Options, the Value of Capital, and Investment

by

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MIT-CEEPR 95-006   June 1995

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Abstract

Capital investment decisions must recognize the limitations on the firm's ability to later sell off or expand capacity. This paper shows how opportunities for future expansion or contraction can be valued as options, how this valuation relates to the q-theory of investment, and how these options affect the incentive to invest. Generally, the option to expand reduces the incentive to invest, while the option to disinvest raises it. We show how these options interact to determine the effect of uncertainty on investment, how these option values change in response to shifts of the distribution of future profitability, and how the q-theory and option pricing approaches to investment are related.

Introduction

When a firm cannot costlessly adjust its capital stock, it must consider future opportunities and costs when making its investment decisions. The literature has interpreted this investment problem in two ways. In the q-theory approach, the firm faces convex costs of adjustment and, along its optimal path, equates the marginal valuation of a unit of capital, measured by q, with the marginal cost of investment.[1] In the irreversible investment literature, which uses option pricing techniques to derive and characterize optimal investment behavior, the firm must consider future opportunities and costs because capital expenditures are at least partly sunk.[2]

This paper links the q-theory and option pricing approaches in a simple model that accounts more generally for the constraints on investment that firms often face. The model reinforces the idea that investment decisions involve the acquisition or exercise of options, and extends it by showing that we must account for a broader set of options. It also shows that options need not always serve to delay investment.

In our model, the firm can disinvest, but the resale price of capital may be less than its current acquisition price, making reversibility costly. Similarly, the firm can continue to invest later, but the future acquisition price of capital may be higher than its current acquisition price, making expandability costly. When future returns are uncertain, these features yield two options. When a firm installs capital which it may later resell (even at a loss), it acquires a put option; if the firm can purchase capital later (even at a price higher than the current price), it has a call option. These two options affect the current incentive to invest. We examine these features of investment and interpret them in two ways: using

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[1]Mussa (1977) demonstrates this result in a deterministic setting and Abel (1983) in a stochastic model.
[2]This literature began with Arrow (1968), and the option interpretation has been emphasized by Bertola (1988), Pindyck (1988, 1991) and Dixit (1991, 1992); see Dixit and Pindyck (1994) for a survey and systematic exposition.

q-theory, where q summarizes the incentive to invest, and using option pricing theory, where each of the options is examined separately.[3]

Besides clarifying the relationship between q-theory and the option pricing approach, we extend the latter by accounting for a richer set of options than found in the existing literature. That literature (see Dixit and Pindyck, 1994) emphasizes the interaction of: (i) uncertainty over future returns to capital, (ii) irreversibility, and (iii) the opportunity to delay the investment. The opportunity to delay gives the firm a call option, whereas complete irreversibility rules out the put option that would arise if the firm could disinvest. In contrast, our model accommodates an arbitrary degree of reversibility, so that in general the firm has a put option to sell capital. Our model also allows for an arbitrary degree of expandability, and we examine the value and characteristics of the call option that this generates.[4]

The irreversible investment literature has typically assumed that the only cost of waiting is the foregone flow of profits. But waiting has an additional cost if the price of capital is expected to increase. Then expandability becomes more costly, reducing the value of the call option on future acquisitions of capital, and increasing the current incentive to invest. Likewise, reversibility is costly when the resale price of capital is less than its purchase price. This reduces the value of the put option associated with selling capital, and thereby reduces the incentive to invest. The net effect of reversibility and expandability on investment depends on the values of these two options.

Irreversibility may be important in practice because of "lemons effects", and because of capital specificity. Even if a firm can resell its capital to other firms, potential

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[3] Abel and Eberly (1994) combine irreversibility and convex adjustment costs and use a q-theoretic model to analyze optimal investment under uncertainty, but do not explore the options associated with reversibility and expandability described here. Abel and Eberly (1995) examine a firm's optimal investment decision when the acquisition price of capital exceeds its resale price. They value the options to purchase and sell capital in an infinite horizon setting using particular forms for the profit function and uncertainty facing the firm.

[4] In models with partial irreversibility, Bertola (1988), Dixit (1989a and 1989b) and Bertola and Bentolila (1990) show that the value function contains both a put and a call option. While allowing for varying degrees of reversibility, these papers assume complete expandability.

buyers may be subject to the same market conditions that induced the firm to want to sell in the first place. (A steel manufacturer will want to sell a steel plant when the steel market is depressed, but that is precisely when no one else will want to pay a price anywhere near its replacement cost.) In this case, even if capital is not firm-specific, the combination of industry-specific capital and industry-specific shocks results in at least partially irreversible investment.[5]

In many industries, the ability to expand capacity is also limited, e.g., because of limited land, natural resource reserves, the need for a permit or license that is in short supply, or the prospect of entry by rivals.[6] Hence, one of our goals is to clarify the implications of both limited reversibility and limited expandability.

