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Abstract

This paper is devoted principally to a summary of the theoretical determination of the characteristics and effects of impulse noise in an idealized but conventional FM receiver. The most significant general conclusion reached is that the theoretical effect of impulse noise (within broad limits of amplitude and duration) on a program signal should be relatively small and of unobjectionable character.



# RESULTS OF TRANSIENT ANALYSIS OF IMPULSE NOISE IN FM RECEIVERS

## 1. Introduction

One of the most serious limitations of the present FM communications system is the lack of an FM receiver which will closely realize the theoretical expectations of the system. In this respect, the correlation between the theoretical response of an ideal FM receiver to impulse noise and the response of the average good FM commercial receiver of the pre-war and immediate post-war period is quite bad. It was the main purpose of the research studies that have resulted in this paper to determine the reasons why theory and practice have differed, and if possible to contribute to the eventual removal of this difference.

Let us consider first an ideal FM detector, which should be insensitive to amplitude variations and highly sensitive to frequency variations. If this is the case, no limiter is necessary since the limiting effect is included in the detector. However, if the FM detector is not ideal, then it will respond to amplitude variations in the carrier as well as to frequency variations and must be preceded by a device which will remove the amplitude variations. This device is commonly called a limiter. It is readily appreciated that the amplitude-insensitive devices, the limiter and detector, of an FM receiver possess important and unique properties which are related, and which depend upon the type of interference against which the devices are to function. For operation against steady-state AM of the carrier most of the limiter detector systems available today are adequate. However, if the FM receiver is to give optimum performance in the presence of impulse noise, then the receiver must be designed from consideration of the characteristics and effects of the amplitude and frequency transients of such interference. Analysis has been made of the transient response of a synchronous single-tuned i-f amplifier to d-c impulses, that is, impulse noise. Analysis has also been made of the transient response of a conventional discriminator to a square-wave modulated carrier. Further, analysis has been made of the effect of the combination of impulse noise in the receiver filter and a steady carrier, in both definite and random phase relationships. These analyses have served to characterize the nature and magnitude of the impulse noise signal in relation to the desired FM signal, and have indicated that the effect of impulse noise on an ideal FM receiver should be relatively small.

## 2. Theoretical Response of an i-f Amplifier to Impulse Noise

The envelope response of an n-stage synchronously tuned amplifier has been computed in several places in the literature. It is sufficient

merely to give the curves of Fig. 1 which show the noise envelopes resulting from an impulse after passing through one, two, and three stages of synchronously tuned circuits. Figure 2 shows the oscillographic verification of these curves. These were obtained with the experimental arrangement to be described later in this report.

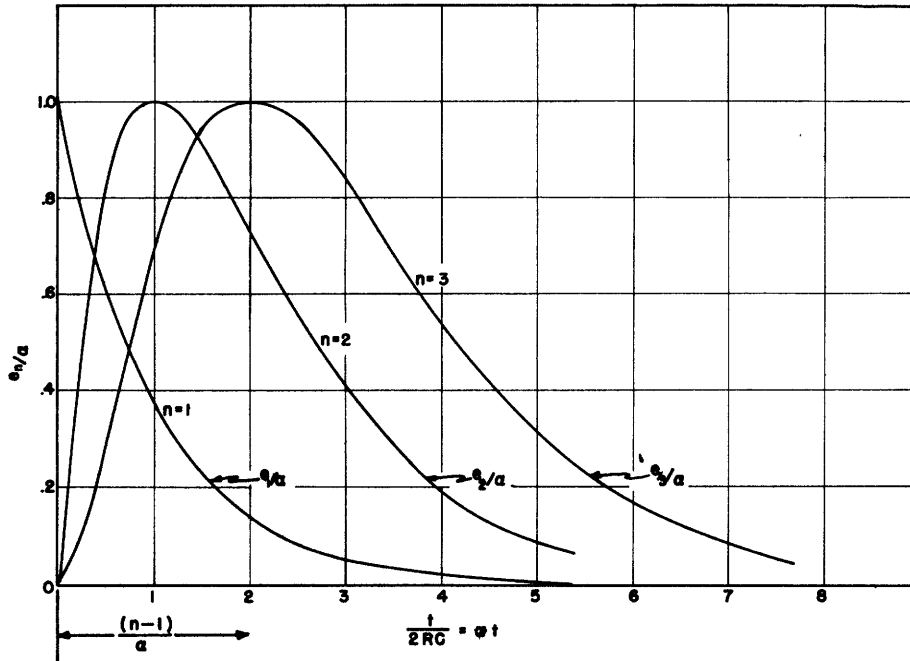
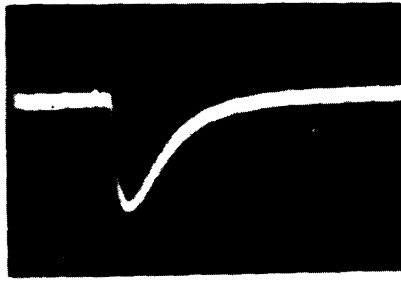


Fig. 1. Normalized impulse transient envelopes with bandwidth per stage kept constant.

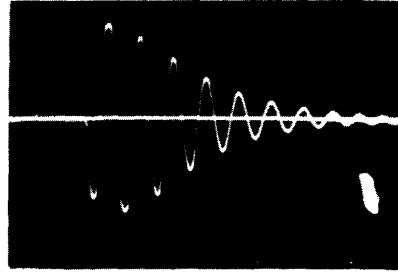
### 3. Transient Response of a Foster-Seeley Discriminator

3.1. Amplitude Response. Figure 3 shows the circuit of the conventional Foster-Seeley discriminator. The amplitude-vs-frequency curve of the Foster-Seeley discriminator circuit can be shown to be the difference between two staggered resonance curves. When either of these equivalent resonant circuits is excited with a square-wave-modulated carrier, of duration long with respect to any of the time constants in the discriminator (actually a unit function modulated carrier was used here), the envelope of the output wave form may be expressed as:

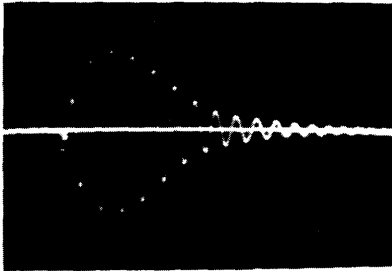
$$a(t) = \sqrt{\left[ A \cos \theta + B e^{-\alpha t} \cos((\omega - p)t + \varphi) \right]^2 + \left[ A \sin \theta + B e^{-\alpha t} \sin((\omega - p)t + \varphi) \right]^2} \quad (1)$$



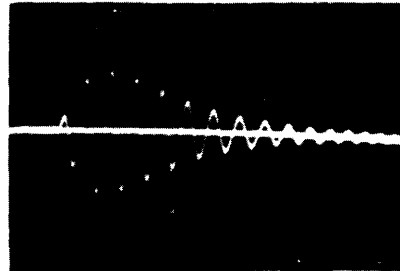
Input d-c pulse



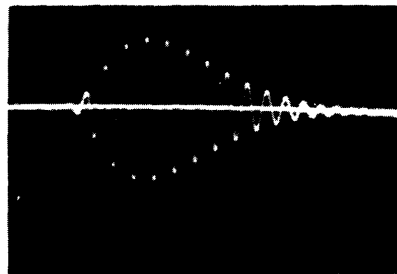
Impulse transient, 1st stage



Impulse transient, 2nd stage



Impulse transient, 3rd stage



Impulse transient, primary  
of discriminator

Fig. 2. Response to impulse noise of synchronously tuned circuits.

