is sometimes used to test the efficiency of future markets. However, the failure of this condition to hold need not imply that the futures market is inefficient. It could instead mean that the spot price deviates from the fundamental present value relation (1). This could, for example, cause the bias between the futures price and the expected spot price to be more complicated than ρP_t, so that (9) would not hold.

Tests of the Model.

Once ρ has been estimated, eqns. (6) and (9), with S_t replacing $\mu S_t'$ on the left-hand side of (6), can be used to test (1). First, note that (9) implies that any variables in the information set at t-1 should be uncorrelated with the residuals of a regression of ΔP_t on S_{t-1}. Hence we can run regressions of the form:

$$\Delta P_t = \alpha_0 + \alpha_1 S_{t-1} + \Sigma_i b_i z_{i,t-1} + \epsilon_t \tag{10}$$

where the z_i's are any variables that might be thought to affect price, including commodity-specific variables such as production and inventory levels, and economy-wide variables such as such as GNP growth and inflation. We can then test whether the coefficients b_1, b_2, etc. are insignificantly different from zero.

This test requires an estimate of ρ to construct S_t; I first use the estimate obtained from the cointegrating regression of $F_{1,t}$ on P_t, and then the sample mean of ρ. A failure of the test could mean that (9) does not hold, or alternatively that the estimate of ρ used to calculate S_t differs substantially from the true value. This second possibility can be ruled out by also running the regression:

$$\Delta P_t = \alpha_0 + \alpha_1 P_{t-1} + \alpha_2 F_{1,t-1} + \Sigma_i b_i z_{i,t-1} + \epsilon_t \tag{11}$$

and again testing that the b_i's are zero.

Second, since $S_t = \mu S_t'$, eqn. (6) implies that S_t should Granger-cause $\Delta\psi_t$. I run Granger causality tests between S_t and $\Delta\psi_t$, again, constructing S_t first using the estimate of ρ from the cointegrating regression, and then using the sample mean.

Finally, as Campbell and Shiller show, eqn. (1) implies constraints on the parameters of a vector autoregression of S_t and $\Delta\psi_t$. Specifically, consider the pth-order vector autoregression:

$$\Delta\psi_t = \sum_{k=1}^{p} \gamma_{11k}\Delta\psi_{t-k} + \sum_{k=1}^{p} \gamma_{12k} S_{t-k} \tag{12a}$$

$$S_t = \sum_{k=1}^{p} \gamma_{21k}\Delta\psi_{t-k} + \sum_{k=1}^{p} \gamma_{22k} S_{t-k} \tag{12b}$$

Note from eqn. (6) that S_t is the present discounted value of the expected future $\Delta\psi_t$'s. This in turn implies that the parameters γ_{ijk} must satisfy the following set of cross-equation restrictions: $\gamma_{21k} = -\gamma_{11k}$, $k = 1, \ldots, p$, $\gamma_{221} = 1/\delta - \gamma_{121}$, and $\gamma_{22k} = -\gamma_{12k}$, $k = 2, \ldots, p$.[15] These restrictions provide another test of the present value model.

The Dynamics of the Percentage Net Basis.

The tests discussed above are based on relationships between spot and futures prices that follow from the present value model of eqn. (1). (The causality tests and vector autoregressions

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[15]These constraints are derived as in Campbell and Shiller (1987) as follows. Define $x_t = [\Delta\psi_t, \ldots, \Delta\psi_{t-p+1}, S_t, \ldots, S_{t-p+1}]'$. Then (12) can be written in the form $x_t = Ax_{t-1} + v_t$, where A is a $2p$ by $2p$ matrix. Also, forecasts from this VAR are given by $E_t x_{t+k} = A^k x_t$. Let g be a column vector whose $p+1$st element is 1 and whose remaining elements are 0, and let h be a column vector whose first element is 1 and whose remaining elements are 0. Then from eqn. (6), $S_t = g'x_t = \Sigma_1^\infty \delta^i h' A^i x_t = h'\delta A(I - \delta A)^{-1} x_t$. This must hold for any x_t, so $g'(I - \delta A) = h'\delta A$, from which the constraints follow.

are based on S_t and $\Delta\psi_t$, but these in turn are functions of P_t and $F_{1,t}$.) An alternative way of studying commodity price dynamics is to work with the differential form of eqn. (1) and look at the components of commodity returns. By imposing some structure on expected returns (e.g., the CAPM), one can constrain the dynamics of the rate of convenience yield, i.e., the ratio of the net convenience yield to price. This ratio is referred to as the *percentage net basis*, and is analogous to the dividend-price ratio for a stock.[16]

Campbell and Shiller (1989) have derived an approximate present value relation for the *log* dividend-price ratio, and have shown that it implies parameter restrictions on a vector autoregression of this ratio and the difference between the expected return and the dividend growth rate. Because the net convenience yield is sometimes negative, I work with a simple ratio, and derive a similar approximate present value relation. This, in turn, yields parameter constraints on a vector autoregression of percentage net basis and the difference between the risk-free rate and a normalized change in convenience yield.