In addition, we use both q-theory and the option pricing approach to examine the effects of changes in the probability distribution of future returns. These two approaches necessarily yield identical results, but they provide distinct insights into the optimal investment decision. For example, we show that an increase in the variance of future returns has an ambiguous effect on the incentive to invest, because greater uncertainty increases the value of the put option, which increases the incentive to invest, and increases the value of the call option, which decreases the incentive to invest. We also show that changing the probabilities within the set of "good states" (when the firm invests) or within the set of "bad states" (when the firm disinvests) does not affect the incentive to invest. This generalizes Bernanke's (1983) "bad news principle" of irreversible investment to what can be thought of as a "Goldilocks principle": like porridge, the only news of interest is that which is neither "too hot" nor "too cold."

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[5] More precisely, if shocks occur at a level of aggregation at least as high as the specificity of capital, then investment is at least partially irreversible. For example, if steel demand fluctuates stochastically, investments in a steel mill will be irreversible, but the steel company's investment in office furniture, which could be used in other industries, is not irreversible.

[6] We offer a simple model where all these considerations are reflected in a higher cost of future expansion. A more complete treatment will endogenize each specific consideration. See Leahy (1993) and Dixit and Pindyck (1994, chapter 8) for the case of a perfectly competitive industry, Smets (1995), Baldursson (1995) and Dixit and Pindyck (1994, chapter 9) for the case of an oligopolistic industry, and Bartolini (1993) for the case of an industry-wide capacity constraint.

We develop a two-period model with costly reversibility and expandability in Section I. The optimal value of the first-period capital stock is derived and interpreted using the q-theory and option pricing approaches, thereby illustrating the equivalence of the two approaches as well as the effects of limited reversibility and expandability. Section II extends the option approach using the "option pricing multiple" emphasized in the irreversible investment literature, and a graphical representation of the options associated with reversibility and expandability is developed in Section III. Section IV examines the effects of shifts in the distribution of returns on the incentive to invest. Section V summarizes our results.

I. Optimal Investment, Reversibility, and Expandability

This section demonstrates the distinct roles played by reversibility and expandability in a dynamic model of optimal investment under uncertainty. We use a simple, two-period framework that incorporates only the necessary features: second-period returns are stochastic (uncertainty), the future purchase price of capital may exceed its current value (costly expandability), and the future resale price of capital may be less than its current value (costly reversibility). First we solve for the optimal first-period capital stock and then we use q-theory to demonstrate the effects of expandability and reversibility. We then show that an option pricing approach yields identical analytical results, but gives new insights into the options generated by expandability and reversibility.

In the first period, the firm installs capital, K_1, at unit cost b and receives total return $r(K_1)$, where $r(K_1)$ is strictly increasing and strictly concave in K_1. In the second period, the return to capital is given by $R(K,e)$, where e is stochastic. Let $R_K(K_1,e) \geq 0$ be continuous and strictly decreasing in K and continuous and strictly increasing in e. Define two critical values of e by

$$R_K(K_1,e_L) = b_L \text{, and } R_K(K_1,e_H) = b_H \tag{1}$$

where $b_L \leq b_H$ denote the resale and purchase prices of capital in the second period, respectively. When $b_L < b$, the resale price of capital is less than its current (period one) price, and we have costly reversibility of investment. Similarly, when $b_H > b$, the second-period purchase price of capital exceeds its current (period one) price, and we have costly expandability of the capital stock.

In the second period, after e becomes known, the capital stock will be adjusted to a new optimal level, which we write as $K_2(e)$. When $e > e_H$, it is optimal to purchase capital to the point where the marginal revenue product of capital equals the new higher purchase price, so $K_2(e)$ is given by $R_K(K_2(e),e) = b_H$. When $e < e_L$, it is optimal to sell capital to the point where the marginal revenue product of capital equals the resale price, so $K_2(e)$ is given by $R_K(K_2(e),e) = b_L$. When $e_L \leq e \leq e_H$, it is optimal to neither purchase nor sell capital, so $K_2(e) = K_1$.

Let $V(K_1)$ denote the expected present value of net cash flow accruing to the firm with capital stock K_1 in period 1, i.e., the value of the firm:

$$\begin{aligned} V(K_1) = r(K_1) + \gamma \int_{-\infty}^{e_L} \left\{ R(K_2(e),e) + b_L[K_1 - K_2(e)] \right\} dF(e) \\ + \gamma \int_{e_L}^{e_H} R(K_1,e) dF(e) + \gamma \int_{e_H}^{\infty} \left\{ R(K_2(e),e) - b_H[K_2(e) - K_1] \right\} dF(e), \end{aligned} \tag{2}$$

where the discount factor γ is positive.[7] The value of the firm is the sum of first- and second-period returns, where second-period returns are calculated in each of three regimes, since e may be less than e_L, between e_L and e_H, or greater than e_H. When $e < e_L$, it is optimal to sell capital so that $K_2(e) < K_1$, and the firm's cash flow consists of the return $R(K_2(e),e)$ plus the proceeds from selling capital, $b_L[K_1 - K_2(e)]$. When e is between e_L and e_H, it is optimal to neither purchase nor sell capital so that $K_2(e) = K_1$, and

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[7]The second period could in principle be much longer than the first period, so the discount factor γ could exceed 1.

the firm's cash flow is simply $R(K_1, e)$. When $e > e_H$, it is optimal to purchase capital so

that $K_2(e) > K_1$, and the firm's cash flow consists of the return $R(K_2(e), e)$ minus the cost of

purchasing capital, $b_H[K_2(e) - K_1]$.

