

**Reserve Asset Values and the "Hotelling  
Valuation Principle"**

by

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"HOTELLING VALUATION PRINCIPLE"

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ABSTRACT

The Hotelling Valuation Principle, that the in-situ value of a mineral unit equals the current net price, is a special case of a more general relation. Tested against a set of recent Canadian sales of oil and gas reserves, the HVP is strongly rejected. The method permits also a demonstration that price expectations were quite different in oil and in gas, confirming industry opinion.

Introduction

An economic classic [Hotelling 1931] was restated by Das Gupta and Heal [1979, p. 158]: the net price of a mineral must rise at the rate of discount. But econometric analysis failed to verify this, according to Miller and Upton [1985a]. Hence they bypassed the rate of discount to restate the rule as the Hotelling Valuation Principle (HVP): the market value of the asset mineral in the ground is equal to its current net price. They compared a sample of oil companies' estimated oil reserve values directly with net prices, and concluded that they were indeed about equal. But estimates they made later [1985b] showed the in-situ value as only half the price; they ascribed this to unsatisfactory data.

Later papers by Adelman [1990] and Watkins [1992] rejected the HVP for reasons

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both of theory and data. The issue is important because long-run changes in the prices of minerals are important to the world economy and to many governments. For example, Boskin et al [1985] estimated the value of the U.S. government's mineral holdings at nearly 20 times what they would be worth if the 1989 price were assumed constant, and incomes discounted at the conventional industry rate. [Adelman 1990]

In this note, we analyze a new data set of recent oil and gas prices and reserve values. We show the Hotelling Valuation Principle to be a special case, which can be tested for, of a more general formula. The estimated in-situ values then give useful insight into expected oil and gas price changes.

### I. The Data

The data consist of the amounts paid for 27 developed oil and gas reserves in Alberta over the period February 1989 to March 1991. (Table 1) All but two purchases were of both oil and gas reserves.

Miller and Upton inferred in situ reserve values from stock market values, after adjusting for liabilities and the value of non-reserve properties. They would avowedly have preferred to use the kind of data we employ here.<sup>2</sup> The transaction values relate solely to the reserves and thus are uncontaminated by other property values.

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<sup>2</sup> See Miller and Upton [1985a], pp. 3 and 15.

Table 1

Canadian Oil and Gas Reserves Transactions  
February 1989 - March 1991

	<u>Transaction Value</u> (MM\$) (1)	<u>Reserves of Oil</u> (MMBBL) (2)	<u>Reserves of Gas</u> (BCF) (3)
1	430	38.0	430
2	70	10.8	114
3	235	20.9	407
4	162	14.0	130
5	105	4.1	144
6	126	12.5	100
7	7	-	10
8	27	3.9	62
9	231	9.9	355
10	55	16.5	2
11	47	5.0	57
12	37	4.0	45
13	57	7.0	65
14	26	3.5	24
15	107	15.0	37
16	112	3.7	253
17	40	8.3	9
18	55	13.5	1
19	275	8.7	200
20	30	1.4	53
21	28	3.8	42
22	80	10.4	95
23	100	5.0	250
24	56	10.0	1
25	109	6.9	179
26	6	2.1	-
27	6	0.9	12

Note: all values are in Canadian dollars. Transactions listed in time sequence.

Source: Columns (1), (2) and (3) - Coles Gilbert and Associates, Ltd.

## II. ANALYSIS OF THE RESERVE TRANSACTION DATA

The data in Table 1 immediately suggest regressing the transaction values on the respective volumes of oil and gas reserves to estimate unit reserve prices. What is the appropriate specification of the function to be estimated?

Conventional cash-flow capital budgeting methods for valuing mineral properties suggest a valuation expression ( $V_T$ ) of the form:

$$V_T = \int_{t=0}^{n_o} (p_{ot} - c_{ot})f_{ot}e^{-it}dt + \int_{t=0}^{n_g} (p_{gt} - c_{gt})f_{gt}e^{-it}dt \quad (1)$$

where

$p_{ot}, c_{ot}$  are prices and costs of oil, respectively, time  $t$

$p_{gt}, c_{gt}$  are prices and costs of gas, respectively, time  $t$

$f_{ot}, f_{gt}$  are quantities of oil and gas production, respectively, time  $t$

$i$  = discount rate

$t$  = (continuous) time; transaction at  $t=1$

$n_o$  = remaining reservoir life for oil,

$n_g$  = remaining reservoir life for gas.

Costs represent all costs, i.e. extraction, future development, if any, income taxes, royalties, etc. If future development costs were anticipated, unit costs could be quite lumpy. All RHS variables are of course expected values.

The linear separation inherent in (1) implies no interaction between oil and gas production and costs. It would hold strictly if the transaction related to an oil reservoir and a non-associated gas reservoir.