Specifically, write the monthly return on the commodity from the beginning of period t to the beginning of period $t+1$ as:

$$q_t = (P_{t+1} - P_t + \psi_t)/P_t \tag{13}$$

Let y_t denote the percentage net basis, i.e., $y_t \equiv \psi_{t-1}/P_t$. Then we can rewrite (13) as:

$$q_t = \psi_t y_t/\psi_{t-1} + \psi_t y_t/\psi_{t-1} y_{t+1} - 1 \tag{14}$$

Now linearize q_t around the sample means $\bar{\psi}$ and $\bar{y}$:

[16] The percentage net basis is $(1+r) - F_{1,t}/P_t$, but note from eqn. (2) that this is just ψ_t/P_t. In what follows, I work with the ratio ψ_{t-1}/P_t.

$$q_t \approx y_t(1 + 1/\bar{y}) + \Delta\psi_t(1+\bar{y})/\bar{\psi} - y_{t+1}/\bar{y} \tag{15}$$

Finally, define $\beta = 1/(1+\bar{y})$, and define the normalized variables $y_t' = y_t/\bar{y}$, and $\psi_t' = \psi_t/\bar{\psi}$.[17] Then (15) can be rewritten as:

$$\beta q_t \approx y_t' - \beta y_{t+1}' + \Delta\psi_t' \tag{16}$$

The solution to this difference equation is a present value relation for the normalized percentage net basis y:

$$y_t' \approx \sum_{j=0}^{\infty} \beta^j(\beta q_{t+j} - \Delta\psi_{t+j}') \tag{17}$$

i.e., the normalized percentage net basis is approximately the present value of the future stream of returns from holding the commodity net of changes in the normalized convenience yield.

This is simply an approximate accounting relationship, but as Campbell and Shiller (1989) have shown, it can be combined with an economic model for expected returns. I will assume that the expected return is the sum of the (time-varying) expected risk-free rate plus the (constant) risk premium ρ: $E_t q_{t+j} = E_t r_{t+j} + \rho$. Then (17) becomes:

$$y_t' \approx E_t \sum_{j=0}^{\infty} \beta^j(\beta r_{t+j} - \Delta\psi_{t+j}') + \frac{\beta\rho}{1-\beta} \tag{18}$$

Eqn. (18) provides another description of a commodity's price in terms of fundamentals. It says that in a steady state equilibrium in which r_t is constant and $E_t\Delta\psi_{t+j} = 0$ for all j, the

[17]For most commodities, $\bar{y}$ is on the order of 1 percent or less, so β is less than but close to 1. Campbell and Shiller (1989) obtain a present value relation for the log dividend-price ratio on a stock by first writing a log-linear approximation to the stock's log gross return, and then assuming that the ratio of the stock price to the sum of price plus dividend is approximately constant. That ratio (which they denote by ρ) is analogous to β in my model. I work with the arithmetic ratio of convenience yield to price, so the only approximation required is that q_t be linearized around $\bar{\psi}$ and $\bar{y}$.

expected return on a commodity ($\mu = r + \rho$) must equal the rate of convenience yield y_t. (To see this, note that if $r_t = r$ and $E_t\Delta\psi_{t+j} = 0$, the equation reduces to $y_t' = \beta\mu/(1 - \beta) = \mu/\bar{y}$, or $\mu = y_t$.) In this case, $E_t\Delta P_{t+1} = 0$ (which also follows from eqns. (4) and (6)). Hence unless the discount rate is expected to change, expected price changes are always due to expected changes in convenience yield.

Earlier we used eqns. (4) and (6) to explain the dynamics of the spread between spot and futures prices in terms of the market's expectations of how convenience yield and the spot price will evolve. Eqn. (18) provides a similar explanation for the dynamics of the percentage net basis, y_t. It says that y_t will be low relative to its average value (i.e., the spot price will be unusually high and/or convenience yield low) if convenience yield is expected to rise. This was the case with gold during 1980 and late 1982 (see Figure 4). Likewise, y_t will be high relative to its average value if convenience yield is expected to fall. This would occur, for example, when inventories are tight because of a strike or other supply disruption that is expected to end (as has been the case periodically with copper).

Eqn. (18) can be used to impose restrictions on the dynamics of the percentage net basis. Specifically, define $\phi_t = \beta r_t - \Delta\psi_t' + \beta\rho$, so that $y_t' = E_t\Sigma_j\beta^j\phi_{t+j}$, and consider the pth-order vector autoregression:

$$y_t' = \sum_{k=1}^{p} \gamma_{11k} y_{t-k}' + \sum_{k=1}^{p} \gamma_{12k}\phi_{t-k-1} \tag{19a}$$

$$\phi_{t-1} = \sum_{k=1}^{p} \gamma_{21k} y_{t-k}' + \sum_{k=1}^{p} \gamma_{22k}\phi_{t-k-1} \tag{19b}$$

Then (18) implies the following cross-equation restrictions on the parameters γ_{ijk}: $\gamma_{211} = 1 - \beta\gamma_{111}$, $\gamma_{21k} = -\beta\gamma_{11k}$, $k = 2, \ldots, p$, and $\gamma_{22k} = -\beta\gamma_{12k}$, $k = 1, \ldots, p$. These restrictions are analogous to, and are derived in the same way, as the restrictions on the VAR of eqns. (12a) and (12b). They provide a test of the present value relation of eqn. (18) for the dynamics of the percentage net basis.

3. The Behavior of Spot and Futures Prices.

In this section I discuss the data set and the calculation of the one-month convenience yield ψ_t. I also discuss the behavior of spot and futures prices, ψ_t, and the spread S_t for each of the four commodities, and show estimates of the expected excess return ρ and expected total return μ obtained from the cointegrating regressions.

Data.