The period-1 decision problem of the firm is

$$\max_{K_1} \ V(K_1) - bK_1. \tag{3}$$

The first-order condition for this maximization is

$$V'(K_1) \equiv r'(K_1) + \gamma b_L F(e_L) + \gamma \int_{e_L}^{e_H} R_K(K_1, e) dF(e) + \gamma b_H[1 - F(e_H)] = b. \tag{4}$$

We examine and interpret this condition in two equivalent ways that provide different

insights.

A q-theory approach

The marginal valuation or the shadow value of capital in period 1, $V'(K_1)$, is related

to Tobin's q,[8] so we use the notation $q(K_1)$ to denote this marginal valuation. Thus,

equation (4) says that the optimal choice of capital in period 1 should equate $q(K_1)$ to the

price of that capital, b.

Equation (4) expresses $q \equiv V'(K_1)$ as the sum of the current marginal revenue

product of capital, $r'(K_1)$, and the expected present value of the marginal revenue product

of capital in the second period, $\gamma \int_{-\infty}^{\infty} R_K(K_2(e), e) dF(e)$, where the second-period marginal

revenue product of capital is evaluated at the optimal level of capital in the second period.

The second-period marginal revenue product of capital is illustrated in Figure 1. The

lower flat segment of the solid line shows that for values of e less than e_L, the firm sells

capital in period 2 until the marginal revenue product of capital equals b_L, the price the

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[8]Tobin defined q as $V(K_1)/(bK_1)$. This ratio is sometimes known as "average q" to distinguish it from "marginal q", which is $V'(K_1)/b$.

firm receives from selling capital in period 2; the probability that e is less than e_L is $F(e_L)$. The upper flat segment illustrates that for values of e greater than e_H, the firm buys capital until the marginal revenue product of capital equals b_H; the probability that e is greater than e_H is $1 - F(e_H)$. For values of e between e_L and e_H, the firm neither buys nor sells capital in the second period, so that the second-period capital stock equals K_1, and the marginal revenue product of capital is $R_K(K_1,e)$.

Notice that

$$\frac{\partial q}{\partial K_1} = r''(K_1) + \gamma \int_{e_L}^{e_H} R_{KK}(K_1,e) dF(e) < 0 \tag{5}$$

Therefore, for any given b, there is a unique value of K_1 that equates $q(K_1)$ and b.

Our expression for $q \equiv V'(K_1)$ in equation (4) allows us to easily determine the effects of changes in b_L and b_H on the incentive to invest in the first period and thus on the optimal value of K_1. Partially differentiating q with respect to b_L and b_H, respectively, we obtain

$$\frac{\partial q}{\partial b_L} = \gamma F(e_L) > 0 \tag{6}$$

and

$$\frac{\partial q}{\partial b_H} = \gamma \left[1 - F(e_H) \right] > 0 \tag{7}$$

Notice that q (and hence the optimal value of K_1) is an increasing function of both the future resale price of capital b_L and the future purchase price of capital b_H. An increase in the future resale price of capital b_L raises the floor below the second-period marginal revenue product of capital (corresponding to the lower flat segment) which increases the expected present value of marginal revenue products. An increase in the future purchase price of capital b_H increases the ceiling on the future marginal revenue product of capital (corresponding to the higher flat segment) and thus increases the

expected present value of marginal revenue products of capital. Thus, increased reversibility (higher b_L) or reduced expandability (higher b_H) increases q, the incentive to invest, and optimal investment.

Equation (4) can be interpreted as the Net Present Value (NPV) rule. The expression for $q \equiv V'(K_1)$ in the equation is the NPV of the marginal revenue product of capital from period 1 onward, accounting for the fact that in period 2 the stock of capital will change, and therefore the marginal revenue product of capital will also change along the optimal path. Although this statement of the NPV rule is theoretically correct, it is very complex to implement in practice. For a manager contemplating adding a unit of capital, it requires rational expectations of the path of the firm's marginal revenue product of capital through the indefinite future. Similar difficulties confront an economist looking to calculate q for a firm or an aggregate of firms.[9] Therefore, practical investment analysis as well as empirical economic research usually works with some proxy for the correct NPV. The one most commonly used treats the marginal unit of capital installed in period 1 as if the capital stock is not going to change again, and calculates the marginal valuation as

$$N(K_1) \equiv r'(K_1) + \gamma \int_{-\infty}^{\infty} R_K(K_1, e) dF(e) \tag{8}$$

In contrast to the correct NPV rule given above, this one can be called the "naive NPV rule", although it is the one commonly used in practice. The relation between these two, and therefore a way of correcting the naive calculation and making it conform to the optimality condition (4), can be seen by using the option approach to investment.

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[9]Under certain conditions, one can correctly measure q using securities market data; see Hayashi (1982).

