

A Note on Competitive Investment under Uncertainty

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Abstract

This paper clarifies how uncertainty affects irreversible investment in a competitive market equilibrium. With free entry, irreversibility affects the distribution of future prices, and thereby creates an opportunity cost of investing now rather than waiting. As with an imperfectly competitive firm, uncertainty can also increase the value of a marginal unit of capital. I show that with an infinite horizon, the opportunity cost is larger than this increase in value, so that uncertainty reduces investment.

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Uncertainty over future output prices or input costs can affect investment by a risk-neutral firm in two opposing ways. First, it can increase the value of the marginal unit of capital, which leads to more investment. This only requires that the stream of future profits generated by the marginal unit be a convex function of the stochastic variable; by Jensen's inequality, the expected present value of that stream is increased. This result was demonstrated by Hartman (1972), and later extended by Abel (1983) and others. In their models, convexity in output price and input costs is due to capital's substitutability with other factors. But even with fixed proportions, convexity is ensured by the ability of the firm to vary output, so that the marginal unit of capital need not be utilized.¹

If investment is irreversible and can be postponed, a second effect of uncertainty is to create an opportunity cost of investing now, rather than waiting for new information to arrive before committing resources. This increases the full cost of the marginal unit of capital, which reduces investment.² Hence the net effect of uncertainty on irreversible investment depends on the size of this opportunity cost relative to the increase in the value of the marginal unit of capital.

Caballero (1991) has recently argued that for a perfectly competitive firm with constant returns to scale, this opportunity cost is zero, so that uncertainty over future demand unambiguously increases current investment, even if that investment is irreversible. This is in contradiction to the negative relationship between uncertainty and irreversible investment found by Pindyck (1988) and Bertola (1989), and is significant because of possible

policy implications. Caballero suggests that such a negative relationship requires either decreasing returns to scale or imperfect competition.

Caballero's result is based on a model with convex adjustment costs. An interesting and innovative aspect of the model is that these costs can be asymmetric, thereby allowing for partial or complete irreversibility; complete irreversibility corresponds to a cost of downward adjustment that is infinite. The model is particularly useful in that it allows one to study the sensitivity of the investment-uncertainty relationship to the extent of asymmetry in adjustment costs.

An important aspect of Caballero's model, shared with Abel's (1983) and other models of this kind, is that the size of the firm would be unbounded were it not for adjustment costs. In fact it is only adjustment costs that determine firm size. As I show below, this role of adjustment costs is crucial to the results of Caballero and earlier authors regarding the effect of uncertainty on investment. While it is helpful for studying the behavior of a firm in isolation, this adjustment cost framework is inconsistent with a competitive market equilibrium, and hence with the behavior of a competitive firm. To study a competitive market equilibrium, one must make price and industry output endogenous, and doing so restores the positive opportunity cost associated with irreversible investment.

1. Adjustment Costs and Irreversibility.

A firm that has constant returns to scale everywhere and faces an infinitely elastic demand curve will have a profit function that is linear in the capital stock. Hence convex costs of some kind are needed to bound the size of the firm; otherwise the firm would expand indefinitely if its marginal profit exceeded the cost of a unit of capital. In Caballero's model, convex adjustment costs serve this role by making the cost of

investment an increasing function of the level of investment. But because these adjustment costs are a function of only the level of investment, investment in each period is independent of investment or the stock of capital in any other period -- there are no "intertemporal links."

This, however, necessarily eliminates irreversibility from the problem. Irreversibility matters when it causes decisions made now to constrain decisions in the future under some states of nature but not under others. For example, a firm that invests a large amount this period would not want to disinvest next period if demand expands, and so would not be constrained by irreversibility. This large investment would lead it to be constrained, however, if demand were to contract, because then it would want to disinvest. This is why irreversibility creates an opportunity cost, which leads the firm to invest somewhat less this period.

This can never arise when the size of the firm is constrained only by adjustment costs. Then investment next period depends only on the realization of demand that period and on the adjustment cost function; it is completely independent of investment this period. Hence the firm need only compare the marginal cost of investing to current and expected future marginal profits. Since uncertainty increases expected future marginal profits, it necessarily increases investment.

Convex adjustment costs may indeed affect the rate at which firms invest (although simple "time to build" and the lumpiness of investment are likely to be more important constraints). It seems unrealistic, however, to treat adjustment costs as the sole or main determinant of firm and industry size in equilibrium. In fact, a pure adjustment cost model is inconsistent with a competitive market equilibrium. In principle, free entry will ensure that a very large number of very small firms come into the industry. (Very

small firms would enter because they would have very small adjustment costs and hence lower total costs.) In the limit, the industry would be composed of an infinite number of infinitesimally small firms, and so each firm would have no adjustment costs.

Even if each firm is constrained to some minimum size, the possibility of entry by new firms or expansion of existing ones will ensure that investment decisions are intertemporally linked. As a result, uncertainty will affect irreversible investment by a competitive firm with constant returns to scale much as it would a noncompetitive firm, or a firm with decreasing returns. The reason is that in each period, if demand increases existing firms will expand or new firms will enter until the market clears. From the point of view of an individual firm, this limits the amount that price can rise under good demand outcomes. However, if investment is irreversible, there is no similar mechanism to prevent price from falling under bad demand outcomes. Each firm takes price as given, but it knows that the distribution of future prices is affected by the irreversibility of investment industry-wide. This reduces its own incentive to invest.