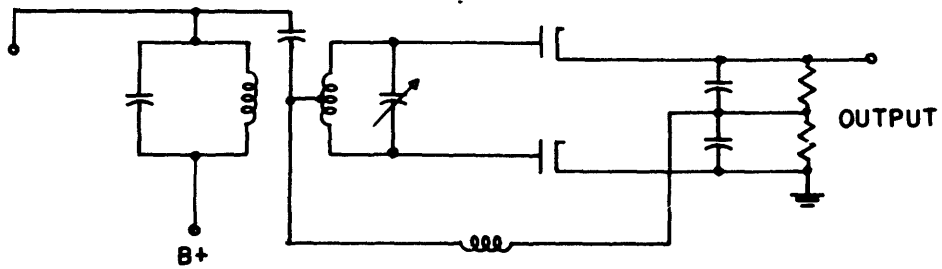


Fig. 3. Foster-Seeley discriminator.

where

$$A = \frac{1}{C} \left[ \frac{p^2}{(\alpha^2 + \omega^2 - p^2)^2 + 4\alpha^2 p^2} \right]^{\frac{1}{2}}$$

$$B = \frac{p}{\omega C} \left[ \frac{\alpha^2 + \omega^2}{(\alpha^2 + \omega^2 - p^2)^2 + 4\alpha^2 p^2} \right]^{\frac{1}{2}}$$

$$\theta = 90^\circ - \tan^{-1} \frac{2\alpha p}{\alpha^2 + \omega^2 - p^2}$$

$$\alpha = \frac{1}{2RC}$$

$$\varphi = \tan^{-1} \frac{\omega}{-\alpha} - \tan^{-1} \frac{-2\alpha\omega}{\alpha^2 - \omega^2 + p^2}$$

$p$  = angular frequency of driving function.

$\omega$  = resonant frequency of either of the equivalent tuned secondary circuits.

By replacing  $\omega$  by either  $\omega_1$  or  $\omega_2$ , the expression of Eq. (1) becomes the detected envelope ( $a_1$  or  $a_2$ ) from either of the two equivalent secondary tuned circuits of the discriminator. Equation (1) was derived by the use of Laplace transform theory. In its present form it is fairly difficult to handle and can best be simplified by the results of quantitative arithmetic studies. These studies indicate that  $A_1$  and  $A_2$  are



very nearly equal to  $B_1$  and  $B_2$ , respectively. By making these approximations, the expression for  $a(t)$  given by Eq. (1) can be simplified to:

$$\begin{aligned} a_1(t) &= A_1 \sqrt{1 + e^{-2\alpha_1 t} + 2e^{-\alpha_1 t} \cos [\theta_1 - \varphi_1 - (\omega_1 - p)t]} \\ a_2(t) &= A_2 \sqrt{1 + e^{-2\alpha_2 t} + 2e^{-\alpha_2 t} \cos [\theta_2 - \varphi_2 - (\omega_2 - p)t]}. \end{aligned} \quad (2)$$

For Eq. (2) to satisfy the initial conditions it must be equal to zero for  $t = 0$ . This is possible only if  $\theta - \varphi = \pm 180^\circ$ . It may be shown that

$$\theta - \varphi = \tan^{-1} \left[ \frac{-\alpha(\omega^2 + p^2 + \alpha^2)(\omega^2 + \alpha^2 - p^2) + 2\alpha\omega(\omega^2 + \alpha^2 - p^2)}{-\omega(\omega^2 - p^2 + \alpha^2)^2 - 2\alpha^2 p(\omega^2 + p^2 + \alpha^2)} \right]. \quad (3)$$

Investigation for an actual case in which it is assumed that  $A_1 = B_1$ ,  $A_2 = B_2$  shows  $\theta - \varphi = 179.96^\circ$ . The difference between this and  $180^\circ$  is due to these assumptions, and is therefore indirectly a measure of this error. If the detector is assumed to be linear, the discriminator output is the difference between  $a_1(t)$  and  $a_2(t)$ , given in Eq. (2) where  $\theta_1 - \varphi_1 = +180^\circ$  and  $\theta_2 - \varphi_2 = -180^\circ$ . The difference between these two envelopes will be very nearly zero if  $A_1 = A_2$  as is actually the case.

**3.2. Frequency Response.** It is also possible to derive an expression for  $\omega_r(t)$ , the transient frequency variation at the plates of the detectors of the discriminator. A convenient way of expressing  $\omega_r(t)$  is in terms of its fractional or relative deviation from the driving frequency  $p$ .

$$\frac{\omega_r(t) - p}{p} = \frac{\omega - p}{p} e^{-\alpha t} \left[ \frac{e^{-\alpha t} - \cos(\omega - p)t + \frac{\alpha}{\omega - p} \sin(\omega - p)t}{1 - 2e^{-\alpha t} \cos(\omega - p)t + e^{-2\alpha t}} \right]. \quad (4)$$

A plot of this fractional deviation as a function of  $\alpha t$  is shown in Fig. 4. In this figure curve A corresponds to the frequency at one side of the discriminator for a given set of conditions, and curve B to the frequency at the other side of the discriminator. As is seen from the figure, these curves are essentially mirror images. This indicates that the net transient