An Option Value Approach

The difference between the correctly calculated period-1 marginal valuation $q(K_1)$ and the commonly-used but naive $N(K_1)$ consists precisely of the marginal call and put options, which arise because of the partial expandability and reversibility of capital in period 2. To illustrate this, we first rewrite equation (2) as

$$V(K_1) = r(K_1) + \gamma \int_{-\infty}^{\infty} R(K_1, e) dF(e)$$
$$+ \gamma \int_{-\infty}^{e_L} \left\{ \left[R(K_2(e), e) - b_L K_2(e) \right] - \left[R(K_1, e) - b_L K_1 \right] \right\} dF(e) \qquad (9)$$
$$+ \gamma \int_{e_H}^{\infty} \left\{ \left[R(K_2(e), e) - b_H K_2(e) \right] - \left[R(K_1, e) - b_H K_1 \right] \right\} dF(e)$$

which we decompose according to

$$V(K_1) = G(K_1) + \gamma P(K_1) - \gamma C(K_1) \qquad (10a)$$

where

$$G(K_1) \equiv r(K_1) + \gamma \int_{-\infty}^{\infty} R(K_1, e) dF(e) \qquad (10b)$$

$$P(K_1) \equiv \int_{-\infty}^{e_L} \left\{ \left[R(K_2(e), e) - b_L K_2(e) \right] - \left[R(K_1, e) - b_L K_1 \right] \right\} dF(e) \qquad (10c)$$

$$C(K_1) \equiv \int_{e_H}^{\infty} \left\{ -\left[R(K_2(e), e) - b_H K_2(e) \right] + \left[R(K_1, e) - b_H K_1 \right] \right\} dF(e) \qquad (10d)$$

The term $G(K_1)$ is the expected present value of revenue in periods 1 and 2 taking the second-period capital stock as given and equal to K_1; that is, it is calculated under the assumption that the firm can neither purchase nor sell capital in period 2, so that K_2 must equal K_1. The term $P(K_1)$ is the value of the put option, i.e., the option to sell capital in period 2 at a price of b_L, which the firm will choose to exercise if $e < e_L$. The term $C(K_1)$ is the value of the call option, i.e., the option to buy capital in period 2 at a price of b_H, which the firm will choose to exercise if $e > e_H$.

The optimal amount of capital in period 1 depends on a comparison of the marginal costs and marginal benefits associated with investment. Recalling that $q(K_1)$ is

the marginal valuation of capital, $V'(K_1)$, which summarizes the incentive to invest, and differentiating equation (10) with respect to K_1 we obtain

$$q = N(K_1) + \gamma P'(K_1) - \gamma C'(K_1) \tag{11a}$$

where

$$N(K_1) \equiv G'(K_1) = r'(K_1) + \gamma \int_{-\infty}^{\infty} R_K(K_1, e) dF(e) > 0 \tag{11b}$$

$$P'(K_1) \equiv \int_{-\infty}^{e_L} [b_L - R_K(K_1, e)] dF(e) > 0 \tag{11c}$$

$$C'(K_1) \equiv \int_{e_H}^{\infty} [R_K(K_1, e) - b_H] dF(e) > 0 \tag{11d}$$

Equation (11a) separates q into three components: (1) the expected present value of marginal revenue products of capital evaluated at the given capital stock K_1; (2) the marginal put option $P'(K_1)$, which equals $E\{\max[b_L - R_K(K_1, e), 0]\}$; and (3) the (negative of the) marginal call option $C'(K_1)$, which equals $E\{\max[R_K(K_1, e) - b_H, 0]\}$.[10]

The optimality condition for first-period capital is still $q(K_1) = b$, which can be rewritten as

$$N(K_1) = b - \gamma P'(K_1) + \gamma C'(K_1) \tag{12}$$

Recall that the left hand side of this equation, $N(K_1)$, is the expected present value of current and future marginal revenue products of capital, evaluated along a path that takes the capital stock as given and therefore does not take account of future purchases or sales of capital.[11] This is exactly the naive NPV, $N(K_1)$, which we discussed above. A

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[10]In general there are many ways to use derivative securities such as options to replicate payoffs. Equivalently, the relationship between $R_K(K_1, e)$ and the second-period marginal revenue product of capital illustrated in Figure 1 corresponds to a "bullish vertical spread" on $R_K(K_1, e)$, as described in Cox and Rubinstein, p. 14, Fig. 1-13. This payoff structure can be obtained by purchasing a call option on $R_K(K_1, e)$ with strike price b_L, and writing (selling) a call option on $R_K(K_1, e)$ with strike price b_H. However, in the context of capital investment, it seems most natural to represent this payoff structure as a claim on $R_K(K_1, e)$ plus a (marginal) put option minus a (marginal) call option.