Most of the literature on irreversible investment has focused on the individual firm,³ as does Caballero in his model of asymmetric adjustment costs. But in a competitive equilibrium, uncertainty affects investment through the feedback of industry-wide capacity expansion and new entry on the distribution of prices.⁴ The following example illustrates this by extending Caballero's two-period model to allow for this feedback.

2. An Example.

As in Caballero's model, each firm is in place two periods, there is no depreciation or discounting, and the production function is Cobb-Douglas:

$$(1) \quad q_i = AL_i^\alpha K_i^{1-\alpha}, \quad 0 < \alpha < 1$$

where q_i is the output of firm i . Market demand is isoelastic:

$$(2) \quad P_t = Q_t^{-1/\epsilon} Z_t$$

where ϵ is the elasticity of demand, and Z_t is a stochastic process, with $Z_1 = 1$. For simplicity, we let Z_2 equal 0 or 2 with equal probability. We will compare this to the certainty case in which $Z_2 = 1$. Also, we will restrict the discussion to the case of complete irreversibility, with no cost of adjusting K_i upward. (In Caballero's notation, $\gamma_1 = 0$ and $\gamma_2 = \infty$.)

Let there be a large number, N , of equal size firms, so that each takes price as given, and $Q = Nq_i$. The profit function for each firm is then:

$$(3) \quad \Pi_i = hP^\eta K_i$$

where $h = (1-\alpha)A^{1/(1-\alpha)}(\alpha/w)^{\alpha/(1-\alpha)}$, and $\eta = 1/(1-\alpha) > 1$. Also, $q_i = BK_i$, where $B = h/(1-\alpha)$. With no loss of generality, we choose A so that $B = 1$. Note that the value of a marginal unit of capital is hP^η , whatever the firm or industry capital stock. This value is convex in P , so its expectation is increased by a mean-preserving spread in P . But as we will see, this need not mean that uncertainty leads the firm to invest more.

First, consider the certainty case. Here, $P_2 = P_1$, and all investment occurs in period 1. Each firm will want to invest an infinite amount if $2hP_1^\eta > k$, and nothing if $2hP_1^\eta < k$, where k is the cost of a unit of capital. Thus in equilibrium, firms invest until price falls to the point that $2hP_1^\eta = k$. Hence $P_1 = P_2 = (k/2h)^{1/\eta}$. Industry investment in the first period is $I_1 = K_1 = Q_1 = (2h/k)^{\epsilon/\eta}$. (Each firm's investment is just $1/N$ of this.)

Now suppose that Z_2 is unknown when firms invest in period 1; it can turn out to be 2 or 0, each with probability .5. Although Z_t is exogenous, P_t is determined as part of the market equilibrium. To find this equili-

brium, we want a distribution for P_t that results from Z_t and from firms' investment decisions, with those decisions based on this same distribution.⁵

We will surmise that equilibrium investment in period 1 is small enough so that in period 2, firms invest some positive amount if $Z_2 = 2$ (whereas they invest nothing if $Z_2 = 0$). After solving for I_1 we will check that this is indeed the case. Then if $Z_2 = 2$, firms will invest in period 2 to the point that the profit from a unit of capital equals its cost, i.e., until $hP_2^\eta = k$, or $P_2 = (k/h)^{1/\eta}$. This implies that K_2 will equal $(P_2/2)^{-\epsilon} = 2^\epsilon (h/k)^{\epsilon/\eta}$. Of course if $Z_2 = 0$, $P_2 = 0$, $I_2 = 0$, and $K_2 = K_1$.

Given this distribution for price in period 2, risk-neutral firms will invest in period 1 to the point that the expected value of a unit of capital equals its cost:

$$(4) \quad hP_1^\eta + E_1[hP_2^\eta] = k,$$

$$\text{or,} \quad hK_1^{-\eta/\epsilon} + .5k = k.$$

Hence $I_1 = K_1 = (2h/k)^{\epsilon/\eta}$. Finally, we check that I_2 is indeed positive if $Z_2 = 2$. If $Z_2 = 2$, $I_2 = K_2 - K_1 = (2^\epsilon - 2^{\epsilon/\eta})(h/k)^{\epsilon/\eta} > 0$, since $\eta > 1$.