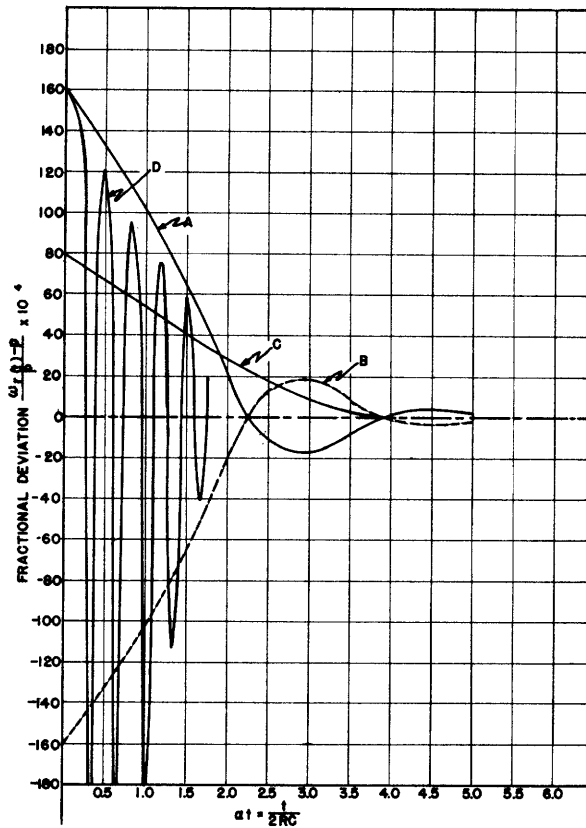


Fig. 4. Transient frequency variations at the plates of the detectors of the discriminator. The fractional or relative deviation from the driving frequency  $p$  is plotted as a function of  $at$ .

response of the discriminator to a unit function modulated carrier at the mid frequency of the discriminator would be very nearly zero. Curve C indicates that as the bandwidth of the discriminator is decreased, the transient oscillations become more damped and of longer period. Curve D indicates that as  $\alpha$  is decreased, the transient oscillations increase in amplitude and take longer to decay. It would appear from curves C and D that by decreasing  $|\omega - p|$  (which is proportional to the bandwidth) and increasing  $\alpha$  one could greatly suppress the transient oscillations. This, however, cannot be done since increasing  $\alpha$  must necessarily increase the bandwidth. One must therefore conclude that the transient response of the Foster-Seeley discriminator is more or less independent of its bandwidth for a given discriminator characteristic.

3.3. Effect of Discriminator Unbalance. The above result has been checked experimentally. However, it was noticed that zero output was obtained only with a very carefully balanced discriminator. Unbalancing the output cir-

cuit of a discriminator results in decreased cancellation of the impulse output, as shown in Fig. 5 for a specific case.

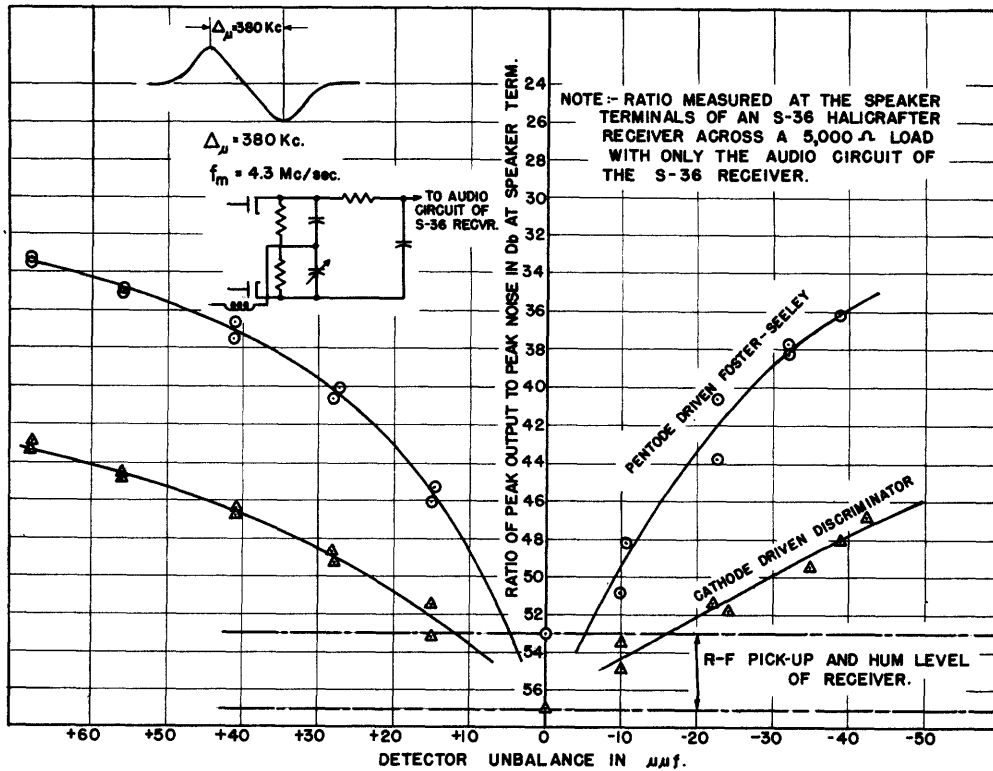


Fig. 5. Ratio of peak output to peak noise vs. detector unbalance.

It should be noted in this figure that curves are drawn for both the pentode-driven Foster-Seeley discriminator and the cathode-driven discriminator. It was observed as an incidental part of the experimental work that the cathode-driven discriminator appeared to have a lower response to impulse noise, roughly 9 db lower, than the conventional pentode-driven Foster-Seeley circuit. An explanation lies in the fact that the normalized characteristic curve for the cathode-driven discriminator is practically independent of both coupling and loading, and hence for the same bandwidth the damping coefficient ( $\alpha$ ) of the cathode-driven discriminator can be made larger than that for the pentode-driven discriminator. Oscillographic techniques have been used to show the effect predicted by the preceding analyses. Figure 6 shows the wave forms at the plates of the detectors in a Foster-Seeley discriminator. The two photographs have been superimposed for relative phase and frequency comparison. It will be noted that, as predicted by the above analyses, the two wave forms start out in phase and are of different frequencies. This frequency difference in addition to imperfect detection accounts for a small spike at the out-

put of even a well-balanced discriminator. This spike is readily suppressed by the de-emphasis network.

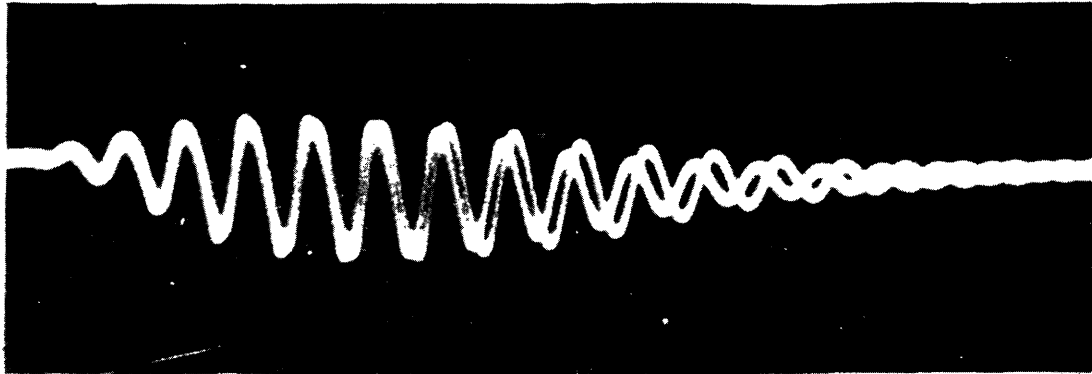


Fig. 6. Transient impulse waveforms at plates of the detector superimposed for relative phase and frequency comparison.