[11]$N(K_1) \equiv G'(K_1)$ corresponds to Pindyck's (1988) "$\Delta V(K)$, the present value of the expected flow of incremental profits attributable to the K+1st unit of capital, which is independent of how much capital the firm has in the future." (footnote 6, p. 972)

practitioner who failed to realize the defect in this calculation would choose K_1 to equate $N(K_1)$ to the cost of purchasing capital, b. However, if the naive NPV calculation is being used, then K_1 will be chosen optimally only if the cost of capital is adjusted as on the right hand side of equation (12). Purchasing an additional unit of capital in period 1 extinguishes the marginal call option to purchase that unit of capital in period 2, and the present value of the cost of extinguishing this option, $\gamma C'(K_1)$, must be added to b. On the other hand, by purchasing an additional unit of capital in period 1, the firm acquires a put option to sell that unit of capital at price b_L in period 2. The acquisition of this marginal put option reduces the effective cost of investment by $\gamma P'(K_1)$.

The effect of a change in the sale price of capital on the value of the marginal put option is easily calculated by differentiating the value of this option with respect to b_L to obtain

$$\frac{\partial P'(K_1)}{\partial b_L} = F(e_L) \geq 0 \tag{13}$$

Increasing the price at which capital can be sold in the future raises the value of the marginal put option to sell capital, and thus reduces the effective cost of capital and increases the optimal value of K_1.

Differentiating the value of the marginal call option with respect to the purchase price b_H yields

$$\frac{\partial C'(K_1)}{\partial b_H} = -\left[1 - F(e_H)\right] \leq 0 \tag{14}$$

Increasing the price at which capital can be purchased in the future reduces the value of the marginal call option that is extinguished and therefore reduces the effective cost of investment. As a result, the optimal value of K_1 increases in response to an increase in b_H. Of course, these results obtained using the option value approach are identical to the results obtained using the q-theory approach.

II. The Option Value Multiple

The literature on irreversible investment has emphasized that optimal behavior is not in general characterized by the equality of the expected present value of marginal revenue products represented by $N(K_1)$ and the marginal cost of investment represented by b. Thus, a naive application of the NPV rule in which K_1 is determined by the equality of $N(K_1)$ and b would not lead to the optimal value of K_1. (Of course, a correct application of the NPV rule equating $q(K_1)$ and b yields the optimal value of K_1.) In the case of irreversible investment, the put option is absent, and thus, at the optimal K_1, $N(K_1)$ exceeds b by $\gamma C'(K_1)$, the present value of the marginal call option. The ratio of $N(K_1)$ to b, which exceeds one in this case, is the "option value multiple" (Dixit and Pindyck, 1994, p. 184). Here we generalize the notion of the option value multiple to include arbitrary degrees of reversibility and expandability.[12]

Define the option value multiple ϕ as

$$\phi \equiv N(K_1)/b \tag{15}$$

where K_1 is the optimal capital stock in period 1. Substituting the optimality condition from equation (12) into the definition of the option value multiple, we obtain

$$\phi = 1 + \gamma \frac{C'(K_1) - P'(K_1)}{b} \tag{16}$$

By definition, the optimal value of K_1 is chosen to satisfy $N(K_1) = \phi b$. We will examine how the option value multiple depends on the degrees of reversibility and expandability in the second period. We scale reversibility and expandability using the two definitions: $z_L \equiv b_L / b$ and $z_H \equiv b / b_H$, and write the option value multiple as $\phi(K_1; z_L, z_H)$

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[12] Since disinvestment may occur in our model, there is an "option value multiple" associated with the decision to disinvest, as well as with the decision to invest. We focus on the option value multiple associated with the investment decision in order to compare our results to those in the existing literature.

to emphasize the dependence of the option value multiple on the price ratios z_L and z_H as well as on the optimal level of the first-period capital stock K_1.

First consider the extreme case in which the capital stock is completely irreversible ($b_L = 0$) and completely unexpandable (infinite b_H) in the second period, which implies $z_L = z_H = 0$. In this case, both the put option and the call option have zero value because it is impossible to either sell or buy capital in the second period. Therefore, $\phi(K_1; 0, 0) = 1$.

Now consider the case in which the capital stock is completely irreversible ($b_L = 0$) but is at least partially expandable (finite b_H) in the second period; this implies that $z_L = 0$ and $z_H > 0$. In this case, the put option still has zero value but the marginal call option will have positive value provided that $F(e_H) < 1$. Therefore, we have $\phi(K_1; 0, z_H) \geq 1$ with strict inequality if $F(e_H) < 1$. This finding is consistent with the literature on irreversible investment which emphasizes that the option value multiple is greater than one, and thus the optimal value of the capital stock is lower than would be obtained by a naive (and incorrect) application of the NPV rule. It is important to note, however, that the option value multiple exceeds one because of the marginal call option associated with expandability, not solely because of irreversibility. Irreversibility eliminates the put option, while expandability generates the call option; both features are needed to produce an option value multiple that unambiguously exceeds one. Indeed, recall the previous case in which investment is irreversible and the option value multiple equals one (because of the absence of expandability).