All of the tests use futures price data for the first Wednesday of each month. In all cases, that day's settlement price is obtained from the *Wall Street Journal*. Occasionally a contract price will be constrained by exchange-imposed limits on daily price moves. In those cases I use prices for the preceding Tuesday. If those prices are likewise constrained by limits, I use prices for the following Thursday, or if those are constrained, the preceding Monday.

To obtain a spot price P_t, whenever possible I use the price on the spot futures contract, i.e., the contract that is expiring in month t. This has the advantage that the spot and futures prices then pertain to exactly the same good, and the time interval between the two delivery times is known exactly.[18] One difficulty with this is that a spot contract does not trade in every

[18]Alternatively, one could use data on cash prices, purportedly reflecting actual transactions over the month. One problem with this is that it results in an average price over the month, as opposed to a beginning-of-month price. A second and more serious problem is that a cash price can apply to a different grade or specification of the commodity (e.g., copper or gold of a different purity), and can include discounts and premiums that result from longstanding relationships between buyers and sellers. Hence a cash price is not directly comparable to a futures price.

month for every commodity. For those months when a spot contract does not trade, I inferred a spot contract price from the nearest active futures contract (i.e., the active contract next to expire, typically a month or two ahead), and the next-to-nearest active contract. This is done by extrapolating the spread between these contracts backwards to the spot month:

$$P_t = F_{1,t}(F_{1,t}/F_{2,t})^{n_{01}/n_{12}} \tag{19}$$

where $F_{1,t}$ and $F_{2,t}$ are the prices on the nearest and next-to-nearest futures contracts, and n_{01} and n_{12} are, respectively, the number of days between t and the expiration of the nearest contract, and between the nearest and next-to-nearest contract.

This approach provides spot prices for every month of the year. It has the disadvantage that errors can arise if the term structure of spreads is very nonlinear. To check that such errors are small, I calculated spot prices using eqn. (19) and compared them to actual spot contract prices for copper (available for 200 out of 223 observations), for lumber (available for 114 out of 226 observations), and for gold (available for 173 out of 194 observations). In all three cases, I found little discrepancy between the two series.[19]

Given a series for P_t, I then calculate the one-month net marginal convenience yield, ψ_t, using the nearest futures contract and the Treasury bill rate that applies to the same day for which the futures prices are measured. In some cases the nearest futures contract has an horizon greater than one month; I then infer a one-month futures price using the spot contract and the nearest contract if the spot contract exists, or else using the nearest and next-to-nearest contracts.

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[19]The RMS percent error and mean percent error for the three series are, respectively, 1.21% and -0.12% for copper, 3.99% and 0.39% for lumber, and 3.40% and 0.12% for gold. The simple correlations are .998 for copper, .983 for lumber, and .999 for gold. No spot contract prices were available for heating oil.

(For example, if in January the nearest futures prices are for March and May and there is no January spot contract, I infer a February price using eqn. (19) with $n_{01} = 28$ and $n_{12} = 61$.)

To test the sufficiency of P_t and F_t in forecasting P_{t+1}, I use the following set of variables in the vector z_t: the change in the exchange value of the dollar against ten other currencies, and the growth rates of the Index of Industrial Production, the Index of Industrial Materials Prices, and the S&P 500 Index. For copper, heating oil, and lumber, z_t also includes the level and change of monthly U.S. production and inventories of that commodity. All of these variables are measured at the end of the month preceding the date for which prices are measured.

Prices and Convenience Yields.

Figures 1 to 4 show spot prices and the percentage net basis for each of the four commodities. Note that for copper, heating oil, and lumber, price and convenience yield tend to move together. For example, there were three periods in which copper prices rose sharply: 1973, 1979-80, and late 1987 to 1989. On each occasion (and especially the first and third), convenience yield also rose sharply, even as a percentage of price. The same was true when lumber prices rose in early 1973, 1977-79, 1983, and 1986-87. For heating oil the comovement is smaller (and much of it is seasonal), but there has still been a tendency for the percentage net basis to move with price. This behavior is consistent with the notion that these periods of high prices were expected to be temporary, i.e., that price (and convenience yield) were expected to fall as supply and demand adjust towards long-run equilibrium levels.

These figures also show that for these three commodities, convenience yield is a quantitatively important part of the commodity's return. There were periods, for example, when the monthly net convenience yield was 5 to 10 percent of the commodity's price. This means that firms were paying 5 to 10 percent *per month* - plus interest and direct storage costs - to maintain stocks.

The behavior of price and convenience yield for gold is quite different. Monthly net convenience yield has always been less than 1 percent of price, and usually less than 0.2 percent. Moreover, except for the brief spike in convenience yield in 1981, there is little comovement with price. This suggests that sharp increases in price (such as those of 1980 and late 1982 - early 1983) were not expected to be temporary. This is consistent with the view that the price of gold follows a speculative bubble, or alternatively that it is based on fundamentals and rose at the time because of an expectation that at some point in the future convenience yield would rise.