4. Analysis of Combination of an Impulse Transient and a Carrier in Definite and Random Phase Relationships

An analysis of impulse noise combined with a carrier in both definite and random phase relationships has been presented in a recent paper by Bradley and Smith.<sup>1</sup> This paper resolves impulse noise into essentially two basic types which are called "clicks" and "pops". A mathematical check of the general conclusions reached has been made by using the method of Laplace transforms. The most significant general conclusion is, that the effect of impulse noise on an FM carrier should be relatively small. Calculations have been made as to the probability of the "pop" variety of impulse noise for various assumed types of noise generators. Certain portions of the earlier work have been reproduced in an effort to make the picture more complete. It is agreed that, for a given disturbance in desired signal, the resulting transient noise-to-signal ratio is essentially directly proportional to the over-all receiver bandwidth.

4.1. Characterization of Impulse Noise. Curves of the instantaneous phase of the sum of signal and impulse are shown in Fig. 7. Since the time derivative of phase is frequency, the disturbance at the output of the receiver will be essentially proportional to the slope of these curves. As is shown in this figure, the noise does divide itself into two categories, namely, the pop and click type. Further, the slopes are steeper in the pop cases than in the click cases, and owing to the effects mentioned

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1. W. E. Bradley and D. B. Smith, "The Theory of Impulse Noise in Ideal Frequency Modulation Receivers", Proc. IRE, 34, 743 (1946).

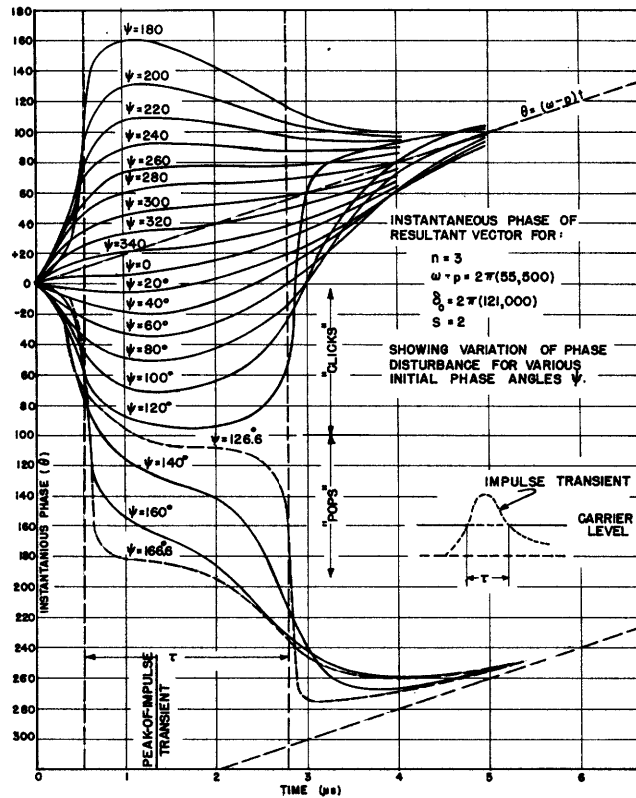


Fig. 7. Instantaneous phase of the sum of signal and impulse transient for various values of initial phase angle  $\psi$ .

by Bradley and Smith, the disturbance is also more lasting in its effect on the listener. Figure 8 shows a photograph of the combination of an impulse transient and a signal with a relative phase at  $180^\circ$  at the peak of the transient. This picture shows two wave forms superimposed, one is taken with relative phase slightly less than  $180^\circ$  and the other with relative phase slightly greater. This presentation gives a time scale and shows the generation of a pop. Figure 9, taken similarly, shows the combination which is in phase at the peak of the transient. The larger wave form has the carrier component tuned to the center frequency of the receiver. As can be seen, very little frequency deviation is present. The small wave form has its carrier component deviated from center frequency and, as can be shown, there is a complete phase reversal of one part of the transient, thus showing the effect of  $f_d$  on the generation of a pop type of noise.

4.2. Effect of Capture Time. One may define a capture time of the signal as that time interval of the impulse during which it is possible to generate a pop, that is, the time during which the transient is greater than the signal. It is interesting to note that this capture time is also the

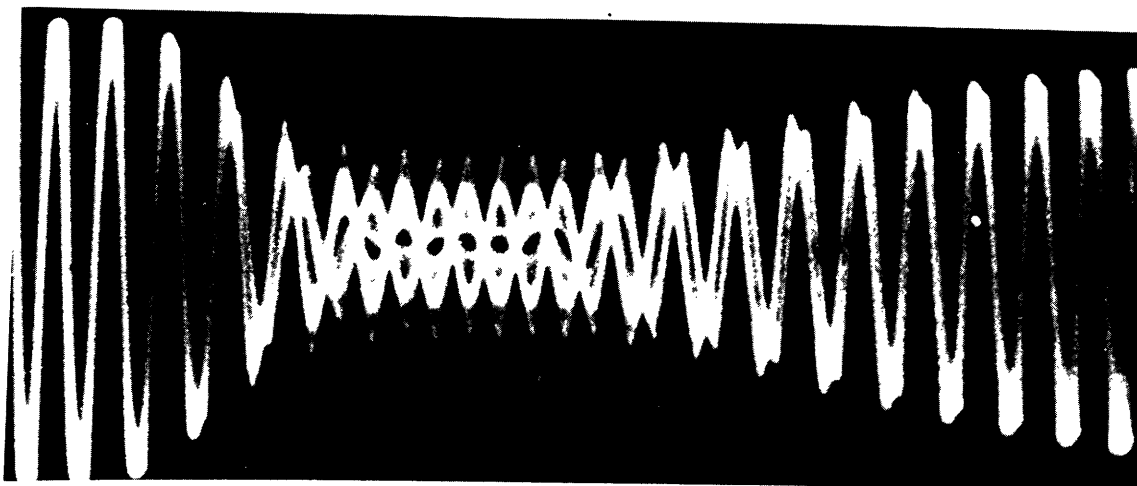


Fig. 8. Impulse transient and signal combined with relative phase  $\approx 180^\circ$  at peak of impulse transient. Two resultant waveforms are superimposed to show generation of a pop.