The option value multiple can also be less than one. Consider the case in which investment is at least partially reversible ($b_L > 0$) but is completely unexpandable (infinite b_H) in the second period, which implies that $z_L > 0$ and $z_H = 0$. With partially reversible investment the put option has positive value provided that $F(e_L) > 0$. With completely unexpandable investment, the call option has zero value. Therefore, we have $\phi(K_1; z_L, 0) \leq 1$ with strict inequality if $F(e_L) > 0$. In this case, capital may be sold at a positive price, but no additional capital may be purchased at a finite price. The firm is therefore more

willing to invest initially than a naive application of the NPV rule would indicate. Note that the presence of at least partial reversibility is necessary for this result; the absence of expandability alone is not sufficient.

Finally, consider the special case of complete reversibility ($b_L = b$) and complete expandability ($b_H = b$) which implies $z_L = z_H = 1$. In this case, the excess of the value of the marginal put option over the value of the marginal call option, $P'(K_1) - C'(K_1)$, equals $b - E\{R_K(K_1,e)\}$ so that $\phi(K_1;1,1) = 1 - \gamma\left(1 - \dfrac{E\{R_K(K_1,e)\}}{b}\right)$. Therefore, $\phi(K_1;1,1)$ could be greater than, equal to, or less than one, depending on whether the value of the marginal put option is less than, equal to, or greater than the value of the marginal call option.

The relationship between ϕ and the degrees of expandability and reversibility is illustrated in Figure 2 which shows various "iso-ϕ" loci. These loci are derived by totally differentiating the expression for ϕ in equation (16) to obtain

$$d\phi = -\frac{\gamma}{b}\left(\frac{\partial P'(K_1)}{\partial b_L}\frac{\partial b_L}{\partial z_L}dz_L - \frac{\partial C'(K_1)}{\partial b_H}\frac{\partial b_H}{\partial z_H}dz_H + \left(P''(K_1) - C''(K_1)\right)dK_1\right) \qquad (17)$$

and then setting $d\phi = 0$. Observe from the definition of ϕ in equation (15) that given b and the distribution of e, changes in the values of z_L and z_H will leave the value of ϕ unchanged if and only if they leave the optimal value of K_1 unchanged. Setting $d\phi = dK_1 = 0$ in the above expression yields

$$\left.\frac{dz_H}{dz_L}\right|_{d\phi=0} = z_H^2 \frac{F(e_L)}{1 - F(e_H)} \geq 0 \text{ with strict inequality when } F(e_L) > 0 \text{ and } b_H < \infty. \qquad (18)$$

Thus the iso-ϕ loci slope upward from left to right as illustrated in Figure 2. (The convexity or concavity of these curves is in general indeterminate.) The value of ϕ is increasing in z_H and decreasing in z_L. The locus $\phi = 1$ passes through the point $z_L = z_H = 0$ because $\phi(K_1; 0, 0) = 1$ as explained earlier. This locus may pass either above, through, or

below the upper right corner of the unit square depending whether $\phi(K_1; 1, 1)$ is less than, equal to, or greater than one.

III. Graphical Illustration of the Put and Call Options

Define the period-2 marginal revenue product of period-1 installed capital as

$$x \equiv R_K(K_1, e).\qquad(19)$$

Given K_1, the distribution of e induces a distribution on x. Let $\Phi(x)$ be the cumulative distribution function induced by $F(e)$ and use integration by parts to obtain expressions for the marginal put and marginal call options, respectively:

$$
\begin{aligned}
P'(K_1) &\equiv \int_{-\infty}^{e_L}\left[b_L - R_K(K_1, e)\right]dF(e) = \int_0^{b_L}\left[b_L - x\right]d\Phi(x) \\
&= \left[b_L - x\right]\Phi(x)\Big|_0^{b_L} + \int_0^{b_L}\Phi(x)dx = \int_0^{b_L}\Phi(x)dx
\end{aligned}
\qquad(20)
$$

and similarly,

$$
\begin{aligned}
C'(K_1) &\equiv \int_{b_H}^{\infty}\left[R_K(K_1, e) - b_H\right]dF(e) = \int_{b_H}^{\infty}\left[x - b_H\right]d\Phi(x) \\
&= \left[x - b_H\right]\left[\Phi(x) - 1\right]\Big|_{b_H}^{\infty} - \int_{b_H}^{\infty}\left[\Phi(x) - 1\right]dx = \int_{b_H}^{\infty}\left[1 - \Phi(x)\right]dx
\end{aligned}
\qquad(21)
$$

The value of the marginal put option is equal to the area under the lower tail of the cumulative distribution function, $\Phi(x)$, to the left of $x = b_L$, as shown in Figure 3. Notice that an increase in the sale price b_L increases this area -- illustrating the corresponding increase in the value of the marginal put option we demonstrated analytically in equation (13). Similarly, the value of the marginal call option is equal to the area to the right of $x = b_H$ between the upper tail of the cumulative distribution and the horizontal line with unit height. An increase in the purchase price of capital b_H reduces this area -- illustrating the reduction in the value of the marginal call option we found earlier in equation (14).