However, it is more likely that the transactions relate to oil reservoirs with some associated gas, or to natural gas reserves containing some liquids. If so, there could be some interaction between oil and gas production, individual costs and joint costs. This

would not matter if such relations were linear, as they would be if production gas-oil ratios (GORs) were constant. But if there were strong non-linear relations between, say, associated gas production and oil production, then a 'reduced form' of expression (1) might thwart a linear regression of asset values on reserve volumes and prevent interpretation of the reserve coefficients in a straightforward way.

However, it is doubtful that any non-linearities among the oil and gas production and cost data would be sufficient to violate the essentially additive nature of the conventional reserves evaluation procedure: estimate the net present value of expected flows of oil production plus the net present value of expected natural gas production from each property under review.

Thus the basic regression expression can be written as:

$$V_T = a_0 + b_1 R_o + b_2 R_G \quad (2)$$

where

$$R_o = \int_0^{n_o} f_{ot} dt \quad \text{and} \quad R_G = \int_0^{n_g} f_{gt} dt$$

If reserves are zero,  $V_T$  is zero, and the intercept term should be constrained as zero.

Then the regression equation becomes

$$V_T = b_1 R_o + b_2 R_G \quad (3)$$

Note that expression (2) can still be useful since the statistical significance of the intercept can indicate the degree of 'cleanliness' of the data.

### III. REGRESSION RESULTS

When the constant term was suppressed we obtained:

$$V_T = 5.2 R_0 + 0.46 R_G \quad (4)$$

(5.6)      (7.1)

$\bar{R}^2 = 0.86$ ,  $DW = 2.09$  (t values in parenthesis).

The degree of (linear) fit is reasonable, both coefficients are highly significant and there is no evidence of autocorrelation. The Goldfeld-Quandt test (and visual inspection of the data) suggested no heteroscedasticity.

Imposition of the intercept yielded:

$$V_T = -5.6 + 5.5 R_0 + 0.47 R_G \quad (5)$$

(-0.5) (5.0)      (7.0)

$\bar{R}^2 = 0.86$ ,  $DW = 2.06$  (t values in parenthesis).

The intercept term is insignificant, and the statistical properties of expression (5) are nearly identical to (4), which we employ hereafter.

### IV. INTERPRETATION OF REGRESSION COEFFICIENTS

A few simplifying and not distortive assumptions provide an interpretation of what the reserve coefficients would represent, viewed as a 'reduced form.'

Suppose expected prices and unit costs were constant, production were set by a fixed production/reserves ratio (thus defining the decline rate), and the reserve life were infinite.

Then expression (1) becomes

$$V_T = (P_0 - c_0) \frac{a_0}{a_0+i} R_0 = (P_g - c_g) \frac{a_g}{a_g+i} R_g \quad (6)$$

where  $a_o$ ,  $a_g$  are the production/reserves ratio respectively for oil and gas (see Adelman [1990, p.6]).

Reverting now to (3), the implication of (6) is that the coefficient  $b_1$  represents  $(p_0 - c_0) \frac{a_o}{a_o + i}$  and the coefficient  $b_2$  represents

$$(P_0 - c_0) \frac{a_g}{a_g + i}$$

The coefficients  $b_1$  and  $b_2$  are estimated reserve values for oil and gas, predicated on assumed values for  $a$ ,  $i$  and  $c_o$  and  $c_g$ . Other combinations could also be tried. For example, values could be assumed for  $p_o$ ,  $a$  and  $i$  to get at the implicit  $c_o$ , and so on. By comparing estimated values with the known data set down in Table 2, we have a check on the structural relations.

Table 2  
Reference Parameters

	<u>Oil</u>	<u>Gas</u>
Wellhead Price (p)	CDN \$21.86/bbl	CDN \$1.45/Mcf
Unit costs (c)	CDN \$ 8.00/bbl	CDN \$0.65/Mcf
P/R ratio (a)	0.1	0.067
Discount rate (i)	15%	15%

The reference oil price (in Canadian dollars) is the average of Alberta wellhead prices for February 1989 to March 1991 (the period of the observed transactions), excluding the months August 1990 to January 1991 when the Gulf crisis noticeably

influenced the market.<sup>3</sup> The reference gas price is the average wellhead price for long-term domestic sales by Western Gas Marketing Limited (WGML), Canada's largest marketer of natural gas, for the months November 1989 to March 1991. Data prior to November 1989 concerning these sales were not publicly available. Further, until November 1990 long-term industrial sales were included in the calculation of the average price. Since then, the price data reflect only WGML sales to Canadian LDC 'core' markets.<sup>4</sup>

Unit costs for both oil and gas include (average) operating costs, royalties and taxes.<sup>5</sup> The P/R ratio for oil is based on the most recent data from Alberta's Energy Resources Conservation Board,<sup>6</sup> that for gas is predicated on the long-term WGML contracts which run from 1988 to 2003. The discount rate of 15 percent is the nominal after-tax rate suggested by previous work on hurdle rates.<sup>7</sup>

We deal first with oil. Each of the four variables embedded in the coefficient attached to oil reserves of 5.2 can be treated in turn as an unknown. The solutions are:

$$P_0 = \$21.00, c_0 = \$8.86, a_0 = 0.09, \text{ and } i = 17\%$$

Since these derived values are all close to the underlying (average) data (see Table 2), the interpretation of the oil coefficient is straightforward. It suggests oil price

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<sup>3</sup> Shell Canada, Crude Oil Price Bulletin, various issues.