Table 1 shows the results of unit root tests for spot and futures prices, convenience yield, and the spreads S_t and S'_t.[20] Note that for all four commodities, spot and futures prices are integrated of order 1, and at least for copper, heating oil, and lumber, are clearly cointegrated. The table also shows the estimates of the expected monthly excess return, ρ, from the cointegrating regression of the futures price on the spot price, as well as the sample means of ρ. The regression estimates of ρ are fairly close to the sample means for heating oil and gold, and imply expected annual excess returns of 11 percent for heating oil, and -12 percent for gold (versus a sample mean for gold of about 0 percent). However, these estimates are unreasonably large for copper and lumber, and imply annual excess returns of about 70 percent and 100

[20]All of the unit root tests on a variable x_t shown in the table are augmented Dickey-Fuller tests that include Δx_{t-1} and Δx_{t-2} on the right-hand side, but do not include a time trend. All of the significance levels are the same when a time trend is included, or if one or three lags of Δx_t are included. Significance levels are based on MacKinnon's (1990) critical values.

percent respectively (versus sample means of about 7 percent and 2 percent). Also, except for gold, S_t is stationary when calculated using either value of ρ. (For gold, we can reject a unit root at the 5 percent level when S_t is calculated using the estimate of ρ from the cointegrating regression, but not when using the sample mean of ρ.) These results are generally consistent with the cointegration of the futures and spot prices, with cointegrating vector $(1 \quad \rho\text{-}1)'$.

On the other hand, we strongly reject a unit root in ψ_t for all four commodities, and a regression of P_t on ψ_t yields estimates of the expected total return μ that are extremely large. (In addition, when S'_t is computed using the estimated value of μ, we fail to reject a unit root for two commodities, and reject at only the 5 percent level for the other two.) Although this is inconsistent with the present value relation of eqn. (1), it may reflect problems of sample size, and are similar to results obtained by Campbell and Shiller (1987) for interest rates and the stock market. Note that when S'_t is computed using the sample mean of μ, we can reject a unit root at the 1 percent level for every commodity but gold. For at least these commodities, it is likely that either P_t is in fact mean-reverting, but the mean reversion is too slow to be detected in samples spanning less than 20 years, or alternatively both P_t and ψ_t are integrated of order 1.[21]

4. Test Results.

Tests of the present value relation (1) are based on eqn. (9), which implies that S_t and P_t are sufficient to forecast P_{t+1}, on eqn. (6), which implies that S_t should Granger-cause $\Delta\psi_t$, and on the cross-equation restrictions on the VAR of eqns. (12a) and (12b). The second and

[21]One might expect the long-run adjustment of supply and demand to be slow so that mean reversion in prices can only be discerned from long time series. However, Agbeyegbe (1991) studies the stochastic behavior of prices for pig iron, copper, lead, and zinc using data for 1871 - 1973, and in each case finds strong evidence of a unit root.

third of these tests require a series for the futures-spot spread, S_t. I calculate S_t first using $\hat{\rho}$ estimated from the cointegrating regression, and then using the sample mean $\bar{\rho}$.

Table 2 shows F-statistics for Wald tests of the restrictions that $b_i = 0$ in regression eqns. (10) and (11). The vector z_t can include any variables that might reasonably be expected to help forecast next period's spot price. In both equations, I include four economy-wide variables that are predictors of industrial commodity demand or supply (the exchange value of the dollar, and the growth rates of the Index of Industrial Production, the S&P 500 Index, and the Index of Industrial Commodity Prices). Also, for copper, heating oil, and lumber, I include the level and change of monthly U.S. production and inventory levels for the respective commodity. (These data were not available for gold.)

Note that the restrictions are not rejected in any version of the equation for copper, heating oil, and gold. However, in the case of lumber, they are rejected at the 1 percent level both for eqn. (11) and for eqn. (10) when S_t is calculated using the sample mean of ρ. This result for lumber could reflect a failure of eqn. (1), or alternatively, inefficiency in the futures market. (The latter possibility is more likely for lumber than the other commodities because of the four, lumber futures are the most thinly traded.) Finally, note that the predictive power of S_t (as measured by the R^2) varies considerably across the commodities. It is very low for copper and gold, but surprisingly high for heating oil.

Table 3 shows the results of Granger causality tests between S_t and $\Delta\psi_t$. Eqn. (9) implies unidirectional causality from S_t to $\Delta\psi_t$, i.e., that we should be able to reject the hypothesis that S_t does not Granger-cause $\Delta\psi_t$, but fail to reject the hypothesis that $\Delta\psi_t$ does not Granger-cause S_t. Using the Akaike Information Criterion to choose the number of lags in these tests was inconclusive; the AIC (and FPE) suggest between 2 and 8 lags, but are fairly flat within this range. Hence I report results for 2, 4, 6, and 8 lags.

For copper, heating oil, and lumber these results are consistent with the present value model; in each case we can clearly reject the hypothesis that S_t does not cause $\Delta\psi_t$. Furthermore, for heating oil and lumber, the causality is unidirectional; we fail to reject the noncausality of $\Delta\psi_t$ to S_t. For gold, the results are more ambiguous. We reject the hypothesis that S_t does not cause $\Delta\psi_t$ with 4, 6, or 8 lags, but not with 2 lags. Also, with any number of lags there is always a much stronger rejection of the hypothesis that $\Delta\psi_t$ does not cause S_t.

Table 3 also shows chi-square statistics for Wald tests of the cross-equation restrictions implied by eqn. (1) on the vector autoregression of S_t and $\Delta\psi_t$. (The results shown are for a 4th-order VAR, but are qualitatively the same for 2nd-order and 6th-order VARs.) These restrictions are strongly rejected for copper, lumber, and gold, irrespective of whether $\hat{\rho}$ or $\bar{\rho}$ is used to calculate S_t. The restrictions are accepted, however, for heating oil.

These results provide mixed evidence on the ability of the present value model to explain commodity prices. The model fits the data well for heating oil, but some of its implications are rejected by the data for copper and lumber. This may be because on average, convenience yield is a larger percentage of price for heating oil than for the other commodities. Hence price movements for heating oil will be tied more closely to expected *near-term* changes in convenience yield, rather than changes that might occur in the more distant future.