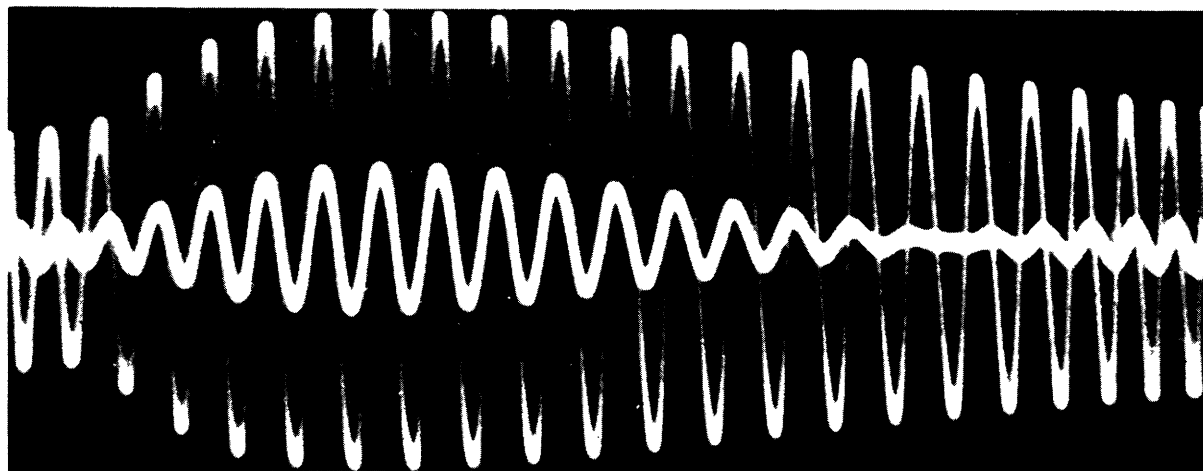


Fig. 9. Impulse transient and signal combined in phase at peak of impulse transient. Larger waveform has carrier component tuned to the center frequency of the receiver. Smaller waveform has its carrier component deviated from center frequency showing effect of instantaneous frequency deviation ( $f_d$ ) on generation of a pop.

time interval during which the maximum phase disturbance of both clicks and popstrokes place. The maximum phase disturbance that can take place instantaneously from the generation of either a click or a pop as is shown in Fig. 7 is  $180^\circ$ . It is exactly  $180^\circ$  for a pop, and on an arbitrarily small angle less than  $180^\circ$  for a click. It is apparent that the concept of capture time is very important in the study of impulse noise in FM. By graphical solution of the transcendental equations given by Bradley and Smith, curves have been plotted showing capture time as a function of signal-to-peak-noise ratio for various number of stages with bandwidth per stage kept

constant. These curves are plotted for two, four, and eight stages in Fig. 10. In this figure the actual quantities plotted are the product of capture time and the damping constant ( $\tau\alpha$ ), as a function of signal-to-noise ratio. In

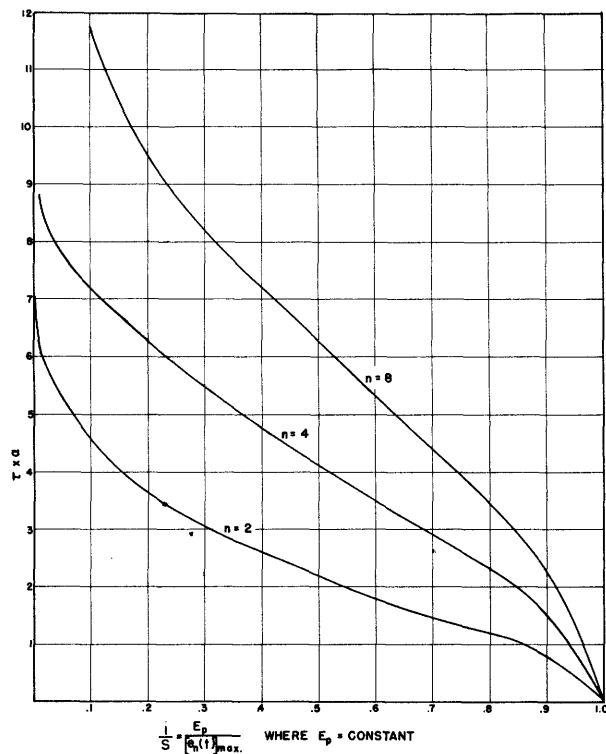


Fig. 10. Product of capture time and damping coefficient versus signal-to-peak-noise ratio with bandwidth for each stage kept constant.

Fig. 11, the product of capture time and  $\delta_0$ , the half bandwidth, is plotted as a function of signal-to-noise ratio, again for two, four, and eight stages with the over-all bandwidth constant. These curves have been computed by graphical methods and have been found to exhibit differences from those computed by Bradley and Smith by a series-expansion method. In particular, the series-expansion method yields a 9 per cent error for a noise-to-signal ratio greater than 20, for four stages of intermediate frequency.

4.3. Effect of De-emphasis. We must also consider the effect of de-emphasis since in actuality it is the de-emphasis circuit that makes the effect of pops noticeable. A further analysis using the Laplace transform has been made resulting in Fig. 12, a graph of peak amplitude of a pop in percent of the amplitude of a 100-per-cent-modulated program signal as a function of discriminator bandwidth, with de-emphasis time constant as a parameter. It should be noted that the longer the de-emphasis time constant, the lower will be the percentage of the signal represented by the pop. Further, high bandwidths give low noise-to-signal ratios as might be expected,

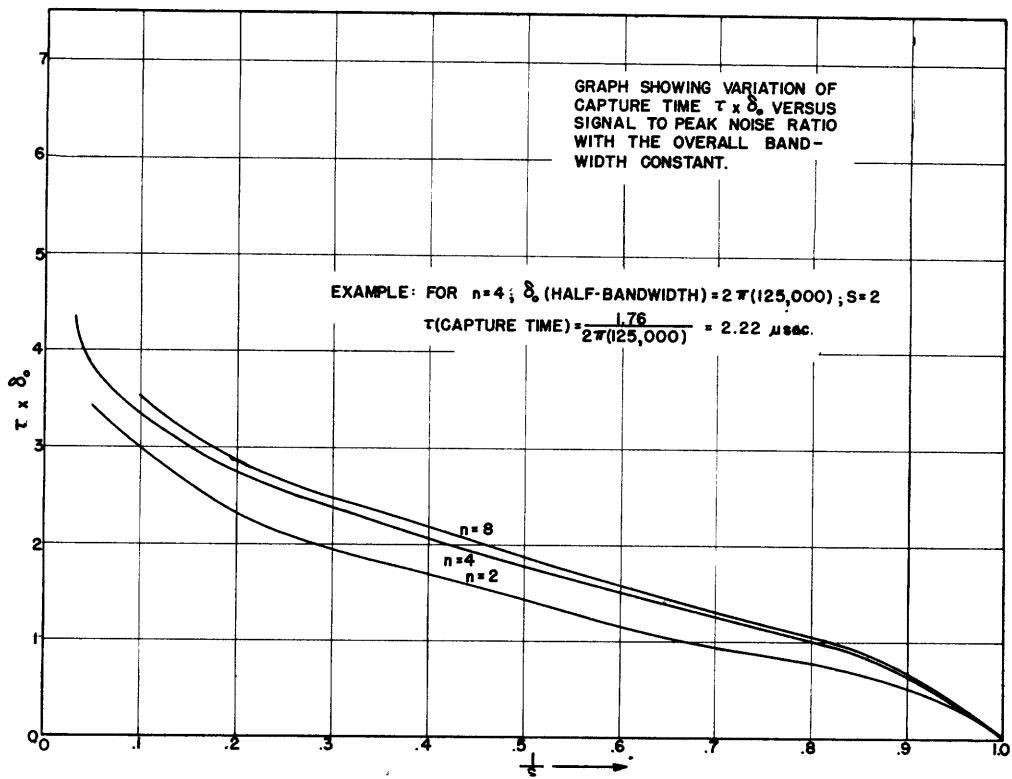


Fig. 11. Product of capture time and half bandwidth vs. signal-to-noise ratio with over-all bandwidth constant.