IV. The Distribution of Future Returns and the Incentive to Invest

In this section we analyze the effects of changes in the distribution of future shocks, namely shifts of the distribution function $F(e)$, on the incentive to invest. While such shifts are often analyzed by parametrizing the distribution in terms of its moments and then doing comparative statics with respect to these parameters, here it is easier to use the concept of *stochastic dominance*. (See Hirshleifer and Riley (1992, pp. 105-116) for a discussion.)

Begin with a first-order increase in the distributions of e and x. This raises the mean of x, and therefore the naive NPV, $N(K_1)$. This by itself increases the incentive to invest, but may be offset by changes in the values of the associated options. From Figure 3, we can visualize that a rightward shift in the cumulative distribution function will reduce the shaded area in the bottom left corner (the value of the marginal put option) and increase the shaded area in the top right corner (the value of the marginal call option). Both these effects act to lower the incentive to invest. Therefore the option approach does not give a clear answer to the question of the balance of these effects; one would have to determine the magnitudes of the effects working in opposite directions.

The q approach gives a clear answer. In Figure 1, call the function shown by the heavy line $M(K_1,e)$. This function is strictly increasing in the range (e_L, e_H), and takes on constant values to the left of e_L and to the right of e_H. Now equation (4) can be written as

$$q(K_1) = r'(K_1) + \gamma \int_{-\infty}^{\infty} M(K_1, e) dF(e) \tag{22}$$

This shows that $q(K_1)$ is the expected value of a non-decreasing function of e. Therefore a first-order shift to the right in the distribution of e cannot lower this expected value. The incentive to invest in period 1 is not lowered on balance. Moreover, the function $M(K_1, e)$ is strictly increasing in e in the range (e_L, e_H), and takes on constant values to the left of e_L and to the right of e_H. Unless the shift of the distribution is

confined entirely to the ranges $(-\infty, e_L]$ and $[e_H, \infty)$, the incentive to invest is actually increased.

The qualification about shifts restricted to these extreme ranges has independent interest. An inspection of equation (4) or (22) shows that $q(K_1)$ is affected by the cumulative probabilities $F(e_L)$ to the left of e_L and $[1-F(e_H)]$ to the right of e_H, but it is not affected by any details of the probability densities in these separate ranges. If a little probability weight shifts from a point just to the right of e_H to another point farther to the right (some good news becomes better news) or vice versa, the value of $q(K_1)$, and therefore the incentive to invest, will remain unchanged. Similarly for any shifts of probability densities confined to the left of e_L: if bad news becomes even worse, that has no effect on the incentive to invest. Details of the probability density function matter only in the middle range (e_L, e_H).

The "tail-events" do not matter because a value of e in either tail induces the firm to buy or sell capital to mitigate the effect of such extreme realizations. A realization of e in the lower tail will induce the firm to sell capital and prevent the marginal revenue product from falling below b_L; a realization of e in the upper tail will induce the firm to purchase capital and prevent the marginal revenue product from rising above b_L.

This is an extension of Bernanke's (1983) "bad-news principle," which applies in the case of completely irreversible investment. (See also the exposition in Dixit (1992, p. 118).) In this case $e_L = -\infty$, and $F(e_L) = 0$. Therefore there is no lower tail of e where M is constant, so all of the details of the probability distribution in this "bad news" region affect the incentive to invest. However, in this case there is also complete expandability ($b_H = b$), so for any realization of e above e_H, the firm will expand its capital stock and set the marginal product of capital equal to its price. The probability mass $[1-F(e_H)]$ could be rearranged arbitrarily in the region $e > e_H$ without affecting the current incentive to invest. Together, these results produce Bernanke's "bad-news principle", since the upper tail of realizations of e does not affect the incentive to invest, but the lower tail (the "bad news")

does. In our more general model, there is a range of low values of e that will lead to disinvestment in period 2, so the probability mass $F(e_L)$ could be rearranged arbitrarily in the region below e_L without affecting the incentive to invest.

In a model that is the mirror-image of Bernanke's, investment is completely unexpandable ($b_H = \infty$), so there will be no upper tail where M is constant. Then the details of the probability distribution throughout the "good news" region will affect the incentive to invest and we will have a "good-news principle". Most generally, for partially expandable and partially reversible investments, we have a "Goldilocks principle"; the only region of the probability distribution of e that affects the incentive to invest is the intermediate part where news is neither "too hot" nor "too cold".

Finally, consider a second-order shift -- a mean-preserving spread -- in the distribution of e. Such a shift has an ambiguous effect on the naive NPV given by equation (8). This shift increases (decreases) the naive NPV if $R_K(K_1,e)$ is a convex (concave) function of e. See Dixit and Pindyck (1994, pp. 199, 371-2) for more on this issue. How does this shift affect the values of the two options? In Figure 3, a mean-preserving spread in e twists the distribution of x clockwise (although the mean of x may not be preserved). Provided the point of crossing between the old and the new distributions lies between b_L and b_H, this will increase both shaded areas, that is, the values of both the marginal call and the put options. Since the marginal call option decreases the incentive to invest and the marginal put option increases it, the net effect on the incentive to invest in period 1 will be ambiguous. The alternative approach based on q cannot resolve the ambiguity.