The strongest rejections of the present value model are for gold; for this commodity, it is not even clear that futures and spot prices are cointegrated, and there is no evidence that the spot price and convenience yield are cointegrated. But if the present value model indeed holds for gold, it must be based on the expectation of increases in convenience yield that are extremely infrequent but very substantial if they occur (i.e., events of the "peso problem" sort). Throughout the 15 year sample, the convenience yield for gold has always been very small relative to price, so the present value model can only explain price movements in terms of changes in market perceptions of either the mean arrival rate of an event, or the probability distribution for the size of the event. Since such changes in market perceptions are unobservable and do not affect current convenience yields, these test results are not surprising.

Table 4 shows statistics for the percentage net basis, $y_t = \psi_{t-1}/P_t$, and the variable $\phi_t = \beta r_t - \Delta\psi'_t + \beta\rho$. Note that $\bar{y}$ is largest for heating oil (about 1.5 percent per month), and extremely small for gold. Also, we can clearly reject a unit root for both y_t and ϕ_t. The table also shows chi-square statistics for Wald tests of the cross-equation restrictions imposed by the present value relation (18) on the vector autoregression of $y'_t = y_t/\bar{y}$ and ϕ_{t-1}. (Again, results are reported for a 4th-order VAR, but are qualitatively the same for 2nd- and 6th-order VARs.) These restrictions are strongly rejected for all four commodities. Although this is not a rejection of eqn. (1), it is troubling because it can be viewed as a rejection of a constant risk premium (recall that (18) was derived by assuming that the expected return $E_t q_{t+j} = E_t r_{t+j} + \rho$), and (1) includes a constant discount rate. Alternatively, this result could be a rejection of the linear approximation of eqn. (15) that was used to derive (18).

5. Serial Correlation of Excess Returns.

Given these mixed results, I turn to an alternative test of market efficiency, and examine the serial correlation of excess returns. Apart from systematic changes in the risk premium, significant serial correlation of returns would suggest temporary deviations of prices from fundamentals. Although these tests have low statistical power, they are useful because we can look for patterns of results across commodities that are similar to the results above for the present value model.

Cutler, Poterba, and Summers (1991) found evidence of strong serial correlation of excess returns that is positive in the short run and negative in the long run for a broad range of assets. They included gold, silver, and an index of industrial metals, but ignored convenience yield when measuring returns. This can lead to substantial measurement errors, at least for the industrial metals, where convenience yield is often a large component of returns. I calculate autocorrelations for excess returns that include convenience yields, and that are measured relative to the rate on three-month Treasury bills. In addition to examining the first twelve individual autocorrelations, I follow Cutler, Poterba, and Summers and also examine the averages of autocorrelations 1 - 12, 13 - 24, 25 - 36, and 37 - 48. As they point out, with limited samples individual autocorrelations may be difficult to distinguish from zero, and persistent deviations may yield stronger evidence of serial dependence.

The autocorrelations of excess returns for each commodity are shown in Table 5. (These autocorrelations are corrected for small sample bias by adding $1/(T - j)$ to the jth correlation, where T is the sample size.) Also shown are Box-Pierce Q statistics that test whether the first K autocorrelations are insignificantly different from zero (as would be the case if the excess returns were white noise). Observe that we can reject a non-zero first-order autocorrelation at the 5 percent level for copper and gold. Of the first 12 autocorrelations, 2 are significantly different from zero at the 5 percent level for copper, four are for lumber, and five are for gold. Overall, gold exhibits the greatest serial dependence of returns. In addition to individual autocorrelations that are high, the Q statistics are significant at below the 0.1 percent level for the first 12, 24, and 48 autocorrelations.[22] For copper and lumber, there is also evidence of serial dependence, although it is weaker. Fewer individual autocorrelations are significant (especially for copper), and the Q statistics are significant for the first 12 or 24 autocorrelations, but not the first 48. Also, for all three of these commodities the serial dependence is positive for short horizons, but becomes negative for longer horizons. This is similar to the patterns observed by Fama and French (1988a) and Poterba and Summers (1988) for stock returns, and is consistent with the notion that prices temporarily drift away from fundamentals.

For heating oil, however, there is no evidence at all of serial dependence of returns. Every individual autocorrelation is within one standard deviation of zero, and the probability levels for the three reported Q statistics are all above .9. This pattern across commodities parallels that in the previous section for tests of the present value model. There, too, the strongest rejections were for gold, results for copper and lumber were mixed, but heating oil exhibited close conformance to the present value model.

6. Conclusions.

The present value model of rational commodity pricing can be viewed as a highly reduced form of a dynamic supply and demand model, and when the commodity is traded on a futures market, it can be tested through the constraints it imposes on the joint dynamics of spot and futures prices. I found a close conformance to the model for heating oil, but not for copper or lumber, and especially not for gold. (For gold, futures and spot prices do not even appear to

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[22]I find much greater serial dependence in excess returns for gold than do Cutler, Poterba, and Summers. Their estimate of ρ_1, for example, is only .020. However, their sample period is 1974 to 1988, while mine is 1975 through the first three months of 1990.

be cointegrated.) The pattern is the same when one looks at the serial dependence of excess returns. For three of the four commodities, these results are consistent with the notion that prices at least temporarily drift away from fundamentals, perhaps because of "fads."