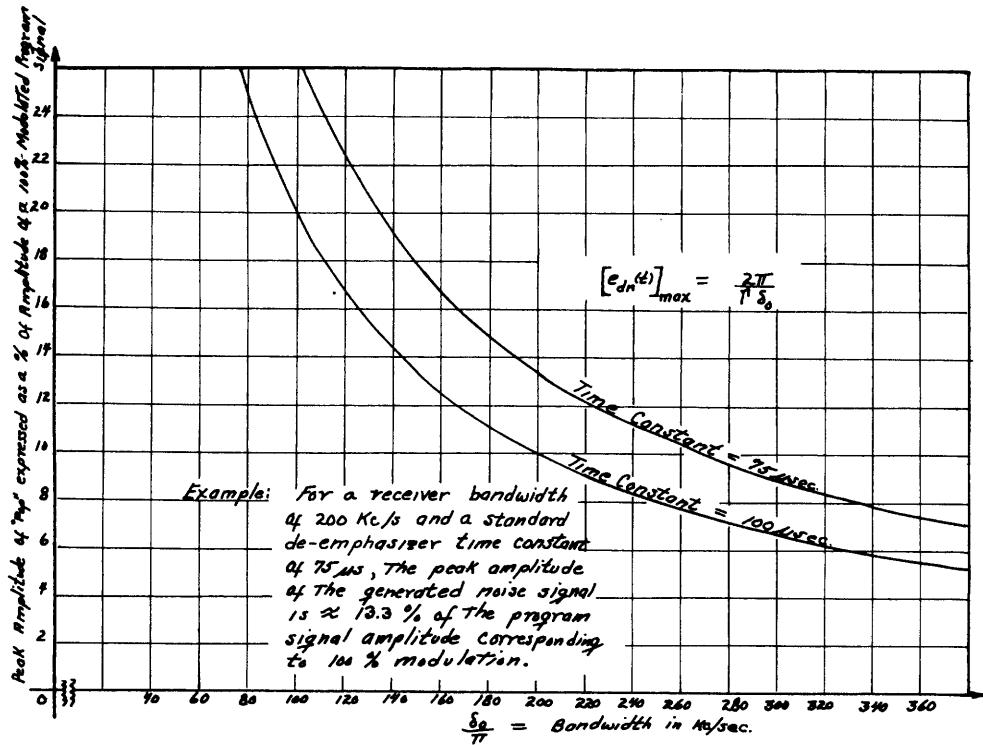


Fig. 12. Peak amplitude of the generated noise signal of a "pop" as a function of receiver bandwidth and de-emphasizer time constant.



but there is little advantage to be gained above a bandwidth of approximately 300 kc. In particular, however, it is noted that maximum pop amplitudes are of the order of 10 per cent, which is not a very annoying interference, especially if it comes at reasonable interval times. One might expect this, since a pop is not generated by every noise impulse.

#### 4.4. Probability of a Pop as a Function of the Type of Impulse Noise. The

effect of a click can be shown to be negligible compared to that of a pop so that analysis of impulse noise can be narrowed down for practical purposes to a study of the pop variety and its frequency of occurrence. The generation of this noise depends upon several factors, most of which can be evaluated only on a probability basis. The factors which determine the probability that a pop will occur are the peak-noise-to-signal ratio, the number of stages in the receiver filter, the over-all receiver bandwidth, the instantaneous frequency deviation of the carrier from the center frequency of the filter at the time of the impulse, and the probability that a noise impulse will have a given amplitude. An additional factor which must be considered is the general type of impulse noise. Three types of impulse noise have been considered: (a) periodic impulse noise of constant amplitude; (b) purely random impulse noise having a distribution approximated by a Gaussian error curve; (c) random impulse noise having a sinusoidal distribution of amplitudes, for example, impulse noise produced by commutator type a-c motors or a-c electric razors. Again recourse has to be made to graphical solutions. For the second or purely random impulse noise Fig. 13 shows the curve of the probability that a pop will occur as a function of  $\overline{S^2}$ , the ratio of the average noise power to average signal power. Although this graph is plotted for noise power up to twice signal power, its significance is questionable when the noise power is beyond eight-tenths of the signal power. This is due to the fact that there is an inherent assumption that there is only one impulse per capture time. If this is not the case, the probability of a pop will increase more rapidly above  $\overline{S^2} \cong 0.8$  through the increased probability at that noise level of more than one impulse per capture time. However, the probability of a pop for this type of impulse noise is in any event relatively low, only about 2 per cent. For case (a), periodic impulse noise at constant amplitude, we may compute an explicit equation for  $\rho$ , the number of pops per second:

$$\rho = \frac{2Nf_d}{\delta_0} \sqrt{2(n-1)\ln S} \sqrt{2^{1/n} - 1} \quad (6)$$

where

N = number of impulses per second.