In contrast to Caballero's result, we have found that period 1 investment is the same when Z_2 is uncertain as it is when $Z_2 = 1$ with certainty. The reason is that while a mean-preserving spread in the distribution of P_2 increases the value of a unit of capital, a mean-preserving spread in Z_2 reduces the expected value of P_2 . The equilibrium response of firms limits price increases under good outcomes of Z_2 , but because of irreversibility, it does not limit price decreases under bad outcomes. In this particular example these two effects just offset each other, so investment is left unchanged.⁶

The Appendix extends this example to n periods and allows Z_t to follow a random walk; it begins at 1 and increases or decreases by 100% in each

period. The Appendix shows that investment in period 1 is lower when Z_t is stochastic, as long as $n \geq 3$. Also, in any period, the difference between investment when future values of Z_t are known and investment when they are stochastic grows with the number of periods remaining. The reason is that the variance of future values of Z_t increases with the time horizon, but industry investment always limits price increases under good outcomes.⁷

3. Concluding Remarks.

The simple example presented above shows how the negative effect of uncertainty on irreversible investment remains even when the firm is perfectly competitive and has constant returns to scale. That effect is mediated by the equilibrium behavior of all firms, and the resulting impact on market price. In the two-period example above, that effect just offsets the increase in the value of a unit of capital that results from the convexity of the marginal profit function, so that period 1 investment is left unchanged by a mean-preserving spread in the demand shift variable. When the number of periods exceeds 2, period 1 investment is lower when future demand is uncertain.

In the example, we found the equilibrium distribution for price and the levels of investment in each period consistent with that distribution. Alternatively, we could have used the fact, demonstrated by Lucas and Prescott (1971), that the competitive equilibrium is the solution to the social planning problem. The social planner will use the downward sloping demand curve to calculate the optimal investment rule. Hence she will solve an optimal investment problem that is identical in structure to that of a monopolist with constant returns to scale. When investment is irreversible, that problem is the one treated by Pindyck (1988) and Bertola (1989), and their results will once again hold.

APPENDIX

This Appendix extends the two-period example to n periods, where $Z_1 = 1$, and then in each succeeding period, Z_t increases or decreases by 100%, with probability 1/2 for each. Thus $Z_2 = 0$ or 2. If $Z_2 = 0$, Z_t remains 0 for all future t , but if $Z_2 = 2$, $Z_3 = 0$ or 4, and so on. As before, there is no depreciation or discounting. Hence in the certainty case ($Z_t = 1$ always), firms invest in the period 1 to the point that

$$(A.1) \quad nhP_1^\eta = nh(K_1^{-1/\epsilon})^\eta = k$$

so $I_1 = K_1 = (nh/k)^{\epsilon/\eta}$, $I_t = 0$ for $t > 1$, and $P_1 = P_2 = \dots = (k/nh)^{1/\eta}$.

We will again find a solution for the stochastic case by surmising that investment is positive in a good state (i.e., when $Z_{t+1} > Z_t$), and then check that this is indeed the case. First, in period n , the good state is that in which $Z_n = 2^{n-1}$. In this state, firms invest until $P_n = (k/h)^{1/\eta}$. Thus $K_n = (P_n/Z_n)^{-\epsilon} = 2^{\epsilon(n-1)}(h/k)^{\epsilon/\eta}$, and $I_n = K_n - K_{n-1}$.

In period $n-1$, in the good state $Z_{n-1} = 2^{n-2}$, and firms invest to the point that $hP_{n-1}^\eta + E_{n-1}[hP_n^\eta] = k$, which implies that:

$$(A.2) \quad h(2^{n-2}K_{n-1}^{-1/\epsilon})^\eta + .5k = k$$

or,
$$K_{n-1} = [2^{\eta(n-2)+1}h/k]^{\epsilon/\eta}$$

Note that since $\eta > 1$, $K_n > K_{n-1}$ in a good state, as we surmised. In period $n-2$, in the good state firms invest to the point that $hP_{n-2}^\eta + E_{n-2}[hP_{n-1}^\eta + hP_n^\eta] = k$, so that $K_{n-2} = [2^{\eta(n-3)+1}h/k]^{\epsilon/\eta}$. In general, in a good state:

$$(A.3) \quad K_{n-m} = [2^{\eta(n-m-1)+1}h/k]^{\epsilon/\eta}$$

Finally, working back to period 1, $I_1 = K_1 = (2h/k)^{\epsilon/\eta}$. Note that this is smaller than the certainty case when $n > 2$.

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Footnotes

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1. Then the marginal profit of capital at each time t in the future is $\max[0, (p_t - c_t)]$, where c_t is variable cost. Thus a unit of capital represents a set of call options on future production, which are worth more the greater the variance of p_t and/or c_t .
 2. For a detailed discussion of this point and a survey of the recent literature on irreversibility and its implications for investment and market evolution, see Pindyck (1990).
 3. Exceptions are Dixit (1989, 1991), Leahy (1990), and Lippman and Rumelt (1985).
 4. This can also be the case under imperfect competition.
 5. This is easy to do for this simple example. Leahy (1990) solves the more general continuous-time problem.
 6. If $Z_2 = 1$ with certainty, $P_2 = (k/2h)^{1/\eta}$, but if $Z_2 = 0$ or 2 , $E_1(P_2) = \frac{1}{2}(k/h)^{1/\eta}$, which is smaller since $\eta > 1$. The expected marginal profit of capital is $k/2$ in both cases.
 7. Like our two-period example, the n -period example in the Appendix ignores depreciation and discounting. Hence if $Z_t = 1$ for all t , the value of a unit of capital grows linearly with n , but it grows less rapidly if Z_t is stochastic expected future prices are lower. Including depreciation and discounting would reduce the depressive effect of uncertainty on current investment.