Earlier studies provide different evidence that commodity prices are not always based on fundamentals. For example, Roll (1984) found that only a very small fraction the price movements for frozen orange juice can be expained by "fundamentals," i.e., by variables such as the weather that in principle should explain a good deal of the variation in price. And Pindyck and Rotemberg (1990a) found high levels of unexplained price correlation across commodities that is also inconsistent with prices following fundamentals. However, both the Roll and Pindyck and Rotemberg results may be suspect because of the possibility that one or more key variables (that affect orange juice supply or demand, or supplies or demands for a broad range of commodities) have been omitted. The present value model, on the other hand, is based entirely on a payoff stream that can be measured from futures market data. The rejections of some of the implications of that model (together with the finding of serially dependent returns) provides additional evidence that the prices of some commodities may be partly driven by fads.

Heating oil prices, however, conform closely to the present value model, and there is no evidence of serial dependence in excess returns. Why does heating oil seem to differ from the other commodities in this respect? It may be that its high average convenience yield makes speculating in heating oil too costly. (A speculative long position sacrifices 1½ percent per month on average in convenience yield.) This would make the odds in other commodities more favorable for speculators.

Table 1. Unit Root Tests and Estimates of ρ

	Copper (1/71-8/89)	Heating Oil (10/80-3/90)	Lumber (4/71-3/90)	Gold (1/75-3/90)
P_t	-2.55	-1.83	-2.92*	-1.55
ΔP_t	-9.21**	-6.89**	-11.87**	-8.24**
F_t	-2.40	-1.86	-2.90*	-1.55
ΔF_t	-8.95**	-6.93**	-11.60**	-8.25**
ψ_t	-4.27**	-5.58**	-3.77**	-5.38**
$\Delta\psi_t$	-12.52**	-7.15**	-11.08**	-13.29**
$S_t(\hat{\rho})$	-4.38**	-5.50**	-3.73**	-2.95*
$S_t(\bar{\rho})$	-4.38**	-5.50**	-3.67**	-2.22
$S_t'(\tilde{\mu})$	-2.88*	-1.92	-3.25*	-1.56
$S_t'(\bar{\mu})$	-4.46**	-5.66**	-3.80**	-2.41
$\hat{\rho}$	.04728	.00892	.06100	-.01080
$\bar{\rho}$	.00579	.00926	.00136	.00011
$\tilde{\mu}$	.10863	.53080	.31263	-.05962
$\bar{\mu}$	.01237	.01673	.00800	.00711

Note: Unit root tests are t-statistics on β in the regression $\Delta x_t = \alpha_0 + \alpha_1\Delta x_{t-1} + \alpha_2\Delta x_{t-2} + \beta x_{t-1}$. Significance levels are based on MacKinnon's (1990) critical values; * denotes significance at 5% level, ** at 1%. $\hat{\rho}$ is estimate of expected monthly excess return ρ from cointegrating regression: $F_t = \alpha_0 + (1-\rho)P_t$; $\bar{\rho}$ is sample mean. $S_t = F_t - (1-\rho)P_t$. $\tilde{\mu}$ is estimate of expected monthly return μ from cointegrating regression $P_t = (1/\mu)\psi_t$; $\bar{\mu}$ is sample mean. $S_t'(\mu) = P_t - (1/\mu)\psi_t$.

Table 2. Sufficiency of F_t and P_t in Forecasting P_{t+1}

Eqn.		**Copper**	**Heating Oil**	**Lumber**	**Gold**
(1), $\rho = \hat{\rho}$	F	0.55	1.82	1.97	1.14
	R^2	.054	.404	.112	.034
(1), $\rho = \bar{\rho}$	F	0.65	1.83	2.75**	1.48
	R^2	.032	.404	.161	.057
(2)	F	1.27	1.78	2.92**	1.47
	R^2	.112	.431	.179	.057

Note: F-statistics test the restrictions $b_i = 0$ in the regressions (1) $\Delta P_t = a_0 + a_1 S_{t-1}(\rho) + \Sigma_i b_i z_{i,t-1}$, where $S_t = F_t - (1-\rho)P_t$, and (2) $\Delta P_t = a_o + a_1 F_{t-1} + a_2 P_{t-1} + \Sigma_i b_i z_{i,t-1}$. For all commodities, z_t includes the change in the exchange value of the dollar against ten other currencies, and the growth rates of the Index of Industrial Production, the Index of Industrial Materials Prices, and the S&P 500 Index. For copper, heating oil, and lumber, z_t also includes the level and change of monthly U.S. production and inventories of that commodity.

Table 3. Causality Tests and Tests of VAR Restrictions

# lags	H_0	Copper	Heating Oil	Lumber	Gold
		A. Causality Tests			
2	$S \not\rightarrow \Delta\psi$	13.90**	14.89**	10.39**	1.88
	$\Delta\psi \not\rightarrow S$	2.42	1.59	0.81	5.66**
4	$S \not\rightarrow \Delta\psi$	7.55**	9.04**	4.77**	2.83*
	$\Delta\psi \not\rightarrow S$	4.07**	1.02	0.33	8.41**
6	$S \not\rightarrow \Delta\psi$	6.32**	5.00**	2.97**	3.74**
	$\Delta\psi \not\rightarrow S$	3.39**	0.26	0.27	12.53**
8	$S \not\rightarrow \Delta\psi$	4.93**	2.23*	3.00**	2.97**
	$\Delta\psi \not\rightarrow S$	4.02**	0.44	0.96	9.88**
		B. Tests of Restrictions on VAR			
4	Restrictions on VAR of $S(\hat{\rho})$ and $\Delta\psi$	40.58**	12.29	40.74**	67.54**
4	Restrictions on VAR of $S(\bar{\rho})$ and $\Delta\psi$	65.03**	12.85	27.61**	43.27**

Note: (A) In causality tests of $y \not\rightarrow x$, F-statistics are shown for tests of restrictions $b_i = 0$ in regressions of $x_t = a_0 + \Sigma_i a_i x_{t-i} + \Sigma_i b_i y_{t-i}$. S is computed using $\hat{\rho}$ from cointegrating regression. (Results are qualitatively the same when $\bar{\rho}$ is used.) (B) χ^2 statistics are shown for Wald tests of restrictions on 4-period VAR of $S_t(\rho)$ and $\Delta\psi_t$. A * denotes significance at 5% level, ** at 1%.