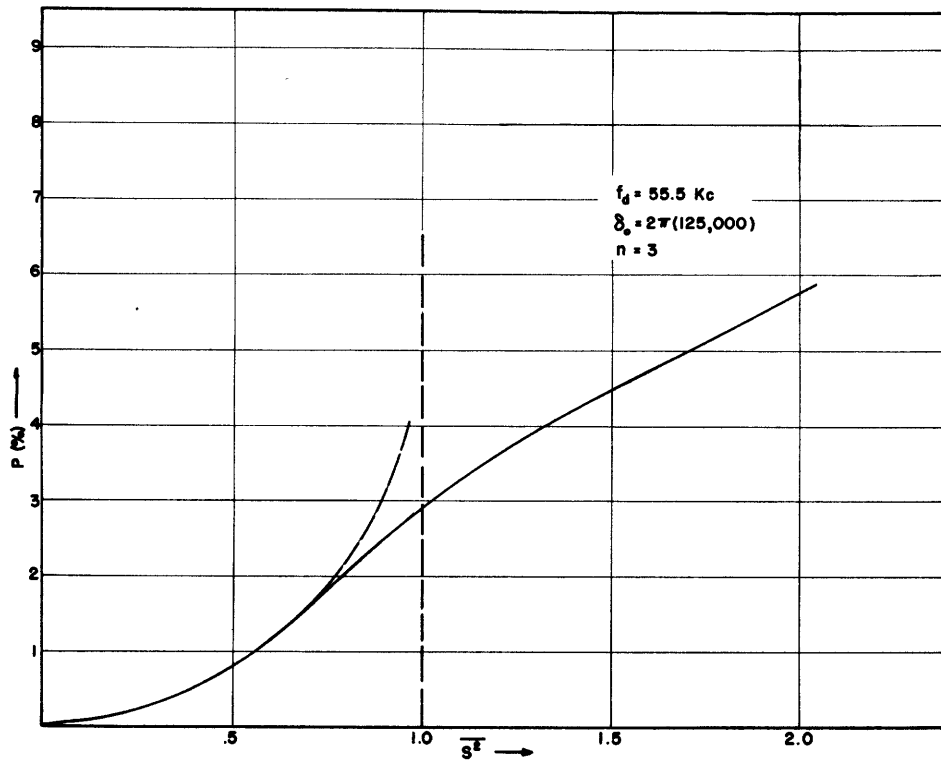


Fig. 13. Probability that a "pop" will occur vs. average power of random impulse noise.

This case is considered of small importance, however. Case (c), random impulse noise with a sinusoidal distribution of amplitude, is more interesting, since it corresponds to one type of impulse noise that has actually been used in some laboratories as a standard, namely, electric razor impulse noise. In Fig. 14, the probability that this noise will produce one pop per impulse transient is given as a function of the amplitude of the noise. As is seen from this figure, again the probability is relatively small, 15 per cent, for most practical cases. One might expect, therefore, that an impulse of this type at 60 spikes per second would produce about 9 pops per second.

One must therefore conclude that the effect of impulse noise on an FM carrier should be relatively small. It should be pointed out further that the effect of an impulse is to subtract a noise signal from the desired signal; that is, an impulse cuts a hole in the modulation of the carrier. This is quite important, since the holes will be minimized by the integrating action of the audio system including the human ear and the de-emphasizer, providing the noise impulses are short enough.

##### 5. Experimental Equipment

Experimentally, it was observed that the theoretical predictions outlined in this paper were not in agreement with the results ob-

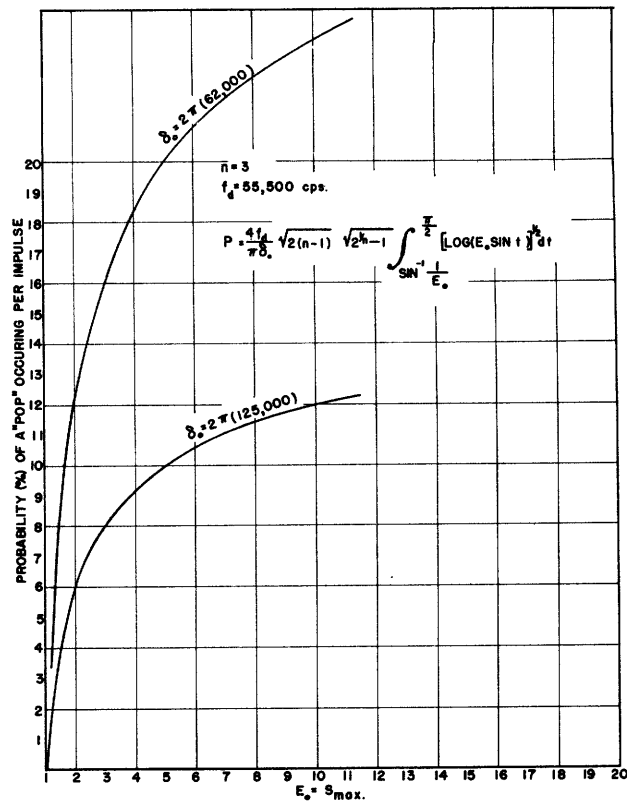


Fig. 14. Probability that random impulse noise having a sinusoidal distribution of amplitudes will cause a "pop" per impulse transient.

served with a good commercial FM receiver, having a double-stage grid-leak limiter. It was observed that agreement between practice and theory could be obtained only over a limited range of impulse-noise amplitudes. As a result, experimental equipment was devised for observing the transient effects of impulse noise in frequency modulation. This is built around a Type 258B A/R Scope and a Radiation Laboratory Mark 3 test pulser. The Mark 3 test pulser provides for a positive input pulse which triggers two separate multivibrator circuits. The outputs of the multivibrators are used to produce a positive or negative short and long d-c pulse and a short and long pulse at about 4.5 Mc/sec. The triggers of the two multivibrators were isolated. The trigger circuit of the A/R scope was used to activate one multivibrator, and the strobe trigger circuit used to activate the other. It was therefore possible to obtain both a sharp d-c pulse and a square-wave-modulated carrier, each adjustable in duration, amplitude, and relative phase. Since the pulsed carrier always starts with the same phase, it is possible to move the d-c pulse through the carrier in time phase, and hence study the effect of relative phase with great ease. This method and technique of transient study is extremely flexible in its application to other noise and interference studies, since the d-c pulse can be used to trigger and gate interfering sources or other types of noise, and

the pulse carrier can be modulated as desired. Figure 15 gives a block diagram of the experimental equipment. Figure 16 is the circuit diagram

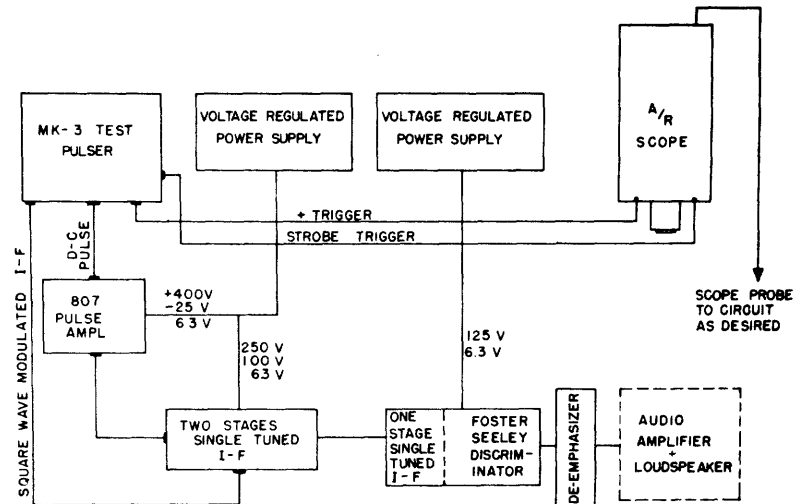


Fig. 15. Block diagram of experimental equipment used in the study of impulse noise in frequency modulation.

of the Mark 3 test pulser as modified for these experiments, and Fig. 17 is a photograph of the experimental equipment.