V. Concluding Remarks

The irreversible investment literature emphasizes that the value of a firm is determined in part by its options to invest. We have shown more generally how the incentive to invest, summarized by q, can be decomposed into the returns to existing capital, ignoring the possibility of future investment and disinvestment, and the marginal value of the options to invest and disinvest. The option to invest (the call option) arises from the expandability of the capital stock, while the option to disinvest (the put option) arises from the reversibility of investment. The call option reduces the firm's incentive to invest; while it adds to the firm's value, it is extinguished by investment. The put option increases the incentive to invest, since it is by investing that the firm acquires this option.

The interaction of these options determines the net effect of expandability and reversibility on the optimal capital stock. The irreversible investment literature has emphasized how uncertainty and irreversibility reduce the incentive to invest. We have shown that irreversibility is not sufficient to reduce the incentive to invest under uncertainty; irreversibility eliminates the put option associated with the possible resale of capital, but it is the call option associated with expandability that causes uncertainty to reduce the optimal capital stock. Likewise, it is the interaction of these two options that determines the net effect of uncertainty on q. Since the values of both options rise with uncertainty, and the two options have opposing effects on the incentive to invest, the net effect of uncertainty is ambiguous. The effect of changes in the distribution of future returns is characterized by the Goldilocks principle: the incentive to invest is unaffected by changes within the upper tail (where news is "too hot") and by changes within the lower tail (where news is "too cold"); only changes within the intermediate range of the distribution (where the news is "just right") affect the incentive to invest.

Finally, we have shown precisely how the usual naive application of the NPV rule fails to characterize optimal behavior. The naive NPV rule evaluates future marginal

revenue products of capital at the current level of the capital stock, rather than at the future optimal levels. To obtain the correct value of the optimal capital stock, the calculation requires an adjustment that is captured by the option value multiple, which may be greater than, equal to, or less than one. Alternatively, one can apply the NPV rule (without an option value multiple) to determine the optimal value of the capital stock if care is taken to evaluate future marginal revenue products of capital at the future optimal levels of the capital stock, as in the q-theory approach. Both the option value approach and the q-theory approach will correctly characterize optimal behavior, yet each offers its own set of distinctive insights about the investment decision.

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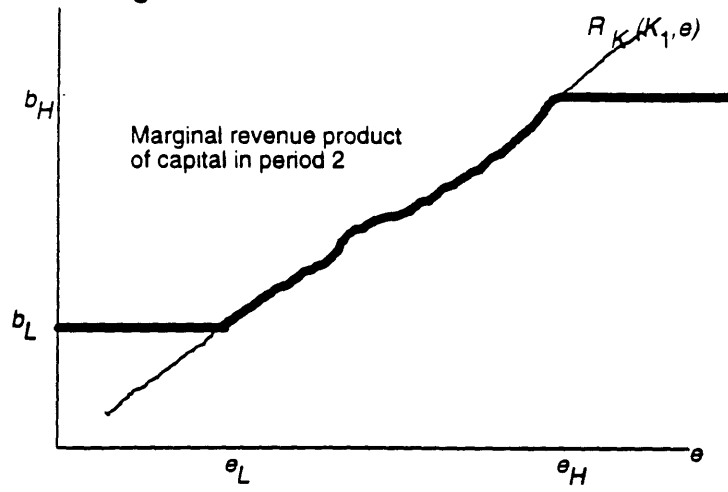
Figure 1

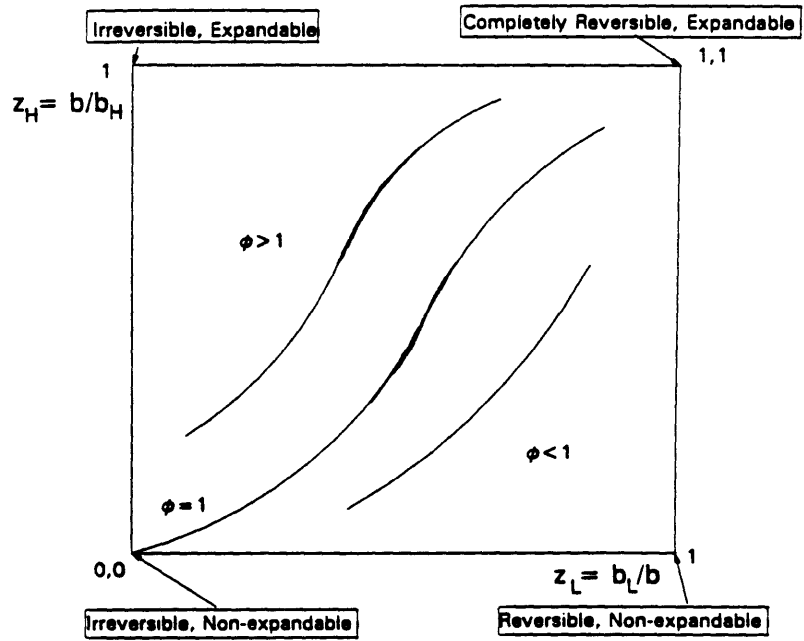## Figure 2

Figure 3