Table 4. Behavior of Percentage Net Basis

	Copper	Heating Oil	Lumber	Gold
	A. Basic Statistics			
$\bar{\psi}$	0.5972	1.0807	0.6850	0.0873
$\bar{P}$	73.488	69.464	166.64	341.51
$\bar{y} = \overline{\psi_{-1}/P}$	.00493	.01468	.00153	.00030
$\beta = 1/(1+\bar{y})$	.9951	.9855	.9985	.9997
$\bar{\phi}$	.00436	-.00439	-.01516	-.01508
	B. Unit Root Tests			
y_t	-3.87**	-4.81**	-3.76**	-5.44**
ϕ_t	-12.52**	-7.15**	-11.08**	-13.29**
	C. Tests of VAR Restrictions			
$\chi^2(8)$	130.94**	95.69**	62.01**	96.48**

Note: $y_t = \psi_{t-1}/P$, $\phi_t = \beta r_t - \Delta\psi'_t + \beta\rho$, and $\psi'_t = \psi_t/\bar{\psi}$. For unit root test, see note to Table 1. χ^2 statistics are Wald tests of restrictions on 4-period VAR of y'_t and ϕ_{t-1}.

Table 5. Autocorrelations of Excess Returns

Autocorrel.	Copper	Heating Oil	Lumber	Gold
ρ_1	.192	-.046	.090	.182
ρ_2	.037	-.055	-.019	-.155
ρ_3	.058	-.062	-.030	.021
ρ_4	.057	.019	.054	.197
ρ_5	.080	-.035	.176	.255
ρ_6	.053	-.044	.160	-.057
ρ_7	-.094	-.061	.053	.022
ρ_8	-.013	.088	.054	.146
ρ_9	.034	.062	.044	-.023
ρ_{10}	.128	.037	.180	.033
ρ_{11}	.141	-.053	.143	.147
ρ_{12}	.102	.071	.092	.064
ρ_{1-12}	.065	-.007	.083	.069
ρ_{13-24}	-.023	.011	.001	-.033
ρ_{25-36}	.022	.007	-.019	.005
ρ_{37-48}	-.012	.002	-.006	-.023
s.e.(ρ_j)	.067	.094	.066	.074
Q(12)	23.04 (P = .027)	4.49 (P = .973)	29.03 (P = .004)	37.16 (P < .001)
Q(24)	36.13 (P = .053)	11.11 (P = .988)	40.74 (P = .018)	59.89 (P < .001)
Q(48)	50.11 (P = .390)	33.71 (P = .941)	60.78 (P = .102)	98.94 (P < .001)

Note: Autocorrelations ρ_j are bias-corrected by adding 1/(T-j). ρ_{1-12} is the average of the first 12 autocorrelations, ρ_{13-24} is the average of the next 12, etc. Q(K) is the Box-Pierce Q statistic for the first K autocorrelations and P is the associated probability level.

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FIG. 1 – COPPER: PRICE AND PERCENTAGE NET BASIS