## 6. Conclusions

(1) The output of a Foster-Seeley discriminator is very nearly zero in both transient and steady state for any type AM wave provided the carrier frequency of the wave is the same as the center frequency of the discriminator.

(2) The transient at the output of a discriminator, mathematically equivalent to an optimum coupled Foster-Seeley discriminator, is proportional to the bandwidth of the discriminator and inversely proportional to the damping coefficient.

(3) The time constants of the output filters of the discriminator must be balanced to within about 6 per cent if impulse noise is to be reduced to 50 db below maximum signal.

(4) Agreement has been reached with the general results of Bradley and Smith and in particular, with their characterization of impulse noise into two basic types.

(5) While the amplitude of a click is largely, and of a pop essentially, independent of the original noise impulse, providing the capture time of the transient is small compared to the time constant of the de-emphasis network, the probability of the occurrence of a pop or click is dependent upon the original noise impulse. In particular it is



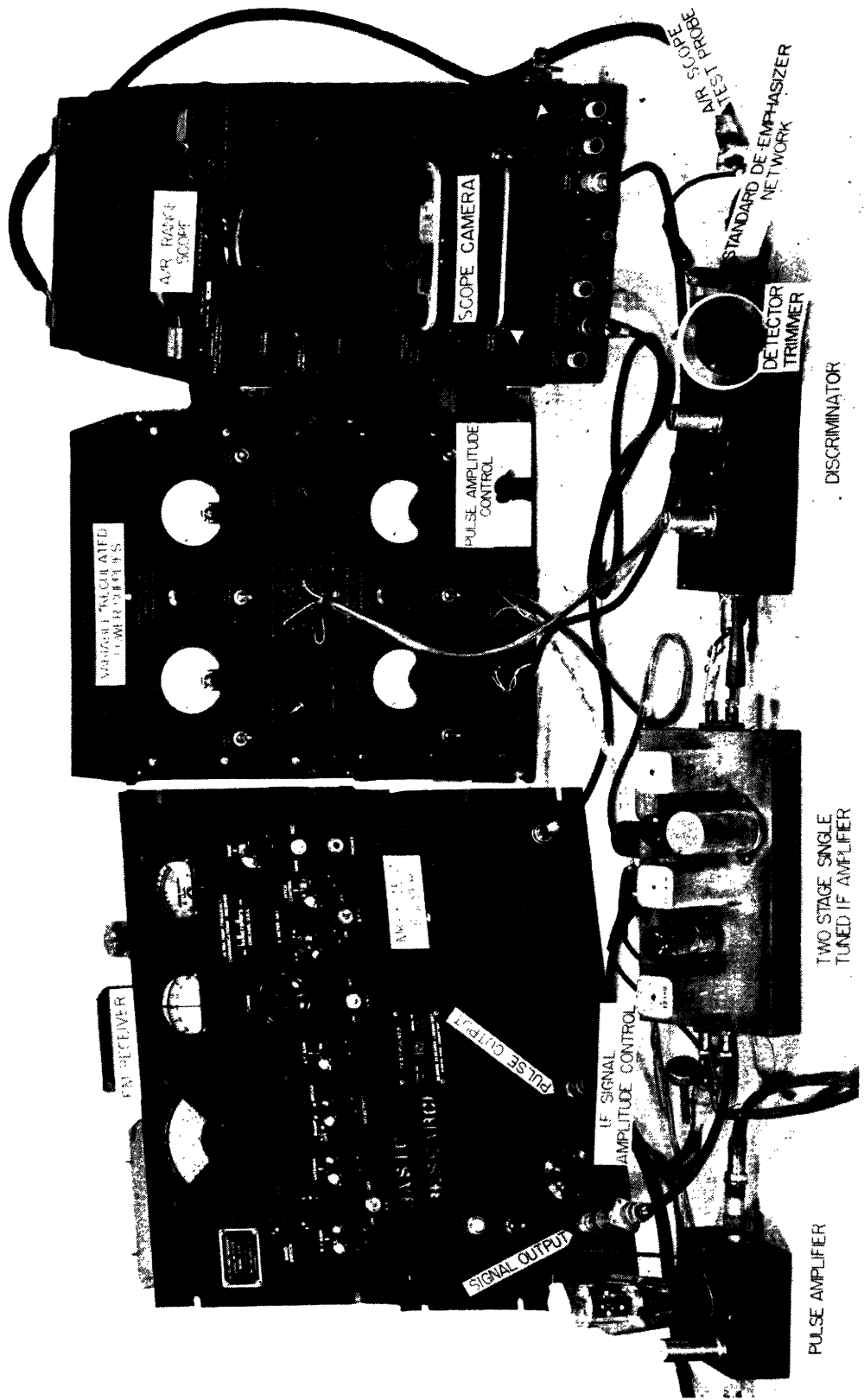


Fig. 17. Experimental equipment used in the study of impulse noise in frequency modulation.

a function of the following parameters:

- (a)  $S$ , the peak-noise-to-signal ratio.
- (b)  $n$ , the number of stages in the i-f amplifier.
- (c)  $\delta_o$ , the over-all half bandwidth.
- (d)  $f_d$ , the instantaneous frequency deviation of the desired signal at the time of the impulse.
- (e)  $P(S)dS$ , the probability that a noise impulse peak amplitude will be between  $S$  and  $S + \Delta S$ .

It is noted that probability increases with an increase in  $S$ ,  $n$ ,  $f_d$ , and  $P(S)dS$ . Probability decreases with an increase in  $\delta_o$ .

(6) The experimental results obtained by Bradley and Smith when using an electric drill or razor as a source of impulse noise are believed to be explained in more detail by the results of Sec. 4.4.

(7) The analyses and experimental work as a whole verify and emphasize a conclusion that has been made by several authors in recent years; that is, the need for accurate tuning and alignment in frequency-modulation receivers. The inherent advantages of FM in noise discrimination cannot be reasonably approached if the alignment and tuning of the receiver are not done with a precise and critical procedure.

(8) The theoretical effect of impulse noise of even the pop variety on a program signal should be negligible.

In general, commercial receivers do not agree with theoretical predictions in their performance against impulse noise. In an effort to discover the reasons for this non-conformity between theory and practice, an investigation has been made of the limiters and discriminators used in FM receivers. This study resulted in placing additional transient functions on the limiter, which when approximated in the laboratory were found to give a large reduction in impulse noise response. These results are included in an additional report.<sup>1</sup>

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1. T. P. Cheatham, Jr., "A Logarithmic Limiter for Use in an FM Receiver in the Presence of Impulse Interference", RLE Tech. Report No. 36, April 3, 1947