<sup>4</sup> Source: Canadian Gas Price Reporter, various issues.

<sup>5</sup> Sources: DataMetrics Limited [1984], DataMetrics Limited [1987], and Petroleum Monitoring Agency Canada, 1990 Monitoring Report, First Six Months.

<sup>6</sup> Energy Resources Conservation Board [1990].

<sup>7</sup> Source: DataMetrics Limited and Coles Gilbert Associates [1989].

expectations underlying the various purchases of oil reserves were flat.

The 'Hotelling Valuation Principle' suggests the oil reserve coefficient would be \$13.86 (the net price), about nine standard deviations away from the regression coefficient.

The case of natural gas is similar, up to a point. The HVP value is \$0.80/Mcf, compared with the \$0.46/Mcf regression value, (about four standard deviations away from it). Again the HVP is strongly rejected.

But when price is treated as the unknown, the implicit value is \$2.14/Mcf, in contrast to the contemporaneous average of \$1.45/Mcf. Solving in turn for the other unknowns yielded values for  $c_g$  of \$-0.04,  $a_g$  of 0.203 and  $i$  of 5 percent. They deviate very considerably from the reference data.

These results suggest that the gas price expectations underlying the valuation of natural gas reserves were considerably above prevailing short-term prices. Indeed, the implicit prices seemingly underlying the transaction values even exceed those for the so-called 'core' market prices under WGML aggregator contracts for Canadian gas buyers east of Alberta. The implication is that buyers of gas reserves were anticipating future price appreciation. This accords with our knowledge of market expectations, at that time.<sup>8</sup>

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<sup>8</sup> For example, see National Energy Board [1988]; in the United States, "the industry had expected a dramatic rise in prices" for 3-5 years. (see Oil and Gas Journal, December 2, 1991, p. 71)

We can test this suggestion, by asking what the results imply about an expected growth rate ( $g$ ) in prices. Adapting expression [6] above, and Adelman [1990, p. 8], we have:

$$b = \frac{a(p-c)}{a+i-g}$$

$$g = i + a \left(1 - \frac{(p-c)}{b}\right) \quad (7)$$

Our results for oil above were:  $(p_0 - c_0) = 2.7b_0$ ; for gas,  $(p_g - c_g) = 1.7b_g$

For further calculation, we try to measure  $a$  more precisely. In general, reserves are cumulative expected output, declining at a constant exponential rate:

$$R = P \int_{t=0}^n e^{-aT} dt = P/a (1 - e^{-aT}) \quad (8)$$

As  $T$  becomes large, convergence is rapid to the limiting value,  $a = P/R$ . But for shorter lives, we need a correction. Since terminal output  $P_f = Pe^{-aT}$ , it follows that:

$$a = P/R (1 - (P_f/P)) \quad (9)$$

We approximate  $P_f/P$  by  $P/R$ , because the higher is the initial rate of output relative to reserves, the higher the level of operating expenditures, hence the higher the abandonment output level. Then  $a = (P/R) - (P/R)^2$ . We adjust from Table 2:

$$a_0 = .1 - .01 = .09 \quad a_g = .067 - .0045 = .0625$$

The implicit expected growth rate for natural gas ( $g_g$ ) and for oil ( $g_0$ ) are

$$g_g = .15 + .0625(1-1.7) = .106$$

$$g_0 = .15 + .09(1-2.7) = -.003$$

We caution that derivation of the  $g$ 's in this way impounds errors in any of the five contributing parameters. However, the contrast between the oil and gas expected growth rates does certainly agree with what we know of representative industry forecasts.

Expression (7) takes the Hotelling Valuation Principle as a special case, for which one can test. If the net price ( $p-c$ ) is equal to the in situ value 'b,' then 'a' becomes irrelevant,  $g=i$ , and the net price grows at the rate of discount. But 'a' has been shown as positively correlated with value. [Adelman 1990] Moreover, when  $(p-c) = 2b$ , as has been approximately true for many years in the USA, (see Adelman, et al. [1991]), then  $g = i - a$ . Since 'i' and 'a' were historically near equality, then  $g$  would approximate zero. If so, it was a good forecast.

## V. CONCLUDING REMARKS

We have examined some pristine transaction data on the sale of developed oil and gas reserves in Alberta over a recent two-year period (February 1989 to March 1991). The Hotelling Valuation Principle (HVP) suggests that the unit values of reserves in the ground will be in the neighborhood of the prevailing net price.

Our estimated reserve values cannot be reconciled with the HVP, since they are considerably below the net wellhead prices. The analysis also suggests that transactors expected little change in oil prices but sustained increases in natural gas prices. It would be useful to see whether natural gas price expectations changed greatly in late 1991.

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