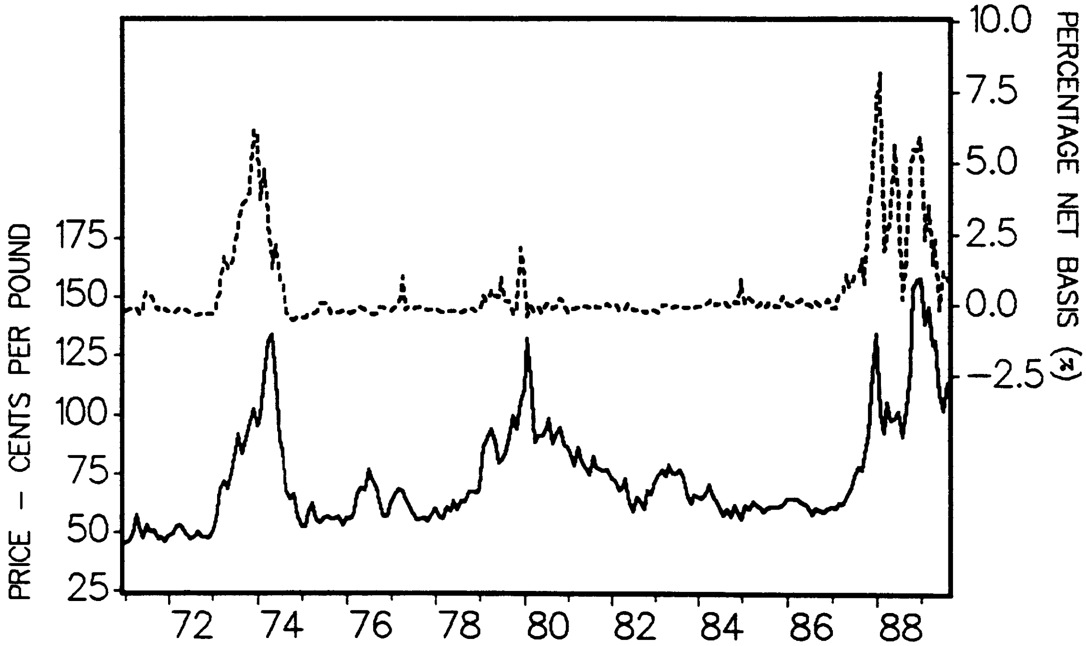

FIG. 2 – HEATING OIL: PRICE AND PERCENTAGE NET BASIS

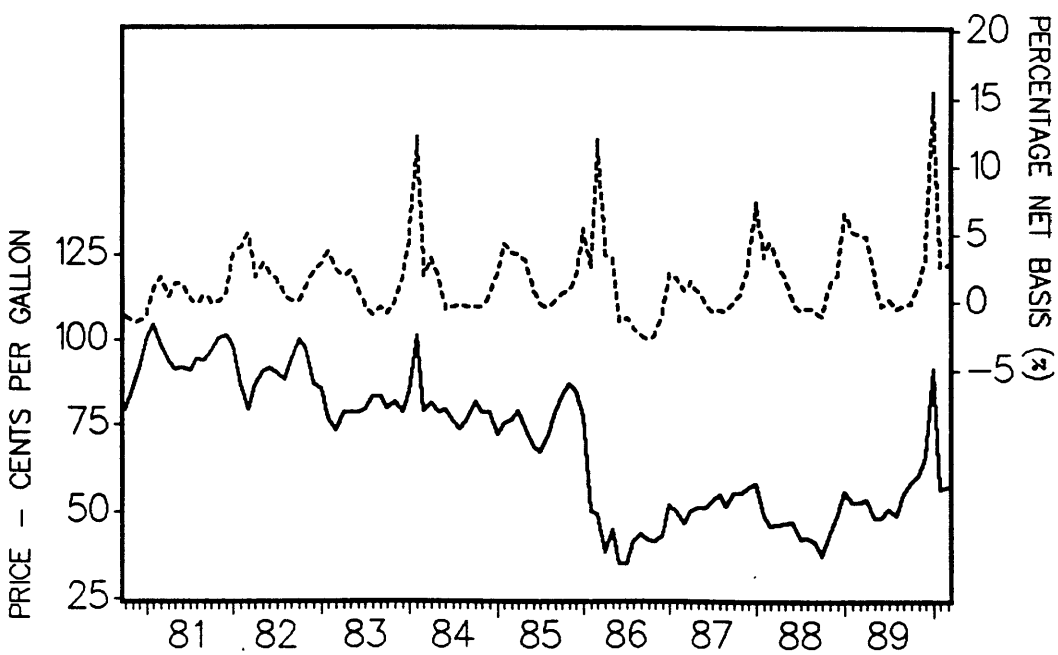

FIG. 3 — LUMBER: PRICE AND PERCENTAGE NET BASIS

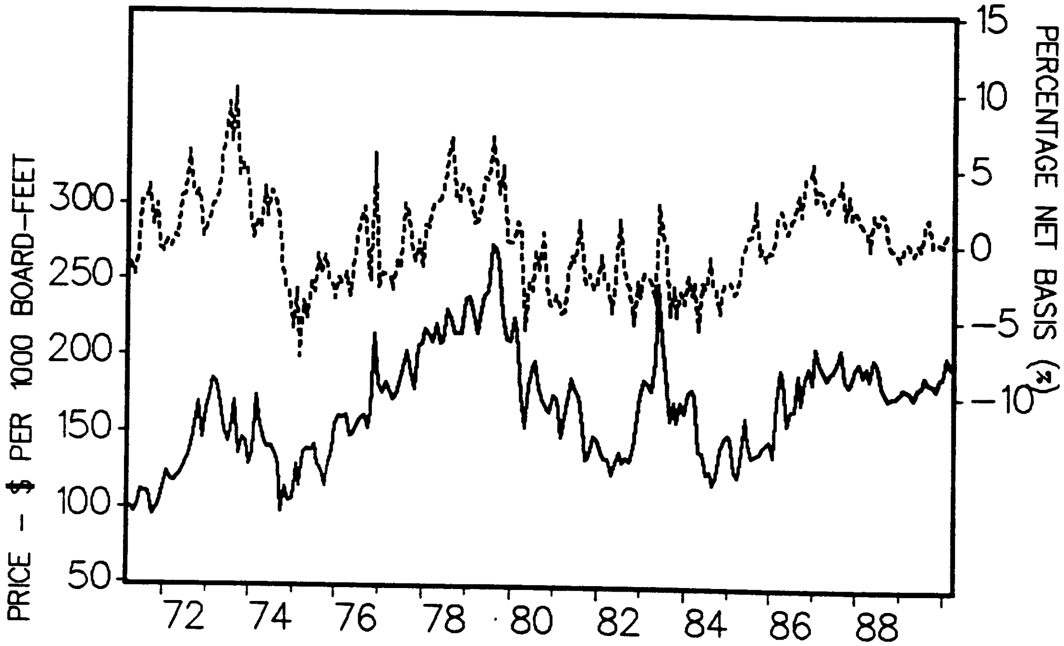

FIG. 4 — GOLD: PRICE AND PERCENTAGE NET BASIS